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San Ecuaciones Diferenciales
Aux: Un buen hombre

La Gran y Última tarea

https://github.com/emiliodd97/ULTIMA-TAREA

15. $2xy'' - y' + 2y = 0$

$2x\sum_{n=0} C_n(n+r)(n+r-1)x^{n+r-2} - \sum C_n(n+r)x^{n+r-1} + 2\sum_{n=0} C_n x^{n+r} = 0$

$2\sum_{n=0} C_n(n+r)(n+r-1)x^{n+r-1} - \sum_{n=0} C_n(n+r)x^{n+r-1} + 2\sum_{n=0} C_n x^{n+r} = 0$

$(2C_0 r(r-1) - C_0 r)x^{r-1} + \sum\left[2C_n(n+r)(n+r-1) - C_n(n+r)\right]x^{n+r-1} + 2\sum C_n x^{n+r}$

$(2r^2-3r)C_0x^{r-1} + \sum\left[2C_{k+1}(k+r+1)(k+r) - C_{k+1}(k+r+1) + 2C_k\right]x^{k+r} = 0$

$(2r^2-3r)C_0 = 0$ $\qquad C_{k+1} = \frac{-2C_k}{(k+r)(2k+2r-3)}$ $\qquad k \geq 1$

$r(2r-3) = 0$

$r=0 \quad r=\frac{3}{2}$

$y_1 = C_1(1+2x-2x^4+4x^5+\dots) \qquad y_2 = C_2x^{3/2}(1-\frac{3}{5}x+\frac{2}{35}x^2+\dots)$

$y = y_1 + y_2$

$y = C_1(1+2x-2x^4+4x^5\dots) + C_2x^{3/2}(1 - \frac{3}{6}x + \frac{2}{35}x^2+\dots)$

17. $2xy'' - y' + 2y = 0$

$2x\sum_{n=0} C_n(n+r)(n+r-1) + \sum C_n(n+r)x^{n+r-1} + 2\sum C_n x^{n+r} = 0$

$2\sum_{n=0} C_n(n+r)(n+r-1)x^{n+r-1} - \sum C_n(n+r)x^{n+r-1}$

$+ 2\sum C_n x^{n+r} = 0$

$(2C_0 r(r-1)+(C_0 r)x^{r-1}+\sum [2C_n(n+r)]x^{n+r-1}+2\sum C_n x^{n+r}=0$

$(2r^2-3r)(C_0 x^{-1}$

$2r^3-3C_0=0$

$r=0 \quad r=3/2$

$y_1=C_1(1+2x-2x^4+4x^5+\dots)$

$y_2=C_2x^{3/2}(1-\frac{3}{6}x+\frac{2}{35}x^2+\dots)$

$y=C_1(1+2x-2x^4+4x^5+\dots)+C_2x^{3/2}(1-3/6x+2/35x^2+\dots)$

19. $3xy''+(2-x)y'+y=0$

$3\sum_{n=0} 3(n+r)(n+r-1)C_nx^{n+r-1}+\sum(n+r)C_nx^{n+r}-\sum(n+r)C_nx^{n+r}$

$-\sum_{n=0} C_nx^{n+r}=0$

$\sum(n+r+2(n+r))C_n^{n+r-1}-\sum_{n=0}[(n+r)+1]C_nx^{n+r}=0$

$r(3r-1)C_0x^{n-1}+\sum_{n=1}(n+r)C_nx^{n+r}=0$

$r(3r-1)C_0x^{n-1}+\sum[(k+r-1)(3r+nr+2)C_k-(k+r+1)C]x^{k}=0$

$y_1=C_0(1+1/2x+1/10x^2+1/80x^3+1/880x^4+\dots)$

$y_2=x^{1/3}C_1(1+1/3x+1/18x^2+1/162x^3+\dots)$

$y=$

21. $0 = 9x^2y'' + 9x^2y' + 2y$

$$9x^2\sum_{n=0} C_n(n+r)(n+r-1)x^{n+r-2} + 9x^2\sum_{n=0} C_n(n+r)x^{n+r-1} + 2\sum C_n x^{n+r}$$

$$9\sum_{n=0} C_n(n+r)(n+r-1)\,x^{n+r} + 9\sum_{n=0} C_n(n+r)x^{n+r+1} + 2\sum_{n=0} C_n x^{n+r}$$

$$\sum_{n=0} 9C_n(n+r)(n+r-1) + 2C_n\,x^{n+r} + 9\sum C_n(n+r)x^{n+r+1}$$

$$(9r^2 - 9r + 2)C_0x^r + \sum 9C_n(n+r)(n+r-1) + 2C_nx^{n+r}$$
$$+ 9\sum C_n(n+r)x^{n+r+1})$$

$K = n$

$(3r-1)(3r-2)$

$r = 1/3 \quad r = 2/3$

$$0 = 9_k(k+r)(k+r-1) + 2c_k + 9c_{k-1}(k+r-1)$$
$$0 = 9(k+r)(k+r-1) + 2C_k + 9c_{k-1}(k+r-1)$$

$$C_k = \frac{-9c_{k-1}(k+r-1)}{9(k+r)(k+r-1)+2}$$

$$y_1 = x^{1/3}C_1\left(1 - \frac{1}{2}x + \frac{1}{5}x^2 - \frac{7}{120}x^3 + \dots\right)$$

$$y^2 = x^{2/3}C_2\left(1 - \frac{1}{2}x + \frac{5}{28}x^2 - \frac{1}{21}x^3 + \dots\right)$$

$$y = y_1 + y_2$$

16. $y = \sum_{n=0}^{\infty} C_n x^{n+r}$, $\quad \sum_{n=0}^{\infty} (n+r)(n+r-1) C_n x^{n+r} - \sum_{n=0}^{\infty} C_n x^{n+r+1} + \frac{2}{9} \sum_{n=0}^{\infty} C_n x^{n+r} = 0 \longrightarrow$

$$\sum_{k=0}^{\infty} (k+r)(k+r-1) C_k x^{k+r} - \sum_{k=1}^{\infty} C_{k-1} x^{k+r} + \frac{2}{9} \sum_{k=0}^{\infty} C_k x^{k+r} = 0 \longrightarrow$$

$$x^r \left[r(r-1)C_0 + \frac{2}{9} C_0 + \sum_{k=1}^{\infty} \left((k+r)(k+r-1) C_k - C_{k-1} + \frac{2}{9} C_k \right) x^k \right] = 0 \longrightarrow$$

$$r(r-1) + \frac{2}{9} = 0 \rightarrow 9r^2 - 9r + 2 = 0 \longrightarrow (3r-1)(3r-2) = 0 \rightarrow r_1 = \frac{1}{3} \ \& \ r_2 = \frac{2}{3}$$

Also, $(k+r)(k+r-1) C_k - C_{k-1} + \frac{2}{9} C_k = 0 \rightarrow C_k = \dfrac{C_{k-1}}{(k+r)(k+r-1) + \frac{2}{9}}$, $k = 1, 2, 3, \cdots$

For r_1: $C_k = \dfrac{C_{k-1}}{(k+\frac{1}{3})(k-\frac{2}{3}) + \frac{2}{9}} = \dfrac{C_{k-1}}{k(k-\frac{1}{3})} = \dfrac{3C_{k-1}}{k(3k-1)}$.

$$C_1 = \frac{3C_0}{2}, \quad C_2 = \frac{3C_1}{2 \cdot 5} = \frac{3 \cdot 3C_0}{2 \cdot 2 \cdot 5}, \quad C_3 = \frac{3 \cdot C_2}{3 \cdot 8} = \frac{3^3 C_0}{2 \cdot 2 \cdot 3 \cdot 5 \cdot 8}, \quad C_4 = \frac{3C_3}{4 \cdot 11} = \frac{3^4 C_0}{2 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 8 \cdot 11},$$

$C_n = \dfrac{3^n C_0}{n!\,(2 \cdot 5 \cdot 8 \cdot 11 \cdots (3n-1))}$. For r_2: $C_k = \dfrac{C_{k-1}}{(k+\frac{2}{3})(k-\frac{1}{3}) + \frac{2}{9}} = \dfrac{3C_{k-1}}{k(3k+1)}$, $k = 1, 2, 3, \ldots$

$$C_1 = \frac{3C_0}{4}, \quad C_2 = \frac{3C_1}{2 \cdot 7} = \frac{3^2 C_0}{2 \cdot 4 \cdot 7}, \quad C_3 = \frac{3C_2}{3 \cdot 10} = \frac{3^3 C_0}{2 \cdot 3 \cdot 4 \cdot 7 \cdot 10}, \quad C_4 = \frac{3C_3}{4 \cdot 13} = \frac{3^4 C_0}{2 \cdot 3 \cdot 4 \cdot 4 \cdot 7 \cdot 10 \cdot 13}$$

$$C_n = \frac{3^n C_0}{n!\,(4 \cdot 7 \cdot 10 \cdot 13 \cdots (3n+1))}.$$

$$y_1(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!\,(2 \cdot 5 \cdot 8 \cdots (3n-1))} x^{n+1/3}, \quad y_2(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!\,(4 \cdot 7 \cdot 10 \cdots (3n+1))} x^{n+2/3}, \quad |x| < \infty.$$

(19) $3xy'' + (2-x)y' - y = 0 \qquad y = \sum_{n=0}^{\infty} C_n x^{n+r}$

$\Leftrightarrow \sum_{n=0}^{\infty} 3(n+r)(n+r-1)C_n x^{n+r-1} + \sum_{n=0}^{\infty} 2(n+r)C_n x^{n+r-1} - \sum_{n=0}^{\infty} (n+r)C_n x^{n+r} - \sum_{n=0}^{\infty} C_n x^{n+r} = 0$

$\Leftrightarrow \sum_{n=0}^{\infty} [3(n+r)(n+r-1) + 2(n+r)] C_n x^{n+r-1} - \sum_{n=0}^{\infty} [(n+r)+1] C_n x^{n+r} = 0.$

$\Leftrightarrow \sum_{n=0}^{\infty} (n+r)(3n+3r-1) C_n x^{n+r-1} - \sum_{n=0}^{\infty} (n+r+1) C_n x^{n+r} = 0.$

$\Leftrightarrow r(3r-1)C_0 x^{r-1} + \sum_{n=1}^{\infty} (n+r)(3n+3r-1)C_n x^{n+r-1} - \sum_{n=0}^{\infty} (n+r+1)C_n x^{n+r} = 0.$

$\Leftrightarrow r(3r-1)C_0 x^{r-1} + \sum_{k=0}^{\infty} (k+r+1)(3k+3r+2)C_{k+1} x^{k+r} - \sum_{k=0}^{\infty} (k+r+1)C_k x^{k+r} = 0$

$\Leftrightarrow r(3r-1)C_0 x^{r-1} + \sum_{k=0}^{\infty} [(k+r+1)(3k+3r+2)C_{k+1} - (k+r+1)C_k] x^{k+r} = 0$

$C_0 \neq 0 \Rightarrow r(3r-1) = 0 \Leftrightarrow \begin{cases} r = 0 \\ r = 1/3 \end{cases} \qquad C_{k+1} = \frac{\cancel{k+r+1}}{\cancel{(k+r+1)}(3k+3r+2)} C_k$

- $r = 0 \Rightarrow C_{k+1} = \frac{k+1}{(k+1)(3k+2)} C_k$

$k=0 \quad C_1 = 1/2\, C_0$

$k=1 \quad C_2 = 1/5\, C_1 = 1/10\, C_0$

$k=2 \quad C_3 = 1/8\, C_2 = 1/80\, C_0$

$k=3 \quad C_4 = 1/11\, C_3 = 1/880\, C_0$

- $r = 1/3$

$\Rightarrow C_{k+1} = \frac{1}{3k+3}$

$k=0 \quad C_1 = 1/3\, C_0$

$k=1 \quad C_2 = 1/6\, C_1 = 1/18\, C_0$

$k=2 \quad C_3 = 1/9\, C_2 = 1/162\, C_0$

$k=3 \quad C_4 = 1/12\, C_3 \quad \ldots$

$y_1 = C_0\left(1 + \frac{1}{2}x + \frac{1}{10}x^2 + \frac{1}{80}x^3 + \frac{1}{880}x^4 + \ldots\right)$

$y_2 = x^{1/3} C_1 \left(1 + \frac{1}{3}x + \frac{1}{18}x^2 + \frac{1}{162}x^3 + \ldots\right)$