Generalized Additive Models & Functional Gradient Boosting with Geometrically Designed (GeD) Splines: Application to Insurance Data

Dimitrina S. Dimitrova[1], Vladimir K. Kaishev[1] and
Emilio Sáenz Guillén (presenter)[1]

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[1] Faculty of Actuarial Science and Insurance, Bayes Business School.
Email: emilio.saenz-guillen@bayes.city.ac.uk

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CITY, UNIVERSITY OF LONDON

Motivation

GeDS estimation method

Generalized Additive Models with GeDS

Functional Gradient Boosting with GeDS

Insurance Data Application

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- GeD spline methodology is extended further by:
  1. **GAM-GeDS**: encompassing **Generalized Additive Models (GAM)**, thereby making GeDS highly multivariate.
  2. **FGB-GeDS**: incorporating **Functional Gradient Boosting (FGB)**, improving the construction of the underlying spline regression model.

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Implemented in the R package **GeDS**, available from CRAN:
https://cran.r-project.org/package=GeDS

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- **STAGE A** constructs a least squares linear spline fit to the data.
  - Starting with a straight-line, LS fit, which is then sequentially "broken" by iteratively introducing knots at those points 'where the fit deviates most from the underlying functional shape determined by the data'.

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- **STAGE B**
  - Builds smoother higher order spline fits using Schoenberg's variation diminishing spline (VDS) approximation to the linear fit from Stage A.
  - For each higher spline order (quadratic, cubic...), compute the *averaging knot location* and re-estimate the spline coefficients by LS.

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GAM-GeDS

The **Generalized Additive Model (GAM)** assumes the response variable, $Y \sim E.F.$, and relates its conditional expectation, $\mu = E\left[Y|X\right]$, to the predictor variables, $X_1, ..., X_P$, via a link function $g(\cdot)$:

$$g(\mu) = \alpha + \sum_{j=1}^{P} f_j(X_j), \text{ with } \mathbb{E}\left[f_j(X_j)\right] = 0, \quad j = 1, ..., P \tag{1}$$

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GAM with GeD Splines: Local-scoring algorithm using GeD splines as the function smoothers, f_j, within the backfitting algorithm.

FGB-GeDS

- **Functional Gradient Boosting** (Friedman, 2001): ensemble machine learning technique that iteratively combines multiple simple models ('weak-learners'), each striving to enhance the performance of the previous accumulative model.

  $\longrightarrow$ ***Component-wise Gradient Boosting*** (Bühlmann and Yu, 2003; Schmid and Hothorn, 2008): boosting algorithm for fitting additive models, inherently performing variable selection; implemented in **mboost** package (boosting with P-splines).

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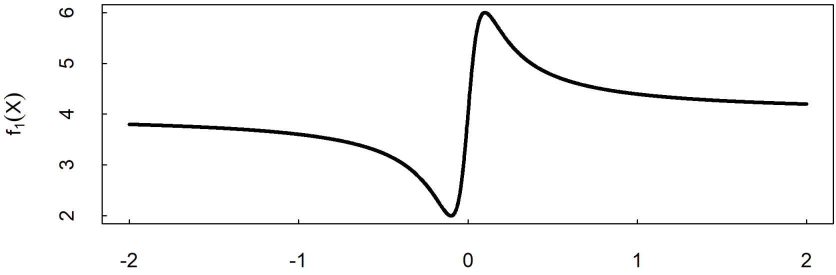
❊ FGB with GEDS base-learners

- Flexible control of the strength of the base-learners:
  1. Weak GeDS initial learner + few boosting iterations with strong GeDS learners.
  2. Boosting iterations with weak GeDS learners based on single knot addition with memory.
- Optimal number of boosting iterations determined by a **stopping rule** based on a ratio of consecutive deviances.
- Final boosted fit expressed as a **single spline model**.

Simulated Data Application

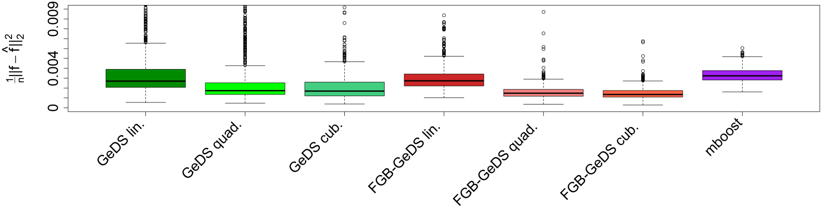
Consider the function:

$$f_1(x) = 40\frac{x}{1+100x^2} + 4 \quad , \quad x \in c(-2, 2)$$

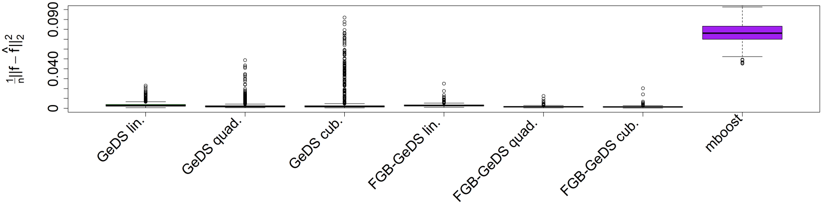


- Generate $1,000$ random samples, $\{X_i, Y_i\}_{i=1}^{N}$ with $Y_i \sim N(\mu_i, \sigma)$ with $\sigma = 0.2$, $\mu_i = \eta_i = f_1(X_i)$ and $X_i \sim U[-2, 2]$, $i = 1, ..., N$, where $N = 500$.

$$\begin{cases} \textbf{GeDS}: 10 \text{ int. knots} \\ \textbf{FGB-GeDS}: \text{initial learner with } 2 \text{ int. knots + } 1 \text{ boosting iter. with } 8 \text{ int. knots} \\ \textbf{mboost}: 10,000 \text{ boosting iter. with } 36 \text{ knots p/iter.} \end{cases}$$



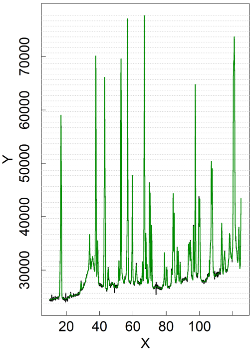
And setting **mboost** to have 10 int. knots p/boosting iter. instead:



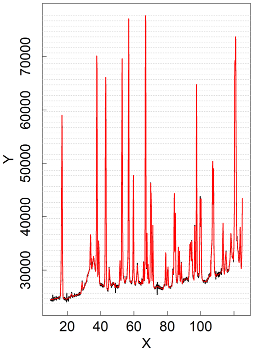
Real Data Application

- High pressure neutron barium-iron arsenide ($BaFe_2As_2$) powder diffraction data (Kimber et al., 2009), with number of observations $N = 1151$.

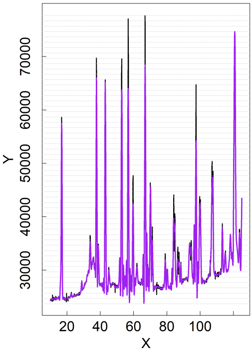
NGeDS
282 knots
MSE: 59,385.34



NGeDSboost
1 knot p/boosting iter.,
284 boosting iter.
MSE: 59,175.96



mboost
285 knots p/boosting iter.,
10,000 boosting iter.
MSE: 2,760,848.65



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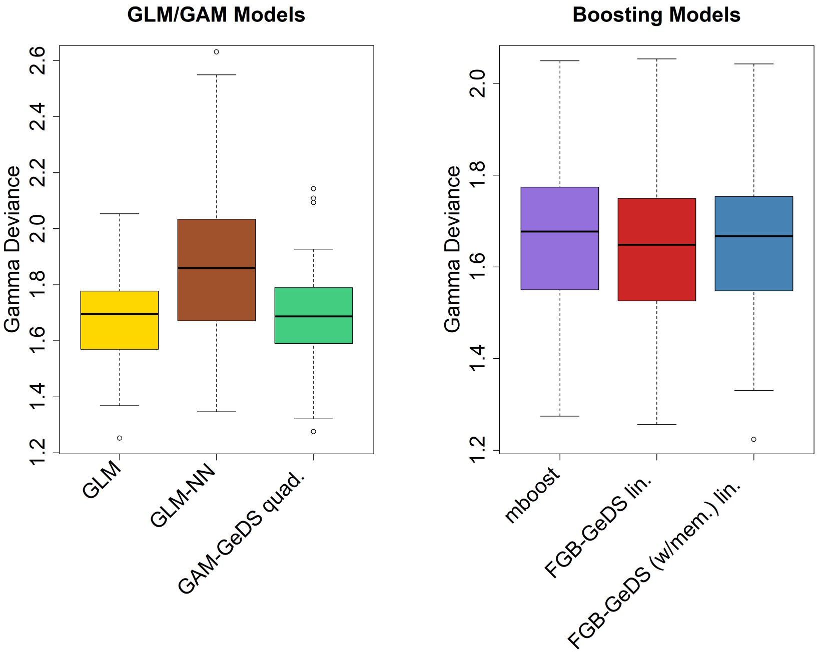
Motorcycle insurance data `swmotorcycle` available through the `R` package `CASdatasets` (Dutang and Charpentier, 2020).

$\longrightarrow$ We follow Delong et al., 2021 and model **gamma claim sizes**:

1. Gamma GLM regression + Gamma Neural Network regression.
2. `mboost`: FGB with P-splines.
3. GAM-GeDS.
4. FGB-GeDS.

- *Response*: `ClaimAmount/ClaimNb`, i.e., the average claim size.
- *Covariates*: `OwnerAge`; `Gender`; `Area`, `RiskClass`; `VehAge`.
- *Train/Test split*: **80%/20%**.
  - Simulate $\underline{100}$ different splits of data.



GLM/GAM Models
Gamma Deviance
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
GLM
GLM-NN
GAM-GeDS quad.
Boosting Models
Gamma Deviance
1.2
1.4
1.6
1.8
2.0
mboost
FGB-GeDS lin.
FGB-GeDS (w/mem.) lin.

Table 1: GLM/GAM Models

	Gamma Deviance			Internal knots
	Train Data	Test Data	Time (sec.)	(OwnerAge+VehAge)
GLM	1.585727	1.694797	0.008708	-
GLM NN	1.719903	**1.859394**	167.224576	-
GAM-GeDS quadratic	1.557612	**1.686492**	0.671260	5

Table 2: Boosting Models

	Gamma Deviance			Internal knots	Boosting
	Train Data	Test Data	Time (sec.)	p/boosting iter. (OwnerAge+VehAge)	iterations
mboost	1.610290	**1.676810**	0.156095	4	100
FGB-GeDS linear (2 starting knots)	1.575972	**1.648345**	0.130963	2	1
FGB-GeDS w/mem. linear (1 starting knot)	1.575536	1.667158	0.129040	1	3

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