IMDS 2025 — Individual Assessment 1

Instructions

You will upload your work electronically as a pdf or doc file via *Gradescope* (accessed through Blackboard). You may:

- Type your work and/or include computer screenshots.
- Do your work by hand on paper and then scan it.
- Use a combination of the above.

You may discuss these questions with classmates, but you should write out your solutions individually. Your solutions should be explained step-by-step in complete sentences.

You may work these problems by hand, and you may additionally use any of the following computer tools:

- Python code based on what we have covered in workshops;
- Desmos or Python for plotting functions.

Your submission should include any code that you use, with sufficient explanation of the code.

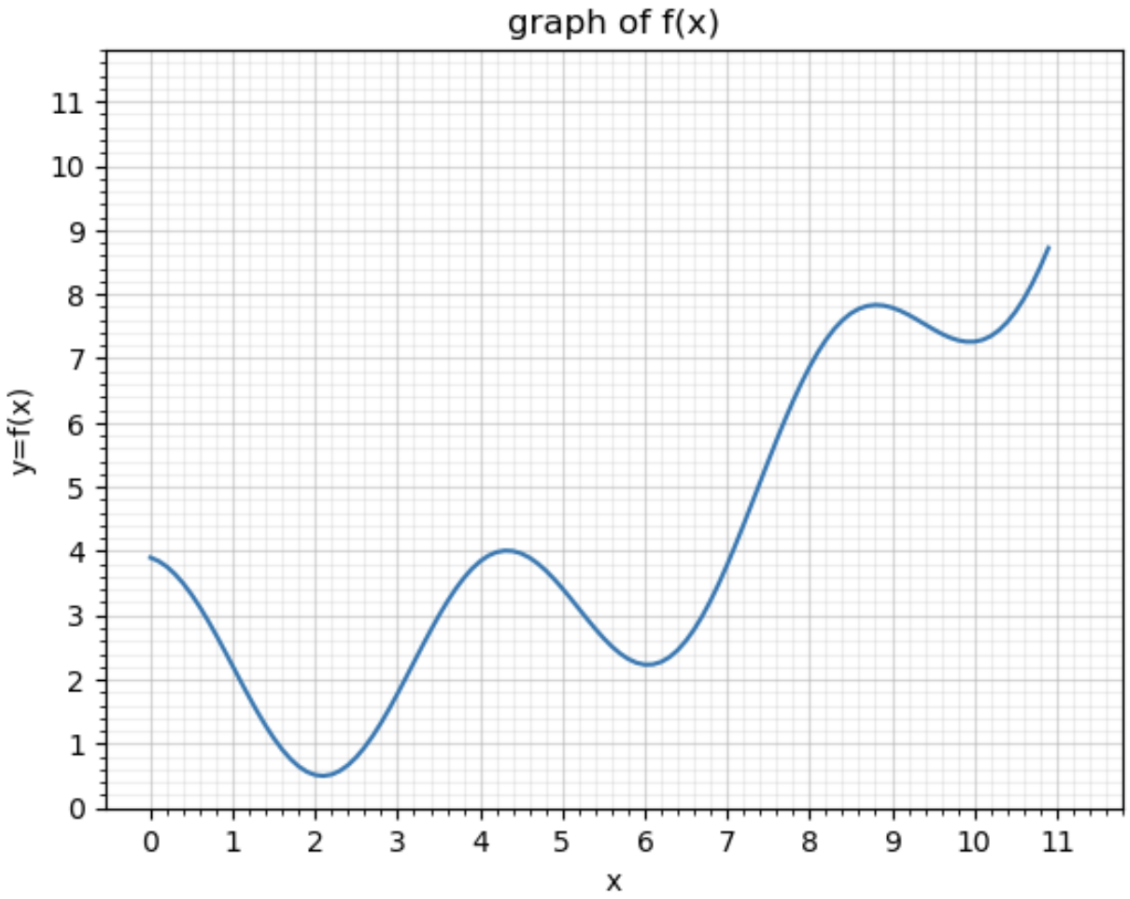
Please ask Prof. Giansiracusa if you need clarification of anything on this assignment.

Question 1 [20 points]

a) A new and dangerous bacterial infection is spreading through the world population. During the summer months it spreads roughly exponentially, but its spread slows to nearly zero in the winter months. Write a function $f(t)$ that could model the number of new cases per unit time. Be sure to explain what units of time you are using and plot your function.

b) Before the year 2000, wildfires in Wales almost never happened. Between 2000 and 2025 they became gradually more common (perhaps due to climate change), and now the frequency has levelled off at around 20 wildfires each year. The frequency of fires peaks in the summers and is nearly zero during the wet winters. Write a function that could model the number of fires per unit time. Be sure to explain what units of time you are using and plot your function.

Question 2 [20 points] Consider the function $f(x)$ whose graph is shown below.



a) Estimate the value of the derivative $f'(x)$ at $x = 5$. Give your answer to the nearest 0.5.

b) Using the graph above, estimate the integral $\int_6^{10} f(x)dx$. Give your answer to the nearest 0.1.

c) Now consider a function $h(x)$ for which the derivative $h'(x)$ is positive, negative, or zero according to the following table:

$x < 0$	$h'(x) > 0$
$x = 0$	$h'(x) = 0$
$0 < x < 1$	$h'(x) < 0$
$x = 1$	$h'(x) = 0$
$1 < x < 2$	$h'(x) < 0$
$x = 2$	$h'(x) = 0$
$2 < x < 3$	$h'(x) > 0$
$x = 3$	$h'(x) = 0$
$3 < x < 4$	$h'(x) > 0$
$x = 4$	$h'(x) = 0$
$x > 4$	$h'(x) < 0$

Suppose also that $h(2) = 0$. Sketch the graph of $h(x)$ between $x = -1$ and $x = 5$. Label any local minima, local maxima, and inflection points.

Question 3 [20 points] Consider the function $g(x, y)$ with values given in the following table.

1.0	-0.900	-0.710	-0.540	-0.390	-0.260	-0.150	-0.060	0.010	0.060	0.090
0.9	-0.999	-0.799	-0.619	-0.459	-0.319	-0.199	-0.099	-0.019	0.041	0.081
0.8	-1.092	-0.882	-0.692	-0.522	-0.372	-0.242	-0.132	-0.042	0.028	0.078
0.7	-1.173	-0.953	-0.753	-0.573	-0.413	-0.273	-0.153	-0.053	0.027	0.087
0.6	-1.236	-1.006	-0.796	-0.606	-0.436	-0.286	-0.156	-0.046	0.044	0.114
0.5	-1.275	-1.035	-0.815	-0.615	-0.435	-0.275	-0.135	-0.015	0.085	0.165
0.4	-1.284	-1.034	-0.804	-0.594	-0.404	-0.234	-0.084	0.046	0.156	0.246
0.3	-1.257	-0.997	-0.757	-0.537	-0.337	-0.157	0.003	0.143	0.263	0.363
0.2	-1.188	-0.918	-0.668	-0.438	-0.228	-0.038	0.132	0.282	0.412	0.522
0.1	-1.071	-0.791	-0.531	-0.291	-0.071	0.129	0.309	0.469	0.609	0.729
0.0	-0.900	-0.610	-0.340	-0.090	0.140	0.350	0.540	0.710	0.860	0.990
y/x	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

a) Use the table to estimate the value of ∇g at the point $(x, y) = (0.2, 0.4)$.

b) Using your estimate of the gradient from the previous part, estimate the value of the directional derivative of g at the point $(x, y) = (0.2, 0.4)$ along the vector $\vec{u} = (7, 8)$.

Question 4 [20 points] Consider the function $F(x, y) = x^2 + y^2 - 6\sin^2(x - y)$. If we start at the point $(x, y) = (-3.2, -3)$ and use gradient descent with step size 0.1, where do we end up after 3 steps? Provide a contour plot and a draw your steps on this plot.

Question 5 [20 points] Consider the vectors $\vec{u} = (9, 12)$ and $v = (-1, 7)$.

a) Calculate the length of $\vec{u}$, the length of $\vec{v}$, and the angle between them.

b) Calculate the orthogonal projection $\text{Proj}_{\vec{v}}(\vec{u})$ of the vector $\vec{u}$ onto the vector $\vec{v}$ and sketch a diagram of $\vec{u}, \vec{v}$ and $\text{Proj}_{\vec{v}}(\vec{u})$.