1 Exercise 1

Solve the system of equations given in Eq (1) using elimination, clearly showing the updated matrix after every row operation.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3 \end{aligned} \tag{1}$$

which gives

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} \tag{2}$$

2 Exercise 2

Solve the following system of equations, showing all relevant steps and stating the method used to solve the system (e.g., LU factorization, direct numerical solution, matrix elimination, etc.,). Furthermore, if a system has no solution state it has no solution and briefly describe why it has no solution.

$$\begin{pmatrix} 1 & 3 & 1 \\ 4 & 0 & 2 \\ 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 3 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 1 & 3 & 1 \\ 4 & 1 & 2 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} 1 & 3 & 1 \\ 4 & 1 & 2 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{5}$$

$$\begin{pmatrix} 0 & 1 & 2 & 0 & 0 \\ 5 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} \tag{6}$$

3 Exercise 3

Describe clearly but briefly the meaning of each term below, including advantages and disadvantages, number of operations, etc., in your own words (not anyone else's words, so not copied from any book, Wikipedia, internet, etc.,) and without using any equations whatsoever.

(a) Direct Elimination

(b) LU Factorization

(c) Inverse Matrices

(d) Symmetric Matrices

(e) Transpose

(f) Permutation

(g) inner product

(h) Singular Matrices

4 Exercise 4

Solve the following system and verify the solution visually by plotting the two systems using Python and annotating the interception point.

$$\begin{aligned} 2x + 3y &= 4 \\ x + 2y &= 5 \end{aligned} \tag{7}$$

5 Exercise 5

Use

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \tag{8}$$

$$B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \tag{9}$$

and

$$C = \begin{pmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{pmatrix} \tag{10}$$

to verify the following

(a) $(AB)^T = B^T A^T$

(b) $\left(A^{-1}\right)^T = \left(A^T\right)^{-1}$

(c) $C^T = C$