1 Exercise 1

As discussed in lecture tri-diagonal solvers are highly efficient solvers used to solve a particular system of equations, including diffusion transport type problems, which we will explore in the following exercises.

(a) Start with a brief description of tri-diagonal solvers, highlighting their advantages and disadvantages.

(b) Using the following three steps

(1) Store four 1D vectors

(2) Elimination

```
for i = 1 to N do
   b(i) = b(i) - c(i-1)*a(i)/b(i-1)
   d(i) = d(i) - d(i-1)*a(i)/b(i-1)
end for
```

(3) Back substitution

```
d(N) = d(N)/b(N)
for i = N-1 to 0 do
   d(i) = (d(i) - c(i)*d(i+1))/b(i))
end for
```

hand write pseudo code for a Python tri-diagonal solver subroutine. Make sure to include a brief description of the boundary treatment in your pseudo code. Upload your ***hand written subroutine*** as part of your submission for this problem.

(c) Use your pseudo code from part(b) to code a tri-diagonal solver that takes in four 1D-vectors and outputs a single 1D solutions vector. Upload a well annotated code with a single verification cases as your submission to this problem.

2 Exercise 2

For this problem we will focus on solving the non-dimensional steady-state heat transfer equation. Starting with the general heat transfer equation, namely

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x \partial x} + \dot{Q} \tag{1}$$

where ρ is the density, c_p is the specific heat, and k is the thermal conductivity. Next, we can properly non-dimensionalize Eq. (1) to get the following

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x \partial x} + \dot{Q} \tag{2}$$

where the left hand side represents the unsteady terms, the first term on the right hand side represents the diffusive transport of energy, and the last term contains the source and sink terms. Furthermore, for steady-state problems, like the one we are interested in, Eq. (2) further reduces to

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = -\dot{Q} \tag{3}$$

Next, we can apply Eq. (3) to the control volume shown in Fig. (1). The control volume shows a solar water heater that draws water from a well at temperature $T_w = 10$ (non-dimensional), and uses the thermal flux from the sun given as $\dot{Q} = 50\sin(2\pi x)$ to heat up water to temperature $T_{in} = 50$. Performing a control volume analysis we get the following ordinary differential equation,

$$\frac{\mathrm{d}^2 T}{\mathrm{d}x^2} = -\dot{Q} \tag{4a}$$

along with the following boundary conditions

$$T(x = 0) = T_w \tag{4b}$$

$$T(x = 0) = T_{in} \tag{4c}$$

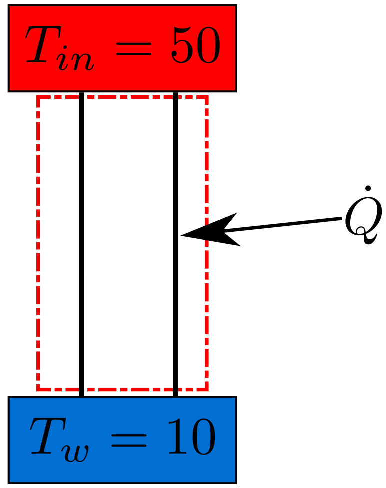

Figure 1: Water heater exchanger

(a) Start by discretizing Eq. (4a) using the second order central difference method

$$\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} \approx \frac{f_{i+1} - f_i + f_{i-1}}{\Delta x^2} \tag{5}$$

and construct the coefficient and solution vectors (refer to pg. 21 of Module 1). Show these matrices for a domain discretized with $M = 8$ equidistant elements.

(b) Solve Eq. (4) using the tri-diagonal solver from the previous exercise on a mesh of $M = 4, 8, 16, 32, 64$ equidistant elements. Furthermore, compare the approximate solution obtained from the tri-diagonal solver to the exact analytical solution for each mesh. Please provide five plots with two curves comparing the results from the analytical and numerical solutions. Lastly, discuss your results.

(c) Conduct a performance study on your tri-diagonal solver by solving Eq. (4) with $M = 10, 100, 1000, 5000, 10000$ elements and plotting mesh size versus time. Briefly discuss your results.

(d) Re-do parts (b) and (c), however instead of using your tri-diagonal solver, solve the system using $T = \mathbf{A}^{-1}\mathbf{b}$. Briefly discuss your results, ensuring to compare the performance and accuracy between the two methods.