Name: ____________________

ASU ID: ____________________

Midterm Exam

MAE/MSE 501 Fall 2020

Instructions:
This is a timed take-home exam. You are allowed 75min starting 7:30am MST. Your work needs to be uploaded to GradeScope within **15min** of the end of the allotted time. Submissions after 9:00am MST will not be accepted. Make sure that your name and ASU ID appear in your GradeScope submission. If you have problems uploading your midterm, contact the instructor immediately.

You may print and work on the exam sheet, write on blank sheets or use a tablet. Either way, you must show all work in a neat and legible fashion in order to receive credit for your answers.

There are **4 problems** in this exam for 40 points total.

Problem 1 [10 pts]
Given the matrix

$$A = \begin{bmatrix} 0 & 0 & 6 \\ 1 & 2 & 3 \\ 0 & 4 & 5 \end{bmatrix}$$

(a) (4pts) Find a permutation P that makes PA upper triangular. (explain your thought process)

(b) (2pt) Is PA invertible? Explain why.

(c) (2pt) Using properties of permutation matrices, find the inverse of P.

(d) (2pt) Calculate A^{-1}.

Problem 2 (10 pts)
Let A be the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 4 & 3 & 2 \\ -1 & 0 & 1 & 2 \end{bmatrix}$$

(a) (4 pts) Calculate the LU factorization of A.

(b) (1 pt) What is the rank of A?

(c) (2 pts) What are the dimensions of the four fundamental subspaces of A?

(d) (3 pts) Give the complete solution to $A\boldsymbol{x} = \boldsymbol{b}$ with

$$\boldsymbol{b} = A \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

Problem 3 (10 pts)
After solving a system of linear equations $A\boldsymbol{x} = \boldsymbol{b}$, we found that the complete solution is

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix} + \alpha \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

(a) (3pt) What is the rank of the matrix A?

(b) (2pt) How many columns and rows does A have? Give all possibilities.

(c) (2 pt) Explain why the particular solution $\begin{bmatrix} 1 \\ 0 \\ -4 \end{bmatrix}$ is not in the row space.

(d) (3 pt) Find a particular solution which is in the row space.

Problem 4 (10 pts)
Let $\mathcal{P}$ be a plane in 3D defined by $2x + 2y - z = 0$.

(a) (3 pts) Find a unitary vector $\boldsymbol{n}$ normal to the plane.

(b) (3 pts) Give the projection matrix P_1 onto the line oriented along $\boldsymbol{n}$. What is the rank of this matrix.

(c) (2 pts) Give the projection matrix P_2 onto the plane $\mathcal{P}$.

(d) (2 pts) What is the rank and column space of P_2?