1 Exercise 1

Solve the system of equations given in Eq (1) using elimination, clearly showing the updated matrix after every row operation.

$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 &= b_2 \\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 &= b_3
\end{aligned} \tag{1}
$$

which gives

$$
\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{2}
$$

2 Exercise 2

Solve the following system of equations, showing all relevant steps and stating the method used to solve the system (e.g., LU factorization, direct numerical solution, matrix elimination, etc.,). Furthermore, if a system has no solution state it has no solution and briefly describe why it has no solution.

$$
\begin{bmatrix} 1 & 3 & 1 \\ 4 & 0 & 2 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 5 \\ 6 \\ 3 \end{bmatrix} \tag{3}
$$

$$
\begin{bmatrix} 1 & 3 & 1 \\ 4 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{4}
$$

$$
\begin{bmatrix} 1 & 3 & 1 \\ 4 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \tag{5}
$$

$$
\begin{bmatrix} 0 & 1 & 2 & 0 & 0 \\ 5 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix} \tag{6}
$$

3 Exercise 3

Describe clearly but briefly the meaning of each term below, including advantages and disadvantages, number of operations, etc., in your own words (not anyone else's words, so not copied from any book, Wikipedia, internet, etc.,) and without using any equations whatsoever.

(a) Direct Elimination

(b) *LU* Factorization

(c) Inverse Matrices

(d) Symmetric Matrices

(e) Transpose

(f) Permutation

(g) Inner product

(h) Singular Matrices

4 Exercise 4

Solve the following system and verify the solution visually by plotting the two systems using Python and annotating the interception point.

$$\begin{aligned} 2x + 3y &= 4 \\ x + 2y &= 5 \end{aligned} \tag{7}$$

5 Exercise 5

Use

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \tag{9}$$

and

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} \end{bmatrix} \tag{10}$$

to verify the following

(a) $(AB)^T = B^T A^T$

(b) $(A^{-1})^T = \left(A^T\right)^{-1}$

(c) $C^T = C$

6 Exercise 6

Find the LU and LDU factorizations by hand for the following matrices.

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 5 & 4 \\ 2 & 4 & 6 \end{bmatrix} \tag{11}$$

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 2 \\ 3 & 4 & 5 \end{bmatrix} \tag{12}$$

$$C = \begin{bmatrix} 2 & 1 & 3 \\ 3 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} \tag{13}$$

7 Exercise 7

Using Python generate a LU factorization tool (subroutine) that takes in a $N \times N$ matrix and outputs both L and U factors. Next, use your factorization tool to compute the LU factors for a matrix with $N = 10, 100, 200, 300, 400, 500, 1000$ and plot N versus time it takes to compute the factorization. Use the pseudo code provided to learn see how to time different blocks of code and generate random matrices.

8 Exercise 8

Find the inverse by hand for the following matrices. If a matrix does not exist briefly state why it cannot be inverted.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

$$B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \tag{15}$$

$$C = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \tag{16}$$

9 Exercise 9

Using Python generate a matrix inverse tool (subroutine) that takes in a $N \times N$ matrix and outputs its inverse. Similarly to the previous assignment, use your inverse tool to compute the A^{-1} for a matrix with $N = 10, 100, 200, 300, 400, 500, 1000$ and plot N versus time it takes to compute the inverse.