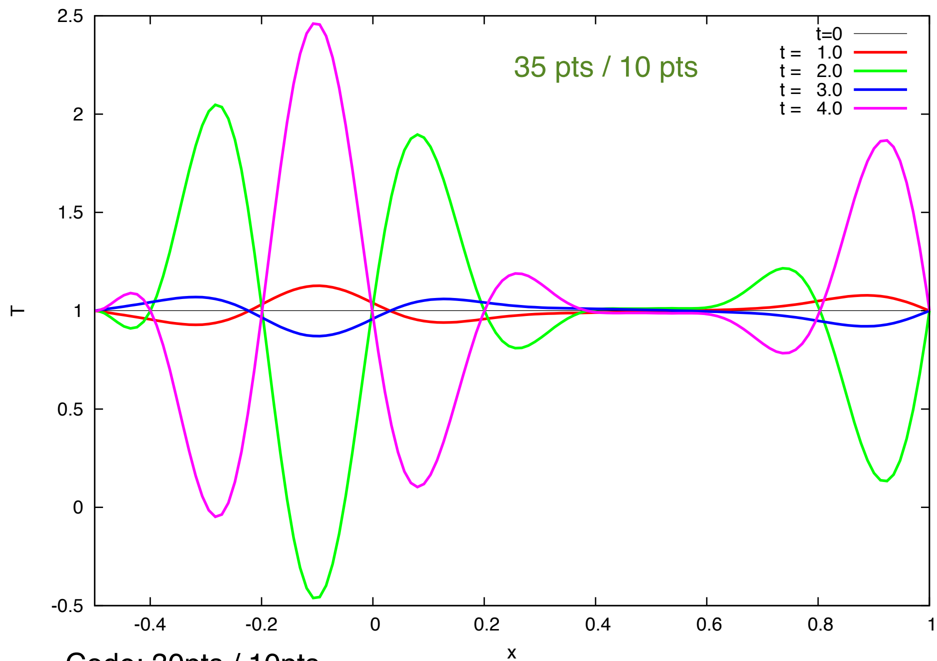
Homework 6 Solution

Problem 1(100 points total)

Task 1 (25 / 5 points)   scan

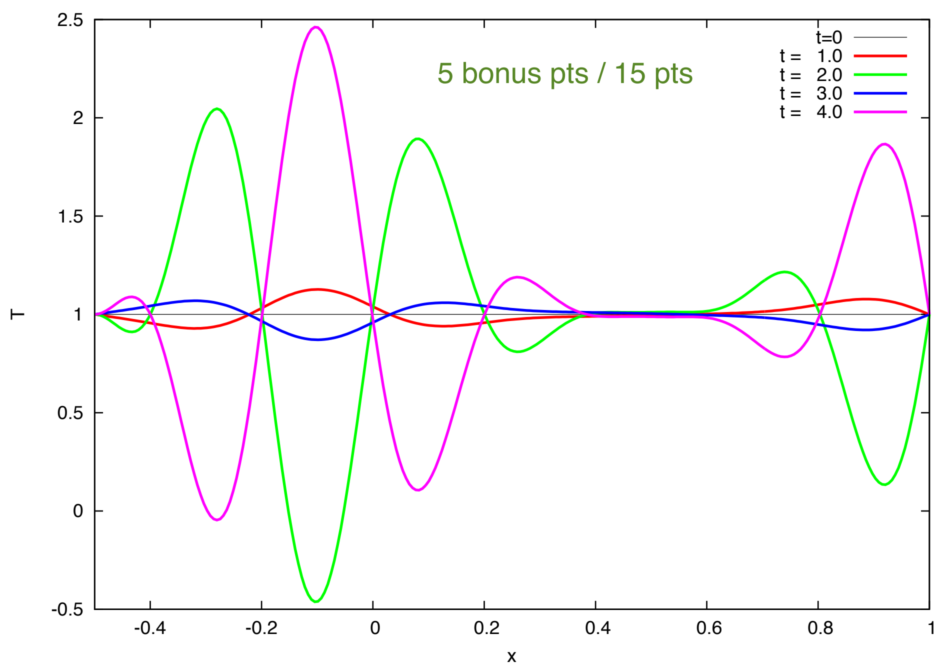
Task 2 (65 / 20 points)



Code: 30pts / 10pts

Task 3 (12 bonus points / 30 points)   scan

Task 4 (10 bonus points / 35 points)



Code: 5 bonus pts / 20pts

Homework 6 Solution

Task 5 (10 points / 10 points)
can use FTCS as well, results can be inconsistent, but need to be documented.

8 points

CN	Tmax (t=100)	p	f(h=0)	GCI	GCI23	asymptotic?
64	2.48776E+00					
128	2.49329E+00					
256	2.49409E+00	2.79895E+00	2.49422E+00	0.0067%	0.0465%	0.9997

Answer: Crank Nicholson, T= 2.49422E+00 +/- 0.0067% (2 points)

Homework 6

(AEG471/MAE561)

1)

FTCS: $$\frac{T_i^{n+1}-T_i^n}{\Delta t} = \frac{\alpha}{h^2}\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + q_i^n$$

$$\Rightarrow \quad T_i^{n+1} = T_i^n + \frac{\alpha\Delta t}{h^2}\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + \Delta t\, q_i^n$$

10/2

ghost cells:

$$T_0^n = 2T(x=-\tfrac{1}{2}, t) - T_1^n \quad \Leftrightarrow \quad T_0^n = 2 - T_1^n$$

5/1 (or Matlab indices)

$$T_{N+1}^n = 2T(x=1, t) - T_N^n \quad \Leftrightarrow \quad T_{N+1}^n = 2 - T_N^n$$

5/1

ay: time step

$$\Delta t_{max} = \frac{1}{2}\frac{h^2}{\alpha}$$

5/1

3) CN:
$$\frac{T_i^{n+1}-T_i^n}{\Delta t} = \frac{1}{2}\left\{\frac{\alpha}{h^2}\left(T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}\right) + \frac{\alpha}{h^2}\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + q_i^{n+1} + q_i^n\right\}$$

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$$\Leftrightarrow \quad T_{i+1}^{n+1} = T_i^n + \frac{\alpha\Delta t}{2h^2}\left(T_{i+1}^{n+1} - 2T_i^{n+1} + T_{i-1}^{n+1}\right) + \frac{\alpha\Delta t}{2h^2}\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right) + \frac{\Delta t}{2}\left(q_i^{n+1} + q_i^n\right)$$

$$\Leftrightarrow \underbrace{-\frac{\alpha\Delta t}{2h^2}}_{=a} T_{i-1}^{n+1} + \underbrace{\left(1 + \frac{\alpha\Delta t}{h^2}\right)}_{=b} T_i^{n+1} \underbrace{-\frac{\alpha\Delta t}{2h^2}}_{=c} T_{i+1}^{n+1} = \underbrace{\frac{\alpha\Delta t}{2h^2} T_{i-1}^n + \left(1 - \frac{\alpha\Delta t}{h^2}\right) T_i^n + \frac{\alpha\Delta t}{2h^2} T_{i+1}^n + \frac{\Delta t}{2}\left(q_i^{n+1} + q_i^n\right)}_{=d}$$

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b.c.

$$i=1: \quad T_0^{n+1} = 2T(x=-\tfrac{1}{2}, t^{n+1}) - T_1^{n+1}$$

$$\Rightarrow \quad -\frac{\alpha\Delta t}{2h^2}\left(2T(x=-\tfrac{1}{2}, t^{n+1}) - T_1^{n+1}\right) + b_1 T_1^{n+1} - c_1 T_2^{n+1} = d_1$$

$$\Leftrightarrow \quad \underbrace{\left(b_1 + \frac{\alpha\Delta t}{2h^2}\right)}_{=b_1} T_1^{n+1} - c_1 T_2^{n+1} = \underbrace{d_1 + \frac{\alpha\Delta t}{h^2} T(x=-\tfrac{1}{2}, t^{n+1})}_{=d_1}$$

$$= 1 + \frac{3}{2}\frac{\alpha\Delta t}{h^2}$$

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$i = M$: $\quad T_{M+1}^{n+1} = 2T(x=1, t^{n+1}) - T_M^{n+1}$

$$\Rightarrow \quad a_M T_{M-1}^{n+1} + b_M T_M^{n+1} - \frac{\alpha \Delta t}{2h^2}\left(2T(x=1, t^{n+1}) - T_M^{n+1}\right) = d_M$$

$$\Leftrightarrow \quad a_M T_{M-1}^{n+1} + \underbrace{\left(b_M + \frac{\alpha \Delta t}{2h^2}\right)}_{\substack{= b_M \\ = 1 + \frac{3}{2}\frac{\alpha \Delta t}{h^2}}} T_M^{n+1} = \underbrace{d_M + \frac{\alpha \Delta t}{h^2} T(x=1, t^{n+1})}_{= d_M}$$

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