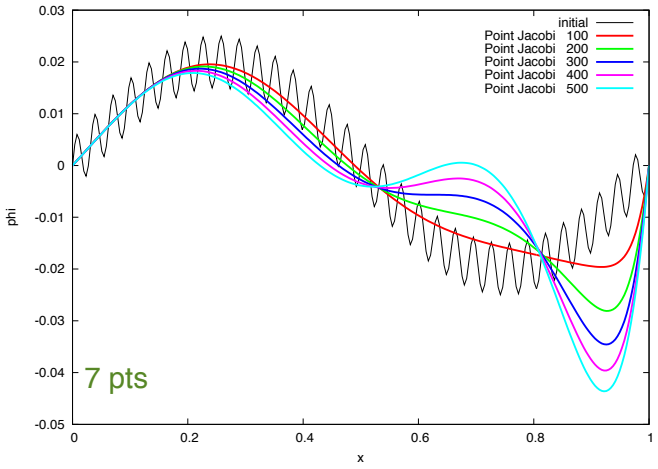
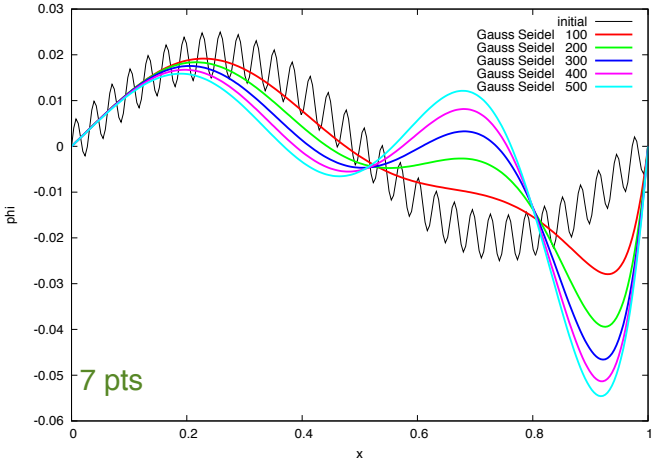
score is % out of 120 points (MAE561) and 100 points (AEE471)

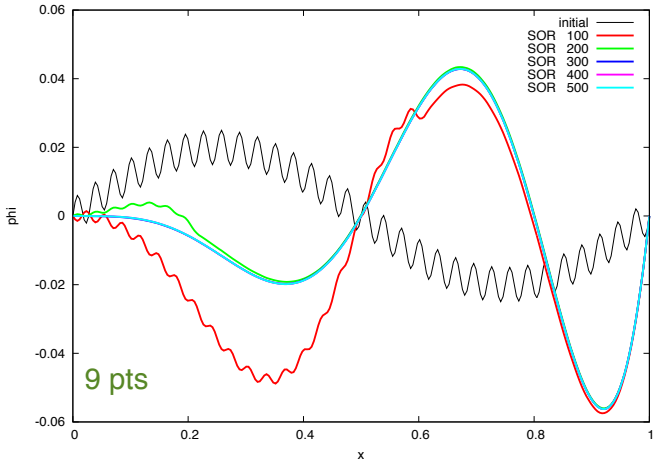
Homework 3 Solution

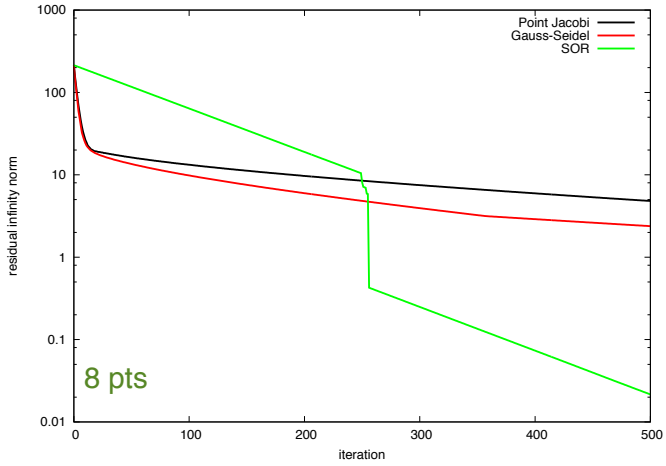
Problem 1 (40 points total)

theoretical optimum omega: 1.9757544535797118 (3 pts)









Discussion:

- higher wavenumber converge faster than lower wavenumber (at least for PJ and GS) (2 points)
- PJ converges slower than GS (2 points)
- SOR, while initially converging slower, in the end converges faster than PJ and GS (2 points)

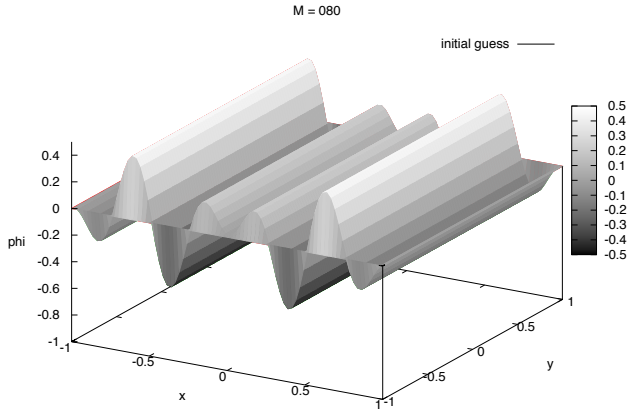
Deductions:

- no SafeAssign code upload: -20pts
- not using optimum omega: -10pts
- if wrong right hand side: -15pts

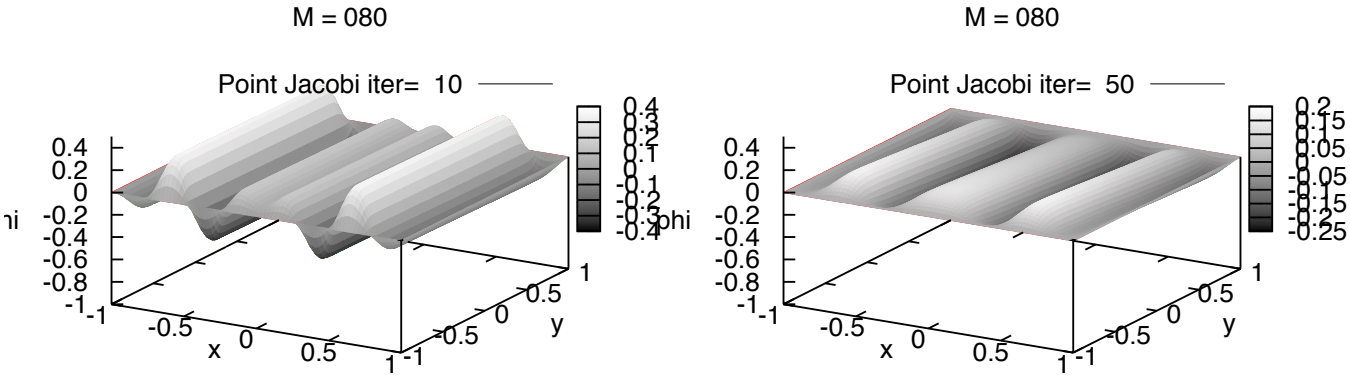
Homework 3 Solution

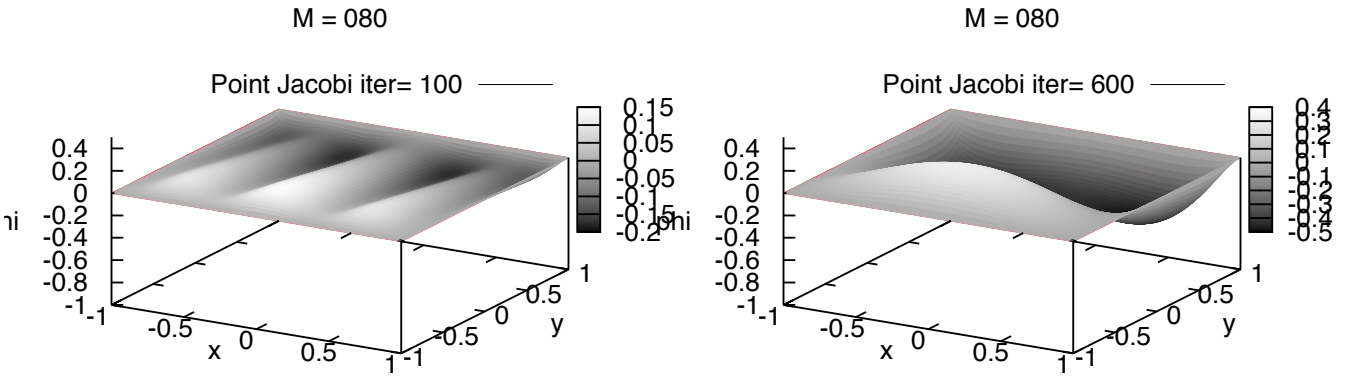
Problem 2 (60 points total)

theoretically optimum omega: 1.9244419120566281 (3 pts)



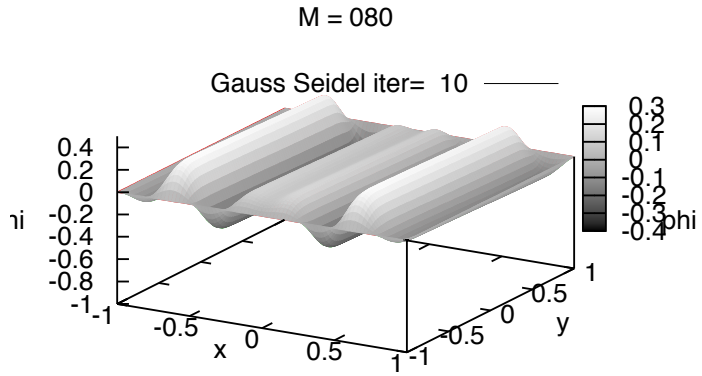
2 pts

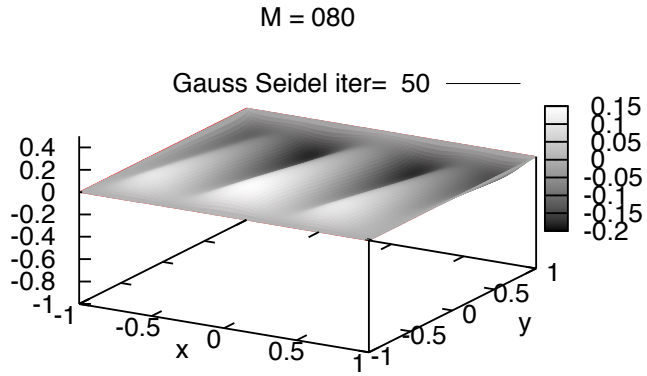


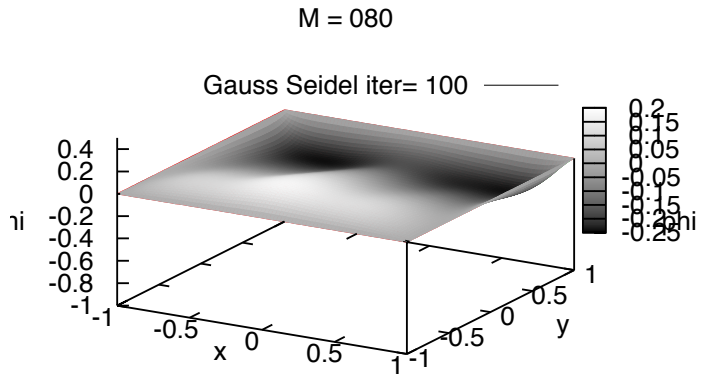


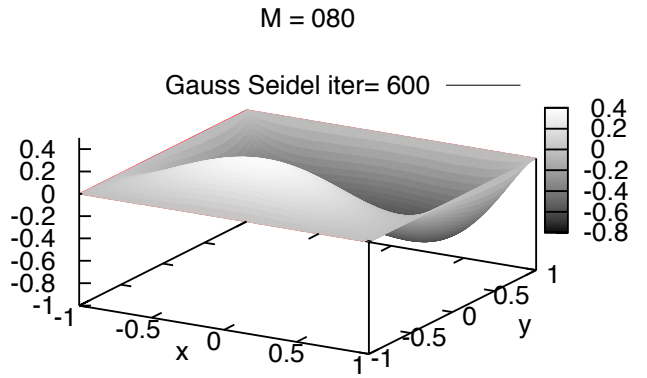
16 pts

Homework 3 Solution

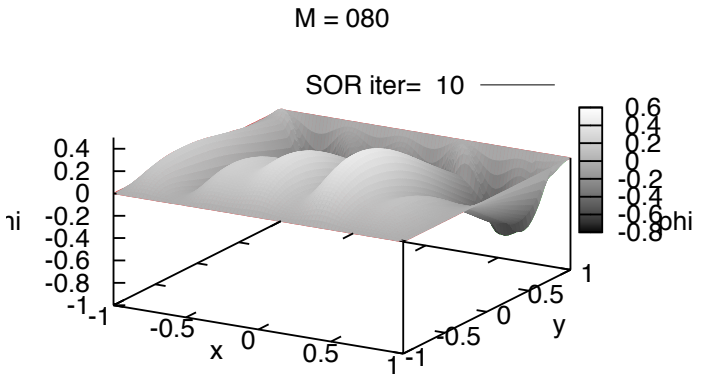


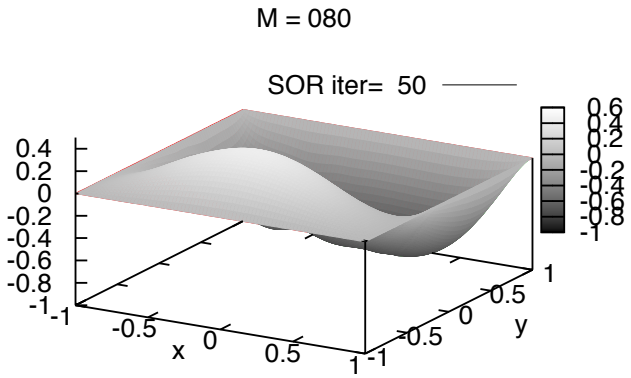


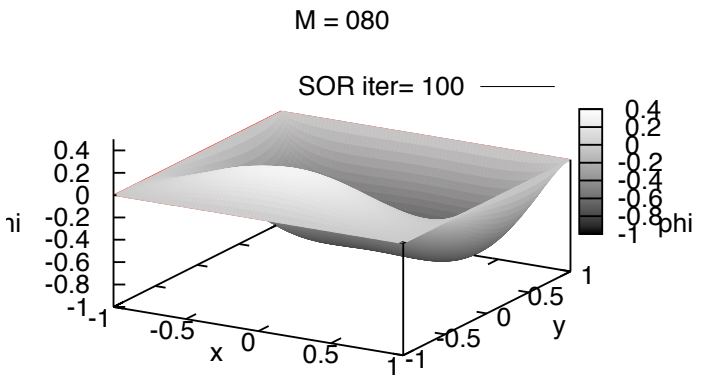


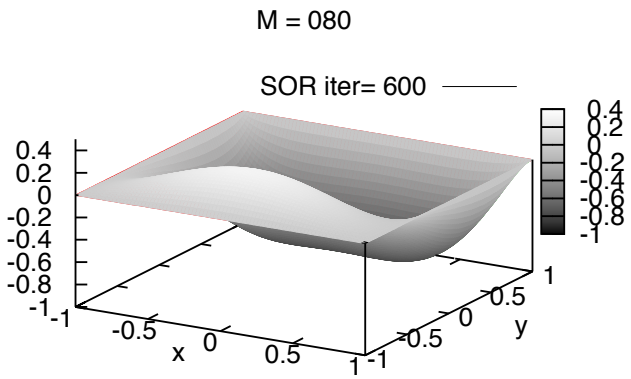


16 pts



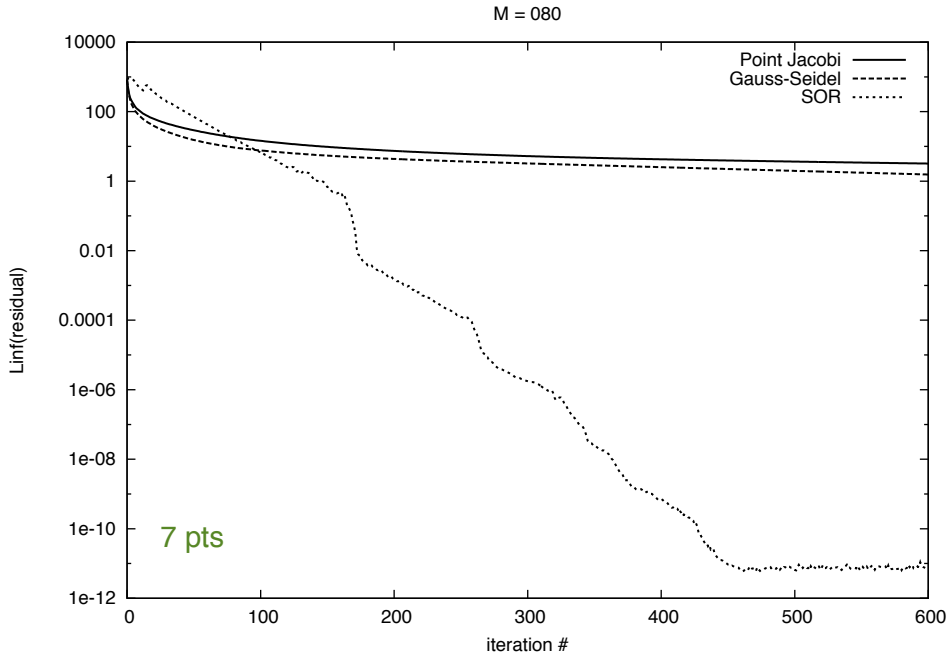






16 pts

Homework 3 Solution



Deductions:
- no SafeAssign code upload: -30pts
- not using optimum omega: -10pts
- if wrong right hand side: -20pts
- if wrong initial condition: -10pts

Problem 3 (20 points total MAE461, 10 bonus points total AEE471) AEE471 half the points indicated

8 pts

M	Linf	L1	L2	order Linf	order L1	order L2
128	9.99186E-03	5.91436E-03	6.61514E-03			
256	3.08295E-02	1.49472E-02	1.78445E-02	-1.6255	-1.3376	-1.4316
512	5.05513E-02	2.30233E-02	2.67020E-02	-0.7134	-0.6232	-0.5815

Discussion:
- the error does not converge. In fact the order is negative indicating the solution diverges. (2 points)
- the observed order is not close to the formal order of 2 at all (2 points)
- the reason is that a fixed number of iterations are performed. On finer meshes, the initial guess and the final solution are better resolved, thus have higher relative wavenumber. Since higher wavenumber converge slower than lower wavenumber, coarser meshes will have converged better at 500 iterations than finer meshes. Thus the error for the coarser mesh will be lower than the error of the finer meshes, resulting in the obtained negative observed order of convergence (6 points)
- to obtain a value close to the formal order, the error due to non-zero residual must be smaller than the error due to Taylor series truncation. This can be achieved by increasing the number of iterations significantly and switching to the SOR method for efficienty(2 points)

Homework 3 Solution

Bonus Problem 4 (10 bonus points)

using 10,000 iterations of SOR method

M	Linf	L1	L2	order Linf	order L1	order L2
128	1.21412E-04	5.55035E-05	6.69246E-05			
256	3.03473E-05	1.39279E-05	1.67607E-05	2.0003	1.9946	1.9975
512	7.58656E-06	3.48863E-06	4.19406E-06	2.0000	1.9972	1.9987