MAE 471/561

Homework #6 - Due: March 18th, at the beginning of class

Please submit result graphs together with either handwritten or printed out descriptions and answers at the beginning of class. Combine all code you used to solve the problems into a single text file, and upload the text file to Blackboard using the SafeAssign mechanism. No credit will be given, if your code is not uploaded as a text file using SafeAssign. Also ensure that your code contains adequate comments. Add a printout of all code as an **appendix** to your submission.

Problem 1 (100 points, AEE 471: Core Course Outcome #2)
Consider the following one-dimensional PDE

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + q(x,t)\,, \tag{1}$$

defined on a domain of size $-0.5 \leq x \leq 1$ with boundary conditions $T(x = -0.5, t) = 1$, $T(x = 1, t) = 1$, initial condition $T(x, t = 0) = 1$, and $\alpha = 0.1$, The source term q is given by

$$\begin{aligned} q(x,t) \quad = \quad & \cos(\omega t)\left[-6\ (x-0.5)\,(1+\cos(\pi\ (x-0.5)))\sin(5\,\pi\,x)\right. \\ & +6\ (x-0.5)^2\,\pi\ \sin(\pi\ (x-0.5))\sin(5\,\pi\,x) \\ & -30\ (x-0.5)^2\,(1+\cos(\pi\ (x-0.5)))\,\pi\ \cos(5\,\pi\,x) \\ & +(x-0.5)^3\,\pi^2\cos(\pi\ (x-0.5))\sin(5\,\pi\,x) \\ & +10\ (x-0.5)^3\,\pi^2\sin(\pi\ (x-0.5))\cos(5\,\pi\,x) \\ & \left.+25\ (x-0.5)^3\,(1+\cos(\pi\ (x-0.5)))\,\pi^2\sin(5\,\pi\,x)\right]\,. \end{aligned} \tag{2}$$

On Blackboard, the file `source.txt` contains the source term in code for Matlab, Fortran, and C.

1. Write in index form for a cell centered mesh the equation to solve the PDE using the FTCS method for cell center i and time level $n+1$, i.e. $T_i^{n+1} = \ldots$, the index equation to calculate ghost cell values, and the equation to determine the maximum stable time step size. Do not substitute in the formula for q, but instead use q in index form as well. No points will be given if the formulas merely appear in code.

2. Using a **cell centered** mesh with $M = 128$ interior elements and the FTCS method, determine and plot in one plot $T(x)$ at $t =$ 0, 1, 2, 3, 4 for $\omega = \pi/2$. Use a time step size Δt that is **half** the maximum stable time step size. To help you debug your code, on Blackboard you will find the file `ftcs.txt` that contains the solution variables for the first 2 time steps using $M = 32$ interior elements.

3. **Required for MAE561, Bonus for AEE471**: Write in index form for a cell centered mesh the equation to solve the PDE using the Crank-Nicholson method for cell center i and time level $n + 1$. **Do not** use the two-step process described in Class 13 v2 slide 11, but use the formulation in Class 13 v2 slide 10 to determine the tridiagonal system to solve. Write in index form equations to determine the 3 diagonal vectors a,b, c of the tridiagonal system and the right hand side vector d. There should be M entries in each vector. Incorporate the boundary conditions for T^{n+1} into the vectors a, b, c, and d and use ghost cells directly for the boundary conditions for T^n. Do not substitute in the formula for q, but instead use q in index form as well. No points will be given if the formulas merely appear in code.

4. **Required for MAE561, Bonus for AEE471**: Using a **cell centered** mesh with $M = 256$ interior elements and the Crank-Nicholson method, determine and plot in one plot $T(x)$ at $t =$ 0, 1, 2, 3, 4 for $\omega = \pi/2$. Use a time step size Δt that is **four times** the time step size of part 1.

5. For $\omega = 0$, using **cell centered** meshes of your choosing and a method of your choosing, determine the maximum steady state temperature within 0.01% accuracy. Document your solution steps and justify why your values have the requested accuracy using Richardson extrapolation and GCI.

Required submission:
- FTCS method in index form, boundary conditions in index form, maximum time step formula;

- 1 clearly annotated plot containing T as a function of x for $t = 0, 1, 2, 3,$ and 4, for $M = 128$ and FTCS and $\omega = \pi/2$;

- Crank-Nicholson method in index form, diagonal and right hand side vectors a,b,c,d in index form incorporating boundary conditions for tridiagonal solve;

- 1 clearly annotated plot containing T as a function of x for $t = 0, 1, 2, 3,$ and 4, for $M = 256$ and Crank-Nicholson and $\omega = \pi/2$;

- For $\omega = 0$, documentation of solution procedure (method chosen, mesh elements used, etc.) and GCI analysis to obtain maximum steady state temperature with requested accuracy;

- As an Appendix, printout of code;

- SafeAssign upload of all used, well commented code.