AEE 471 / MAE 561

Project #1 – due April 22nd, beginning of class

AEE471 Core Outcomes #3 & #5

A two-dimensional flow is described by the following set of equations:

$$\frac{\partial u}{\partial t} + \frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y} = \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{1}$$

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial vv}{\partial y} = \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{2}$$

These are not the Navier-Stokes equations, since there is no continuity equation and no pressure! Solve the equations over the domain $0 \leq x \leq 1$ and $0 \leq y \leq 1$ and for $0 \leq t \leq 20$ with the following boundary conditions for $u(x, y, t)$ and $v(x, y, t)$,

$$\begin{aligned}
u(0, y, t) &= -\sin(\omega t) && (3)\\
v(0, y, t) &= 0 && (4)\\
u(1, y, t) &= 0 && (5)\\
v(1, y, t) &= 0 && (6)\\
u(x, 0, t) &= \sin 2\pi x && (7)\\
v(x, 0, t) &= \cos(2\omega t) && (8)\\
u(x, 1, t) &= 1 && (9)\\
v(x, 1, t) &= 0 && (10)
\end{aligned}$$

using a **cell centered** mesh. Comment: the above boundary conditions give inconsistent values for the corners of the domain, however, these values are never used in the actual algorithm. The viscosity of the fluid is $\nu = 0.015$. The frequency is $\omega = \pi/2$. The flow inside the domain is initially at rest.

Monitor the time history of the kinetic energy K in the solution domain by plotting versus time

$$K = \frac{1}{2}\int_0^1 \int_0^1 (u^2 + v^2)\,dy\,dx\,. \tag{11}$$

You are free to choose any scheme covered in class to solve the problem. You must submit a written report on paper containing all the requested parts with your code in the appendix, in addition to uploading your code on Blackboard using the SafeAssign mechanism.

Assignment (100 points):

1) Document the scheme and solution strategy you have chosen (see required submission).

2) Discuss the stable time step for the scheme you chose. (Again: the above equations are not the Navier-Stokes equations!) Your code must contain either dynamic Δt calculation, or a check for every time step that your chosen Δt is stable.

3) Determine the maximum value of the kinetic energy K in the time interval $10 < t < 14$ up to 0.03% accuracy. Justify your results.

4) Plot the time history of the kinetic energy $K(t)$ for $0 \leq t \leq 20$ for a simulation that satisfies task 3).

5) Plot the velocity vectors (u, v) in the interior of the domain as **vector plots** at those times in the interval $10 < t < 14$ where the kinetic energy has a local minima or maxima. Note: If you use Matlab make sure that your data and vectors are plotted in the correct direction. No credit will be given if the axis directions and vectors are not consistent. Should your mesh resolution be so high that individual vectors overlap, plot only every n-th vector so that individual vectors can still be discerned.

Required content of your report:
- name of chosen scheme and all index formulas used;
- formulas to determine the stable time step both in writeup and in the code used;
- grid refinement /GCI study to solve part 3); maximum value of the kinetic energy within 0.03% for $10 < t < 14$*;*
- clearly annotated plot of time history of kinetic energy (K *vs.* t*) for part 4);*
- clearly annotated vector plots of velocity at local extrema in the kinetic energy history for $10 < t < 14$*;*
- discussion of the observed flow;
- printout of well commented code as an appendix;
- SafeAssign upload of all used, well commented code.

Bonus Assignment (15 points): In addition to solving the two-dimensional flow above, solve the following transport equation for an immiscible dye mass fraction Y,

$$\frac{\partial Y}{\partial t} + u\frac{\partial Y}{\partial x} + v\frac{\partial Y}{\partial y} = 0, \tag{12}$$

with the following boundary conditions:

$$\begin{aligned}
Y(0,y,t) &= g(y) \;\text{ if }\; \sin(\omega t) < 0 && (13)\\
\frac{\partial Y}{\partial x}(0,y,t) &= 0 \;\text{ if }\; \sin(\omega t) \geq 0 && (14)\\
\frac{\partial Y}{\partial x}(1,y,t) &= 0 && (15)\\
Y(x,0,t) &= g(x) \;\text{ if }\; \cos(2\omega t) > 0 && (16)\\
\frac{\partial Y}{\partial y}(x,0,t) &= 0 \;\text{ if }\; \cos(2\omega t) \leq 0 && (17)\\
\frac{\partial Y}{\partial y}(x,1,t) &= 0 && (18)
\end{aligned}$$

and

$$g(a) = \begin{cases} 0 & : \quad a < 0.4 \\ 1 & : \quad 0.4 \leq a < 0.6 \\ 0 & : \quad a \geq 0.6 \end{cases} \tag{19}$$

for the simulation satisfying task 3). Initially there is no dye in the domain. Document the scheme you use to solve the equations and plot the dye mass fraction Y as a contour/color plot at appropriate times in the interval $0 \leq t \leq 20$ and generate a movie of the dye mass fraction evolution.

Required additional content in your report:
- name of chosen scheme and all index formulas used for dye mass fraction equation;
- clearly annotated plots of dye mass fraction at appropriate times for $0 \leq t \leq 20$
- uploaded movie of dye mass fraction evolution for $0 \leq t \leq 20$ *using the Blackboard project link.*
- all well commented additional code as an appendix in the report.
- SafeAssign upload of all used, well commented code.