AEE 471 / MAE 561

Homework #4 - Due: Wednesday, February 25th, at the beginning of class

Combine all code you used to solve the problems into a single text file, and upload the text file to Blackboard using the SafeAssign mechanism. No credit will be given, if your code is not uploaded as a text file using SafeAssign. Also ensure that your code contains adequate comments.

Problem 1 (AEE 471: 100 points (Core Course Outcome #2), MAE561: 40 points)
Consider the homework 3 problem 1 second-order differential equation

$$\frac{d^2\phi}{dx^2} = \sin(2\pi x)\left(-5\pi^2 x^4 + 4\pi^2 x^3 + 15x^2 - 6x\right) + \cos(2\pi x)\left(20\pi x^3 - 12\pi x^2\right), \tag{1}$$

with boundary conditions $\phi(x = 0) = \phi(x = 1) = 0$. Solve this equation on the interval $0 \leq x \leq 1$ on a node based mesh using 257 equally spaced points ($x_0 = 0$, $x_{256} = 1$). As initial guess $\phi^{(0)}(x)$ use

$$\phi^{(0)}(x) = \frac{1}{50}\sin(2\pi x) + \frac{1}{200}\sin(64\pi x). \tag{2}$$

Solve the problem using both a Gauss Seidel and a dual grid Multigrid method (just 2 grid levels). One each dual grid level, perform one Gauss-Seidel iteration. Use averaging for restriction (option # 2 on class 9, slide 3).

Note: On Blackboard, you will find a text file *variablesdualgrid.txt* containing the results of each operation for the first cycle of the dual grid Multigrid method. Use this file to help you debug your code.

a) Plot the initial guess and then plot the solutions after 1, 2, 50, and 100 iterations for Gauss Seidel into one plot.

b) Plot the initial guess and then plot the solutions after 1, 2, 50, and 100 iterations for dual grid Multigrid into one plot.

c) In one additional graph, plot the infinity norm of the residual for both methods vs the iteration number (0-100). Use a log scale for the residual and a linear scale for the iteration number. Discuss the results and compare to the plot you obtained in problem 1 of homework 3.

d) Discuss the performance/cost of the method as compared to the methods employed in problem 1 of homework 2.

Required submission:
- 1 clearly annotated plot containing Gauss Seidel solution after 0, 2, 50, and 100 iterations;
- 1 clearly annotated plot containing dual grid Multigrid solution after 0, 2, 50, and 100 iterations;
- 1 clearly annotated log-linear plot containing L_∞ of residual vs iteration number for Gauss Seidel and dual grid Multigrid methods for 0-100 iterations;
- discussion of results incl. comparison to Point Jacobi, Gauss Seidel, and SOR method results from problem 1 of homework3;
- printout of code used
- SafeAssign upload of all used, well commented code.

Problem 2 (**required for MAE561**: 60 points, **extra credit for AEE471** (Core Course Outcome #2): 20 bonus points)
Solve Problem 1 by a full V-cycle Multigrid method. Perform one Gauss-Seidel iteration step on each V-node. Use averaging for restriction.

a) Plot the initial guess and then plot the solutions after 1, 2, 50, and 100 iterations into one plot.

b) Discuss the solution you obtained in comparison to the solution of problem 1 and problem 1 of homework 3.

c) Add to the problem 1, part c) graph the infinity norm of the residual vs the iteration number (0-100). Discuss the results and compare to the plot you obtained in problem 1 and problem 1 of homework 3.

d) Discuss the performance/cost of the method as compared to the methods employed in problem 1 and problem 1 of homework 3.

Note: On Blackboard, you will find a text file *variablesvcycle.txt* containing the results of each operation for the first cycle of the V-cycle Multigrid method. Use this file to help you debug your code.

Required submission:
- 1 clearly annotated plot containing V-cycle Multigrid solution after 0, 2, 50, and 100 iterations;
- 1 clearly annotated log-linear plot containing L_∞ of residual vs iteration number for Gauss Seidel, dual grid Multigrid, and V-cycle Multigrid methods for 0-100 iterations;
- discussion of results incl. comparison to Point Jacobi, Gauss Seidel, SOR, and dualgrid method results from problem 1 and problem 1 of homework 3;
- printout of code;
- SafeAssign upload of all used, well commented code.

Bonus Problem 3 (10 bonus points) AEE471: Core Course Outcomes #1 & #2:
Solve problem 1 using 129, 513, and 1025 equally spaced points employing the V-cycle Multigrid method using an appropriate number of iterations to demonstrate the formal order of the discretization used. Calculate the L_∞, L_1, and L_2 error norms of the solution for the 129, 257, 513, 1025 point nodal meshes, if the exact solution is given by

$$\phi_{exact}(x) = \sin(2\pi x)\left(\frac{5}{4}x^4 - x^3\right). \tag{3}$$

In a table present the number of mesh points, together with the 3 error norms and their observed order of convergence for the finer three meshes. Discuss the error norms and the observed order of convergence.

Required submission:
- table containing number of mesh points, L_∞, L_1, and L_2 error norms and their observed order of convergence;
- discussion of the obtained results;
- SafeAssign upload of all used, well commented code.