AEE 471 / MAE 561

Homework #9 - Due: April 8th, at the beginning of class

Please submit result graphs together with either handwritten or printed out descriptions, equations, and answers at the beginning of class. Combine all code you used to solve the problems into a single text file, and upload the text file to Blackboard using the SafeAssign mechanism. No credit will be given, if your code is not uploaded as a text file using SafeAssign. Also ensure that your code contains adequate comments. Add a printout of all code as an **appendix** to your submission.

Problem 1 (AEE 471: Core Course Outcome #2)
Consider the 1D Burgers equation for the unknown $u = u(x,t)$,

$$\frac{\partial u}{\partial t} + \frac{\partial \left(\frac{u^2}{2}\right)}{\partial x} = 0\ , \quad 0 \leq x \leq 2, \tag{1}$$

with initial condition

$$u(x,0) = 0.25 + 0.5\sin(\pi x) \tag{2}$$

and periodic boundary conditions on a cell centered mesh of M equally sized elements.

(a) Determine $u(x, t = 0.15)$ using a first order TVD scheme for $M = 40$. Plot the correct weak solution vs x together with the initial solution, and give the solution at $t = 0.15$ in a table together with x_i. To help you debug your code, the file `tvd1.txt` available on Blackboard contains data for the first 4 time steps using $M = 20$ and a time step Δt that is 10% of the maximum stable time step.

(b) Determine the correct weak solution for $u(x, t = 2)$ using a first order TVD scheme for $M = 40$. Add the solution to the plot and table of a).

(c) Demonstrate the spatial order accuracy of the first order TVD scheme by performing a mesh refinement study for the solution at $t = 0.15$ based on the L_∞ and L_1 norms of the error. Present the error norms together with the observed order of convergence, M, and Courant number (or Δt) in a table. Make sure that errors from the root finding algorithm for the exact solution and temporal errors do not mask spatial errors. Use at least 3 different meshes with $r = 2$. The exact solution to Eq. (1) up to the point where the shock forms is given by

$$u_{ex}(x,t) = u(x - ut, 0)\ , \tag{3}$$

which can be solved by any standard root finding algorithm (see MAE384).

(d) Required for MAE561, Bonus for AEE471: Repeat tasks (a) and (c) using a 2nd order TVD scheme. To help you debug your code, the file `tvd2.txt` available on Blackboard contains data for the first 4 time steps using $M = 20$ and a time step Δt that is 10% of the maximum stable time step.

Required submission:
- 1 clearly annotated plot of u as a function of x at $t = 0,\ 0.15$, and 2 using a 1st-order TVD method for $M = 40$;
- 1 table containing x and the solution u at $t = 0,\ 0.15$, and 2 using a first order TVD method for $M = 40$;
- 1 table containing results of mesh refinement study (at least 3 meshes with $r = 2$) including M, Courant number or Δt, L_∞ and L_1 norms of the error, and observed order of convergence for each norm.
- all code used uploaded to Blackboard's SafeAssign link. The code must contain a stable time step calculation.
MAE561:
- 1 clearly annotated plot of u as a function of x at $t = 0,\ 0.15$, and 2 using a 2nd-order TVD method for $M = 40$;
- 1 table containing x and the solution u at $t = 0,\ 0.15$, and 2 using a 2nd-order TVD method for $M = 40$;
- 1 table containing results of mesh refinement study (at least 3 meshes with $r = 2$) including M, Courant number or Δt, L_∞ and L_1 norms of the error, and observed order of convergence for each norm for 2nd-order TVD method.