Project 1 Solution

Task 1 (23 points)

1) different options, for example

FTCS ① 1 point

$$\frac{u_{i,j}^{n+1}-u_{i,j}^{n}}{\Delta t}+\frac{(u_{i+1,j}^{n})^2-(u_{i-1,j}^{n})^2}{2\Delta x}+\frac{(uv)_{i,j+1}^{n}-(uv)_{i,j-1}^{n}}{2\Delta y}=\nu\left(\frac{u_{i+1,j}^{n}-2u_{i,j}^{n}+u_{i-1,j}^{n}}{\Delta x^2}+\frac{u_{i,j+1}^{n}-2u_{i,j}^{n}+u_{i,j-1}^{n}}{\Delta y^2}\right)$$ 7 points

$$\frac{v_{i,j}^{n+1}-v_{i,j}^{n}}{\Delta t}+\frac{(uv)_{i+1,j}^{n}-(uv)_{i-1,j}^{n}}{2\Delta x}+\frac{(v_{i,j+1}^{n})^2-(v_{i,j-1}^{n})^2}{2\Delta y}=\nu\left(\frac{v_{i+1,j}^{n}-2v_{i,j}^{n}+v_{i-1,j}^{n}}{\Delta x^2}+\frac{v_{i,j+1}^{n}-2v_{i,j}^{n}+v_{i,j-1}^{n}}{\Delta y^2}\right)$$ 7 points

boundary conditions:

$u_{0,j}^{n}=-2\sin(\omega t^{n})-u_{1,j}^{n}$ 1 point

$v_{0,j}^{n}=-v_{1,j}^{n}$ 1 point

$u_{M+1,j}^{n}=-u_{M,j}^{n}$ 1 point

$v_{M+1,j}^{n}=-v_{M,j}^{n}$ 1 point

$u_{i,0}^{n}=2\sin(2\pi x_i)-u_{i,1}^{n}$ 1 point

$v_{i,0}^{n}=2\cos(2\omega t^{n})-v_{i,1}^{n}$ 1 point

$u_{i,M+1}^{n}=2-u_{i,M}^{n}$ 1 point

$v_{i,M+1}^{n}=-v_{i,M}^{n}$ 1 point

Task 2 (12 points)

2) $\Delta t=\min\left(\frac{h}{\max_{i,j}(2|u_{i,j}^{n}|+|v_{i,j}^{n}|)}, \frac{h}{\max_{i,j}(2|v_{i,j}^{n}|+|u_{i,j}^{n}|}, \frac{1}{4}\frac{h^2}{\nu}\right)\cdot CFL$

3 points 3 points 3 points

and $Re_c=\frac{\max\left[\max_{i,j}(2|u_{i,j}^{n}|+|v_{i,j}^{n}|), \max_{i,j}(2|v_{i,j}^{n}|+|u_{i,j}^{n}|)\right]h}{\nu}\le 2$

3 points

Project 1 Solution

Task 3 (20 points)

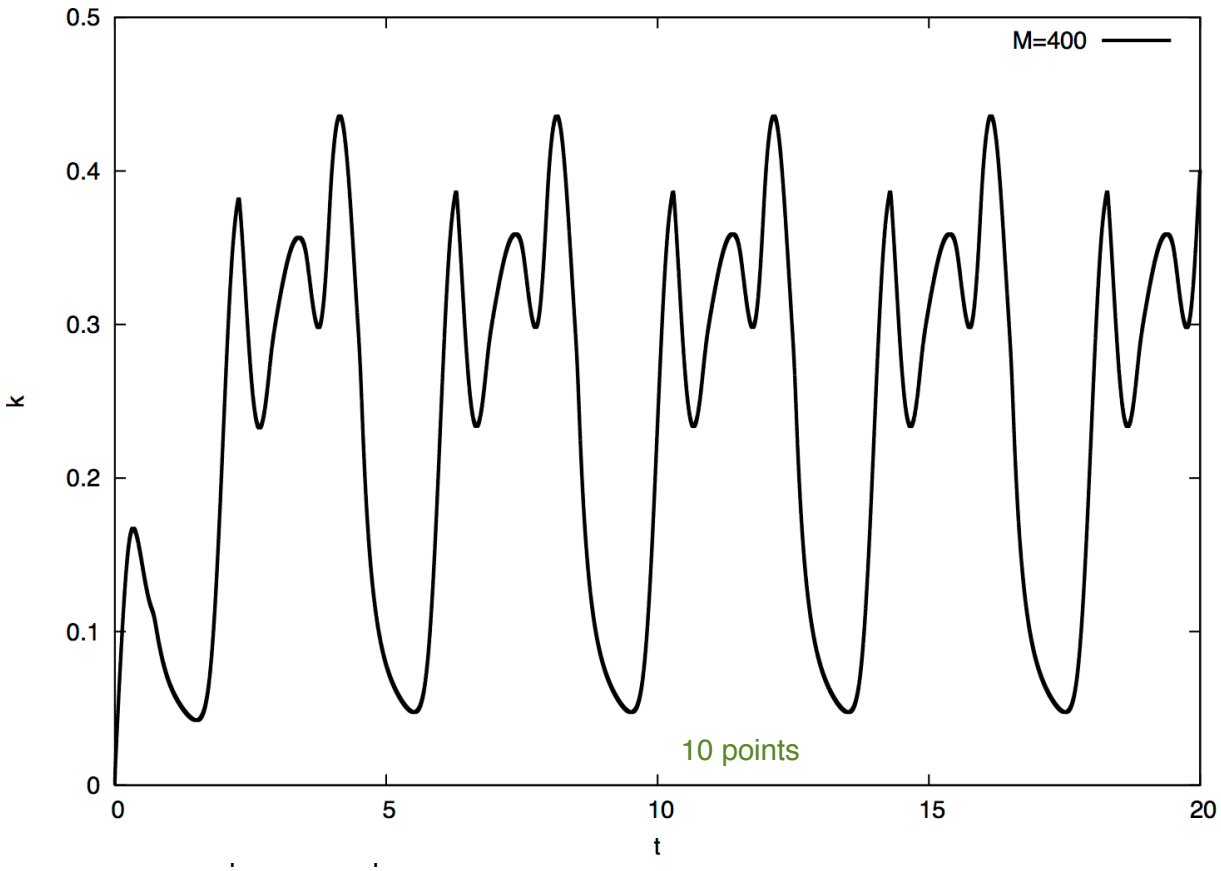
10 points

FTCS	Tmax (t=100)	p	f(h=0)	GCI	GCI23	asymptotic?
100	4.36043E-01					
200	4.35793E-01					
400	4.35709E-01	1.56262E+00	4.35666E-01	0.0124%	0.0367%	1.0002

maximum kinetic energy = 0.435666 with an error band of 0.0124% (10 points)

< 0.435 or > 0.436 : 5 points
< 0.41 or > 0.46 : 2 points

Task 4 (10 points)

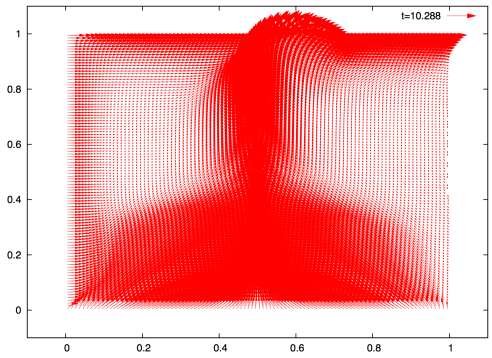


10 points

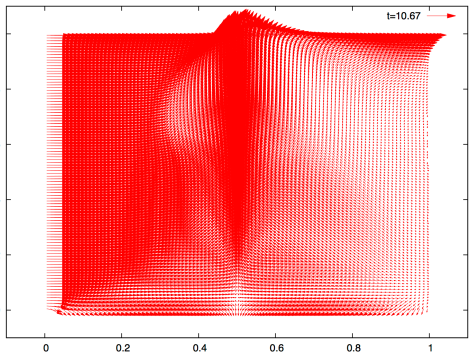
Project 1 Solution

Task 5 (12 points)

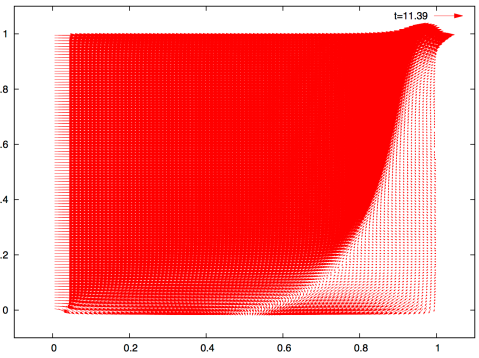
2 points

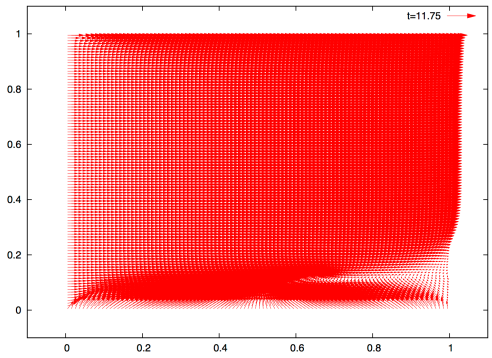


2 points

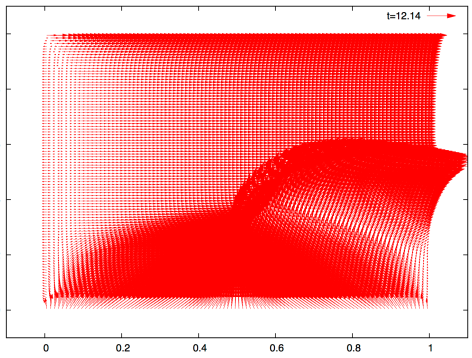


2 points

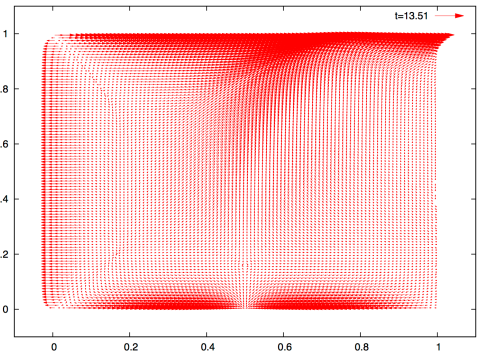




2 points



2 points



2 points

Code: 23 points

Bonus Assignment (15 points)

name: 1pt
index formulas: 4 pts
dye contour plots: 5 pts
dye movie: 5 pts