Problem 1 (i) The only system that is not in SS form

is system 2 $\begin{cases} \dot{\chi}_1 = \chi_2 \\ \dot{\chi}_2 = -(\chi_2 - g(1 - (\frac{\chi_d}{\chi_1})^k)) \end{cases}$ b) 1) $\chi_1 = 0 \Rightarrow \alpha u_1 = b - \sqrt{\chi_1} \Rightarrow \chi_1 = \left(\frac{\alpha u_1}{b}\right)^2$ 2=0 => U.(U2-72)+ (U3-22)=0 => U,U2+CU3=U,X2+CX2 => 72 = U,U2 +CU3 $=> x^* = \begin{bmatrix} \alpha u_1 \\ b \end{bmatrix}$ $=> x^* = \begin{bmatrix} u_1 u_2 + cu_3 \\ u_1 c \end{bmatrix}$ $= \begin{bmatrix} u_1 u_2 + cu_3 \\ u_1 c \end{bmatrix}$ 2) $\mathcal{H}_{2}=0$ => $1=(\mathcal{H}_{\mathcal{H}_{1}})^{k}$ => $\mathcal{H}_{1}=\mathcal{H}_{d}$ => x* = \[\frac{1}{2} \, u* = u

b) 3)
$$\dot{x} = 0$$
 => $\left[x \ y\right] = \left[00\right]$

or $\ln\left(\sqrt{x^2 \mu y^2}\right) y = x$, $\sqrt{x^2 \mu y^2} \neq 1$

In polar coordinates we get

 $x = r \cdot \cos(x)$
 $y = r \cdot \sin(x)$

=> $\ln(r) \cdot \sin(x) \cdot r = \cos(x) \cdot r$, $r \neq 1$

=> $\ln(r) = \frac{\cos(x)}{\sin(x)} = \frac{1}{\tan(x)}$
 $\dot{y} = 0$

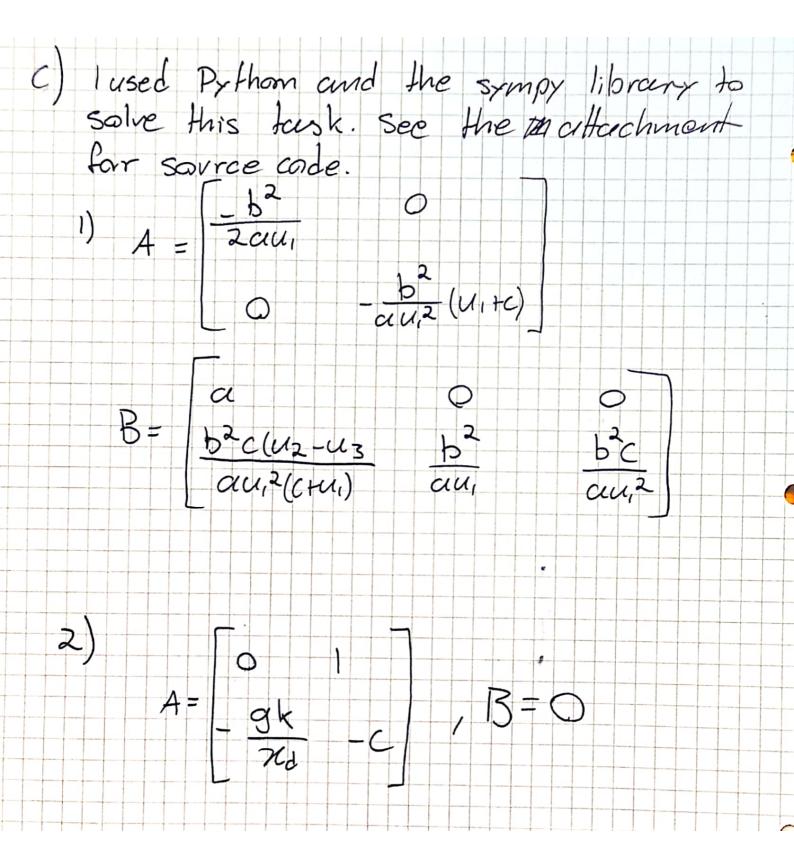
=> $\ln(r) = \frac{\sin(x)}{\sin(x)} = -\tan(x)$, $r \neq 1$

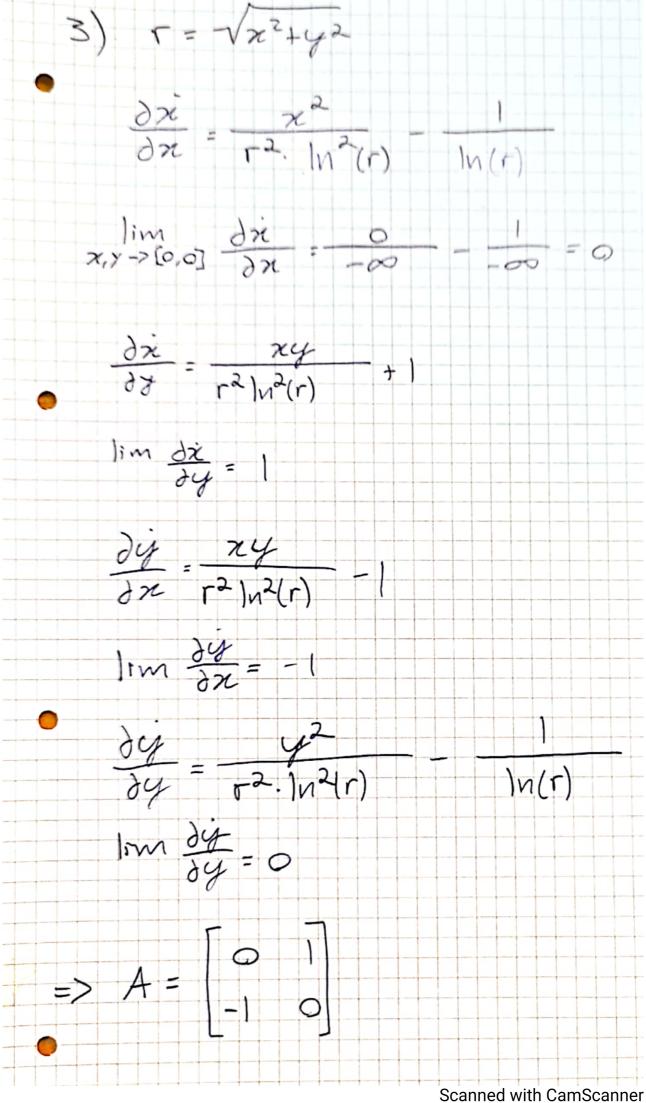
finally

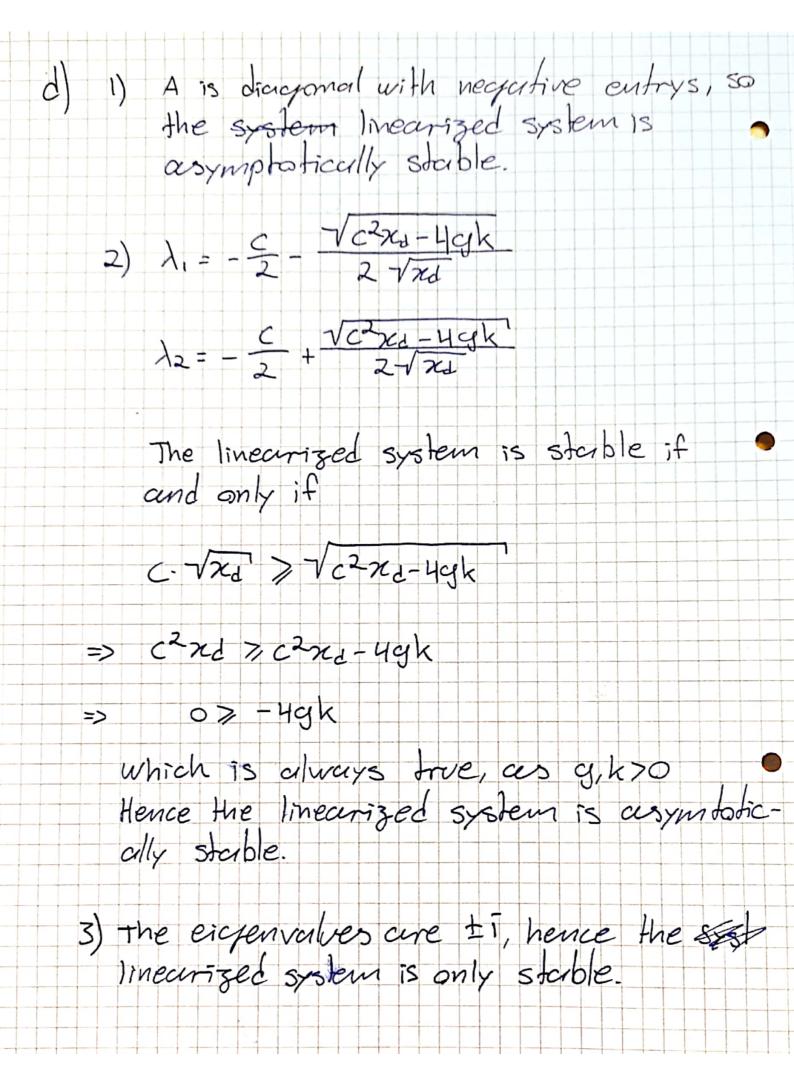
=> $\tan(x) = -\tan(x) = 1 - -\tan(x)$

This is clearly not possible, so the only solution is

 $\left[x \ y\right] = \left[0 \ 0\right]$







Problem 2 a) The first simulation goes toward as when t goes toward I. It throws an error: unable to meet integration tolerances ... " The second simulation is a for all t. b) The first opE is given by it = f(n) = x2 fix) is not globally Lipshitz, so no global Solution exists A local solution can be found us using senciration of variables. dx = x2 $\Rightarrow \int \frac{\pi(t)}{n^2} dn = \int$ 72(to) => x(to) x(t) 7(to) => x(t)=-(t-to)+x(to) 2(to)(t-to)

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with to= and ruto)=1 we get 72(t)= It is clear to see xet) goes to o when t goes to 1. ode45 is an adoptive solver and due to numerical precision, it is not capable of solving the system when t goes to I anditis abviously incapable of solving the system for t>1 The second ODE is given by zi= (zi)= vizil. f(n) = 1/(2 V/n1) sign(n), and is well defined for xx0. Thus, the ODE Was how a unique solution if and only if a never is o.

dx = V121 · Sign (21) $\chi(t)$ Vizel · Sign(x) Cx = t-to $2\sqrt{121} \cdot \text{Sign}(\pi) = t-t_0$ VIX(+) - Sign(x(+)) = 2 + to + VIX.01 Sign(x6) f(n) is manofone, sign(n(t)) = sign(n0) => V|x(t) = t-to Sign(x0) + V|x0| $\pi(t) = \left(\frac{t-t_9}{2} \cdot \text{Sign}(\pi_0) + \sqrt{|\pi_0|}\right)^2 \cdot \text{Sign}(\pi_0)$ With $t_0=0$, $\chi_0=0$ we get $\chi(t)=\frac{1}{4}t^2$ As $t_0=0$ cmy function $\chi(t)=C\cdot \eta\cdot t^2$, $C\in\mathbb{R}$ is valid. Ode 45 finds the trivial solution $\chi(t)=0$. Problem 3 a) The system can be modeled as H = (b-d)H-bdH2-iHZ $I = -(\alpha + d)II + iHZ$ Z = aI+rD-nHZ D = d(H+I)- rD+nHZ H, I, Z, D 20, a, be, d, r, n 70 The new system can be modeled ces b) I = - (a+d+q;)I+iHZ Z = aI+rD-nHZ-43Z D = d(H+I) - rD+nHZ+dqQ Q = q; I + qz Z - dqQ