Problem 1

a) $R_1, R_2 \in SO^3$, so the columns, and rows should have norm 1 and be orthogonal to eachother. Also $\det(R_i) = 1$.

$$R_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$R_2 = \begin{bmatrix} \frac{5}{13} & 0 & -\frac{12}{13} \\ 0 & 1 & 0 \\ \frac{12}{13} & 0 & \frac{5}{13} \end{bmatrix}$$

This task was solved using the python lib sympy. See handin for source code.

b) The columns of R_b^a are the coordinates of $\vec{b}_{1,2,3}$ in the a frame. If we have a vector in the b frame, u_b it will have coordinates u_a in the a frame where

$$u_a = R_b^a u_b$$

We know that b_1 expressed in the a frame is $[b_1 \cdot a_1, b_1 \cdot a_2, b_1 \cdot a_3]^T$, and same for b_2 and b_3.

Thus:
$$R_b^a = \begin{bmatrix} b_1 \cdot a_1 & b_2 \cdot a_1 & b_3 \cdot a_1 \\ b_1 \cdot a_2 & b_2 \cdot a_2 & b_3 \cdot a_2 \\ b_1 \cdot a_3 & b_3 \cdot a_3 & b_3 \cdot a_3 \end{bmatrix}$$

c) $(u^a)^T v^a = (R_b^a u^b)^T R_b^a v^b$

$$= (v^b)^T R_b^{a\,T} R_b^a v^b$$

$$= (v^b)^T R_a^b R_b^a v^b$$

$$= (v^b)^T v^b$$

Scalar product is invariant to rotation

d)

$$u^a \times v^a = u^{a\times} v^a = (R_b^a u_b)^\times R_b^a v^b$$

$$= R_b^a u_b^\times R_b^{a\,T} R_b^a v^b$$

$$= R_b^a u_b^\times v^b = R_b^a (u^b \times v^b)$$

Cross product of rotated vectors is equal to the rotated cross product.

Problem 2

a) The cube rotates as expected

b) The cube still rotates as expected. The DCM method seems better as it is more direct and use simpler functions, but euler uses fewer states. quaternions is probably the best.

Problem 3

a) $Rk = R_{k\theta} k = \left(\cos(\theta) I + \sin(\theta) k^{\times} + (1-\cos(\theta)) k k^T\right) k$

$= \cancel{\cos(\theta)} k + (1 - \cancel{\cos(\theta)}) k k^T k \quad , \quad (k^{\times} k) = \vec{0}$

$= k$

b) The obtained results appear to be reasonable.

the first rotation yields:

$\theta_1 = \pi, \; k_1 = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}^T$

the second yields

$\theta_2 = 1{,}1760, \quad k_2 = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$