

> # Mean free path calculation for neutrons bombarding a slab
restart;

References

Reed, section 2.1

Slab calculations

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Reactions per second

$R[0]$ = rate of neutron bombardment (neutrons / ($m^2 * s$))

σ = effective cross-sectional area of nucleus

Σ = cross-sectional area (m^2)

n = number density of nuclei ($1/m^3$)

s = depth of slab (m)

$R[N] := R[0] * \Sigma * s * n * \sigma$; # ($1/s$)

Probability of a reaction

$P[react] := R[N] / (R[0] * \Sigma)$; # (-)

Probability of escape

$P[escape] := 1 - P[react]$; # (-)

Block calculations

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Neutrons that emerge from a block made up of m slabs

x = depth of block (m)

$N[0]$ = number of neutrons incident on block

$m := x / s$;

Equation 1

$eqn1 := N[escape] = N[0] * P^m$;

Substitute escape probability in the equation

$N[escape] := \text{subs}(P = P[escape], \text{rhs}(eqn1))$;

restart;

Limit as $z \rightarrow 0$

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Define $z = -s * n * \sigma$, then we have:

$eqn2 := N[escape] = N[0] * (1 + z) ^ (-\sigma * n * x / z)$;

Therefore:

$N[escape] := N[0] * [(1 + z) ^ (1 / z)] ^ (-\sigma * n * x)$;

Let $z \rightarrow 0$

$k := \text{limit}((1 + z) ^ (1 / z), z = 0)$;

Final expression for N[escape] as z approaches 0

$N[\text{escape}] := N[0] * \exp(-\sigma * n * x);$

Reactions that occurred

$N[\text{react}] := N[0] - N[\text{escape}];$

Probability a neutron will travel a distance x

$P[\text{directEscape}] := N[\text{escape}] / N[0];$

$P[\text{react}] := 1 - N[\text{escape}] / N[0];$

Probability density function for reaction

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$p[\text{react}] := \text{diff}(P[\text{react}], x);$

Mean value of x

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$xm := \text{simplify}(\text{int}(x * p[\text{react}], x = 0 .. L), \text{symbolic})$ assuming $\sigma :: \text{positive}, n :: \text{positive};$

Number density

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$n := 10^6 * (\rho * N[A] / At);$ # (1/m^3)

Data for U235

$\rho := 18.71;$ # (g/cm^3) Bulk density

$N[A] := 6.022e23;$ # (1/mole) Avogadro's number

$At := 235.04;$ # (g/mole) Atomic weight

$\sigma := 1.235e-28;$ # (m^2) Neutron cross-sectional area

Neutron mean free path

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$x[\text{meanFreePath}] := \text{limit}(xm, L = \text{infinity});$ # (m)

Plotting results

$\text{plot}(xm, L = 0 .. 2);$

$\text{plot}(P[\text{directEscape}], x = 0 .. 2);$

$\text{plot}(P[\text{react}], x = 0 .. 2);$

$$R_N := R_0 \Sigma s n \sigma$$

$$P_{\text{react}} := s n \sigma$$

$$P_{\text{escape}} := -s n \sigma + 1$$

$$m := \frac{x}{s}$$

$$\text{eqn1} := N_{\text{escape}} = N_0 P^{\frac{x}{s}}$$

$$N_{escape} := N_0 \left(-s \, n \, \sigma + 1 \right)^{\frac{x}{s}}$$

$$eqn2 := N_{escape} = N_0 \left(1 + z \right)^{-\frac{\sigma \, n \, x}{z}}$$

$$N_{escape} := N_0 \left[\left(1 + z \right)^{\frac{1}{z}} \right]^{-\sigma \, n \, x}$$

$$k := e$$

$$N_{escape} := N_0 \, e^{-\sigma \, n \, x}$$

$$N_{react} := N_0 - N_0 \, e^{-\sigma \, n \, x}$$

$$P_{directEscape} := e^{-\sigma \, n \, x}$$

$$P_{react} := 1 - e^{-\sigma \, n \, x}$$

$$p_{react} := \sigma \, n \, e^{-\sigma \, n \, x}$$

$$xm := \frac{-L \, e^{-\sigma \, n \, L} \, n \, \sigma - e^{-\sigma \, n \, L} + 1}{\sigma \, n}$$

$$n := \frac{1000000 \, \rho \, N_A}{A t}$$

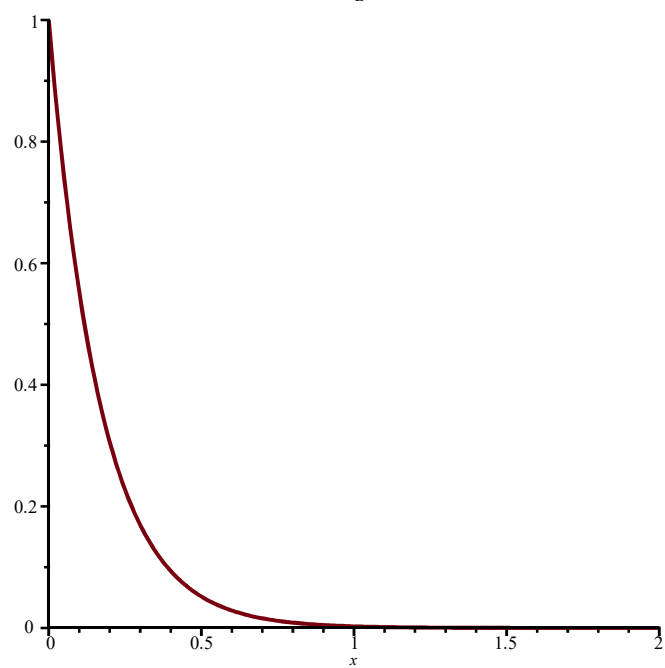
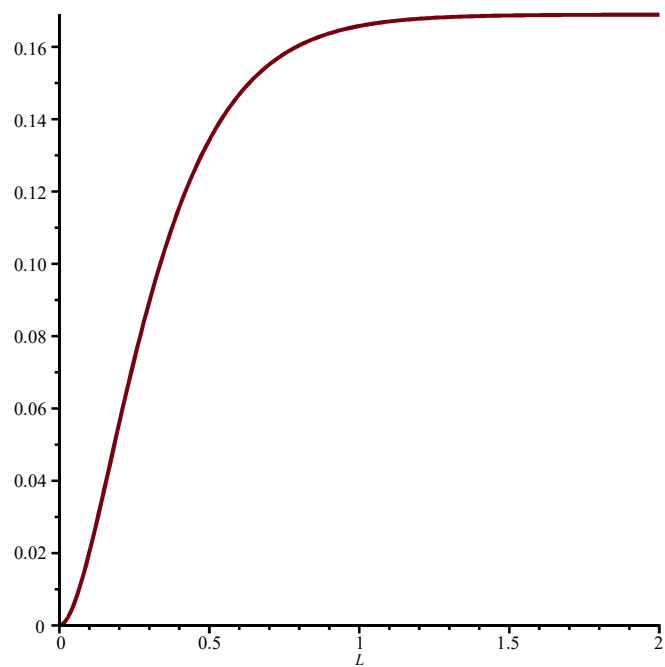
$$\rho := 18.71$$

$$N_A := 6.022 \times 10^{23}$$

$$A t := 235.04$$

$$\sigma := 1.235 \times 10^{-28}$$

$$x_{meanFreePath} := 0.1689119136$$



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