ECE 785

Project 1: Optimizing Spherical Geometry Code

Report

Initial Performance Analysis:

The given code was optimized to compute the closest weather station to all the state capitals of United States. In all nearest waypoint for 50 test cases were computed. The main aim of the project is to optimize the code for speed using optimizations like eliminating pipeline stalls due to the hardware floating point support and by using less accurate but faster trigonometric functions.

To start with, less accurate single precision cosf() & sinf() functions are used for computation instead of double precision cos() and sin() functions. The output using single precision was then validated against that of double precision functions.

Output with validation:

```
debian@beaglebone:~/Project1$ ./sg
Vaildation:
Error in distance!Expected:9.588253 Actual:9.837340 Error:-2.597839 for Annapolis , Maryland
Error in distance!Expected:33.576939 Actual:33.504810 Error:0.214815 for Boston , Massachusetts
Error in distance!Expected:84.079651 Actual:84.108429 Error:-0.034227 for Dover , Delaware
Error in distance!Expected:149.145386 Actual:149.129166 Error:0.010875 for Lansing , Michigan
Error in distance!Expected:91.095810 Actual:91.069244 Error:0.029162 for Trenton , New Jersey
Output
0.Closest waypoint for Albany , New York is NYC ENTR. 254.707184 km away at bearing 181.550583 degrees
1.Closest waypoint for Annapolis , Maryland is THOMAS POINT, MD. 9.837340 km away at bearing 213.130646 degrees
2.Closest waypoint for Atlanta , Georgia is GRAYS REEF. 421.426270 km away at bearing 232.451050 degrees
3.Closest waypoint for Augusta , Maine is PORTLAND. 93.222275 km away at bearing 161.085648 degrees
4.Closest waypoint for Austin , Texas is PORT ARANSAS, TX. 279.349121 km away at bearing 194.268921 degrees
5.Closest waypoint for Baton Rouge, Louisiana is BILOXI. 231.263412 km away at bearing 260.404968 degrees
6.Closest waypoint for Bismarck , North Dakota is DEVILS ISLAND, WI. 772.230591 km away at bearing 259.330780 degrees
7.Closest waypoint for Boise , Idaho is NEWPORT, OR. 634.778564 km away at bearing 77.240952 degrees
8.Closest waypoint for Boston , Massachusetts is BOSTON. 33.504810 km away at bearing 292.475555 degrees
9.Closest waypoint for Carson City, Nevada is SAN FRANCISCO. 309.820831 km away at bearing 119.219414 degrees
10.Closest waypoint for Charleston , West Virginia is W LK ERIE. 376.041748 km away at bearing 9.754081 degrees
11.Closest waypoint for Cheyenne , Wyoming is DEVILS ISLAND, WI. 1300.065674 km away at bearing 305.208618 degrees
12.Closest waypoint for Columbia , South Carolina is FOLLY ISLAND, SC. 180.354263 km away at bearing 216.451447 degrees
13.Closest waypoint for Columbus , Ohio is SOUTH BASS ISLAND, OH. 185.942719 km away at bearing 355.881439 degrees
14.Closest waypoint for Concord , New Hampshire is ISLE OF SHOALS, NH. 80.402351 km away at bearing 250.083450 degrees
15.Closest waypoint for Denver , Colorado is DEVILS ISLAND, WI. 1407.890137 km away at bearing 310.096191 degrees
16.Closest waypoint for Des Moines, Iowa is SHEBOYGAN, WI. 540.894043 km away at bearing 298.330109 degrees
17.Closest waypoint for Dover , Delaware is THOMAS POINT, MD. 84.108429 km away at bearing 109.976585 degrees
18.Closest waypoint for Frankfort , Kentucky is SOUTH BASS ISLAND, OH. 418.856201 km away at bearing 336.318695 degrees
19.Closest waypoint for Harrisburg , Pennsylvania is THOMAS POINT, MD. 156.831635 km away at bearing 193.910110 degrees
20.Closest waypoint for Hartford , Connecticut is MONTAUK PT. 130.731613 km away at bearing 203.853012 degrees
21.Closest waypoint for Helena , Montana is WEST POINT, WA. 795.980713 km away at bearing 77.659920 degrees
22.Closest waypoint for Honolulu , Hawaii is W HAWAII. 385.112244 km away at bearing 129.342636 degrees
23.Closest waypoint for Indianapolis , Indiana is S LK MICHIGAN. 328.521057 km away at bearing 12.689395 degrees
24.Closest waypoint for Jackson , Mississippi is BILOXI. 283.118286 km away at bearing 209.237030 degrees
25.Closest waypoint for Jefferson City, Missouri is S LK MICHIGAN. 629.953918 km away at bearing 317.949829 degrees
26.Closest waypoint for Juneau , Alaska is FIVE FINGERS, AK. 123.929001 km away at bearing 202.528458 degrees
27.Closest waypoint for Lansing , Michigan is LAKE ST. CLAIR. 149.129166 km away at bearing 259.276031 degrees
28.Closest waypoint for Lincoln , Nebraska is SHEBOYGAN, WI. 807.715393 km away at bearing 296.838989 degrees
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.455139 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 156.104843 km away at bearing 299.329956 degrees
```

As we can see, we obtained 5 wrong readings for distance, while no errors were obtained for waypoint names and bearing calculations.

Output without validation:

```
9.Closest waypoint for Carson City, Nevada is SAN FRANCISCO. 309.820831 km away at bearing 119.219414 degrees
10.Closest waypoint for Charleston , West Virginia is W LK ERIE. 376.041748 km away at bearing 9.754081 degrees
11.Closest waypoint for Cheyenne , Wyoming is DEVILS ISLAND, WI. 1300.065674 km away at bearing 305.208618 degrees
12.Closest waypoint for Columbia , South Carolina is FOLLY ISLAND, SC. 180.354263 km away at bearing 216.451447 degrees
13.Closest waypoint for Columbus , Ohio is SOUTH BASS ISLAND, OH. 185.942719 km away at bearing 355.881439 degrees
14.Closest waypoint for Concord , New Hampshire is ISLE OF SHOALS, NH. 80.402351 km away at bearing 250.083450 degrees
15.Closest waypoint for Denver , Colorado is DEVILS ISLAND, WI. 1407.890137 km away at bearing 310.096191 degrees
16.Closest waypoint for Des Moines, Iowa is SHEBOYGAN, WI. 540.894043 km away at bearing 298.330109 degrees
17.Closest waypoint for Dover , Delaware is THOMAS POINT, MD. 84.108429 km away at bearing 109.976585 degrees
18.Closest waypoint for Frankfort , Kentucky is SOUTH BASS ISLAND, OH. 418.856201 km away at bearing 336.318695 degrees
19.Closest waypoint for Harrisburg , Pennsylvania is THOMAS POINT, MD. 156.831635 km away at bearing 193.910110 degrees
20.Closest waypoint for Hartford , Connecticut is MONTAUK PT. 130.731613 km away at bearing 203.853012 degrees
21.Closest waypoint for Helena , Montana is WEST POINT, WA. 795.980713 km away at bearing 77.659920 degrees
22.Closest waypoint for Honolulu , Hawaii is W HAWAII. 385.112244 km away at bearing 129.342636 degrees
23.Closest waypoint for Indianapolis , Indiana is S LK MICHIGAN. 328.521057 km away at bearing 12.689395 degrees
24.Closest waypoint for Jackson , Mississippi is BILOXI. 283.118286 km away at bearing 209.237030 degrees
25.Closest waypoint for Jefferson City, Missouri is S LK MICHIGAN. 629.953918 km away at bearing 317.949829 degrees
26.Closest waypoint for Juneau , Alaska is FIVE FINGERS, AK. 123.929001 km away at bearing 202.528458 degrees
27.Closest waypoint for Lansing , Michigan is LAKE ST. CLAIR. 149.129166 km away at bearing 259.276031 degrees
28.Closest waypoint for Lincoln , Nebraska is SHEBOYGAN, WI. 807.715393 km away at bearing 296.838989 degrees
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.455139 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 156.104843 km away at bearing 299.329956 degrees
31.Closest waypoint for Montgomery , Alabama is ORANGE BCH. 282.013397 km away at bearing 154.293076 degrees
32.Closest waypoint for Montpelier , Vermont is PORTLAND. 211.340836 km away at bearing 248.053284 degrees
33.Closest waypoint for Nashville , Tennessee is DAUPHIN ISLAND, AL. 668.485779 km away at bearing 169.332596 degrees
34.Closest waypoint for Oklahoma City, Oklahoma is SABINE, TX. 723.811646 km away at bearing 207.938629 degrees
35.Closest waypoint for Olympia , Washington is WEST POINT, WA. 76.643372 km away at bearing 333.737518 degrees
36.Closest waypoint for Phoenix , Arizona is SAN CLEMENTE. 565.678833 km away at bearing 99.226471 degrees
37.Closest waypoint for Pierre , South Dakota is DEVILS ISLAND, WI. 803.695923 km away at bearing 295.413635 degrees
38.Closest waypoint for Providence , Rhode Island is BUZZARDS BAY,MA. 57.319054 km away at bearing 214.821564 degrees
39.Closest waypoint for Raleigh , North Carolina is ONSLOW BAY. 189.359787 km away at bearing 221.099457 degrees
40.Closest waypoint for Richmond , Virginia is CHESAPEAKE LIGHT, VA. 170.051636 km away at bearing 246.203003 degrees
41.Closest waypoint for Sacramento , California is SAN FRANCISCO. 148.266769 km away at bearing 126.205139 degrees
42.Closest waypoint for Saint Paul, Minnesota is DEVILS ISLAND, WI. 299.000031 km away at bearing 323.221375 degrees
43.Closest waypoint for Salem , Oregon is NEWPORT, OR. 89.569160 km away at bearing 113.121010 degrees
44.Closest waypoint for Salt Lake, Utah is SAN FRANCISCO. 997.996887 km away at bearing 105.949211 degrees
45.Closest waypoint for Santa Fe, New Mexico is SAN CLEMENTE. 1165.089355 km away at bearing 104.183609 degrees
46.Closest waypoint for Springfield , Illinois is S LK MICHIGAN. 388.604004 km away at bearing 326.534698 degrees
47.Closest waypoint for Tallahassee , Florida is KEATON BEACH, FL. 96.135429 km away at bearing 223.236084 degrees
48.Closest waypoint for Topeka , Kansas is S LK MICHIGAN. 832.105530 km away at bearing 301.781250 degrees
49.Closest waypoint for Trenton , New Jersey is NYC ENTR. 91.069244 km away at bearing 280.769745 degrees
Average     69.458 us
Minimum     67.760 us
debian@beaglebone:~/Project1$
```

An average time of 69.458us was obtained for the base version of the code.

Function-Profile:

```
Samples: 148K of event 'cycles:ppp', Event count (approx.): 36324502008
Overhead  Command   Shared Object       Symbol
  66.78%  sg        sg                  [.] Find_Nearest_Waypoint
  15.66%  sg        sg                  [.] __cosf
   8.89%  sg        sg                  [.] __kernel_cosf
   3.17%  sg        sg                  [.] __ieee754_rem_pio2f
   2.28%  sg        sg                  [.] __kernel_sinf
   0.52%  sg        sg                  [.] __sincosf
   0.42%  sg        sg                  [.] __ieee754_sqrtf
   0.39%  sg        sg                  [.] __ieee754_atan2f
   0.28%  sg        sg                  [.] __atanf
   0.24%  sg        [kernel.kallsyms]   [k] __softirqentry_text_start
   0.23%  sg        [kernel.kallsyms]   [k] _raw_spin_unlock_irqrestore
   0.21%  sg        sg                  [.] __ieee754_acosf
   0.11%  sg        sg                  [.] __mcount_internal
   0.11%  sg        [kernel.kallsyms]   [k] _raw_spin_unlock_irq
   0.07%  sg        sg                  [.] main
   0.06%  sg        sg                  [.] strcmp
   0.05%  sg        [kernel.kallsyms]   [k] setup_sigframe
   0.04%  sg        [kernel.kallsyms]   [k] restore_sigframe
   0.03%  sg        sg                  [.] __gnu_mcount_nc
   0.02%  sg        [kernel.kallsyms]   [k] sys_clock_gettime
   0.02%  sg        [kernel.kallsyms]   [k] rcu_process_callbacks
Tip: For tracepoint events, try: perf report -s trace_fields
```

From the function profile, we can conclude that most time is spent in Find_Nearest_Waypoint function and cosf() function.

Instruction-Level Profile:

```
Find_Nearest_Waypoint   /home/debian/Project1/sg
        |53       return p1->SinLat * p2->SinLat +
  0.74  |         vldr   s15, [r4, #-44] ; 0xffffffd4
        |54        p1->CosLat * p2->CosLat*
        |         vmul.f s14, s17, s14
        |53       return p1->SinLat * p2->SinLat +
  1.58  |         vmul.f s15, s18, s15
        |         vmla.f s15, s14, s0
        |56    Find_Nearest_Waypoint():
        |93        closest_i = i;
  3.26  |         vcmpe. s16, s15
  0.51  |         vmrs   APSR_nzcv, fpscr
 89.49  |         it     mi
        |         movmi  r6, r5
  0.35  |         it     mi
        |         vmovmi s16, s15
        |96        i++;
  0.35  |         adds   r5, #1
        |         b.n    109c0 <Find_Nearest_Waypoint+0x44>
        |         .word  0x45c71800
        |         .word  0x42652ee0
        |         .word  0x43b40000
        |         .word  0x0008c666
Press 'h' for help on key bindings
```

Optimization 1:

This optimization was aimed at finding the fastest less accurate trigonometric cos functions in sincos.c file. Different cos functions with different accuracies were used in the Calc_Closeness function and the speed and number of errors obtained for each function was noted.

Sin_32/Cos_32:

Speed of 47.529us was obtained with cos_32 function but there were lot of errors obtained for distance calculations.

Output:

```
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 613.194519 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 223.104385 km away at bearing 299.329956 degrees
31.Closest waypoint for Montgomery , Alabama is ORANGE BCH. 338.878815 km away at bearing 154.293076 degrees
32.Closest waypoint for Montpelier , Vermont is PORTLAND. 263.615448 km away at bearing 248.053284 degrees
33.Closest waypoint for Nashville , Tennessee is DAUPHIN ISLAND, AL. 693.256653 km away at bearing 169.332596 degrees
34.Closest waypoint for Oklahoma City, Oklahoma is SABINE, TX. 746.535889 km away at bearing 207.938629 degrees
35.Closest waypoint for Olympia , Washington is WEST POINT, WA. 167.658615 km away at bearing 333.737518 degrees
36.Closest waypoint for Phoenix , Arizona is SAN CLEMENTE. 592.830872 km away at bearing 99.226471 degrees
37.Closest waypoint for Pierre , South Dakota is DEVILS ISLAND, WI. 815.358826 km away at bearing 295.413635 degrees
38.Closest waypoint for Providence , Rhode Island is BUZZARDS BAY,MA. 174.254120 km away at bearing 214.821564 degrees
39.Closest waypoint for Raleigh , North Carolina is ONSLOW BAY. 261.041656 km away at bearing 221.099457 degrees
40.Closest waypoint for Richmond , Virginia is CHESAPEAKE LIGHT, VA. 243.794388 km away at bearing 246.203003 degrees
41.Closest waypoint for Sacramento , California is SAN FRANCISCO. 227.646103 km away at bearing 126.205139 degrees
42.Closest waypoint for Saint Paul, Minnesota is DEVILS ISLAND, WI. 335.367920 km away at bearing 323.221375 degrees
43.Closest waypoint for Salem , Oregon is NEWPORT, OR. 179.951324 km away at bearing 113.121010 degrees
44.Closest waypoint for Salt Lake, Utah is SAN FRANCISCO. 1008.759094 km away at bearing 105.949211 degrees
45.Closest waypoint for Santa Fe, New Mexico is SAN CLEMENTE. 1174.900391 km away at bearing 104.183609 degrees
46.Closest waypoint for Springfield , Illinois is S LK MICHIGAN. 421.891357 km away at bearing 326.534698 degrees
47.Closest waypoint for Tallahassee , Florida is KEATON BEACH, FL. 213.210327 km away at bearing 223.236084 degrees
48.Closest waypoint for Topeka , Kansas is S LK MICHIGAN. 845.917175 km away at bearing 301.781250 degrees
49.Closest waypoint for Trenton , New Jersey is NYC ENTR. 190.836304 km away at bearing 280.769745 degrees
Average    47.529 us
Minimum    45.998 us
debian@beaglebone:~/Project1$
```

Sin_52/Cos_52:

A speed of 51.540us was obtained but lot of errors were obtained for distance calculations.

Output:

```
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.778320 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 157.001251 km away at bearing 299.329956 degrees
31.Closest waypoint for Montgomery , Alabama is ORANGE BCH. 282.699097 km away at bearing 154.293076 degrees
32.Closest waypoint for Montpelier , Vermont is PORTLAND. 211.992462 km away at bearing 248.053284 degrees
33.Closest waypoint for Nashville , Tennessee is DAUPHIN ISLAND, AL. 668.768555 km away at bearing 169.332596 degrees
34.Closest waypoint for Oklahoma City, Oklahoma is SABINE, TX. 724.066223 km away at bearing 207.938629 degrees
35.Closest waypoint for Olympia , Washington is WEST POINT, WA. 78.267639 km away at bearing 333.737518 degrees
36.Closest waypoint for Phoenix , Arizona is SAN CLEMENTE. 565.969910 km away at bearing 99.226471 degrees
37.Closest waypoint for Pierre , South Dakota is DEVILS ISLAND, WI. 803.804626 km away at bearing 295.413635 degrees
38.Closest waypoint for Providence , Rhode Island is BUZZARDS BAY,MA. 59.878838 km away at bearing 214.821564 degrees
39.Closest waypoint for Raleigh , North Carolina is ONSLOW BAY. 190.315735 km away at bearing 221.099457 degrees
40.Closest waypoint for Richmond , Virginia is CHESAPEAKE LIGHT, VA. 171.044754 km away at bearing 246.203003 degrees
41.Closest waypoint for Sacramento , California is SAN FRANCISCO. 149.388535 km away at bearing 126.205139 degrees
42.Closest waypoint for Saint Paul, Minnesota is DEVILS ISLAND, WI. 299.428741 km away at bearing 323.221375 degrees
43.Closest waypoint for Salem , Oregon is NEWPORT, OR. 91.069244 km away at bearing 113.121010 degrees
44.Closest waypoint for Salt Lake, Utah is SAN FRANCISCO. 998.089417 km away at bearing 105.949211 degrees
45.Closest waypoint for Santa Fe, New Mexico is SAN CLEMENTE. 1165.164551 km away at bearing 104.183609 degrees
46.Closest waypoint for Springfield , Illinois is S LK MICHIGAN. 388.990051 km away at bearing 326.534698 degrees
47.Closest waypoint for Tallahassee , Florida is KEATON BEACH, FL. 98.226692 km away at bearing 223.236084 degrees
48.Closest waypoint for Topeka , Kansas is S LK MICHIGAN. 832.239563 km away at bearing 301.781250 degrees
49.Closest waypoint for Trenton , New Jersey is NYC ENTR. 92.806091 km away at bearing 280.769745 degrees
Average     51.540 us
Minimum     49.913 us
debian@beaglebone:~/Project1$
```

Sin_73/Cos_73:

A speed of 59.408us was obtained but the number of errors reduced to just 7 for distance calculations.

Output:

```
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.455139 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 156.120331 km away at bearing 299.329956 degrees
31.Closest waypoint for Montgomery , Alabama is ORANGE BCH. 282.013397 km away at bearing 154.293076 degrees
32.Closest waypoint for Montpelier , Vermont is PORTLAND. 211.352295 km away at bearing 248.053284 degrees
33.Closest waypoint for Nashville , Tennessee is DAUPHIN ISLAND, AL. 668.489380 km away at bearing 169.332596 degrees
34.Closest waypoint for Oklahoma City, Oklahoma is SABINE, TX. 723.815002 km away at bearing 207.938629 degrees
35.Closest waypoint for Olympia , Washington is WEST POINT, WA. 76.674934 km away at bearing 333.737518 degrees
36.Closest waypoint for Phoenix , Arizona is SAN CLEMENTE. 565.683105 km away at bearing 99.226471 degrees
37.Closest waypoint for Pierre , South Dakota is DEVILS ISLAND, WI. 803.698975 km away at bearing 295.413635 degrees
38.Closest waypoint for Providence , Rhode Island is BUZZARDS BAY,MA. 57.319054 km away at bearing 214.821564 degrees
39.Closest waypoint for Raleigh , North Carolina is ONSLOW BAY. 189.372559 km away at bearing 221.099457 degrees
40.Closest waypoint for Richmond , Virginia is CHESAPEAKE LIGHT, VA. 170.051636 km away at bearing 246.203003 degrees
41.Closest waypoint for Sacramento , California is SAN FRANCISCO. 148.283081 km away at bearing 126.205139 degrees
42.Closest waypoint for Saint Paul, Minnesota is DEVILS ISLAND, WI. 299.008118 km away at bearing 323.221375 degrees
43.Closest waypoint for Salem , Oregon is NEWPORT, OR. 89.569160 km away at bearing 113.121010 degrees
44.Closest waypoint for Salt Lake, Utah is SAN FRANCISCO. 997.999329 km away at bearing 105.949211 degrees
45.Closest waypoint for Santa Fe, New Mexico is SAN CLEMENTE. 1165.089355 km away at bearing 104.183609 degrees
46.Closest waypoint for Springfield , Illinois is S LK MICHIGAN. 388.610229 km away at bearing 326.534698 degrees
47.Closest waypoint for Tallahassee , Florida is KEATON BEACH, FL. 96.160591 km away at bearing 223.236084 degrees
48.Closest waypoint for Topeka , Kansas is S LK MICHIGAN. 832.105530 km away at bearing 301.781250 degrees
49.Closest waypoint for Trenton , New Jersey is NYC ENTR. 91.095810 km away at bearing 280.769745 degrees
Average     59.408 us
Minimum     57.135 us
debian@beaglebone:~/Project1$
```

Sin_121/Cos_121:

A speed of 68.619us was obtained with number of errors reducing considerably to just 1-2 for distance calculations.

Output:

```
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.450928 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 156.120331 km away at bearing 299.329956 degrees
31.Closest waypoint for Montgomery , Alabama is ORANGE BCH. 282.004822 km away at bearing 154.293076 degrees
32.Closest waypoint for Montpelier , Vermont is PORTLAND. 211.340836 km away at bearing 248.053284 degrees
33.Closest waypoint for Nashville , Tennessee is DAUPHIN ISLAND, AL. 668.489380 km away at bearing 169.332596 degrees
34.Closest waypoint for Oklahoma City, Oklahoma is SABINE, TX. 723.811646 km away at bearing 207.938629 degrees
35.Closest waypoint for Olympia , Washington is WEST POINT, WA. 76.643372 km away at bearing 333.737518 degrees
36.Closest waypoint for Phoenix , Arizona is SAN CLEMENTE. 565.678833 km away at bearing 99.226471 degrees
37.Closest waypoint for Pierre , South Dakota is DEVILS ISLAND, WI. 803.695923 km away at bearing 295.413635 degrees
38.Closest waypoint for Providence , Rhode Island is BUZZARDS BAY,MA. 57.319054 km away at bearing 214.821564 degrees
39.Closest waypoint for Raleigh , North Carolina is ONSLOW BAY. 189.372559 km away at bearing 221.099457 degrees
40.Closest waypoint for Richmond , Virginia is CHESAPEAKE LIGHT, VA. 170.051636 km away at bearing 246.203003 degrees
41.Closest waypoint for Sacramento , California is SAN FRANCISCO. 148.266769 km away at bearing 126.205139 degrees
42.Closest waypoint for Saint Paul, Minnesota is DEVILS ISLAND, WI. 299.000031 km away at bearing 323.221375 degrees
43.Closest waypoint for Salem , Oregon is NEWPORT, OR. 89.569160 km away at bearing 113.121010 degrees
44.Closest waypoint for Salt Lake, Utah is SAN FRANCISCO. 997.999329 km away at bearing 105.949211 degrees
45.Closest waypoint for Santa Fe, New Mexico is SAN CLEMENTE. 1165.089355 km away at bearing 104.183609 degrees
46.Closest waypoint for Springfield , Illinois is S LK MICHIGAN. 388.610229 km away at bearing 326.534698 degrees
47.Closest waypoint for Tallahassee , Florida is KEATON BEACH, FL. 96.135429 km away at bearing 223.236084 degrees
48.Closest waypoint for Topeka , Kansas is S LK MICHIGAN. 832.102600 km away at bearing 301.781250 degrees
49.Closest waypoint for Trenton , New Jersey is NYC ENTR. 91.095810 km away at bearing 280.769745 degrees
Average     68.619 us
Minimum     67.154 us
debian@beaglebone:~/Project1$
```

Conclusion:

Cos_73 was selected based on a trade-off between speed and accuracy.

Output with validation:

```
debian@beaglebone:~/Project1$ sudo perf record ./sg
[sudo] password for debian:
Vaildation:
Error in distance!Expected:9.588253 Actual:9.837340 Error:-2.597839 for Annapolis , Maryland
Error in distance!Expected:84.079651 Actual:84.108429 Error:-0.034227 for Dover , Delaware
Error in distance!Expected:130.731613 Actual:130.750107 Error:-0.014146 for Hartford , Connecticut
Error in distance!Expected:123.929008 Actual:123.948524 Error:-0.015748 for Juneau , Alaska
Error in distance!Expected:76.643372 Actual:76.674934 Error:-0.041181 for Olympia , Washington
Error in distance!Expected:148.266769 Actual:148.283081 Error:-0.011002 for Sacramento , California
Error in distance!Expected:96.135429 Actual:96.160591 Error:-0.026173 for Tallahassee , Florida
Output:
0.Closest waypoint for Albany , New York is NYC ENTR. 254.716675 km away at bearing 181.550583 degrees
1.Closest waypoint for Annapolis , Maryland is THOMAS POINT, MD. 9.837340 km away at bearing 213.130646 degrees
2.Closest waypoint for Atlanta , Georgia is GRAYS REEF. 421.432037 km away at bearing 232.451050 degrees
3.Closest waypoint for Augusta , Maine is PORTLAND. 93.222275 km away at bearing 161.085648 degrees
4.Closest waypoint for Austin , Texas is PORT ARANSAS, TX. 279.357788 km away at bearing 194.268921 degrees
5.Closest waypoint for Baton Rouge, Louisiana is BILOXI. 231.263412 km away at bearing 260.404968 degrees
6.Closest waypoint for Bismarck , North Dakota is DEVILS ISLAND, WI. 772.233704 km away at bearing 259.330780 degrees
7.Closest waypoint for Boise , Idaho is NEWPORT, OR. 634.778564 km away at bearing 77.240952 degrees
8.Closest waypoint for Boston , Massachusetts is BOSTON. 33.576942 km away at bearing 292.475555 degrees
9.Closest waypoint for Carson City, Nevada is SAN FRANCISCO. 309.820831 km away at bearing 119.219414 degrees
10.Closest waypoint for Charleston , West Virginia is W LK ERIE. 376.048218 km away at bearing 9.754081 degrees
11.Closest waypoint for Cheyenne , Wyoming is DEVILS ISLAND, WI. 1300.067627 km away at bearing 305.208618 degrees
12.Closest waypoint for Columbia , South Carolina is FOLLY ISLAND, SC. 180.381088 km away at bearing 216.451447 degrees
13.Closest waypoint for Columbus , Ohio is SOUTH BASS ISLAND, OH. 185.942719 km away at bearing 355.881439 degrees
14.Closest waypoint for Concord , New Hampshire is ISLE OF SHOALS, NH. 80.402351 km away at bearing 250.083450 degrees
15.Closest waypoint for Denver , Colorado is DEVILS ISLAND, WI. 1407.891968 km away at bearing 310.096191 degrees
16.Closest waypoint for Des Moines, Iowa is SHEBOYGAN, WI. 540.894043 km away at bearing 298.330109 degrees
17.Closest waypoint for Dover , Delaware is THOMAS POINT, MD. 84.108429 km away at bearing 109.976585 degrees
18.Closest waypoint for Frankfort , Kentucky is SOUTH BASS ISLAND, OH. 418.861938 km away at bearing 336.318695 degrees
19.Closest waypoint for Harrisburg , Pennsylvania is THOMAS POINT, MD. 156.847061 km away at bearing 193.910110 degrees
20.Closest waypoint for Hartford , Connecticut is MONTAUK PT. 130.750107 km away at bearing 203.853012 degrees
21.Closest waypoint for Helena , Montana is WEST POINT, WA. 795.983765 km away at bearing 77.659920 degrees
22.Closest waypoint for Honolulu , Hawaii is W HAWAII. 385.118530 km away at bearing 129.342636 degrees
23.Closest waypoint for Indianapolis , Indiana is S LK MICHIGAN. 328.528412 km away at bearing 12.689395 degrees
24.Closest waypoint for Jackson , Mississippi is BILOXI. 283.126831 km away at bearing 209.237030 degrees
25.Closest waypoint for Jefferson City, Missouri is S LK MICHIGAN. 629.953918 km away at bearing 317.949829 degrees
26.Closest waypoint for Juneau , Alaska is FIVE FINGERS, AK. 123.948524 km away at bearing 202.528458 degrees
27.Closest waypoint for Lansing , Michigan is LAKE ST. CLAIR. 149.145386 km away at bearing 259.276031 degrees
28.Closest waypoint for Lincoln , Nebraska is SHEBOYGAN, WI. 807.715393 km away at bearing 296.838989 degrees
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.455139 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 156.120331 km away at bearing 299.329956 degrees
```

Function Profile:

```
Samples: 127K of event 'cycles:ppp', Event count (approx.): 31126561547
Overhead  Command   Shared Object       Symbol
  51.43%  sg        sg                  [.] Find_Nearest_Waypoint
  21.47%  sg        sg                  [.] __mcount_internal
  18.93%  sg        sg                  [.] cos_73
   4.40%  sg        sg                  [.] __gnu_mcount_nc
   0.61%  sg        sg                  [.] __sincosf
   0.51%  sg        sg                  [.] __ieee754_sqrtf
   0.40%  sg        sg                  [.] __ieee754_atan2f
   0.34%  sg        sg                  [.] __kernel_cosf
   0.31%  sg        sg                  [.] __atanf
   0.27%  sg        sg                  [.] __ieee754_acosf
   0.25%  sg        [kernel.kallsyms]   [k] _raw_spin_unlock_irqrestore
   0.25%  sg        [kernel.kallsyms]   [k] __softirqentry_text_start
   0.10%  sg        [kernel.kallsyms]   [k] _raw_spin_unlock_irq
   0.08%  sg        sg                  [.] __kernel_sinf
   0.07%  sg        sg                  [.] main
   0.05%  sg        [kernel.kallsyms]   [k] restore_sigframe
   0.05%  sg        [kernel.kallsyms]   [k] setup_sigframe
   0.04%  sg        sg                  [.] strcmp
   0.02%  sg        [kernel.kallsyms]   [k] sys_clock_gettime
   0.02%  sg        [kernel.kallsyms]   [k] rcu_process_callbacks
   0.02%  sg        [kernel.kallsyms]   [k] vector_swi
Tip: Compare performance results with: perf diff [<old file> <new file>]
```

Instruction-level Profile:

```
Find_Nearest_Waypoint  /home/debian/Project1/sg
            |           return p1->SinLat * p2->SinLat +
   7.06     |             vcvt.f  d8, s16
            |               p1->CosLat * p2->CosLat*
            |             vcvt.f  d9, s18
            |           return p1->SinLat * p2->SinLat +
   0.52     |             vmla.f  d8, d9, d0
            |             vcvt.f  s16, d8
            |         Find_Nearest_Waypoint():
            |             closest_i = i;
   0.57     |             vcmpe.  s20, s16
   0.53     |             vmrs    APSR_nzcv, fpscr
  81.87     |             it      mi
            |             movmi   r7, r6
   0.50     |             it      mi
            |             vmovmi  s20, s16
            |               i++;
   0.54     |             adds    r6, #1
            |             b.n     10b24 <Find_Nearest_Waypoint+0x40>
            |             nop
            |             .word   0x45c71800
            |             .word   0x42652ee0
            |             .word   0x43b40000
Press 'h' for help on key bindings
```

Optimization applied to trigonometric functions:

The code was optimized for the cos functions by removing the calculation of fmod for the input value and 2*pi which was done to bring the input values within a range of 2*pi. This was unnecessary since all input values were within the range.The following changes were made to the code:

```
double cos_73(double x){

        int quad;                                    // what quadrant are we in?

        if(x<0)x=-x;                      // cos(-x) = cos(x)

        quad=(int)(x * two_over_pi);           // Get quadrant # (0 to 3) we're in

        //if(x>twopi)

        //x=fmod(x, twopi);                  // Get rid of values > 2* pi

        if(quad==0)

                return cos_73s(x);

        else if (quad==1)

                return -cos_73s(DP_PI-x);

        else if (quad==2)

                return -cos_73s(x-DP_PI);

        else if (quad==3)

                return  cos_73s(twopi-x);

 return 0.0;

}
```

Optimization 2:

This optimization was aimed at using a fast innacurate functions for the calculation of the value,c and storing the values in an array. These values stored in array were then compared to see if these values fell in the range of the maximum value of c obtained using inaccurate function and the apparent error in the maximum value obtained which depends on Epsilon. Those values which were found to be in the range were recalculated using an accurate function and accurate maximum c and hence accurate calculations were obtained while speed was increased considerably. The fastest function used was cos_12 , a second degree cos function and the accurate function used was cos_73.This was decided based on the testing with various cos functions in sincos.c.

Function	Degree	Approximation	Epsilon (range)	# of points in pass 2	Execution time
cos_12	2	$c_1 + c_2x^2$	0.075557	1-12	54.485
cos_32	4	$c_1 + c_2x^2 + c_3x^4$	0.015557	1-6	56.841
cos_52	6	$c_1 + c_2x^2 + c_3x^4 + c_4x^6$	0.000157	1-2	60.201
cos_73	8	$c_1 + c_2x^2 + c_3x^4 + c_4x^6 + c_5x^8$	0.000001	0-1	68.19

Output with validation:

```
debian@beaglebone:~/Project1$ ./sg
Vaildation:
Error in distance!Expected:9.588253 Actual:9.837340 Error:-2.597839 for Annapolis , Maryland
Error in distance!Expected:84.079651 Actual:84.108429 Error:-0.034227 for Dover , Delaware
Error in distance!Expected:130.731613 Actual:130.750107 Error:-0.014146 for Hartford , Connecticut
Error in distance!Expected:123.929008 Actual:123.948524 Error:-0.015748 for Juneau , Alaska
Error in distance!Expected:668.489380 Actual:696.772461 Error:-4.230895 for Nashville , Tennessee
Error in bearing!Expected:169.332611 Actual:331.899628 Error:-162.567017 for Nashville , Tennessee
Error in name!Expected:DAUPHIN ISLAND, AL Actual:SOUTH BASS ISLAND, OH for Nashville , Tennessee
Error in distance!Expected:76.643372 Actual:76.674934 Error:-0.041181 for Olympia , Washington
Error in distance!Expected:148.266769 Actual:148.283081 Error:-0.011002 for Sacramento , California
Error in distance!Expected:96.135429 Actual:96.160591 Error:-0.026173 for Tallahassee , Florida
Output:
0.Closest waypoint for Albany , New York is NYC ENTR. 254.716675 km away at bearing 181.550583 degrees
1.Closest waypoint for Annapolis , Maryland is THOMAS POINT, MD. 9.837340 km away at bearing 213.130646 degrees
2.Closest waypoint for Atlanta , Georgia is GRAYS REEF. 421.432037 km away at bearing 232.451050 degrees
3.Closest waypoint for Augusta , Maine is PORTLAND. 93.222275 km away at bearing 161.085648 degrees
4.Closest waypoint for Austin , Texas is PORT ARANSAS, TX. 279.357788 km away at bearing 194.268921 degrees
5.Closest waypoint for Baton Rouge, Louisiana is BILOXI. 231.263412 km away at bearing 260.404968 degrees
6.Closest waypoint for Bismarck , North Dakota is DEVILS ISLAND, WI. 772.233704 km away at bearing 259.330780 degrees
7.Closest waypoint for Boise , Idaho is NEWPORT, OR. 634.778564 km away at bearing 77.240952 degrees
8.Closest waypoint for Boston , Massachusetts is BOSTON. 33.576942 km away at bearing 292.475555 degrees
9.Closest waypoint for Carson City, Nevada is SAN FRANCISCO. 309.820831 km away at bearing 119.219414 degrees
10.Closest waypoint for Charleston , West Virginia is W LK ERIE. 376.048218 km away at bearing 9.754081 degrees
11.Closest waypoint for Cheyenne , Wyoming is DEVILS ISLAND, WI. 1300.067627 km away at bearing 305.208618 degrees
12.Closest waypoint for Columbia , South Carolina is FOLLY ISLAND, SC. 180.381088 km away at bearing 216.451447 degrees
13.Closest waypoint for Columbus , Ohio is SOUTH BASS ISLAND, OH. 185.942719 km away at bearing 355.881439 degrees
14.Closest waypoint for Concord , New Hampshire is ISLE OF SHOALS, NH. 80.402351 km away at bearing 250.083450 degrees
15.Closest waypoint for Denver , Colorado is DEVILS ISLAND, WI. 1407.891968 km away at bearing 310.096191 degrees
16.Closest waypoint for Des Moines, Iowa is SHEBOYGAN, WI. 540.894043 km away at bearing 298.330109 degrees
17.Closest waypoint for Dover , Delaware is THOMAS POINT, MD. 84.108429 km away at bearing 109.976585 degrees
18.Closest waypoint for Frankfort , Kentucky is SOUTH BASS ISLAND, OH. 418.861938 km away at bearing 336.318695 degrees
19.Closest waypoint for Harrisburg , Pennsylvania is THOMAS POINT, MD. 156.847061 km away at bearing 193.910110 degrees
20.Closest waypoint for Hartford , Connecticut is MONTAUK PT. 130.750107 km away at bearing 203.853012 degrees
21.Closest waypoint for Helena , Montana is WEST POINT, WA. 795.983765 km away at bearing 77.659920 degrees
22.Closest waypoint for Honolulu , Hawaii is W HAWAII. 385.118530 km away at bearing 129.342636 degrees
23.Closest waypoint for Indianapolis , Indiana is S LK MICHIGAN. 328.528412 km away at bearing 12.689395 degrees
24.Closest waypoint for Jackson , Mississippi is BILOXI. 283.126831 km away at bearing 209.237030 degrees
25.Closest waypoint for Jefferson City, Missouri is S LK MICHIGAN. 629.953918 km away at bearing 317.949829 degrees
26.Closest waypoint for Juneau , Alaska is FIVE FINGERS, AK. 123.948524 km away at bearing 202.528458 degrees
27.Closest waypoint for Lansing , Michigan is LAKE ST. CLAIR. 149.145386 km away at bearing 259.276031 degrees
28.Closest waypoint for Lincoln , Nebraska is SHEBOYGAN, WI. 807.715393 km away at bearing 296.838989 degrees
```

Output without validation:

```
29.Closest waypoint for Little Rock, Arkansas is SABINE, TX. 584.455139 km away at bearing 163.670013 degrees
30.Closest waypoint for Madison , Wisconsin is SHEBOYGAN, WI. 156.120331 km away at bearing 299.329956 degrees
31.Closest waypoint for Montgomery , Alabama is ORANGE BCH. 282.013397 km away at bearing 154.293076 degrees
32.Closest waypoint for Montpelier , Vermont is PORTLAND. 211.352295 km away at bearing 248.053284 degrees
33.Closest waypoint for Nashville , Tennessee is SOUTH BASS ISLAND, OH. 696.772461 km away at bearing 331.899628 degrees
34.Closest waypoint for Oklahoma City, Oklahoma is SABINE, TX. 723.815002 km away at bearing 207.938629 degrees
35.Closest waypoint for Olympia , Washington is WEST POINT, WA. 76.674934 km away at bearing 333.737518 degrees
36.Closest waypoint for Phoenix , Arizona is SAN CLEMENTE. 565.683105 km away at bearing 99.226471 degrees
37.Closest waypoint for Pierre , South Dakota is DEVILS ISLAND, WI. 803.698975 km away at bearing 295.413635 degrees
38.Closest waypoint for Providence , Rhode Island is BUZZARDS BAY,MA. 57.319054 km away at bearing 214.821564 degrees
39.Closest waypoint for Raleigh , North Carolina is ONSLOW BAY. 189.372559 km away at bearing 221.099457 degrees
40.Closest waypoint for Richmond , Virginia is CHESAPEAKE LIGHT, VA. 170.051636 km away at bearing 246.203003 degrees
41.Closest waypoint for Sacramento , California is SAN FRANCISCO. 148.283081 km away at bearing 126.205139 degrees
42.Closest waypoint for Saint Paul, Minnesota is DEVILS ISLAND, WI. 299.008118 km away at bearing 323.221375 degrees
43.Closest waypoint for Salem , Oregon is NEWPORT, OR. 89.569160 km away at bearing 113.121010 degrees
44.Closest waypoint for Salt Lake, Utah is SAN FRANCISCO. 997.999329 km away at bearing 105.949211 degrees
45.Closest waypoint for Santa Fe, New Mexico is SAN CLEMENTE. 1165.089355 km away at bearing 104.183609 degrees
46.Closest waypoint for Springfield , Illinois is S LK MICHIGAN. 388.610229 km away at bearing 326.534698 degrees
47.Closest waypoint for Tallahassee , Florida is KEATON BEACH, FL. 96.160591 km away at bearing 223.236084 degrees
48.Closest waypoint for Topeka , Kansas is S LK MICHIGAN. 832.105530 km away at bearing 301.781250 degrees
49.Closest waypoint for Trenton , New Jersey is NYC ENTR. 91.095810 km away at bearing 280.769745 degrees
Average      54.447 us
Minimum      52.187 us
debian@beaglebone:~/Project1$
```

Function Profile:

```
Samples: 116K of event 'cycles:ppp', Event count (approx.): 28525890381
Overhead   Command   Shared Object       Symbol
  45.75%   sg        sg                  [.] Find_Nearest_Waypoint_Pass
  26.24%   sg        sg                  [.] __mcount_internal
  18.20%   sg        sg                  [.] cos_12
   5.03%   sg        sg                  [.] __gnu_mcount_nc
   0.76%   sg        sg                  [.] cos_73
   0.60%   sg        sg                  [.] __sincosf
   0.56%   sg        sg                  [.] __ieee754_sqrtf
   0.50%   sg        sg                  [.] __ieee754_atan2f
   0.38%   sg        sg                  [.] __kernel_cosf
   0.32%   sg        sg                  [.] __atanf
   0.32%   sg        sg                  [.] __ieee754_acosf
   0.27%   sg        [kernel.kallsyms]   [k] _raw_spin_unlock_irqrestore
   0.18%   sg        [kernel.kallsyms]   [k] __softirqentry_text_start
   0.10%   sg        [kernel.kallsyms]   [k] _raw_spin_unlock_irq
   0.08%   sg        sg                  [.] __kernel_sinf
   0.07%   sg        sg                  [.] main
   0.06%   sg        sg                  [.] strcmp
   0.05%   sg        [kernel.kallsyms]   [k] restore_sigframe
   0.05%   sg        [kernel.kallsyms]   [k] setup_sigframe
   0.03%   sg        [kernel.kallsyms]   [k] sys_clock_gettime
   0.03%   sg        [kernel.kallsyms]   [k] rcu_process_callbacks
Tip: Show user configuration overrides: perf config --user --list
```

Instruction-Level Profile:

```
Find_Nearest_Waypoint_Pass   /home/debian/Project1/sg
               return p1->SinLat * p2->SinLat +
  8.63         vmov.f  s15, s16
  1.30         vmla.f  s15, s17, s0
            Find_Nearest_Waypoint_Pass():
                     c[i] = Calc_Closeness_fastest(ref, &(waypoints[i]) );
                     if (c[i]>max_c) {
               vcmpe.  s18, s15
                     c[i] = Calc_Closeness_fastest(ref, &(waypoints[i]) );
  0.73         vstmia r9!, {s15}
                     if (c[i]>max_c) {
  1.43         vmrs    APSR_nzcv, fpscr
 47.97         it      mi
               vmovmi s18, s15
  0.67         it      mi
               movmi  fp, r5
                         max_c = c[i];
                         closest_i2=i;
                             }
                          i++;
  0.69         adds    r5, #1
               while (strcmp(waypoints[i].Name, "END")) {
               ldrb    r0, [r4, #16]
Press 'h' for help on key bindings
```

Conclusion:

The code was optimized to run at an execution time of 54.447us using two passes, where in the first pass cos_12 function was used and in the second pass cos_73 function was used. Cos_73 function was selected based on a trade-off between speed and accuracy.The errors in first pass was eliminated by the use of accurate cos_73 function. It was observed that greater the degree of accuracy, greater is the number of computations performed and hence slower will be the execution time.