Student Information

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Answer 1

a)

For computing 95% confidence interval on the difference between the means, first we should write available information:

$$\alpha = 0.05$$

For people with age 40 and above:

$$n = 19,\ \bar{X} = 3.375,\ s_X = 0.96$$

For people under age 40:

$$m = 15,\ \bar{Y} = 2.05,\ s_Y = 1.12$$

I assume population variances are equal:

$$\sigma_X^2 = \sigma_Y^2 = \sigma^2 \tag{1}$$

So, confidence interval is:

$$\left[\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}} \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}, \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}} \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right] \tag{2}$$

The pooled standard deviation is:

$$\begin{aligned} s_p &= \sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}} \\ &= \sqrt{\frac{18 \cdot (0.96)^2 + 14 \cdot (1.12)^2}{19+15-2}} \\ &= \sqrt{\frac{16.589 + 17.562}{32}} \\ &= \sqrt{1.067} \\ &= 1.033 \end{aligned} \tag{3}$$

Degrees of freedom:

$$d.f. = n + m - 2 = 19 + 15 - 2 = 32 \tag{4}$$

So, we can obtain the critical value from Table A5 on Textbook with $d.f. = 32$:

$$t_{0.025} = 2.037$$

So,

$$\begin{aligned}
&\left[\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}} \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}, \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}} \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right] \\
=&\left[3.375 - 2.05 - 2.037 \cdot 1.033 \cdot \sqrt{\frac{1}{19} + \frac{1}{15}}, 3.375 - 2.05 + 2.037 \cdot 1.033 \cdot \sqrt{\frac{1}{19} + \frac{1}{15}}\right] \\
=&[0.5982, 2.0518]
\end{aligned} \tag{5}$$

b)

For computing 90% confidence interval on the difference between the means, first we should write available information:

$$\alpha = 0.1$$

For people with age 40 and above:

$$n = 19,\ \bar{X} = 3.375,\ s_X = 0.96$$

For people under age 40:

$$m = 15,\ \bar{Y} = 2.05,\ s_Y = 1.12$$

I assume population variances are equal:

$$\sigma_X^2 = \sigma_Y^2 = \sigma^2 \tag{6}$$

So, confidence interval is:

$$\left[\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}} \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}, \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}} \cdot s_p\sqrt{\frac{1}{n} + \frac{1}{m}}\right] \tag{7}$$

The pooled standard deviation is:

$$\begin{aligned}
s_p &= \sqrt{\frac{(n-1)s_X^2 + (m-1)s_Y^2}{n+m-2}} \\
&= \sqrt{\frac{18 \cdot (0.96)^2 + 14 \cdot (1.12)^2}{19 + 15 - 2}} \\
&= \sqrt{\frac{16.589 + 17.562}{32}} \\
&= \sqrt{1.067} \\
&= 1.033
\end{aligned} \tag{8}$$

Degrees of freedom:

$$d.f. = n + m - 2 = 19 + 15 - 2 = 32 \tag{9}$$

So, we can obtain the critical value from Table A5 on Textbook with $d.f. = 32$:

$$t_{0.05} = 1.694$$

So,

$$\begin{aligned}
&\left[\bar{X} - \bar{Y} - t_{\frac{\alpha}{2}} \cdot s_p \sqrt{\frac{1}{n} + \frac{1}{m}}, \bar{X} - \bar{Y} + t_{\frac{\alpha}{2}} \cdot s_p \sqrt{\frac{1}{n} + \frac{1}{m}}\right] \\
= &\left[3.375 - 2.05 - 1.694 \cdot 1.033 \cdot \sqrt{\frac{1}{19} + \frac{1}{15}}, 3.375 - 2.05 + 1.694 \cdot 1.033 \cdot \sqrt{\frac{1}{19} + \frac{1}{15}}\right] \\
= &[0.7206, 1.9294]
\end{aligned} \tag{10}$$

c)

We should test

$$H_0 : \mu_X = 3$$
$$H_A : \mu_X > 3$$

We have **one-sided right-tail** alternative.
So, $\mu_0 = 3$
Available information:

$$n = 19,\ \bar{X} = 3.375,\ s = 0.96$$

Degrees of freedom and α is:

$$d.f. = 18 \text{ and } \alpha = 0.05$$

So,

$$t_\alpha = 1.734 \tag{11}$$

Computing the T-statistic:

$$\begin{aligned}
t &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\
&= \frac{3.375 - 3.0}{\frac{0.96}{\sqrt{19}}} \\
&= 1.703
\end{aligned} \tag{12}$$

We can accept null hypothesis because $t < t_\alpha$. So we cannot say people with age 40 and above supports BREXIT with 95% confidence level.

Answer 2

a)

$$H_0 : \mu = 20 \tag{13}$$

b)

$$H_A : \mu \neq 20 \tag{14}$$

c)

$n = 11$, $\mu_0 = 20$, $s = 0.07$, $\bar{X} = 20.07$.
Significance level: $\alpha = 0.01$.
Degrees of freedom is $n - 1 = 11 - 1 = 10$.
We will use **two-sided alternative**. So,

$$t_{\frac{\alpha}{2}} = 3.169 \tag{15}$$

Compute the T-statistic,

$$\begin{aligned} t &= \frac{\bar{X} - \mu_0}{s/\sqrt{n}} \\ &= \frac{20.07 - 20.00}{0.07/\sqrt{11}} \\ &= 3.3166 \end{aligned} \tag{16}$$

The accepted region is $A = (-3.169, 3.169)$
The rejection region is $R = (-\infty, -3.169] \cup [3.169, \infty)$.
The null hypothesis rejected because, t is not in accepted region. They should stop the line. The corresponding graph are uploaded as another visual file.

Answer 3

a)

$$\mu_X = \mu_Y \tag{17}$$

b)

$$\mu_X < \mu_Y \tag{18}$$

c)

We have **one-sided left-tail alternative**.
Available information:

$$\alpha = 0.05$$

So, $z_\alpha = 1.645$.
For new painkiller:

$$n = 68,\ \bar{X} = 2.8,\ \sigma_X = 1.7$$

For current painkiller:

$$m = 68,\ \bar{Y} = 3.0,\ \sigma_Y = 1.4$$

$$\begin{aligned} Z &= \frac{\bar{X} - \bar{Y}}{\sqrt{\frac{\sigma_X^2}{n} + \frac{\sigma_Y^2}{m}}} \\ &= \frac{2.8 - 3.0}{\sqrt{\frac{(1.7)^2}{68} + \frac{(1.4)^2}{68}}} \\ &= \frac{-0.2}{\sqrt{0.0425 + 0.0288}} \\ &= \frac{-0.2}{0.267} \\ &= -0.749 \end{aligned} \tag{19}$$

The accepted region is: $A = (-1.645, +\infty)$
The rejection region is: $R = (-\infty, -1.645]$
The Null hypothesis rejected, because Z is not in accepted region. So, current painkiller is better.
The corresponding graph are uploaded as another visual file.