MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

CENG 384

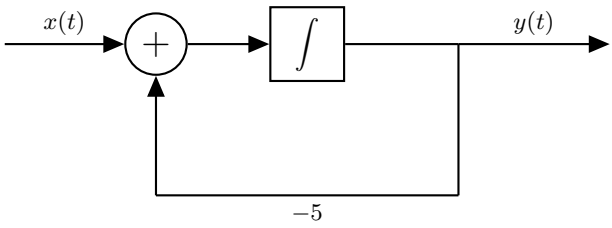
Spring 2020
Homework 2

Regulations:

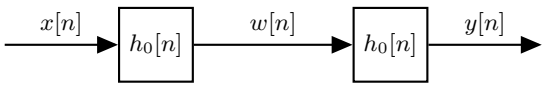
- **Grouping:** You are allowed to work in pairs.
- **Submission:** We provide a latex template for your solutions. Use that template and create a hw2.tar.gz file that includes hw2.tex and all other related files. Tar.gz file should not contain any directories and should create a *hw2.pdf* file with the following commands, otherwise you will get zero;
  tar xvzf hw2.tar.gz
  pdflatex hw2.tex
  Submit hw2.tar.gz to the odtuclass page of the course.
- **Deadline:** 23:55, 9 March, 2020 (Monday).
- **Late Submission:** Not allowed.

---

1. (12 pts) Analyze whether the following systems have these properties: *memory, stability, causality, linearity, invertibility, time-invariance*. Provide your answer in detail.
   (a) (6 pts) $y[n] = \sum_{k=1}^{\infty} x[n-k]$
   (b) (6 pts) $y(t) = tx(2t+3)$
2. (13 pts) Consider an LTI system given by the following block diagram:



   (a) (3 pts) Find the differential equation which represents this system.
   (b) (10 pts) Find the output $y(t)$, when the input $x(t) = (e^{-t} + e^{-3t})u(t)$. Assume that the system is initially at rest.
3. (15 pts) Evaluate the following convolutions.
   (a) (10 pts) Given $x[n] = 2\delta[n] + \delta[n+1]$ and $h[n] = \delta[n-1] + 2\delta[n+1]$, compute and draw $y[n] = x[n] * h[n]$.
   (b) (5 pts) Given $x(t) = u(t-1) + u(t+1)$ and $h(t) = e^{-t}\sin(t)u(t)$, calculate $y(t) = \frac{dx(t)}{dt} * h(t)$.
4. (20 pts) Evaluate the following convolutions.
   (a) (10 pts) Given $h(t) = e^{-2t}u(t)$ and $x(t) = e^{-t}u(t)$, find $y(t) = x(t) * h(t)$.
   (b) (10 pts) Given $h(t) = e^{3t}u(t)$ and $x(t) = u(t) - u(t-1)$, find $y(t) = x(t) * h(t)$.
5. (20 pts) Solve the following homogeneous difference and differential equations with the specified initial conditions.
   (a) (10 pts) $2y[n+2] - 3y[n+1] + y[n] = 0$, $y[0] = 1$ and $y[1] = 0$.
   (b) (10 pts) $y^{(3)}(t) - 3y''(t) + 4y'(t) - 2y(t) = 0$, $y''(0) = 2$, $y'(0) = 1$ and $y(0) = 3$.
6. (20 pts) Consider the following discrete time LTI system which is initially at rest:



   where $w[n] - \frac{1}{2}w[n-1] = x[n]$.
   (a) (10 pts) Find $h_0[n]$.
   (b) (5 pts) Find the overall impulse response, $h[n]$, of this system.
   (c) (5 pts) Find the difference equation which represents the relationship between the input $x[n]$ and the output $y[n]$.