

How to model both vertex and edge weights?

- Split node into two (in/out)
- add weight to edge between them.

Multiple sources/sinks?

- Add node connected to them

Real-time systems

- Job
- start time
- critical path (longest exec. time)
- deadline

Longest paths in edge-weighted DAGs

- Formulate as a shortest path
 - Negate all weights
 - Calculate shortest path
 - Negate result.
- Used for parallel job scheduling

Negative weights

- Dijkstra does not work
- Negative cycle: directed cycle, sum of edge weights < 0 .
 - weight is lowered for each cycle
- Bellman-Ford algorithm
 - Initialize $\text{distTo}[s] = 0$ and $\text{distTo}[v] = \infty$ for all other vertices
 - Repeat V times
 - Relax all edges
 - Quadratic in theory
 - can be terminated when nothing changes.

Dynamic programming algorithm computes SPT in time proportional to $V \times E$

Divide-and-Conquer: Break large problems into smaller subproblems. Combine the smaller solutions

Dynamic programming: Remember past results to find new results. Generally used for optimization where you need to find the best of multiple solutions.

Greedy algorithm: Make choices that seem to be the best right there and then and hope that it will lead you on the right path

Eager algorithm: Same as greedy

Bellman-Ford: If $\text{distTo}[v]$ does not change during pass i , no need to relax any edges pointing from v in pass $i+1$

If negative cycles exist, Bellman-Ford gets stuck in loop.

Finding negative cycle:

If any vertex v is updated in pass V , there exists a negative cycle (and we can trace back $\text{edgeTo}[v]$ entries to find it)

Arbitrage problem can be described as a negative cycle problem.

(Vertex = currency, Edge = transaction)

Remaining part (not covered in book)

CFG (Context free grammars): Defined by a set of production rules.
- see presentation

Parsing, building parse tree

Halting problem

- Can we write a program which can determine if a given program will terminate.

Not solvable:

Proof sketch: `if(halts()) while(true);`

Master theorem (not on exam)

- How can we derive complexity of recursive methods?
- Repeated Substitution.

Let $a \geq 1$ and $b > 1$ be constants, let $f(n)$ be a function, and let $T(n)$ be a function over the positive numbers defined by the recurrence.

$T(n) = aT(n/b) + f(n)$. If $f(n) = \Theta(n^d)$, where $d \geq 0$, then

- $T(n) = \Theta(n^d)$ if $a < b^d$
- $T(n) = \Theta(n^d \log n)$ if $a = b^d$
- $T(n) = \Theta(n^{\log_b a})$ if $a > b^d$

Sustainability

Efficient algorithms:

- saves energy
- Allows longer life span for computers, smart phones, tablets...
- Allows less costly solutions to problems
- May give more people access to information.