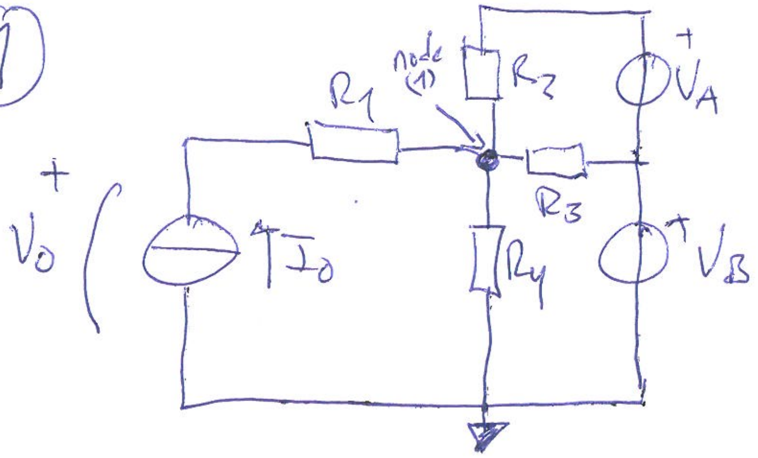
1



$R_1 = 1\,k\Omega$ $\quad V_A = 1V$

$R_2 = 3\,k\Omega$ $\quad V_B = 5V$

$R_3 = 2\,k\Omega$ $\quad I_0 = 1mA$

$R_4 = 6\,k\Omega$

KCL in node 1: $I_0 + \dfrac{V_A+V_B-V_1}{R_2} + \dfrac{V_B-V_1}{R_3} + \dfrac{0-V_1}{R_4} = 0$

$$\Rightarrow V_1\left[\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right] = I_0 + \frac{V_A+V_B}{R_2} + \frac{V_B}{R_3}$$

$$\Rightarrow V_1\left[\frac{1}{3}+\frac{1}{2}+\frac{1}{6}\right] = 1 + \frac{6}{3} + \frac{5}{2} \Rightarrow V_1[2+3+1] = 6+12+15$$

$$\Rightarrow V_1 = \frac{33}{6} = 5{,}5\,V$$

$$V_1 + I_0R_1 = V_0 \Rightarrow V_0 = 5{,}5 + 1\cdot 1 = 6{,}5\,V$$

Answer : $V_0 = 6{,}5\,V$

I_2 ~~is~~ ~~the~~ current trough R_2

(2) (A) $P_2 = I_2 \cdot V_2 = R_2 I_2^2 = \left\{ I_2 = \frac{V_2 - V_1}{R_2} \right\} = \frac{(V_2 - V_1)^2}{R_2} =$

$= \left\{ \begin{matrix} V_2 = 4V \\ V_1 = 2V \quad R_2 = 8k\Omega \end{matrix} \right\} = \frac{2^2}{8 \cdot 10^3} = 0{,}5 \text{ mW}$

(B) Power in resistors = Power in sources.

Power in R_1: $P_1 = \frac{V_1^2}{R_1} = \frac{2^2}{2} = 2 \text{ mW}$

Power in R_3: $P_3 = \frac{V_2^2}{R_3} = \frac{4^2}{4} = 4 \text{ mW}$

Total power in sources is $P_1 + P_2 + P_3 = \cancel{6{,}5}\ 0{,}5 + 2 + 4 =$

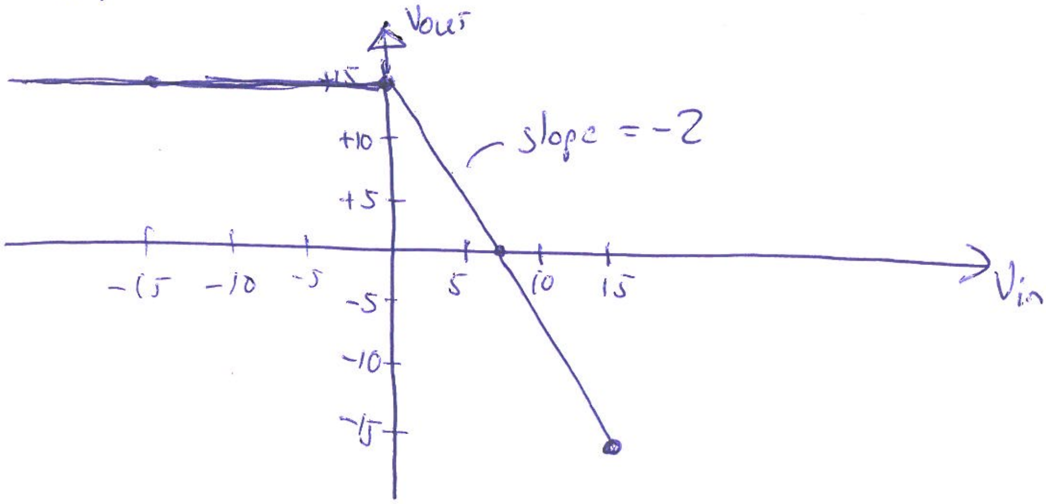
$= 6{,}5 \text{ mW}$

4 A

$$\frac{V_{in}-5}{R_1}+\frac{V_{OUT}-5}{R_2}=0 \Rightarrow \frac{V_{OUT}}{R_2}=\frac{5}{R_2}+\frac{5}{R_1}-\frac{V_{in}}{R_1}$$

$$\Rightarrow V_{OUT}=5+\frac{R_2}{R_1}\cdot 5-\frac{R_2}{R_1}V_{in}$$

B $\frac{R_2}{R_1}=2 \Rightarrow V_{OUT}=15-2V_{in}$



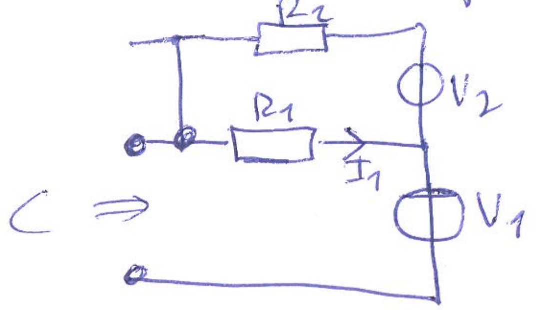
5. $$V_c(t) = V_c(\infty) + \left[V_c(0) - V_c(\infty)\right] e^{-\frac{t-t_0}{\tau}}$$

$t_0 = 0\,s$

at $t < 0\,s$: The capaciter is chaged to $V_c = V_1 + V_2$

$$\Rightarrow V_c(0) = V_1 + V_2 = 1 + 6 = 7V$$

at $t > 0\,s$: The capacitor sees the circuit:



$$V_{TH} = V_1 + I_1 \cdot R_1 = V_1 + \frac{V_2}{R_1 + R_2} \cdot R_1 =$$

$$= 1 + 6 \cdot \frac{10}{25} = 3{,}4\,V$$

$$R_{TH} = R_1 // R_2 = \frac{10 \cdot 15}{10 + 15} = 6\,k\Omega$$

$$V_c(\infty) = V_{TH} = 3{,}4\,V$$

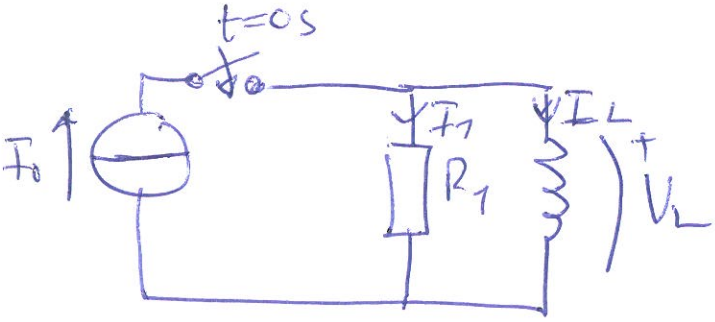
$$\tau = R_{TH} \cdot C = 6 \cdot 10^3 \cdot 1 \cdot 10^{-9} = 6\,\mu s.$$

$$\Rightarrow V_c(t) = 3{,}4 + [7 - 3{,}4]\, e^{-\frac{t}{6\mu s}} = 3{,}4 + 3{,}6 \cdot e^{-\frac{t}{6\mu s}}$$

$$V_c(6\mu s) = 3{,}4 + 3{,}6\, e^{-1} = 4{,}7\,V.$$

Answer: $V_c(t = 6\mu s) = 4{,}7\,V$

6. A $\quad I_1 = \frac{V_L}{R_1} \qquad V_L = L \frac{dI_L}{dt}$



$$I_L = I_L(\infty) + \left[I_L(0) - I_L(\infty)\right] e^{-\frac{t-t_0}{\tau}}$$

$$\left(\begin{array}{l} t_0 = 0\,s \\ \tau = \frac{L}{R_1} = \frac{10^{-3}}{10^3} = 1\,\mu s \end{array} \right)$$

$I_L(0) = 0\ A$

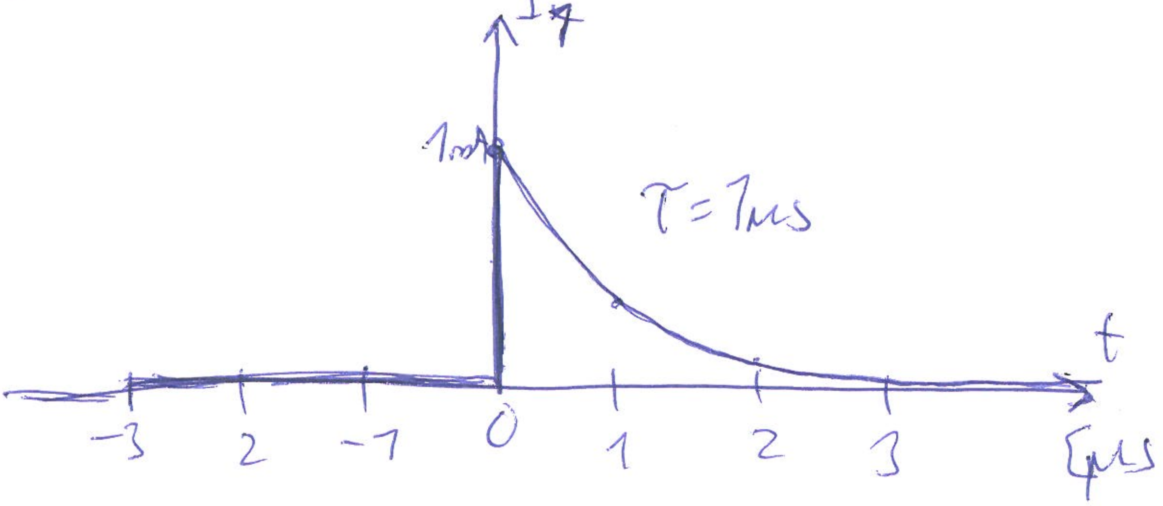
$I_L(\infty) = I_0 \quad \Rightarrow \quad I_L = I_0\left(1 - e^{-\frac{t}{\tau}}\right) \qquad \tau = \frac{L}{R_1}$

$$\Rightarrow V_L = L \cdot \frac{dI_L}{dt} = L\left(-I_0 e^{-\frac{t}{\tau}} \cdot \left(-\frac{1}{\tau}\right)\right) = \frac{L I_0}{\tau} e^{-\frac{t}{\tau}} = \left\{\tau = \frac{L}{R_1}\right\} =$$

$$= R_1 I_0 e^{-\frac{t}{\tau}}$$

$$\Rightarrow \boxed{I_1(t) = \frac{V_L}{R_1} = I_0 e^{-\frac{t}{\tau}}}$$

B $\quad \tau = 1\,\mu s \quad I_0 = 1\,mA$



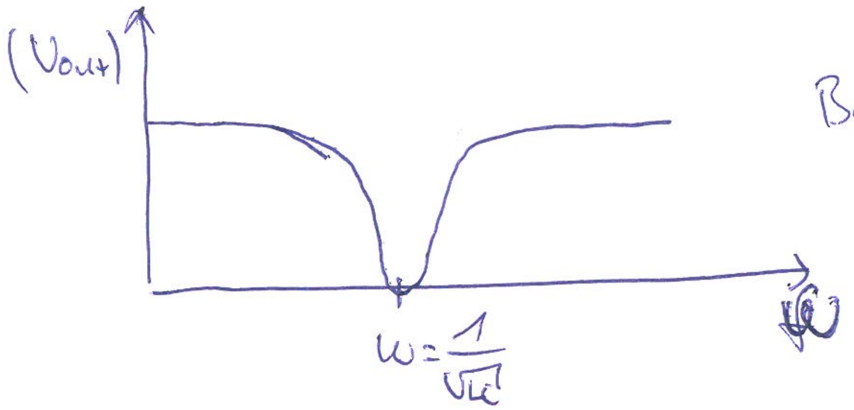
7. (A) $\hat{V}_{out} = \hat{V}_{in} \dfrac{R}{Z_1 + R}$ where $Z_1 = j\omega L // \dfrac{1}{j\omega C}$

$$Z_1 = \frac{j\omega L \frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L} = \frac{j\omega L}{1-\omega^2 LC} \qquad Z_1 \to \infty \text{ when } \omega \to \frac{1}{\sqrt{LC}}$$

$\Rightarrow \hat{V}_{out} \to 0$ Answer: $V_{out}(t) = 0$ at $\omega = \dfrac{1}{\sqrt{LC}}$

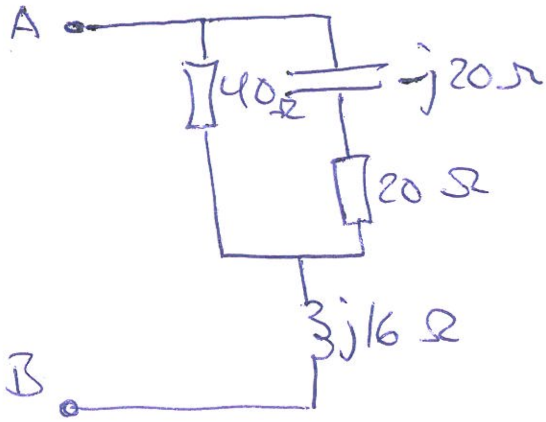
(B) $\omega \to 0 \Rightarrow j\omega L \to 0 \Rightarrow V_{out} = V_{in}$

$\omega \to \infty \Rightarrow \dfrac{1}{j\omega C} \to 0 \Rightarrow V_{out} = V_{in}$



Band-reject filter.

8. Determine Z_{TH}: Zero current source and find $Z_{AB} = Z_{TH}$



$Z_{TH} = Z_1 + j16$ when

$Z_1 = 40 // (20-j20) =$

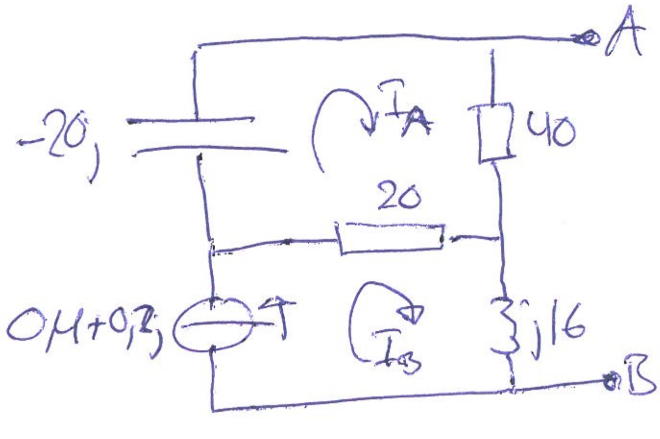
$= \frac{40 \cdot 20(1-j)}{60-j20} = 40\frac{1-j}{3-j} =$

$= 40\frac{(1-j)(3+j)}{9+1} = 4(3+j-3j+1) = 16-8j\ \Omega$

$Z_{TH} = 16-8j + j16 = 16+8j.$

Find V_{TH}

$V_{TH} = V_{AB}$ when open between A-B



$I_B = 0{,}4+0{,}2j$ A

I_a: $-(-j20)I_A - 40I_A - 20(I_A - I_B) = 0$

$j20I_A - 40I_A - 20I_A + 20(0{,}4+0{,}2j) = 0$

$-I_A(60-20j) + (8+4j) = 0 \Rightarrow I_A = \frac{8+4j}{60-20j} =$

$= \frac{2+j}{15-5j} = \frac{(2+j)(15+5j)}{15^2+5^2} = \frac{1}{250}(30+10j+15j-5) = \frac{25+25j}{250} =$

$= 0{,}1+0{,}1j$ A

cont. →

$$V_{TH} = 40\, I_A + j16\, I_B = 40\,(0{,}1 + 0{,}1j) + 16j\,(0{,}4 + 0{,}2j) =$$

$$= 4 + 4j + 6{,}4j - 3{,}2 = 0{,}8 + 10{,}4j$$

$$\Rightarrow V_{TH} = 10{,}4 \angle 85{,}6^\circ \quad \Rightarrow V_{TH} = 10{,}4 \cos(\omega t + 85{,}6^\circ)$$