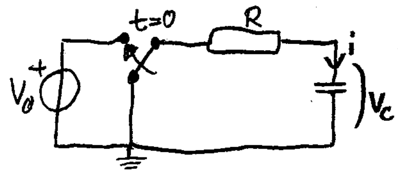
Inbyggd elektronik 2018-04-19 #9

Ex 1

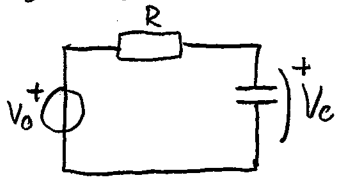


We want to know i, V, P, E in the circuit as a function of time.

I choose V_c as variable since it is continous.

for $t<0$ $i=0$ since the capacitor is not charged
$\Rightarrow V_c(t<0) = 0\,V$

At $t=0$



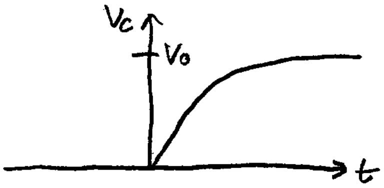
What happens: A current flows into the capacitor and charges the plates up. Finally it is fully charged $Q=CV_0$. Then current is zero and the capacitor is "open".

KVL: $V_0 - Ri - V_c = 0$ $\left\{ i = C\frac{dV_c}{dt} \right\} \Rightarrow V_0 - RC\frac{dV_c}{dt} - V_c = 0$

$$\Rightarrow \frac{dV_c}{dt} + \frac{1}{RC}V_c = \frac{V_0}{RC}$$

Solve $V_c(t) = Ae^{-\frac{t}{RC}} + B \Rightarrow \underbrace{-\frac{A}{RC}e^{-\frac{t}{RC}} + \frac{1}{RC}Ae^{-\frac{t}{RC}}}_{=0} + \frac{B}{RC} = \frac{V_0}{RC}$ $B = V_0$

at $t=0 \Rightarrow A + V_0 = 0 \Rightarrow A = -V_0 \Rightarrow \boxed{V_c(t>0) = V_0\left[1 - e^{-\frac{t}{RC}}\right]}$ $= \left\{ \begin{array}{l} RC = \tau \\ RC \text{ time constant} \end{array} \right\}$



t	$\left[1-e^{-\frac{t}{\tau}}\right]$
τ	0,64
2τ	0,86
3τ	0,95
⋮	
5τ	0,99

Stored energy in C $E(t) = \frac{1}{2}CV_c^2(t)$ can be calculated for all t.

$i(t<0) = i(0^-) = 0\,A$
$i(t>0) = C\frac{dV_c(t)}{dt} = \left(\frac{V_0}{RC}\right)e^{-\frac{t}{RC}} = \frac{V_0}{R}e^{-\frac{t}{\tau}}$

$i(0^+) = \frac{V_0}{R}$ $i(\infty) = 0$

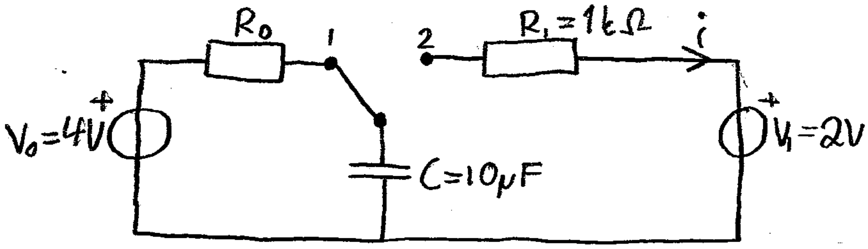
Ex $R = 1k\Omega$, $C = 1\mu F \Rightarrow \tau = 1ms$

$R = 1k\Omega$, $C = 1nF \Rightarrow \tau = 1\mu s$

For first order like RC or RL (only one C or one L) the solution can be written down directly since C or L is connected to Thevenin equivalent.

- Choose V_c for C and i_L for L as variables

$V_c = V_c(\infty) + [V_c(t_0) - V_c(\infty)]e^{-\frac{(t-t_0)}{\tau}}$ See ch. 7.4
t_0 is when the circuit changes (that is the switch moves) often $t_0 = 0$
$V_c(t_0)$ is the value of V_c when switch moves.
$V_c(\infty)$ is the asymptotic final value of V_c (in practise $t > 5\tau$)



At $t = 2ms$ the switch moves to position 2.
Plot $i(t)$ for $t > 0s$

$$V_C = V_C(\infty) + [V_C(t_0) - V_C(\infty)]\, e^{-\frac{t - 2\cdot 10^{-3}}{\tau}} \quad \text{for } t > t_0 = 2ms$$

$$V_C(t_0) = \{\text{at } t < t_0 \text{ } C \text{ is fully charged by } V_0\} = 4V$$

$$V_C(\infty) = \{\text{at } t > \infty \text{ } C \text{ is fully charged by } V_1\} = 2V$$

At $t > t_0$ $\tau = R_1 C = 10ms$

$$V_C(t > t_0) = 2 + [4 - 2]\, e^{\frac{-t+2\cdot 10^{-3}}{10\cdot 10^{-3}}} = 2\left(1 + e^{\frac{-t+2\cdot 10^{-3}}{10\cdot 10^{-8}}}\right)$$