Inbyggd elektronik 2018-05-09 #13

Phasor notation (Complex I, V or in Swedish jw-method)

From math Euler's identity $e^{j\theta} = \cos\theta + j\sin\theta$

Thus $V = V_m\cos(\omega t + \phi) = V_m \mathbb{R}\{e^{j(\omega t+\phi)}\} = \mathbb{R}\{\underbrace{V_m e^{j\phi}}\, e^{j\omega t}\}$

Real part

This is called the phasor representation of complex $\hat{V}$.

$\hat{V} = V_m e^{j\phi}$ contains the amplitude and phase angle information of a given sinusoidal voltage.

The complex number domain is also called the frequency domain since the result depends on frequency ω.

The notation $V_m \angle \phi^\circ$ is very common. It is the same as $V_m e^{j\phi}$

Resistor (R): Assume $i = I_m\cos(\omega t + \phi)$ $\quad \hat{I} = I_m e^{j\phi}$

$V = \underbrace{R I_m}_{V_m}\cos(\omega t + \phi) \qquad \hat{V} = R \cdot I_m e^{j\phi} = R\hat{I}$ Ohm's law.

Inductor L: Assume $i = I_m\cos(\omega t + \phi)$

$$V = L\frac{di}{dt} = L\, I_m(-\sin(\omega t + \phi))\cdot\omega = -L I_m \omega\cos(\omega t + \phi - \tfrac{\pi}{2})$$

$$\Rightarrow \hat{V} = -\omega L I_m e^{j(\phi - \frac{\pi}{2})} = -\omega L I_m e^{j\phi}(\cos\tfrac{\pi}{2} - j\sin\tfrac{\pi}{2}) = j\omega L \cdot I_m e^{j\phi} =$$

$= j\omega L\hat{I}$ {Like Ohm's law, but instead of R we have $j\omega L$}.

Capacitor (C): $v = V_m\cos(\omega t + \phi) \quad \hat{V} = V_m e^{j\phi}$

$$i = C\frac{dv}{dt} = C\, V_m(-\sin(\omega t + \phi))\omega = -C\omega V_m\cos(\omega t + \phi - \tfrac{\pi}{2})$$

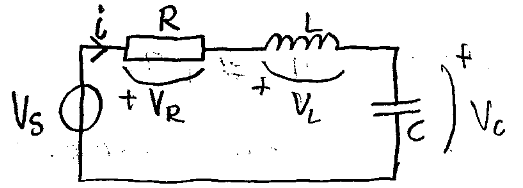
$$\Rightarrow \hat{I} = -\omega C V_m e^{j\phi} e^{-j\frac{\pi}{2}} = j\omega C\underbrace{V_m e^{j\phi}}_{\hat{V}} = j\omega C\hat{V}$$

$$\Rightarrow \hat{V} = \frac{1}{j\omega C}\hat{I}$$

Like Ohm's law but instead of R we have $\frac{1}{j\omega C}$

In general we have $\hat{V} = Z\hat{I}$ where Z is called impedance

Element	Z
Resistor	R
Inductor	$j\omega L$
Capacitor	$\frac{1}{j\omega C}$

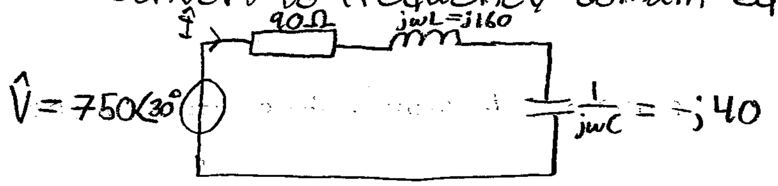


$V_S = 750\cos(5000t + 30°)$

$R = 90\,\Omega \quad L = 32\,mH \quad C = 5\,\mu F$

Find i

Convert to frequency domain equivalent circuit



Impedance seen by the source $\hat{V}$ $\quad Z = R + j\omega L + \frac{1}{j\omega C}$

$$\hat{I} = \frac{\hat{V}}{Z} = \frac{750\angle 30°}{90 + j160 - j40} = \frac{750\angle 30°}{90 + j120} = \left\{ \begin{array}{l} \sqrt{90^2 + 120^2} = 150 \\ \arctan(\frac{120}{90}) = 53{,}1° \end{array} \right\} = \frac{750\angle 30°}{150\angle 53{,}1°} = 5\angle -23{,}1°$$

$\Rightarrow i(t) = 5\cos(5000t - 23{,}1°)$.

Find V_c: $\hat{V}_c = \frac{1}{j\omega C}\hat{I} = -j40 \cdot (5\angle -23{,}1°) = 40\angle -90° \cdot 5\angle -23{,}1° = 200\angle -113{,}1°$

$\Rightarrow V_c = 200\cos(5000t - 113{,}1°)$

Find V_L: $\hat{V}_L = j\omega L \cdot \hat{I} = j160 \cdot (5\angle -23{,}1°) = 160\angle 90° \cdot 5\angle -23{,}1° = 800\angle 66{,}9°$

$\Rightarrow V_L = 800\cos(5000t + 66{,}9°)$

Find V_R: $\hat{V}_R = R\hat{I} = 90\angle 0° \cdot 5\angle -23{,}1° = 450\angle -23{,}1°$

$\Rightarrow V_R = 450\cos(5000t - 23{,}1°)$