Tips Kvadratkomp. i allmänhet

$$x^2 \pm kx = \left(x \pm \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2$$

Kontroll:

$$\left(x+\frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2 = x^2 + 2x \cdot \frac{k}{2} + \left(\frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2$$

$$= x^2 + kx \quad \text{najs}$$

Ex. $\underbrace{x^2 + 10x}_{\text{kvadratkomp.}} - 30$

$$= \underline{\underline{(x+5)^2 - 25}} - 30$$

$$= (x+5)^2 - 55 \qquad \boxed{}$$

Följdfråga

$$\int_3^{\infty} \frac{2}{x^2-4x+5}\,dx = \Big[2\arctan(x-2)\Big]_3^{\infty}$$

$$= \lim_{R\to\infty} \Big[2\arctan(x-2)\Big]_3^{R}$$

$$= \lim_{R\to\infty} 2\underbrace{\arctan(R-2)}_{\to \frac{\pi}{2}} - 2\underbrace{\arctan 1}$$

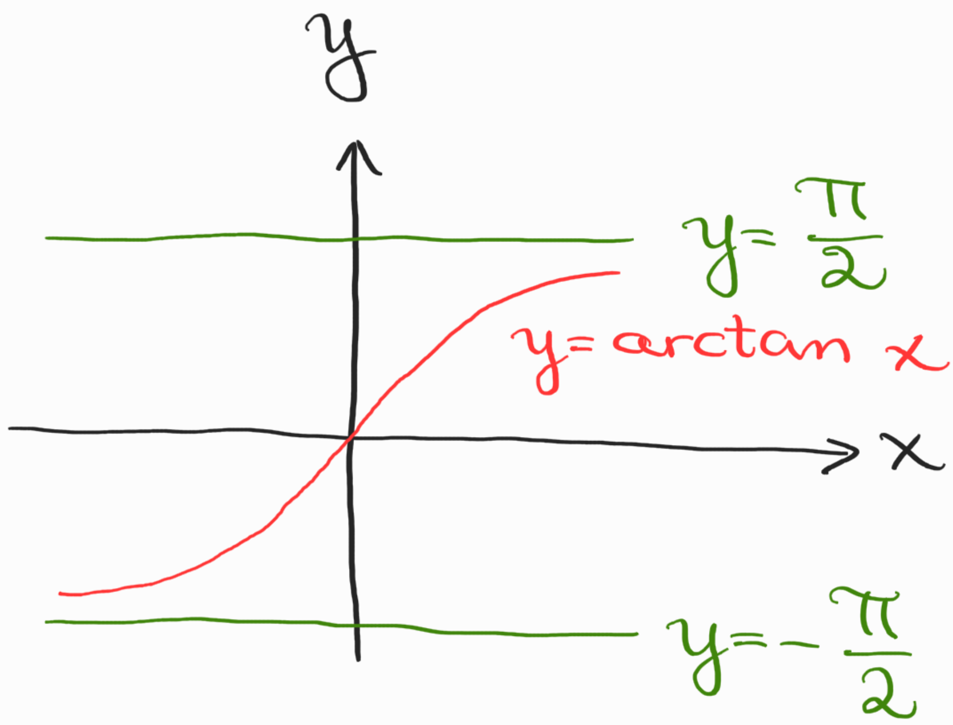
$\tan 45^\circ = 1$

$\Rightarrow \tan\frac{\pi}{4} = 1$

$\Rightarrow \arctan 1 = \frac{\pi}{4}$

$$= 2\cdot\frac{\pi}{2} - 2\cdot\frac{\pi}{4} = \pi - \frac{\pi}{2}$$

$$= \frac{\pi}{2}$$ // Svar



Kändis $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$

$$\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2}$$

Variabelsubstitution

2017.01.11 #4 #UU

a) Beräkna $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$

Steg 1 Betrakta $\displaystyle\int \frac{\cos x}{1+\sin^2 x}\,dx$.

Byt ut det som ställt till med problem, t.ex. $u = \sin x$.

Då fås $\displaystyle\frac{du}{dx} = \cos x$

$$\Downarrow$$

$$du = \cos x\,dx$$

$$\Downarrow$$

$$dx = \frac{1}{\cos x}\,du$$

Alltså

$$\int \frac{\cos x}{1+\sin^2 x}\,dx = \int \frac{\cos x}{1+u^2}\cdot\frac{1}{\cos x}\,du$$

$$= \int \frac{1}{1+u^2}\,du = \arctan u + C$$

$$= \arctan(\sin x) + C$$

Steg 2 Sätt in gränserna:

$$\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx = \Big[\arctan(\sin x)\Big]_0^{\pi/2}$$

$$= \arctan\big(\underbrace{\sin\tfrac{\pi}{2}}_{=1}\big) - \arctan\big(\underbrace{\sin 0}_{=0}\big)$$

$$= \arctan 1 - \arctan 0$$

$$= \frac{\pi}{4} - 0$$

$$= \frac{\pi}{4}$$ // Svar

Anmärkning Om vi vill byta gränser:

$u = \sin x$

$x_1 = 0$ blir $u_1 = \sin 0 = 0$

$x_2 = \pi/2$ blir $u_2 = \sin \pi/2 = 1$