Partiell integration

$$\int f \cdot g \, dx = F \cdot g - \int F \cdot g' \, dx$$

där F står för <u>en</u> primitiv funktion till f.

Anm.

$$\int_a^b f \cdot g \, dx = \left[F \cdot g \right]_a^b - \int_a^b F \cdot g' \, dx$$

Kändis

osynlig etta

$$\int \ln x \, dx = \int \underbrace{1}_{f(x)} \cdot \underbrace{\ln x}_{g(x)} \, dx$$

$$= F \cdot g - \int F \cdot g' = x \cdot \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= x \cdot \ln x - \int 1 \, dx$$

$$= x \cdot \ln x - x + C \qquad \boxtimes$$

2017.08.22 #1 #LiU

b) Beräkna $\int \ln(x^2+1)\,dx$

$$= \int \underbrace{1}_{f(x)} \cdot \underbrace{\ln(x^2+1)}_{g(x)}\,dx$$ partiell integration

$$= x \cdot \ln(x^2+1) - \int x \cdot \underbrace{\frac{1}{x^2+1} \cdot 2x}_{g'(x)}\,dx$$

$$= x \cdot \ln(x^2+1) - 2\int \underbrace{\frac{x^2}{x^2+1}}_{\substack{\text{polynomdivision då} \\ x^2 \approx x^2+1}}\,dx$$

$$= x \cdot \ln(x^2+1) - 2\int \left(\frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}\right)dx$$

$$= x \cdot \ln(x^2+1) - 2\int \left(1 - \frac{1}{x^2+1}\right)dx$$

$$= x \cdot \ln(x^2+1) - 2(x - \arctan x) + C$$

$\boxtimes$

Partialbråksuppdelning

Bakgrund

Ex. $\displaystyle\int \frac{1}{x^2-5x+4}\,dx$ Svår att integrera direkt

Men om vi kan dela upp bråket

$$\int \frac{1}{x^2-5x+4}\,dx = \int \underbrace{\left(\frac{1/3}{x-4} - \frac{1/3}{x-1}\right)}_{\text{lättare att integrera nu}} dx$$

$$= \frac{1}{3}\ln|x-4| - \frac{1}{3}\ln|x-1| + C$$

2018.01.13 # 2 # Lill

Beräkna $\displaystyle\int_3^{\infty} \frac{1}{x^3-3x^2+2x}\,dx$

Steg 1 Betrakta

$$\int \frac{1}{\underbrace{x^3-3x^2+2x}_{\text{faktorisera nämnaren}}}\,dx = \int \frac{1}{x\underbrace{(x^2-3x+2)}}\,dx$$

nollställen: $x_1 = 1$
$x_2 = 2$

$x^2-3x+2 = (x-x_1)(x-x_2)$

$$= \int \frac{1}{x(x-1)(x-2)}\,dx$$

Genomför PBU med ansats:

$$\frac{1}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

Multiplicera båda led med $x(x-1)(x-2)$:

$$1 = A(x-1)(x-2) + Bx(x-2) + Cx(x-1)$$

$$\Rightarrow 1 = \underline{\underline{Ax^2}} \underline{\underline{- 3Ax}} + 2A + \underline{\underline{Bx^2}} \underline{\underline{- 2Bx}} + \underline{\underline{Cx^2}} \underline{\underline{- Cx}}$$

$$\Rightarrow 1 = \underline{\underline{(A+B+C)x^2}} + (\underline{\underline{-3A-2B-C)x}} + 2A$$

Matcha koefficienterna:

$$\begin{cases} A+B+C = 0 \\ -3A-2B-C = 0 \\ 2A = 1 \end{cases} \Rightarrow \dots \Rightarrow \begin{cases} A = 1/2 \\ B = -1 \\ C = 1/2 \end{cases}$$

<u>Alltså</u>

$$\int \frac{1}{x(x-1)(x-2)}\,dx = \int \left(\frac{1/2}{x} + \frac{-1}{x-1} + \frac{1/2}{x-2} \right) dx$$

$$= \frac{1}{2}\ln|x| - \ln|x-1| + \frac{1}{2}\ln|x-2| + C$$