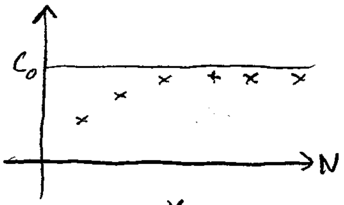
SF1626 2018-10-09 #19

Decide wether $\iint_{\mathbb{R}^2} \frac{x^2}{(1+x^2)(x^2+y^2)^{3/2}}\,dxdy$ is convergent or divergent

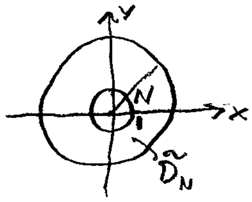
- Convergence criteria for a sequence of numbers.

$0 < S_N$, $S_N \le S_{K+K}$, $K>0$, $S_N \nearrow$

If $S_N \le C_0$ for every N then $\lim_{N\to\infty} S_N$ exists



for every $\varepsilon>0$, $\exists M_\varepsilon > 0$ such that $|S_N - S| < \varepsilon$ whenever $N > M_\varepsilon$



$$\tilde{S}_N = \overbrace{\iint_{D_N} \frac{1}{(x^2+y^2)^{3/2}}\,dxdy}$$

$$S_N = \iint_{\tilde{D}_N} \frac{x^2}{(1+x^2)(x^2+y^2)}\,dxdy \le \iint_{D_N} \frac{\cancel{x^2}}{\cancel{x^2}(x^2+y^2)^{3/2}}\,dxdy \le \iint_{D_N} \frac{1}{(x^2+y^2)^{3/2}}\,dxdy$$

WRITE THE INTEGRAL IN POLAR COORDINATES

$x^2+y^2 = r^2$

$$\tilde{S}_N = \iint_{D_N} \frac{1}{r^3}\,r\,dr\,d\theta = \int_0^{2\pi}\int_{(1)}^{N} \frac{1}{r^2}\,dr\,d = 2\pi\left[(-1)\frac{1}{r}\right]^N \le 2\pi$$

$S_N < 2\pi$ $\quad S_N \nearrow$

$$\iint_{\mathbb{R}^2} f(x,y)\,dxdy \leftarrow \underbrace{\iint_{\substack{\text{unit}\\\text{disk}}} f(x,y)\,dxdy}_{\text{Has a value}} + \underbrace{\iint_{\tilde{D}_N} f(x,y)\,dxdy}_{\le 2\pi}$$

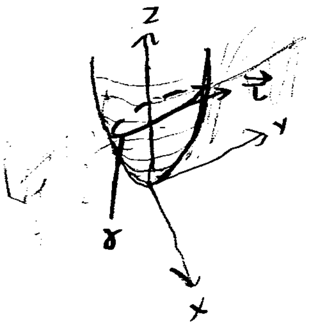
Parametrization of a line:

a point $\begin{pmatrix}a\\b\\c\end{pmatrix}$ and a tangent vector $\vec{\tau} = \begin{pmatrix}\tau_1\\\tau_2\\\tau_3\end{pmatrix}$

$$t \to \begin{pmatrix}a\\b\\c\end{pmatrix} + t\cdot\vec{\tau} =: \gamma(t)$$

e.g. $f(x,y) = 10x^2+y^2$, $\quad g(x,y) = x+y$

Find the tangent vectors of the curve lying at the intersection of the graph of f and the graph of g.



$$\vec{\tau} \perp \vec{n}_f,\ \vec{\tau} \perp \vec{n}_g \implies \vec{\tau} \parallel \vec{n}_f \times \vec{n}_g$$

$$\vec{n}_f(x,y) \ne \nabla f(x,y) \qquad \vec{n}_f(x,y) = \begin{pmatrix}-f_x(x,y)\\-f_y(x,y)\\1\end{pmatrix} = \nabla_{x,y,z} F(x,y,z) \qquad F(x,y,z) = z - f(x,y)$$

$$\vec{n}_g(x,y) = \begin{pmatrix}-1\\-1\\1\end{pmatrix} \qquad \begin{pmatrix}-f_x(x,y)\\-f_y(x,y)\\1\end{pmatrix} = \begin{pmatrix}-10x\\-y\\1\end{pmatrix}$$

$$x+y = 10x^2+y^2 \implies \dots \implies x = \pm\frac{1}{20} + \frac{1}{10}\left(y-y^2+\frac{1}{40}\right)$$

Eq'n of a plane $\quad$ A POINT $\begin{pmatrix}a\\b\\c\end{pmatrix}$ and a normal vector $\vec{n} = \begin{pmatrix}n_1\\n_2\\n_3\end{pmatrix}$ $\quad \begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}a\\b\\c\end{pmatrix} \perp \vec{n}$

$$\left(\begin{pmatrix}x\\y\\z\end{pmatrix} - \begin{pmatrix}a\\b\\c\end{pmatrix}\right)\cdot\vec{n} = 0 \implies n_1(x-a) + n_2(y-b) + n_3(z-c) = 0$$