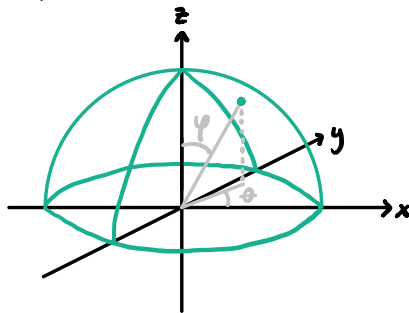


Ex

$$\iint_Y z^2 dS, \text{ om } Y \text{ ges av } x^2 + y^2 + z^2 = 1, \quad z \geq 0$$



Parametriseras av $\vec{r}(\varphi, \theta) = (\sin \varphi \cos \theta, \sin \varphi \sin \theta, \cos \varphi)$
Sfäriska koord., där $R = 1$

Ytelementet, dS :

$$dS = \underbrace{|\vec{r}_\varphi \times \vec{r}_\theta|}_{\sin \varphi} d\varphi d\theta$$

$$\iint_Y z^2 dS = \int_0^{2\pi} \left(\int_0^{\pi/2} \cos^2 \varphi \sin \varphi d\varphi \right) d\theta = \{ \text{variabelsubstitution} \} = 2\pi \left[-\frac{\cos^3 \varphi}{3} \right]_0^{\pi/2} = \underline{\underline{\frac{2\pi}{3}}}$$