F 12 VARIABELSUBSTITUTION I TRIPPELINTEGRAL

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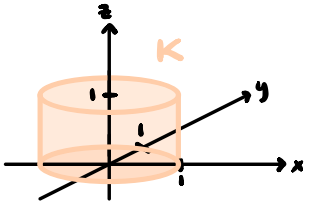
Ex $\iiint_K (x^2+y^2+z^2)\,dx\,dy\,dz$

$K = \{(x,y,z): x^2+y^2 \le 1,\ 0 \le z \le 1\}$



Välj nya variabler så att området blir så enkelt som möjligt.

Integrera över cylinder ⇒ upplagt för cylindriska koordinater:

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$0 \le r \le 1, \quad 0 \le \theta \le 2\pi, \quad 0 \le z \le 1$

$$\det\begin{pmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} = r$$

$$\iiint_K (x^2+y^2+z^2)\,dx\,dy\,dz = \int_0^1\int_0^{2\pi}\int_0^1 (r^2+z^2)\,r\,dr\,d\theta\,dz =$$

$$= \int_0^1 2\pi\left[\frac{r^4}{4} + \frac{z^2r^2}{2}\right]_0^1 dz = 2\pi\int_0^1 \left(\frac{1}{4}+\frac{z^2}{2}\right)dz = 2\pi\left[\frac{z}{4}+\frac{z^3}{6}\right]_0^1 = 2\pi\left(\frac{1}{4}+\frac{1}{6}\right) =$$

$$= 2\pi\,\frac{5}{12} = \frac{5\pi}{6}$$