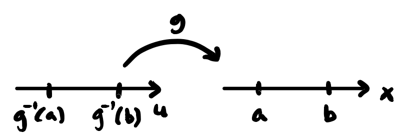
f11 – VARIABELSUBSTITUTION I DUBBELINTEGRALER

Lars Filipsson

Linnéa Gustafsson
linneag2@kth.se

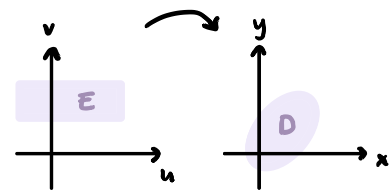
ENVARIABEL

$$\int_a^b f(x)\,dx = \left\{\begin{matrix} x = g(u) \\ dx = g'(u)\,du \end{matrix}\right\} = \int_{g^{-1}(b)}^{g^{-1}(a)} f(g(u))\,g'(u)\,du$$



FLERVARIABEL

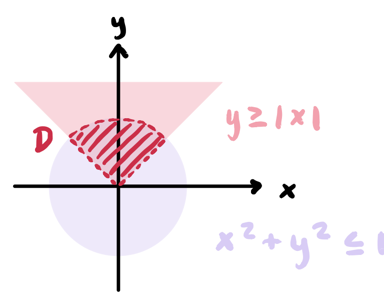
$$\iint_D f(x,y)\,dx\,dy = \iint_E f(x(u,v),\, y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| du\,dv$$



$$\begin{cases} x = x(u,v) \\ y = y(u,v) \end{cases}, \quad \left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \det \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix}$$

Ex

$$D = \{(x,y) : x^2+y^2 \le 1 \text{ och } y \ge |x|\}$$



$$\iint_D (x^2+y^2)\,dx\,dy = \Bigg\{$$

D uttrycks bäst i polära koordinater: $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

$$\left.\det\begin{pmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{pmatrix} = \det\begin{pmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{pmatrix} = r\cos^2\theta + r\sin^2\theta = r \right\} =$$

$$= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\int_0^1 \underbrace{r^2\, r}_{r^3}\, dr \right) d\theta = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left[\frac{r^4}{4} \right]_{r=0}^{r=1} d\theta = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{4}\, d\theta = \left[\frac{\theta}{4} \right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = \frac{3\pi}{16} - \frac{\pi}{16} = \underline{\underline{\frac{\pi}{8}}}$$