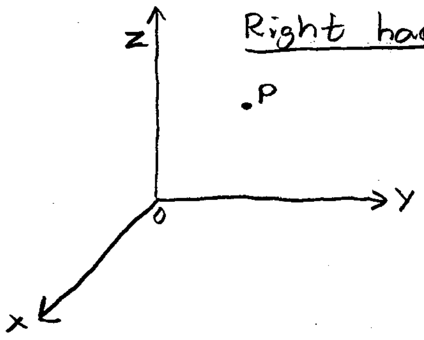
SF1626 2018-08-27 #1

Given a point $P \in \mathbb{R}^3$ ($\mathbb{R}^3$ is the physical space), we need 3 coordinates to locate it. Visually we use a <u>cartesian</u> system.



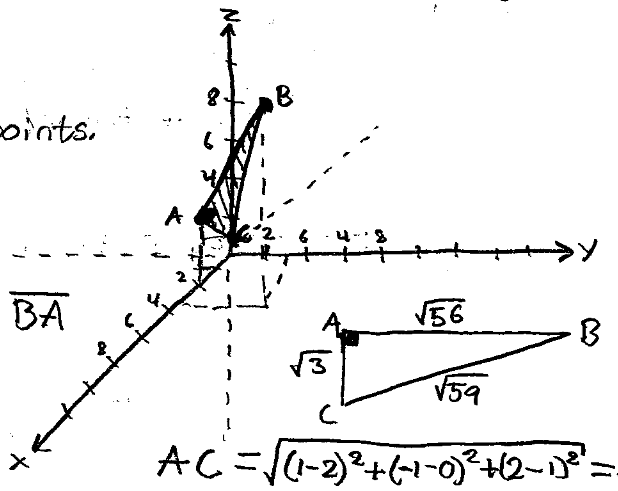
<u>Right hand rule</u>

$P=(x_0, y_0, z_0)$

<u>Exercise</u>: Given three points

$A=(1,-1,2)$
$B=(3,3,8)$
$C=(2,0,1)$

Show that A B C form a triangle with a $90°$ angle.



Rule #1: Distance between two points.

$A=(x_0, y_0, z_0)$
$B=(x_1, y_1, z_1)$

$$\overline{AB} := \sqrt{(x_0-x_1)^2+(y_0-y_1)^2+(z_0-z_1)^2} = \overline{BA}$$

$$AC = \sqrt{(1-2)^2+(-1-0)^2+(2-1)^2} = \sqrt{3}$$

$$AB = \sqrt{(1-3)^2+(-1-3)^2+(2-8)^2} = \sqrt{56}$$

$$CB = \sqrt{(3-2)^2+(3-0)^2+(8-1)^2} = \sqrt{59}$$

1st <u>Remark</u>: A line in $\mathbb{R}^3$ is the intersection of two planes.

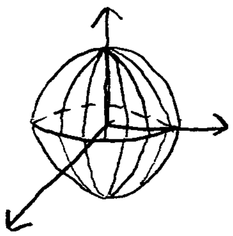
<u>ex. 1</u>: the z-axes $\begin{cases} x=0 \\ y=0 \end{cases}$

The general equation for a plane in $\mathbb{R}^3$ is:

$$\boxed{ax+by+cz=d}$$

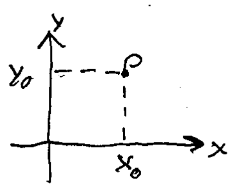
where a, b, c and d are numbers.

$x^2+y^2+z^2=1$ is the set of all points (x,y,z) with distance $=1$ from the origin. (shell)



$x^2+y^2+z^2 \leq 1$ is the set of all points (x,y,z) with distance ≤ 1 from the origin.

$$\boxed{x^2+y^2+z^2=R^2}$$



$P=(x_0,y_0)$ = (d, θ)

cartesian coordinates

polar coordinates

distance from the origin

angle with the x-axes (positive)

In $\mathbb{R}^3$ we have "two kinds of polar coordinates"

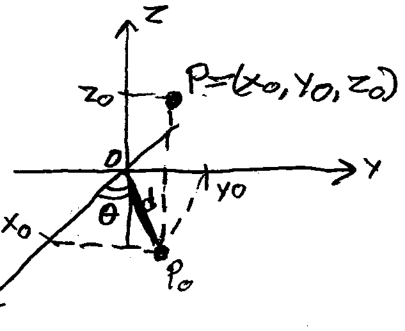
$\boxed{\text{cylindrical}}$ and $\boxed{\text{spherical}}$

cylindrical coordinates:

$(x_0, y_0, z_0) \to (d, \theta, z_0)$

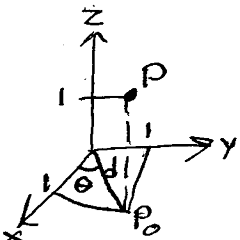
angle between the $\vec{OP}$ and the positive part of the x-axis.

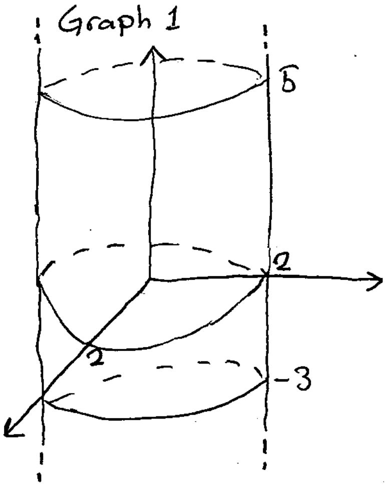
distance of the projection of P into the (x,y)-plane from the origin.



Example: Write $P=(1,1,1)$ into cylindrical coordinates

$d = \sqrt{1^2+1^2} = \sqrt{2} \qquad \theta = \frac{\pi}{4} \quad z = 1 \quad \Rightarrow \quad P = (\sqrt{2}, \frac{\pi}{4}, 1)$

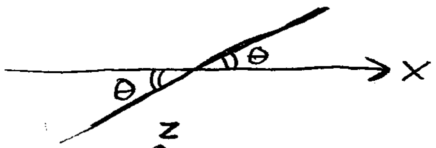


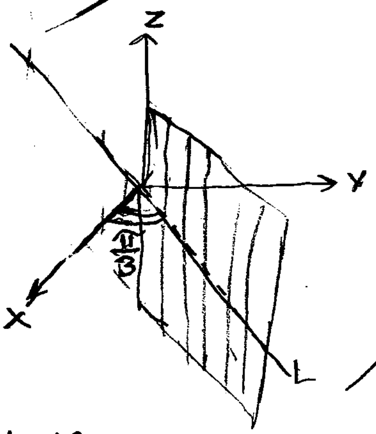


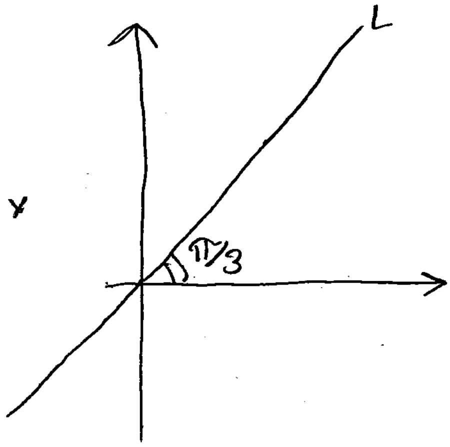
Equation of the cylinder in graph 1 in cylindrical coordinates:
$d = 2$

Finite cylinder: $d=2 \quad -3 \leq z \leq 5$

Draw the surface $\theta = \pi/3$







Question: Write the half plane $\theta = \pi/3$ in cartesian coordinates

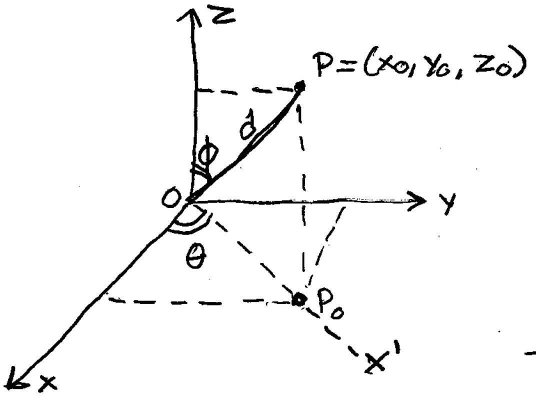
Equation of the line L is:

$(0,0) \quad (\cos(\pi/3), \sin(\pi/3)) = (\frac{1}{2}, \frac{\sqrt{3}}{2})$

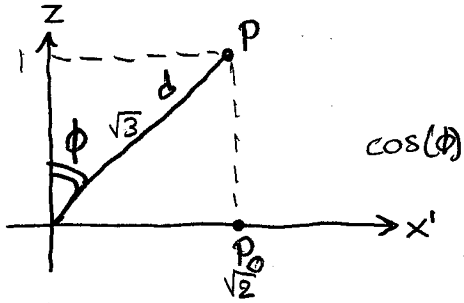
line passing through $(0,0)$ and $(\frac{1}{2}, \frac{\sqrt{3}}{2})$, we get:

$$\frac{x-0}{\frac{1}{2}-0} = \frac{y-0}{\frac{\sqrt{3}}{2}-0} \Rightarrow 2x = \frac{2}{\sqrt{3}} y \Rightarrow \boxed{y = \sqrt{3}x} \quad \oplus \; y \geq 0 \text{ and } x \geq 0$$

Spherical coordinates:



$(x_0, y_0, z_0) \rightarrow (d, \theta, \phi)$



$\cos(\phi) = \frac{1}{\sqrt{3}}$

Write $P = (1,1,1)$ in spherical coordinates

$d = \sqrt{3}$, $\theta = \pi/4$ $\phi = \arccos\left(\frac{1}{\sqrt{3}}\right)$

$$\boxed{\left(\sqrt{3}, \pi/4, \arccos\left(\tfrac{1}{\sqrt{3}}\right)\right)}$$