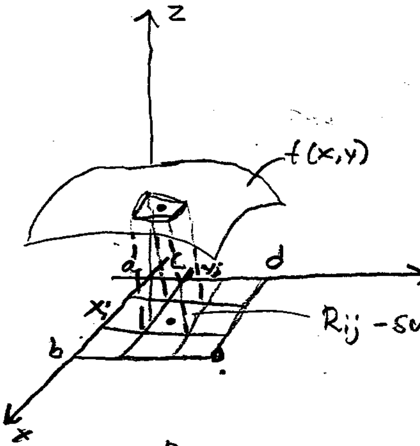
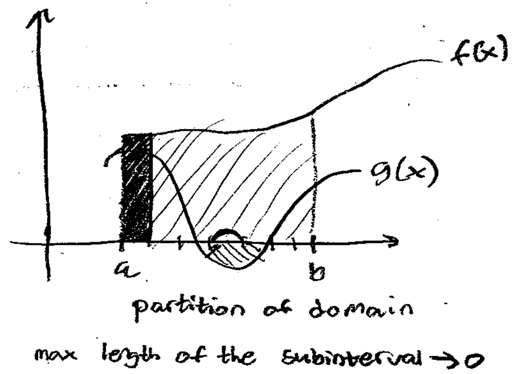
SF1626 2018-09-17 #10

Riemann Sum

Objective: Approximate the volume between the graph of $f(x,y)$ and the x-y plane. Volume can be negative if $f(x,y) \leq 0$.



Defined by:
- A rectangular domain D
- An integrant $f(x,y)$
- A partition P of D



Riemann sum: $$R(f,P) := \sum_{i=1}^{m}\sum_{j=1}^{n} f(x^*_{ij}, y^*_{ij})\,\Delta A_{ij},$$

- (x^*_{ij}, y^*_{ij}) is an arbitrary point in the subrectangle R_{ij} whose lower left corner is grid node (i,j)
- ΔA_{ij} is the aria of that rectangle

Integrability

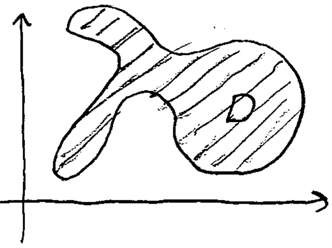
f is integrable over D if there is a number denoted by $I = \iint_D f(x,y)\,dA$ such that for every $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$

$$|R(f,P) - I| < \varepsilon$$

for every partition of D satisfying $\|P\| < \delta(\varepsilon)$, and for all choices of the points (x^*_{ij}, y^*_{ij}) $\quad \|P\| := \max(R_{ij}) := \max_{i,j} \sqrt{(x_i - x_{i-1})^2 + (y_j - y_{j-1})^2}$

You need to know how to define integration of a function in a domain which is not a square.

$$\iint_D f(x,y)\,dxdy$$



$$\tilde{f}(x,y) = \begin{cases} f(x,y), & (x,y) \in D \\ 0, & \text{otherwise} \end{cases}$$

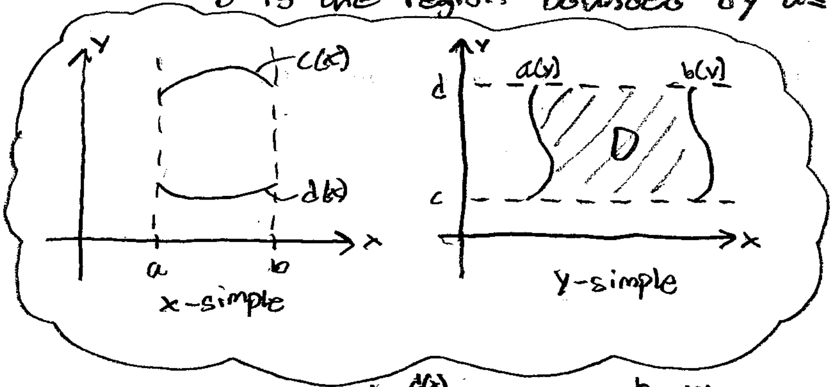
You need to know some important properties of the double integrals.

- Under which conditions on D and f is f integrable?
  - D is closed and bounded and whose boundary consists of finite number of curves of finite length
  - f is continous on D.

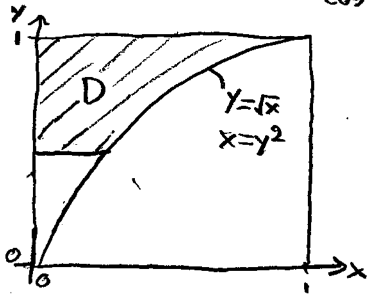
(These are of course only sufficient conditions)
- other important properties are summarized in the book

Iteration of double integrals in Cartesian Coordinates.
Reduce a double integral into a series of one-variable integrals

D is the region bounded by $a \leq x \leq b$ and $c(x) \leq y \leq d(x)$ (D is an x-simple domain)



$$\iint_D f(x,y)\,dA \equiv \int_a^b dx \int_{c(x)}^{d(x)} f(x,y)\,dy = \int_a^b \left(\int_{c(x)}^{d(x)} f(x,y)\,dy\right) dx$$



$$\iint_D e^{y^3}\,dA = \int_0^1 \int_{\sqrt{x}}^1 e^{y^3}\,dy\,dx \qquad (A)$$

$$= \int_0^1 \int_0^y e^{y^3}\,dx\,dy \qquad (B)$$

$$= \int_0^1 e^{y^3} y^2\,dy \qquad z = y^3,\ dz = 3y^2 dy$$

$$= \int_0^1 e^z \tfrac{1}{3}\,dz$$