F12 Trippelintegral

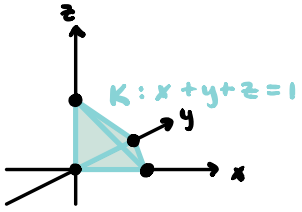
LARS FILIPSSON

Linnéa Gustafsson
linneag2@kth.se

Ex

$$\iiint_K y\,dV, \quad K: \{(x,y,z): x+y+z=1,\ x\geq 0,\ y\geq 0,\ z\geq 0\}$$

Tetraeder med hörn i $(0,0,0)$, $(1,0,0)$, $(0,1,0)$, $(0,0,1)$



$$\iiint_K y\,dV = \int_0^1 \int_0^{1-x} \underbrace{\int_0^{1-x-y} y\,dz}_{y(1-x-y)}\,dy\,dx = \int_0^1 \underbrace{\int_0^{1-x} y(1-x-y)\,dy}_{}\,dx =$$

$$= \int_0^1 \left[\frac{y^2}{2}(1-x) - \frac{y^3}{3}\right]_0^{1-x} dx = \int_0^1 \underbrace{\left(\frac{(1-x)^3}{2} - \frac{(1-x)^3}{3}\right)}_{\frac{(1-x)^3}{6}} dx =$$

$$= \left[-\frac{(1-x)^4}{24}\right]_0^1 = \frac{1}{24}$$