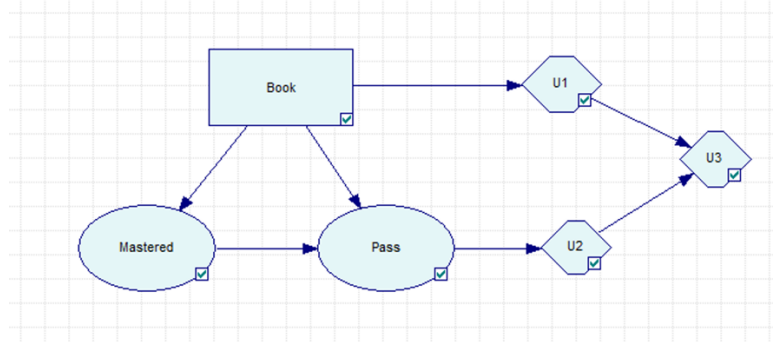


TDT4171 – Assignment 3

Task 1

Problem a)



$P(M | B)$

B	M	Value
true	true	0.9
true	false	0.1
false	true	0.65
false	false	0.35

$P(P | B, M)$

B	M	P	Value
true	true	true	0.9
true	true	false	0.1
true	false	true	0.4
true	false	false	0.6
false	true	true	0.7
false	true	false	0.3
false	false	true	0.2
false	false	false	0.8

$U1(P)$

B	Value
true	-150
false	0

$U2(M)$

P	Value
true	2100
false	0

We have that

$$U(x_1, \dots, x_n) = \sum_{i=1}^n U_i(x_i)$$

$$EU(A_j | e) = \sum_i P(\text{Result}_i(A_j) | do(A_j), e) \cdot U(\text{Result}_i(A_j))$$

$$EU(a) = \sum_x [P(x, a) \cdot \sum_i^n U(x_i, a)]$$

$$P(P, M, B) = P(P, M | B) \cdot P(B)$$

$$P(P, M, B) = P(P | B, M) \cdot P(B, M) = P(P | B, M) \cdot P(M | B) \cdot P(B)$$

$$\rightarrow P(P, M | B) \cdot P(B) = P(P | B, M) \cdot P(M | B) \cdot P(B)$$

$$\rightarrow P(P, M | B) = P(P | B, M) \cdot P(M | B) \quad (1)$$

$$\rightarrow P(\bar{P}, M | B) = P(\bar{P} | B, M) \cdot P(M | B) \quad (2)$$

$$\rightarrow P(P, \bar{M} | B) = P(P | B, \bar{M}) \cdot P(\bar{M} | B) \quad (3)$$

$$\rightarrow P(\bar{P}, \bar{M} | B) = P(\bar{P} | B, \bar{M}) \cdot P(\bar{M} | B) \quad (4)$$

$$EU(B) = P(P, M | B) \cdot U(P, M, B) + P(\bar{P}, M | B) \cdot U(\bar{P}, M, B)$$

$$+ (P, \bar{M} | B) \cdot U(P, \bar{M}, B) + (\bar{P}, \bar{M} | B) \cdot U(\bar{P}, \bar{M}, B)$$

If we insert (1), (2), (3) and (4), we get that

$$EU(B) = P(P | B, M) \cdot P(M | B) \cdot U(P, B) + P(\bar{P} | B, M) \cdot P(M | B) \cdot U(\bar{P}, B)$$

$$+ P(P | B, \bar{M}) \cdot P(\bar{M} | B) \cdot U(P, B) + P(\bar{P} | B, \bar{M}) \cdot P(\bar{M} | B) \cdot U(\bar{P}, B)$$

$$= 0.9 \cdot 0.9 \cdot 1950 + 0.1 \cdot 0.9 \cdot (-150) + 0.4 \cdot 0.1 \cdot 1950 + 0.6 \cdot 0.1 \cdot (-150)$$

$$= 1635$$

$$EU(\bar{B}) = P(P | \bar{B}, M) \cdot P(M | \bar{B}) \cdot U(P, \bar{B}) + P(\bar{P} | \bar{B}, M) \cdot P(M | \bar{B}) \cdot U(\bar{P}, \bar{B})$$

$$+ P(P | \bar{B}, \bar{M}) \cdot P(\bar{M} | \bar{B}) \cdot U(P, \bar{B}) + P(\bar{P} | \bar{B}, \bar{M}) \cdot P(\bar{M} | \bar{B}) \cdot U(\bar{P}, \bar{B})$$

$$= 0.7 \cdot 0.65 \cdot 2100 + 0.3 \cdot 0.65 \cdot 0 + 0.2 \cdot 0.35 \cdot 2100 + 0.8 \cdot 0.35 \cdot 0$$

$$= 1102.5$$

$$\underline{EU(B) > EU(\bar{B}) \rightarrow \text{Geir should buy the book}}$$

Task 2 – Decision making in a football game

Problem description

It is the last football game of the season. Gunnar is battling with another team-mate about the top-scorer title, and he is one goal ahead before the game. Additionally, Gunnar's team is in a position to win the league if they win the last game. But a win in the last game is not enough, and they are also dependent on some results from other games to go their way. In the early stage of the game, Gunnar comes in a position where he has to make a decision about whether to pass the ball to the team-mate who can rob him of the top-scorer title, or to shoot and try to score by himself. Gunnar wants the top-scorer title because he knows that clubs playing at a higher level is watching him and might want to give him an offer to join them next season.

As the game is approaching half-time, Gunnar gets a feeling of an old back injury. He then has to make a decision on whether to inform the coach or play on without telling him. He knows that it is more likely that the coach will substitute him if he tells the coach about the injury, and it is a risk that he might lose the top-scorer title race against the team-mate. Additionally, Gunnar's doctor has told him to be careful with his back. He doesn't make enough money just playing football, so he works as a carpenter for an extra job. In the past, the back injury has made him unable to work, so playing on might be a big risk.

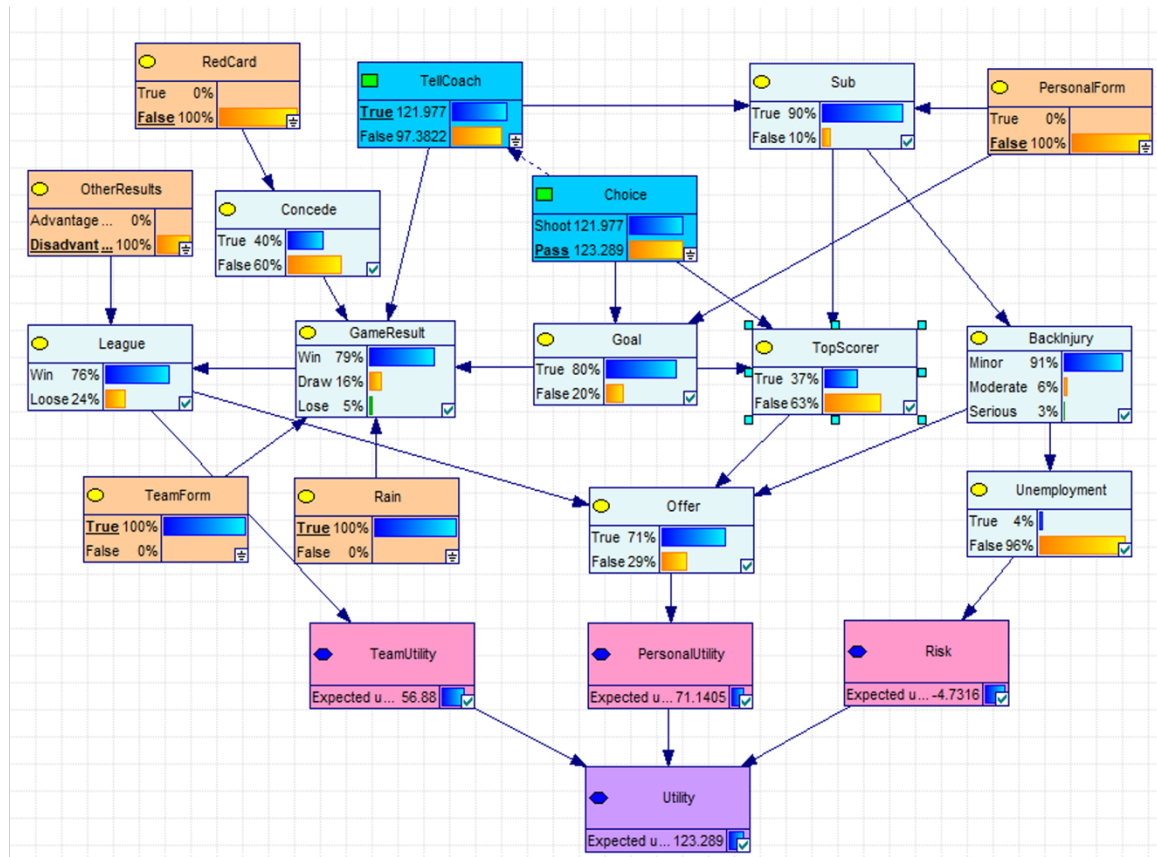
Modeling

The decision problem can be modeled with two decision nodes. One for the decision in the goal-scoring opportunity, **Choice**, which can take the values; Choice=shoot and Choice=pass. The other decision node **TellCoach** indicates whether to tell the coach about the injury or not.

The decision network is modeled with the following chance nodes:

- **Goal**, indicating whether the team scores a goal or not on the goal-scoring opportunity.
- **TopScorer**, indicating whether Gunnar becomes the top-scorer or not.
- **GameResult**, indicating whether the team win, draw or lose.
- **League**, indicating whether the team wins the league or not.
- **Sub**, indicating whether Gunnar gets substituted or not.
- **Offer**, indicating whether Gunnar gets an offer from another club or not.
- **Concede**, indicating whether Gunnar's team concede a goal or not.
- **RedCard**, indicating whether the team gets a red card or not.
- **PersonalForm**, indicating whether Gunnar has a good or bad day.
- **TeamForm**, indicating whether the team has a good or bad day.
- **Rain**, indicating whether it's raining or not on the game-day or not.
- **BackInjury**, indicating whether Gunnar gets a back-injury from playing the game or not.
- **Unemployment**, indicating whether the back-injury makes him unable to work or not

- **OtherResults**, indicating whether the results in the other games go in their favor of Gunnar's team or not.



Goal is dependent on **Choice** and **PersonalForm**. The team is more likely to score if Gunnar passes the ball to the team-mate who is in a better position to score. Furthermore, if Gunnar has a good day, it is more likely that his shot/pass ends up in a goal.

Choice	Shoot		Pass	
	True	False	True	False
PersonalForm				
True	0.6	0.4	0.9	0.8
False	0.4	0.6	0.1	0.2

Figure 1: CPT for Goal

TopScorer is dependent on **Choice**, **Sub**, and **Goal**. I.e., it is a good chance that Gunnar becomes the top-scorer if he chooses to shoot, and it results in a Goal. It is less likely that he becomes the top-scorer if he chooses to pass, and it ends up in a goal. If he gets substituted the team-mate is in a better position to regain the leap.

Sub	True				False			
	True		False		True		False	
Goal	True		False		True		False	
Choice	Shoot	Pass	Shoot	Pass	Shoot	Pass	Shoot	Pass
True	0.6	0.3	0.4	0.5	0.9	0.6	0.6	0.7
False	0.4	0.7	0.6	0.5	0.1	0.4	0.4	0.3

Figure 2: CPT for TopScorer

Sub is dependent on **TellCoach** and **PersonalForm**. Gunnar is more likely to get substituted if he tells the coach about the injury, or if he isn't inform on the gameday.

TellCoach		True		False	
PersonalForm		True	False	True	False
►	True	0.7	0.9	0.2	0.6
	False	0.3	0.1	0.8	0.4

Figure 3: CPT for Sub

GameResult is dependent on **TellCoach**, **Goal**, **TeamForm**, **Rain**, **Concede**. Gunnar assumes it to be more likely for the team to win/draw if he tells the coach about the injury because the coach is then more likely to make good decisions. Gunnar's team is from a small town in Western Norway, where rain is common when they play home games. He therefore assumes a good result is more likely if it rains.

Concede is dependent on **RedCard**, as Gunner knows that his team is far more likely to concede if his team gets a red card.

RedCard		True	False
►	True	0.9	0.4
	False	0.1	0.6

Figure 4: CPT for Concede

League is dependent on **OtherResults** and **GameResult**, and assumes the following probabilities based on the results in the other game.

GameResult		Win		Draw		Lose	
OtherResults		Advantage...	Disadvanta...	Advantage...	Disadvanta...	Advantage...	Disadvanta...
►	Win	1	0.9	0.7	0.3	0.2	0
	Loose	0	0.1	0.3	0.7	0.8	1

Figure 5: CPT for League

BackInjury is dependent on **Sub**, as Gunnar is more likely to incur serios damage if he plays on.

Sub		True	False
►	Minor	0.99	0.2
	Moderate	0.009	0.5
	Serious	0.001	0.3

Figure 6: CPT for BackInjury

Offer is dependent on **League**, **TopScorer** and **BackInjury**. Gunnar knows that it is a good chance that he gets an offer from a club at a higher level if the team wins the league and he becomes the top-scorer. But if the back injury is serious enough, they might not want to sign him.

TopScorer	True						False					
BackInjury	Minor		Moderate		Serious		Minor		Moderate		Serious	
League	Win	Loose	Win	Loose	Win	Loose	Win	Loose	Win	Loose	Win	Loose
True	0.95	0.9	0.65	0.6	0.3	0.25	0.65	0.6	0.35	0.3	0.05	0
False	0.05	0.1	0.35	0.4	0.7	0.75	0.35	0.4	0.65	0.7	0.95	1

Figure 7: CPT for Offer

Unemployment is dependent on **BackInjury**, as a serious back injury may make him unable to work as a carpenter. He assumes that he will still be able to work with a high probability if the injury is moderate.

BackInjury	Minor	Moderate	Serious
True	0	0.2	0.9
False	1	0.8	0.1

Figure 8: CPT for Unemployment

Gunnar's team won the league last year, and because Gunnar wants to play for a better club at a higher level, he sees it as more important to get an offer than for the team to win the league. Additionally, he knows that a serious back injury can affect both his job as a carpenter and his future football career. The problem is modeled with an additive utility node **Utility** which consists of three parts:

- $U_1(\text{Offer} = \text{true}) = 100$ and $U_1(\text{Offer} = \text{false}) = 0$
- $U_2(\text{League} = \text{true}) = 75$ and $U_2(\text{League} = \text{false}) = 0$
- $U_3(\text{Unemployment} = \text{true}) = -120$ and $U_3(\text{Unemployment} = \text{false}) = 0$

The utility function Gunnar wants to maximize is then given $U = U_1 + U_2 + U_3$