TDT4171 - Exercise 1

Problem 1)

X: number of siblings

P(X=0) = 0.15
P(X=1) = 0.49
P(X=2) = 0.27
P(X=3) = 0.06
P(X=4) = 0.02
P(X>=5) = 0.01

a)
$A: \ At\ most\ two\ siblings$

$P(A) = P(X <= 2) = P(X = 0) + P(X = 1) + P(X = 2) = 0.15 + 0.49 + 0.27 = 0.91$

b)
$A: \ More\ than\ two\ sibling$
$B: \ At\ least\ one\ sibling$

$P(A) = P(X > 2) = 1 - P(X <= 2) = 1 - 0.15 - 0.49 - 0.27 = 0.09$
$P(B): \ P(X >= 1) = 1 - P(X = 0) = 1 - 0.15 = 0.85$

$P(A|B) = P(A, B)/P(B) = P(B \mid A) * P(A) / P(B) = 1 * 0.09 / 0.85 = 0.106$

c)
Combined three siblings of three non-siblings
Possible distribution of siblings:

$A: \ P(X = 1 \wedge Y = 1 \wedge Z = 1) = 0.49 * 0.49 * 0.49 = 0.1176$

$B: \ P(X = 2 \wedge Y = 1 \wedge Z = 0) = 0.27 * 0.49 * 0.15 = 0.0198$
$C: \ P(X = 2 \wedge Y = 0 \wedge Z = 1) = 0.27 * 0.15 * 0.49 = 0.0198$
$D: \ P(X = 3 \wedge Y = 0 \wedge Z = 0) = 0.06 * 0.15 * 0.15 = 0.00135$

$E: \ P(X = 1 \wedge Y = 2 \wedge Z = 0) = 0.49 * 0.27 * 0.15 = 0.0198$
$F: \ P(X = 0 \wedge Y = 2 \wedge Z = 1) = 0.15 * 0.27 * 0.49 = 0.0198$
$G: \ P(X = 0 \wedge Y = 3 \wedge Z = 0) = 0.15 * 0.06 * 0.15 = 0.00135$

$H: \ P(X = 1 \wedge Y = 0 \wedge Z = 2) = 0.49 * 0.15 * 0.27 = 0.0198$
$I: \ P(X = 0 \wedge Y = 1 \wedge Z = 2) = 0.15 * 0.49 * 0.27 = 0.0198$

$J: P(X = 0 \wedge Y = 0 \wedge Z = 3) = 0.15 * 0.15 * 0.06 = 0.00135$

$P(X + Y + Z = 3) = P(A \cup B \cup C \cup D \cup E \cup F \cup G \cup H \cup I \cup J) =$

$= 0.1176 + 0.0198 + 0.0198 + 0.00135 + 0.0198 + 0.0198 + 0.00135 + 0.0198 + 0.0198 + 0.00135$

$= 0.1176 + 0.0198 * 6 + 0.00135 * 3 = 0.240$

d)

$A: Emma\ has\ no\ siblings$

$B: Jacob\ and\ Emma\ has\ three\ siblings\ together$

$P(A) = 0.15$

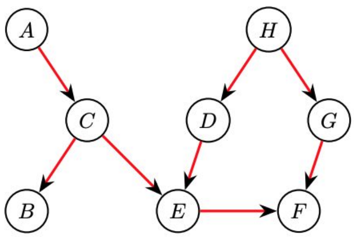
$P(B) = P(X = 3 \wedge Y = 0) + P(X = 2 \wedge Y = 1) + P(X = 1 \wedge Y = 2) + P(X = 0 \wedge Y = 3)$

$= 0.06 * 0.15 + 0.27 * 0.49 + 0.49 * 0.27 + 0.15 * 0.06 = 0.2826$

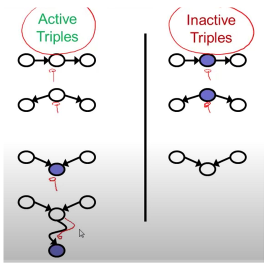
$P(A, B) = P(X + Y = 3 \wedge Y = 0) = P(X = 3 \wedge Y = 0) = 0.06 * 0.15 = 0.009$

$P(A \mid B) = P(A, B) / P(B) = 0.009/0.2826 = 0.0318$

Problem 2)



[1]



a)

$p(A, B, C, D, E, F, G, H) = [1, 2, 2, 2, 4, 4, 2, 1]$

$sum(p) = 18$

$\Rightarrow true$

b)

$G \perp\!\!\!\perp A$

$Triple\ GFE\ and\ DEC\ are\ inactive \Rightarrow the\ two\ possible\ paths\ are\ inactive$

$\Rightarrow No\ active\ paths\ between\ G\ and\ A$

$\Rightarrow True$

c)

$E \perp\!\!\!\perp H \mid \{D, G\}$

$Triple\ EDH\ and\ FGH\ are\ inactive \Rightarrow the\ two\ possible\ paths\ are\ inactive$

[1] https://www.youtube.com/watch?v=yDs_q6jKHb0&ab_channel=PieterAbbeel

$\Rightarrow No\ active\ paths\ between\ E\ and\ H$

$\Rightarrow True$

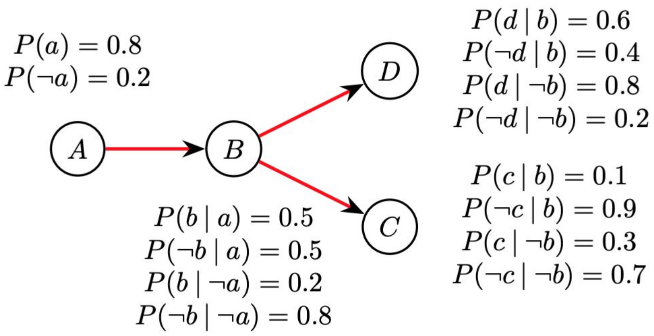
d)

$E \perp\!\!\!\perp H \mid \{C, D, F\}$

$Triple\ EFG\ and\ FGH\ are\ active \Rightarrow there\ exists\ an\ active\ path\ from\ E\ to\ H\ -\ EFGH)$

$\Rightarrow False$

Problem 3)



a)

$$P(b) = P(b \mid a) * P(a) + P(b \mid \neg a) * P(\neg a) = 0.5 * 0.8 + 0.2 * 0.2 = 0.44$$

b)

$$P(d) = P(d \mid b) * P(b) + P(d \mid \neg b) * P(\neg b) = 0.6 * 0.44 + 0.8 * (1 - 0.44) = 0.712$$

c)

$$P(c \mid \neg d) = P(c, \neg d) / P(\neg d) = \alpha * P(c, \neg d)$$

$$= \alpha * \sum P(b, c, \neg d) = \alpha\, [\, P(b) * P(c \mid b) * P(\neg d \mid b) + P(\neg b) * P(c \mid \neg b) * P(\neg d \mid \neg b)\,]$$

$$= (1/0.288) * [0.44 * 0.1 * 0.4 + 0.56 * 0.3 * 0.2] = 0.1778$$

d)

$$P(a \mid \neg c, d) = P(a, \neg c, d) / P(\neg c, d) = \alpha * P(a, \neg c, d)$$

$$= \alpha * \sum P(a, b, \neg c, d)$$

$$= \alpha\, [\, P(a) * P(b \mid a) * P(\neg c \mid b) * P(d \mid b) + P(a) * P(\neg b \mid a) * P(\neg c \mid \neg b) * P(d \mid \neg b)]$$

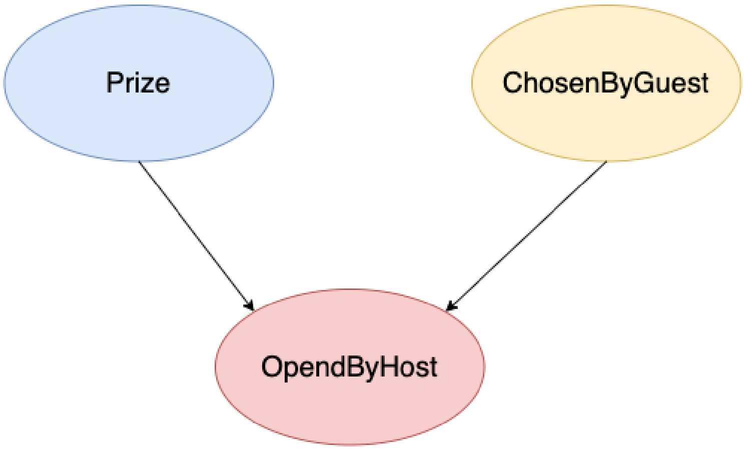
$$P(\neg c, d) = \sum P(b, \neg c, d) = P(b) * P(\neg c \mid b) * P(d \mid b) + P(\neg b) * P(\neg c \mid \neg b) * P(d \mid \neg b)$$

$$= 0.44 * 0.9 * 0.6 + 0.56 * 0.7 * 0.8 = 0.551$$

$$\Rightarrow P(a \mid \neg c,\ d)\ = (1/0.551)\ * [0.8 * 0.5 * 0.9 * 0.6\ +\ 0.8 * 0.5 * 0.7 * 0.8]\ = 0.7983$$

Problem 4)

	Prize
D1	0.333
D2	0.333
D3	0.333



	CBG
D1	0.333
D2	0.333
D3	0.333

	P=D1, CBG=D1	P=D1, CBG=D2	P=D1, CBG=D3	P=D2, CBG=D1	P=D2, CBG=D2	P=D2, CBG=D3	P=D3, CBG=D1	P=D3, CBG=D2	P=D3, CBG=D3
D1	0	0	0	0	0.5	1	0	1	0.5
D2	0.5	0	1	0	0	0	1	0	0.5
D3	0.5	1	0	1	0.5	0	0	0	0

$$P(Prize \mid ChosenByGuest\ =\ 1,\ OpendByHost\ =\ 3\ )$$

	CBG = 1, OBH = 3
D1	0.3333
D2	0.6667
D3	0

It would be to my advantage to switch my choice, as the probability of the prize being behind door 2 is greater than the probability of the prize being behind door 1, given the evidence.