Discrete Mathematics - II

Module I

Graphs and Graph models
Graph terminology and special types of graph representing graphs and graph isomorphism. Connectivity, euler and hamilton path

Module II

Introduction to trees, application of trees, tree traversal and spanning trees

Module III

Boolean Algebra, Boolean Functions, Representing Boolean Functions and Logic Gates

Module IV

matrices definitions and examples of symmetric, skew symmetric, Conjugate, Hermitial, Skew Hermitial matrices, Rank of matrix, determination of rank by row canonical form and normal form, linear equations, solution of

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non - homogenous equation, using augmented matrix and by cramers rule, homogenous equations, characteristic equation, characteristic rules and characteristic vectors of matrix, Cayley hamilton theorem and applications

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MATRICES

A matrix is a rectangular presentation of numbers arranged systematically in rows and columns. Matrix is therefore a rectangular array of numbers.

Leading diagonal

Elements a_{11} a_{22} a_{33} ... form the leading diagonal. Consider the matrix $A = a_{11} = 3$ $a_{22} = 6$, $a_{33} = 8$, the leading diagonal numbers are 3, 6, 8

Diagonal matrix

A square matrix in which all the elements except those in leading are zero is called a diagonal matrix.

eg:- $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{pmatrix}$

Scalar matrix

A diagonal matrix in which all the diagonal elements are equal is called a scalar matrix.

$$A = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Unit Matrix / Identity Matrix

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A diagonal matrix in which each of the diagonal element is unity is said to be a unit matrix. This is also known as an identity matrix and is denoted by I

Triangular Matrix

If every element above or below the leading diagonal is zero the matrix is called a triangular matrix.

Triangular matrix may be upper triangular or lower triangular. A square matrix a_{ij} is upper triangler if $a_{ij} = 0$ when $i > j$, $a_{ij} \neq 0$ when $i \leq j$. Similarly a square matrix a_{ij} is lower triangular if $a_{ij} = 0$ when $i < j$, $a_{ij} \neq 0$ when $i \geq j$

a_{12}

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 8 & 5 \\ 0 & 0 & 2 \end{bmatrix} \qquad \begin{bmatrix} 5 & 0 & 0 \\ 2 & 7 & 0 \\ 1 & 5 & 3 \end{bmatrix}$$

upper   lower

$a_{31} = 0$

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Transpose of a matrix

A matrix obtained from any given matrix A by interchanging its rows and columns is called its transpose and is denoted by A' or A^t.

If $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 2 & 5 \end{bmatrix}$

$A' = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 2 \\ 4 & 1 & 5 \end{bmatrix}$

Symmetric Matrix

Any square matrix A is said to be symmetric if it is equal to its transpose

ie A is symmetric if $A = A'$

$A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ $\quad A' = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$

Here $A = A'$

$\therefore$ A is symmetric matrix

Skew Symmetric matrix

Any square matrix A is said to be skew symmetric if it is equal to its negative transpose. ie if $A = -A^t$ then A is skew symmetric

eg:-

$$A = \begin{bmatrix} 0 & 3 & 5 \\ -3 & 0 & -2 \\ -5 & 2 & 0 \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & -3 & -5 \\ 3 & 0 & 2 \\ 5 & -2 & 0 \end{bmatrix}.$$

$$-A' = \begin{bmatrix} 0 & 3 & 5 \\ -3 & 0 & -2 \\ -5 & 2 & 0 \end{bmatrix}$$

$A = -A'$ OR $-A = A'$

$\therefore$ A is skew symmetric matrix

Idempotent Matrix

A square matrix A is said to be idempotent if A is equal to A^2.

eg:- ~~Show that~~

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eg:- Show that $\begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 4 \end{bmatrix}$ is symmetric

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

Since $A = A'$

A is a symmetric matrix

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Ques If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$

Find $6A - 3B$

$$6A = 6\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 18 \\ 6 & 12 \end{bmatrix}$$

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$$3B = \begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix}$$

$$6A - 3B = \begin{bmatrix} 12 & 18 \\ 6 & 12 \end{bmatrix} - \begin{bmatrix} 12 & 6 \\ 3 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 12 \\ 3 & 3 \end{bmatrix}$$

Conjugate matrix

Let a and b be two real numbers and $i = \sqrt{-1}$, then $z = a + bi$ is called a complex number.

The complex numbers $a + bi$ and $a - bi$ are called conjugates, each being the conjugate of the other.

If $z = a + bi$ its conjugate is denoted by

$$\bar{z} = \overline{a + bi}$$

When A is a matrix having complex numbers as elements, the matrix obtained from A by replacing each elements by its conjugate is called the conjugate of A and

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is denoted by $\bar{A}$.

If $A = \begin{bmatrix} 1+2i & i \\ 3 & 2-3i \end{bmatrix}$

then $\bar{A} = \begin{bmatrix} 1-2i & -i \\ 3 & 2+3i \end{bmatrix}$

Properties

1. $\overline{(\bar{A})} = A$

2. $\overline{(K \cdot A)} = \bar{K} \cdot \bar{A}$

Hermitian Matrix

A square matrix A represented by $[a_{ij}]$ such that $A = \overline{A^T}$ is called Hermitian.

eg:- $A = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 1+i & 2 \\ 1-i & 3 & -i \\ 2 & i & 0 \end{bmatrix}$

$\overline{A^T} = \begin{bmatrix} 1 & 1-i & 2 \\ 1+i & 3 & i \\ 2 & -i & 0 \end{bmatrix}$ $\therefore$ A is a hermitean matrix

Skew Hermitian Matrix

A square matrix is represented by $A=(a_{ij})$ such that $\overline{A'} = -A$ is called a skew hermitian matrix.

eg:- $A = \begin{bmatrix} i & 1-i & 2 \\ -1-i & 3i & i \\ -2 & i & 0 \end{bmatrix}$

$A^T = \begin{bmatrix} i & -1-i & -2 \\ 1-i & 3i & i \\ 2 & i & 0 \end{bmatrix}$

$\overline{A^T} = \begin{bmatrix} -i & -1+i & -2 \\ 1+i & -3i & -i \\ 2 & -i & 0 \end{bmatrix}$

$-\overline{A^T} = \begin{bmatrix} i & +1-i & 2 \\ -1-i & 3i & i \\ -2 & i & 0 \end{bmatrix}$

Since $A = -\overline{A^T}$; A is Skew hermitian

The Rank of a matrix (ρ)

A non-zero matrix A is said to have ranked r if atleast one of its r-square minors is different from zero. and all minors of order more than r is zero. Therefore the rank of a matrix is said to be true if all miners of order 3 or more are zero. and atleast one minor of order 2 is not zero.

* A zero matrix is said to have rank zero.

* Minimum rank of non-zero matrix is one.

* If the given matrix is rectangular matrix of order $m \times n$ than the ~~rank~~ $\rho(A) \leq \min(m, n)$

Ques Find the rank of $\begin{bmatrix} 5 & 2 & 1 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \end{bmatrix}$

$$|A| = 5 \begin{vmatrix} 1 & 3 \\ 1 & 0 \end{vmatrix} - 2 \begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix} + 1 \begin{vmatrix} 0 & 1 \\ 2 & 1 \end{vmatrix}$$

$$= 5(0-3) - 2(0-6) + 1(0-2)$$
$$= -15 + 12 - 2$$
$$= -15 + 10$$
$$|A| = -5$$

Since this is non-zero, the $\rho(A) = 3$

Ques Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 6 & 9 \\ 2 & 4 & 6 \end{bmatrix}$

$$\bar{A} = 1 \begin{vmatrix} 6 & 9 \\ 4 & 6 \end{vmatrix} - 2 \begin{vmatrix} 3 & 9 \\ 2 & 6 \end{vmatrix} + 3 \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix}$$

$$= 1(36-36) - 2(18-18) + 3(12-12)$$
$$= 1(0) - 2(0) + 3(0)$$
$$\bar{A} = \underline{\underline{0}}$$

$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} = 1 \times 6 - 2 \times 3$
$= 6 - 6 = \underline{\underline{0}}$

$\begin{pmatrix} 2 & 3 \\ 6 & 9 \end{pmatrix} = 2 \times 9 - 3 \times 6$
$= 18 - 18$
$= \underline{\underline{0}}$

$\begin{pmatrix} 3 & 6 \\ 2 & 4 \end{pmatrix} = 3 \times 4 - 6 \times 2$
$= \underline{\underline{0}}$

$\begin{pmatrix} 6 & 9 \\ 4 & 6 \end{pmatrix} = 6 \times 6 - 9 \times 4$
$= 36 - 36$
$= \underline{\underline{0}}$

$\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} = 1 \times 4 - 2 \times 2 = 4 - 4$
$= \underline{\underline{0}}$

$\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} = 2 \times 6 - 3 \times 4$
$= 12 - 12 = \underline{\underline{0}}$

The rank is 1.

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(3×4)

Ques Find the rank of $\begin{bmatrix} 1 & 2 & 0 & 5 \\ 3 & 1 & 2 & 2 \\ 2 & 4 & 0 & 10 \end{bmatrix}$

$$\begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 2 \\ 2 & 4 & 0 \end{pmatrix} = 1\begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} + 0\begin{vmatrix} 3 & 1 \\ 2 & 4 \end{vmatrix}$$

$$= 1(0-8) - 2(0-4) + 0(12-2)$$

$$= \cancel{-8+8+10} \qquad = -8+8$$

$$= \cancel{-16+10 = -6} \qquad = \underline{\underline{0}}$$

2×2

$$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix} = 1\times1 - 2\times3$$

$$= 1-6$$

$$= \underline{\underline{-5}}$$

~~It~~ Ans The rank is 2.

Elementary Transformation

I The~~se~~ ~~are~~ following operations are called elementary transformation on a matrix which donot change its order ~~of~~ or its rank.

(a) The interchange of i and j rows is denoted by R_{ij}

(b) The interchange of ith and jth columns denoted by C_{ij}

II (a) The multiplication of every element of the ith row by a non-zero scalar K denoted by ~~KRij~~ KR_i

(b) The multiplication of every element of the ith column by a non-zero scalar K denoted by KC_i

III (a) The addition of the elements of the ith row of K scalar times, the corresponding elements of the jth row is denoted by $R_{ij}(K)$

(b) The addition of the elements of the ith column of K scalar times, the corresponding elements of the jth row is denoted by $C_{ij}(K)$

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V The transformation R are called elementary row transformation and C are called elementary column transformation.

(only row transformation)

Row Canonical form / Row Echelon Form

A matrix A is said to be in Row Echelon form if its satisfies :-

(a) every zero row matrix occurs below a non-zero row

(b) the first non-zero number from the left of a non-zero row is 1. This is called

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leading 1.

(c) for non-zero row the leading one appears to the right and below any leading 1 in the preceding rows.

* The rank of a matrix in the echelon form is the number of non-zero rows of ~~that~~ matrix.

eg:- $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ $\rho(A) = 2$

eg:- $B = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\rho(B) = 3$

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~~Ques~~ Find the rank of the matrix by using row echelon form

$A = \begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

$R_1 \to R_{13}$

$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 5 & 3 & 14 & 4 \end{bmatrix}$

$-4+-4$

$R_3 \to R_3 - 5R_1$

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 8 & 4 & 4 \end{bmatrix}$$

$R_3 \to R_3 - 8R_2$

$4-16$

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & -12 & -4 \end{bmatrix}$$

$R_3 \to R_3/-12$

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1/3 \end{bmatrix}$$

$\rho(A)$ = no. of non-zero rows = 3

Ques

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$4-4$

$3-6$

$2-$

$R_2 \to R_2 - 2R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

2 - 6

1 - 9

3 -

8 - 12

7 - 18

$R_4 \to R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_2 \to R_{23}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \to R_2 + R_4$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & -19 & 8 \end{bmatrix}$$

$R_2 \to R_2/-4$

$R_2 \to R_2/-4$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

~~$R_3 \to R_3 + 4R$.~~

$R_4 \to R_4 + 4R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & +3 & +2 \end{bmatrix}$$

$-4+4$

$-11+8$

$R_4 \to R_4 + R_3$

$-3+3$ 0

$-3+3$ $2-2$

$5 + -\frac{3}{4} \times 4$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$

Ques

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 2 & 6 \\ 3 & 3 & 7 \\ 2 & 4 & 5 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 4 & 2 & 6 \\ 3 & 3 & 7 \\ 2 & 4 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 4R_1$

2 - 4

6 - 8

6 - 12

6 - 8

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -2 \\ 3 & 3 & 7 \\ 2 & 4 & 5 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_1$

3 - 3

7 - 6

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 1 \\ 2 & 4 & 5 \end{bmatrix}$$

$R_4 \rightarrow R_4 - 2R_1$

4 - 2

5 - 4

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

$R_2 \rightarrow R_2 / -2$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$$

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$R_4 \to R_4 - R_2$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_3$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 3$

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$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & 6 & 8 & 4 \\ 5 & 7 & 3 & 2 \\ 3 & 9 & 12 & 6 \end{bmatrix}$$

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$R_2 \to R_2 - 2R_1$

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 5 & 7 & 3 & 2 \\ 3 & 9 & 12 & 6 \end{bmatrix}$$

6 - 6

8 - 8

4 - 2

$R_4 \to R_{42}$

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 3 & 9 & 12 & 6 \\ 5 & 7 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 5 & 7 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3 - 20

2 - 10 17

$R_3 \to R_3 - 5R_1$

7 - 15

0

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & -8 & -17 & -8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_{23}$

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & -8 & -17 & -8 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2/-8$

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & 1 & 17/8 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\cancel{R}$ $\underline{\underline{\rho(A) = 2}}$

Ques find ρ $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1$ 3 - 4

1 - 6

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 3 & 1 & 2 \end{bmatrix}$$

$R_3 \to R_3 - 3R_1$ 1 - 6

2 - 9

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -5 \\ 0 & -5 & -7 \end{bmatrix}$$

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$R_2 \to R_2/-1$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & -5 & -7 \end{bmatrix}$$

$R_3 \to R_3 + 5R_2$

$-7+25$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 18 \end{bmatrix}$$

$\underline{\underline{\rho(A) = 3}}$

Ques

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$R_1 \to R_{12}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}$$

$1-6$

$R_3 \to R_3 - 3R_1$

$1-9$

$-8+10$

-2

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix}$$

$R_3 \to R_3 + 5R_2$

$$A = \begin{pmatrix} \cancel{1} & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{pmatrix}$$

~~$R_3 \to R_3$~~.

$\rho(A) = 3$

Ques

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{pmatrix}$$

$R_2 \to R_2 - R_1$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 2 & 6 & 5 \end{pmatrix}$$

$0/2 = 0$

$2/0 = \infty$

$R_3 \to R_3 - 2R_1$

$6-4$

$5-6$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{pmatrix}$$

$R_3 \to \cancel{R_3} \; R_3 - R_2$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

$R_2 \to R_2/2$

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rho(A) = 2$$

Rank of a matrix by reducing to its normal form

Every $m \times n$ matrix of rank r can be reduced to one of the following forms by a finite chain of elementary transformation.

Normal form of any matrix is

$$[I_r] \quad \left[\begin{array}{c} I_r \\ \hline 0 \end{array}\right] \quad \left[\begin{array}{c|c} I_r & 0 \end{array}\right] \quad \left[\begin{array}{c|c} I_r & 0 \\ 0 & 0 \end{array}\right]$$

where I_r is the unit matrix of order r

Rank of matrix = order of the identity matrix

Ques Reduce the matrix to its normal form and hence determine the rank

$$\begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

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$$A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

$R_1 \to R_{12}$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 3 & 2 & 5 & 7 & 12 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & -2 & -3 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

2 - 3

5 - 15

7 - 9

12 - 15

$R_3 \to R_3 - 3R_1$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2/-1$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - R_2$

$$A = \begin{bmatrix} 1 & 0 & +1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

OR

$\rho(A) = 2$

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$$A = \begin{bmatrix} 3 & 2 & 5 & 7 & 12 \\ 1 & 1 & 2 & 3 & 5 \\ 3 & 3 & 6 & 9 & 15 \end{bmatrix}$$

Apply $R_3 - R_1$ and R_{12}

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 3 & 2 & 5 & 7 & 12 \\ 0 & 1 & 1 & 2 & 3 \end{bmatrix}$$

$R_2 =$

Apply $R_2 - 3R_1$

$$A = \begin{bmatrix} 1 & 1 & 2 & 3 & 5 \\ 0 & -1 & -1 & -2 & -3 \\ 0 & 1 & 1 & 2 & 3 \end{bmatrix}$$

Apply $R_1 = R_1 - R_3$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & -2 & -3 \\ 0 & 1 & 1 & 2 & 3 \end{bmatrix}$$

Apply $R_3 \to R_3 + R_2$

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & -1 & -1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \Rightarrow -R_2$.

$$A = \begin{bmatrix} 1 & 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

I_2

$\rho(A) = 2$

Ques

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 3R_1$

$R_4 \rightarrow R_4 - 6R_1$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$$

$R_4 \rightarrow R_4 - R_2$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_{23}$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$R_2 \to R_2/-4$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \to R_3/-3$

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$0 - 2 \times \frac{-3}{4}$

$R_1 \to R_1 - 2R_2$

$$A = \begin{bmatrix} 1 & 0 & -1 & +3/2 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3/2 + -2$

$\frac{-3}{2} + \frac{-2}{3}$

$= \frac{-9 + -4}{6}$

$R_1 \to R_1 + R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & +5/6 \\ 0 & 1 & 2 & -3/4 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\frac{2}{3} \quad \frac{-2}{3}$

$+\frac{3}{2} + \frac{-2}{3}$

$\frac{9 + -4}{6}$

$R_2 \to R_2 - 2R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 & +5/6 \\ 0 & 1 & 0 & 7/12 \\ 0 & 0 & 1 & -2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$-3/4 -$

$\frac{-3}{4} + \frac{4}{3}$

$\frac{-9 + 16}{12}$

$\underline{\rho(A) = 3}$

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Normal form column transformation

Ques

$$A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$$

$C_1 \to C_{12}$

$$A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 1 & 2 & 3 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$A = \begin{bmatrix} 1 & 0 & 2 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$C_3 \to C_3 - 2C_1$

$$A = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$C_4 \to C_4 + 2C_1$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 2 & 6 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2/2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 3 \\ 0 & 2 & 1 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$C_2 \to C_{32}$

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad \begin{matrix} I_2 & 0 \\ 0 & 0 \end{matrix}$$

$$\rho(A) = 2$$

$$A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_1 \to R_{12}$

$$A = \begin{pmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_2 \to R_2 - 2R_1$

$$A = \begin{pmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 1 & 7 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}$$

$R_3 \to R_3 - 3R_1$

$$A = \begin{pmatrix} 1 & -1 & -2 & -4 \end{pmatrix}$$

Ques

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_1 \to R_{12}$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 4 & 0 & 1 & -6 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1 \quad R_3 \to R_3 - 3R_1 \quad R_4 \to R_4 - 4R_1$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & +7 \\ 0 & 4 & 9 & 10 \\ 0 & 4 & 9 & 10 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - R_3$

$$A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -2 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + R_2$

$$A = \begin{pmatrix} 1 & 0 & -8 & -6 \\ 0 & 1 & -6 & -2 \\ 0 & 4 & 9 & 10 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$R_3 \to R_3 - 4R_2$

$$A = \begin{pmatrix} 1 & 0 & -8 & -6 \\ 0 & 1 & -6 & -2 \\ 0 & 0 & 33 \ \cancel{16} & 18 \\ 0 & 0 & 0 \ \cancel{0} & 0 \end{pmatrix}$$

$C_3 \to C_3 - C_4$

$$A = \begin{pmatrix} 1 & 0 & -2 & -6 \\ 0 & 1 & -4 & -2 \\ 0 & 0 & 15 & 18 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$C_3 \to C_3 + 2C_4$

$$A = \begin{pmatrix} 1 & 0 & 10 & -6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 21 & 18 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$R_3 \to R_3/21$

$$A = \begin{pmatrix} 1 & 0 & 10 & -6 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 18/21 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$R_1 \to R_1 - 10R_3$

$$\cancel{\rho(A)}\ A = \begin{pmatrix} 1 & 0 & 0 & 306/21 \\ 0 & 1 & 0 & \cancel{30}\ -2 \\ 0 & 0 & 1 & 18/21 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\rho(A) = 3$$

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Wednesday

Solving Linear Equations Using Elementary Transformation

Consider the system of equations given below

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

The matrix equation corresponding to the given system is

$$\underset{A}{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Coefficient of matrix — Unknown matrix — Known Column matrix

$$A X = B$$

Homogeneous System

In the system of equation representing in matrix form ie $AX = B$ where $B = 0$ is called homogeneous system of equation

eg:-

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eg:- $6x + 2y + 3z = 0$
$2x + y + z = 0$
$3x + 6y + 7z = 0$

$$\begin{bmatrix} 6 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Non - Homogeneous System

In the system of equations in matrix representation $AX = B$ if atleast one of the elements in B is non-zero, the system is called non-homogeneous system of equation.

eg:- $6x + 2y + 3z = 2$
$2x + y + z = 0$
$3x + 6y + 7z = 3$

$$\begin{bmatrix} 6 & 2 & 3 \\ 2 & 1 & 1 \\ 3 & 6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

Consistent system and Inconsistent system

~~Consider~~ A system of equation is said to be consistent if it has one or more solutions.

A system of equation is said to be

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inconsistent if it has no solutions.

Procedure to solve a system of equation

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

coefficient of matrix — unknown matrix — known column matrix

Ques

$$AX = B$$

The augmented matrix is given by

$$K = \begin{bmatrix} a_1 & b_1 & c_1 & : & d_1 \\ a_2 & b_2 & c_2 & : & d_2 \\ a_3 & b_3 & c_3 & : & d_3 \end{bmatrix}$$

Then there are 3 possibilities

(a) If $\rho(A) \neq \rho(K)$, the system is inconsistent.

(b) If $\rho(A) = \rho(K)$, the system is consistent.

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(c) If $\rho(A) = \rho(K)$ which is equal to number of unknowns, then the system has unique solution.

(d) If $\rho(A) = \rho(K)$ less than the number of unknowns, then the system has infinite number of solution.

(e) If $B = 0$ ie $X = 0$ is always a solution and is known as null solution or trivial solution. Here each unknown has the value zero.

Ques ~~Does~~ Test whether the following equations are consistent, if consistent find the solution

$$x + y - z = 8$$
$$x - y + 2z = 6$$
$$3x + 5y - 7z = 14$$

The coefficient matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 2 \\ 3 & 5 & -7 \end{bmatrix}$$

The augmented matrix

$$K = \begin{bmatrix} 1 & 1 & -1 & : & 8 \\ 1 & -1 & 2 & : & 6 \\ 3 & 5 & -7 & : & 14 \end{bmatrix}$$

$$K = \left[\begin{array}{ccc:c} 1 & 1 & -1 & 8 \\ 1 & -1 & 2 & 6 \\ 3 & 5 & -7 & 14 \end{array}\right]$$

5 - 3

-1 - 1

2 - -7 + 1

$R_2 \to R_2 - R_1$; $R_3 \to R_3 - 3R_1$ $-7+3$

$$K = \left[\begin{array}{ccc:c} 1 & 1 & -1 & 8 \\ 0 & -2 & 3 & -2 \\ 0 & 2 & -4 & -10 \end{array}\right]$$

-2 + 10

-12

$\cancel{C_2 \to C_{23}}$

$$\cancel{K = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 1 & -2 & -2 \\ 0 & -10 & 2 & -10 \end{array}\right]}$$

$R_3 \to R_3 + R_2$

$$K = \left[\begin{array}{ccc:c} 1 & 1 & -1 & 8 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & -1 & -12 \end{array}\right]$$

$R_2 \to R_2 / -2$

$$K = \left[\begin{array}{ccc:c} 1 & 1 & -1 & 8 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & -1 & -12 \end{array}\right]$$

$\cancel{C_2 \to C_2 - C_1}$

$$\cancel{K = \left[\begin{array}{ccc:c} 1 & 0 & 1 & 8 \\ 0 & 1 & -1/2 & 1 \\ 0 & 0 & -9 & -12 \end{array}\right]}$$

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$R_3 \to -R_3$

$$K = \left[\begin{array}{ccc:c} 1 & 1 & 1 & 8 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & 1 & 12 \end{array}\right]$$

$R_1 \to R_1 - R_2$

$$K = \left[\begin{array}{ccc:c} 1 & 0 & 1/2 & 7 \\ 0 & 1 & -3/2 & 1 \\ 0 & 0 & 1 & 12 \end{array}\right]$$

$R_1 \to R_1 - 1/2 R_3$

$R_2 \to R_2 + 3/2 R_3$

$$K = \left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 19 \\ 0 & 0 & 1 & 12 \end{array}\right]$$

Rank of K = Rank of A = 3

= No. of unknowns

∴ The given system is consistant and it has unique solution

$$X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 1 \\ 19 \\ 12 \end{bmatrix}$$

$x = 1$

$y = 19$

$z = 12$

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Thursday

Ques:-

$$x + 2y + z = 3$$
$$2x + 3y + 2z = 5$$
$$3x - 5y + 5z = 2$$
$$3x + 9y - z = 4$$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 3 & -5 & 5 \\ 3 & 9 & -1 \end{bmatrix} = A$$

$$K = \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 2 & 3 & 2 & : & 5 \\ 3 & -5 & 5 & : & 2 \\ 3 & 9 & -1 & : & 4 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$K = \begin{bmatrix} 1 & 2 & 1 & : & 3 \\ 2 & 3 & 2 & : & 5 \\ 3 & -5 & 5 & : & 2 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 3R_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & -11 & 2 & -4 \\ 0 & 3 & -4 & -5 \end{array}\right]$$

$R_2 \to -R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & -11 & 2 & -4 \\ 0 & 3 & -4 & -5 \end{array}\right]$$

$R_3 \to R_3 + 11R_2$
$R_4 \to R_4 - 3R_2$

$$\left[\begin{array}{ccc:c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & -4 & -8 \end{array}\right]$$

$C_3 \to C_3 - C_1$

$$\left[\begin{array}{ccc:c} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & -4 & -8 \end{array}\right]$$

$R_4 \to R_4/2$

$$\left[\begin{array}{ccc:c} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & -2 & -4 \end{array}\right]$$

$R_4 \to R_4 + R_3$

$$\left[\begin{array}{ccc:c} 1 & 2 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 7/2 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

$R_3 \to R_3/2$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 7/2 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

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$\rho(K) = 4, \rho(A) = 3$

$\therefore$ system R is inconsistent;

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Solution of non-homogenous equation using Cramer's rule

Ques

$3x + 2y + z = 6$

$2x - 3y + 3z = 2$

$x + y + z = 3$

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & -3 & 3 \\ 1 & 1 & 1 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix} B$$

$$A_1 = \begin{pmatrix} 6 & 2 & 1 \\ 2 & -3 & 3 \\ 3 & 1 & 1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 3 & 6 & 1 \\ 2 & 2 & 3 \\ 1 & 3 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} 3 & 2 & 6 \\ 2 & -3 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

$$x = \frac{|A_1|}{|A|} \quad y = \frac{|A_2|}{|A|} \quad z = \frac{|A_3|}{|A|}$$

$$|A| = 3\begin{vmatrix} -3 & 3 \\ 1 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$$

$= 3(-3-3) - 2(2-3) + 1(2 - -3)$

$= (3 \times -6) - (2 \times -1) + (1 \times 5)$

$= -18 + 2 + 5$

$= -18 + 7$

$= \underline{\underline{-11}}$

$|A_1| = 6 \begin{vmatrix} -3 & 3 \\ 1 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & -3 \\ 3 & 1 \end{vmatrix}$

$= 6(-3+3) - 2(2-9) + 1(2 - -9)$

$= (6 \times -6) - (2 \times -7) + (1 \times 11)$

$= -36 + 14 + 11$

$= -36 + 25$

$= \underline{\underline{-11}}$

$\begin{array}{r} 14 \\ 11 \\ \hline 25 \end{array}$ $\begin{array}{r} 36 \\ 25 \\ \hline 11 \end{array}$

$|A_2| = 3 \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} - 6 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix}$

$= 3(2-9) - 6(2-3) + 1(6-2)$

$= (3 \times -7) - (6 \times -1) + (1 \times 4)$

$= -21 + 6 + 4$

$= -21 + 10$

$= \underline{\underline{-11}}$

$\begin{array}{r} 21 \\ 10 \\ \hline 11 \end{array}$

$|A_3| = 3 \begin{vmatrix} -3 & 2 \\ 1 & 3 \end{vmatrix} - 2 \begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} + 6 \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$

$= 3(-9-2) - 2(6-2) + 6(2 - -3)$

$= (3 \times -11) - (2 \times 4) + (6 \times 5)$

$= -33 - 8 + 30$

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$= -3-8$
$= -11$

$x = \frac{|A_1|}{|A|} = \frac{-11}{-11} = 1$

$y = \frac{|A_2|}{|A|} = \frac{-11}{-11} = 1$

$z = \frac{|A_3|}{|A|} = \frac{-11}{-11} = 1$

Ques $5x - 6y + 4z = 15$
$7x + 4y - 3z = 19$
$2x + y + 6z = 46$

$A = \begin{pmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 15 \\ 19 \\ 46 \end{pmatrix}$

$A_1 = \begin{pmatrix} 15 & -6 & 4 \\ 19 & 4 & -3 \\ 46 & 1 & 6 \end{pmatrix} \quad A_2 = \begin{pmatrix} 5 & 15 & 4 \\ 7 & 19 & -3 \\ 2 & 46 & 6 \end{pmatrix}$

$A_3 = \begin{pmatrix} 5 & -6 & 15 \\ 7 & 4 & 19 \\ 2 & 1 & 46 \end{pmatrix}$

$$|A| = 5\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} + 6\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix}$$

$= 5(24 - {}^{-}3) + 6(42 - {}^{-}6) + 4(7 - 8)$

$= 5 \times 27 + 6 \times 48 - 4 \times 1$

$= 135 + 288 - 4$

$= \underline{\underline{419}}$

$$|A_1| = 15\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} + 6\begin{vmatrix} 19 & -3 \\ 46 & 6 \end{vmatrix} + 4\begin{vmatrix} 19 & 4 \\ 46 & 1 \end{vmatrix}$$

$= 15(24 - {}^{-}3) + 6(114 - {}^{-}138) + 4(19 - 184)$

$= 15 \times 27 + 6 \times 252 - 4 \times 165$

$= 405 + 1512 - 660$

$= \underline{\underline{1257}}$

$$|A_2| = 5\begin{vmatrix} 19 & -3 \\ 46 & 6 \end{vmatrix} - 15\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 7 & 19 \\ 2 & 46 \end{vmatrix}$$

$= 5(114 - {}^{-}138) - 15(42 - {}^{-}6) + 4(322 - 38)$

$= 5 \times 252 - 15 \times 48 + 4 \times 284$

$= 1260 - 720 + 1136$

$= \cancel{690 + 1136}\ 540 + 1136$

$= \cancel{1826} = \underline{\underline{1676}}$

$\begin{array}{r} 19 \\ \times 2 \\ \hline 38 \end{array}$

$$|A_3| = 5\begin{vmatrix} 4 & 19 \\ 1 & 46 \end{vmatrix} + 6\begin{vmatrix} 7 & 19 \\ 2 & 46 \end{vmatrix} + 15\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix}$$

$= 5(184 - 19) + 6(322 - 38) + 15(7 - 8)$

$= 5 \times 165 + 6 \times 284 + 15 \times 1$

$= 825 + 1704 - 15$

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$|A_3| = \cancel{25}\cancel{\#}4$

$x = \dfrac{|A_1|}{|A|} = \dfrac{1257}{419} = 3$

$y = \dfrac{|A_2|}{|A|} = \dfrac{1676}{419} = 4$

$z = \dfrac{|A_3|}{|A|} = \dfrac{2514}{419} = 6$

$x = 3, \; y = 4, \; z = 6$

$(x, y, z) = (3, 4, 6)$

Characteristic Equation.

If A is a square matrix of order n and λ is any real number, consider the matrix $[A - \lambda I]$ where I is the unit matrix of order n, the determinant of the matrix $|A - \lambda I| = 0$ is known as the characteristic equation.

Eigen values/ characteristic ~~roots~~. roots

The roots of the characteristic equation is called Eigen values / characteristic roots. The value of λ for which $|A - \lambda I| = 0$ are the Eigen values

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Eigen Vectors / Characteristic Vectors

Consider the matrix equation $AX = \lambda I X$

ie $AX - \lambda I X = 0$

ie $(A - \lambda I)X = 0$

for each value of λ this equation possesses a non zero solution of x and is called Eigen vector corresponding to the Eigen values of λ

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Ques Find the Eigen vectors and Eigen values of the matrix $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$

The characteristic equation is $|A - \lambda I| = 0$

$$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 0$$

$$\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} = 0$$

$$\begin{bmatrix} (5-\lambda) & 4 \\ 1 & (2-\lambda) \end{bmatrix} = 0$$

$$\begin{vmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$
$$(10 - 5\lambda - 2\lambda + \lambda^2) - 4 = 0$$
$$(10 - 7\lambda + \lambda^2) - 4 = 0$$
$$\lambda^2 - 7\lambda + 6 = 0$$

$S = -7$

$P = 6$

-1×-6

$$\lambda^2 - \lambda - 6\lambda + 6 = 0$$

$$\lambda(\lambda - 1) - 6(\lambda - 1) = 0$$
$$(\lambda - 1)(\lambda - 6) = 0$$

$$\underline{\lambda = 1, \ \lambda = 6}$$

<u>Case 2</u>

when $\lambda = 1$

$$(A - \lambda I)X = 0$$
$$(A - 1I)X = 0$$
$$(A - I)X = 0$$

$$\left(\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) X = 0$$

$$\begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix} X = 0$$

$$\begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$$4x + 4y = 0$$
$$x + y = 0$$

$x = -y$

$(1, -1)$ is an Eigen vector

Case 2

when $\lambda = 6$

$(A - 6I)\, x = 0$

$$\left(\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix} - \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix} \right) x = 0$$

$$\begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix} x = 0$$

$$\begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 0$$

$-x + 4y = 0$

$x - 4y = 0$

$(4, -1)$ is an Eigen vector.

ues $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix}$ $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$A - \lambda I = 0$

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = 0$$

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$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = 0$$

$$\begin{pmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 3 & 2 & 3-\lambda \end{pmatrix} = 0$$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 2-\lambda & 1 \\ 3 & 2 & 3-\lambda \end{vmatrix} = 0$$

$$= (1-\lambda)\begin{vmatrix} 2-\lambda & 1 \\ 2 & 3-\lambda \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 3 & 3-\lambda \end{vmatrix} + 1\begin{vmatrix} 1 & 2-\lambda \\ 3 & 2 \end{vmatrix} = 0$$

$$= (1-\lambda)\left((2-\lambda)(3-\lambda)-2\right) - 1\left((3-\lambda)-3\right) + 1\left(2-3(2-\lambda)\right) = 0$$

$$= (1-\lambda)(6-2\lambda-3\lambda+\lambda^2-2) - 1(0-\lambda) + (2-6+3\lambda) = 0$$

$$\cancel{= (1-\lambda)(\lambda^2-5\lambda-4\lambda) - 9 + 3\lambda + 4 - 3\lambda} = 0$$

$$= (1-\lambda)(\lambda^2-5\lambda+4) + \lambda + 3\lambda - 4 = 0$$

$$= (\lambda^2 - 5\lambda + 4 - \lambda^3 + 5\lambda^2 + 4\lambda) + 4\lambda - 4 = 0$$

$$= -\lambda^3 + 6\lambda^2 - 5\lambda = 0$$

$$= -\lambda(\lambda^2 - 6\lambda + 5) = 0$$

$$\cancel{= \lambda^2 - 6\lambda - 5 = 0}$$

$$\cancel{\lambda^2 - 1\lambda - 5\lambda - 5 = 0}$$

$$\cancel{\lambda(\lambda - 1) - 5(\lambda - 1)}$$

$$\lambda^2 - 6\lambda + 5 = 0$$

$$\lambda^2 - 1\lambda - 5\lambda + 5 = 0$$

$$\lambda(\lambda - 1) - 5(\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda - 5) = 0$$

$$\lambda = 1, \quad \lambda = 5$$

$\cancel{S = 6}$, $\cancel{P = -5}$

$S = -6$

$P = 5$

1×5

Case 1

when $\lambda = 1$

$$(A - \lambda I)X = 0$$

$$(A - 1I)X = 0$$

$$\left(\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) X = 0$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 2 \end{pmatrix} x = 0$$

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 3 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$$0x + 1y + 1z = 0$$
$$1x + 1y + 1z = 0$$
$$3x + 2y + 2z = 0$$

$$\begin{array}{r} 0x + 1y + 1z = 0 \\ - \quad 1x + 1y + 1z = 0 \\ \hline -x = 0 \end{array}$$

$$\underline{x = 0}$$

$$\begin{array}{r} 3x + 3y + 3z = 0 \\ - \quad 3x + 2y + 2z = 0 \\ \hline 0x + y + z = 0 \end{array}$$

$$y = 0 - z$$

$$0x + 1y + 1z = 0$$
$$0x + 0 - z + 1z = 0$$
$$\cancel{1x + 0 - z + 1z = 0}$$

$(0, 1, -1)$ is an Eigen vector.

Case 2

when $\lambda = 5$

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$(A - \lambda I)x = 0$
$(A - 5I)X = 0$

$$\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 3 & 2 & 3 \end{bmatrix} - \begin{pmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{pmatrix}\right) X = 0$$

$$\begin{bmatrix} -4 & 1 & 1 \\ 1 & -3 & 1 \\ 3 & 2 & -2 \end{bmatrix} X = 0$$

$$\begin{bmatrix} -4 & 1 & 1 \\ 1 & -3 & 1 \\ 3 & 2 & -2 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$$

$-4x + 1y + 1z = 0$
$1x - 3y + 1z = 0$
$3x + 2y - 2z = 0$

$-4x + 1y + 1z = 0$
$- \underline{1x - 3y + 1z = 0}$
$-5x + 4y + 0z = 0$

$-5x + 4y = 0$
$-5x = -4y$
$x = +\frac{4}{5}y$

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monday

Cofactor of an Element

Cofactor of an element is obtained by multiplying the minor of that element with $(-1)^{i+j}$ where i is the number of raw and j is the number of column of that element.

eg. - Suppose $A = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 4 & 5 \\ -3 & 7 & 0 \end{bmatrix}$

cofactor of a_{23} is $\alpha_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ -3 & 7 \end{vmatrix}$

$= -(7+6) = -13$

Adjoint Matrix

Adjoint of a given square matrix is the transpose of the matrix formed by cofactors of elements of a given square matrix.

Ques Find the adjoint of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\alpha_{11} = (-1)^2 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$

$= (2-3) = -1$

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$$\alpha_{12} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= -(1-9)$$

$$= \underline{\underline{8}}$$

$$\alpha_{13} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 1-6$$

$$= \underline{\underline{-5}}$$

$$\alpha_{21} = (-1)^3 \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix}$$

$$= -(1-2)$$

$$= \underline{\underline{1}}$$

$$\alpha_{22} = (-1)^4 \begin{vmatrix} 0 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= (0-6)$$

$$= \underline{\underline{-6}}$$

$$\alpha_{23} = (-1)^5 \begin{vmatrix} 0 & 1 \\ 3 & 1 \end{vmatrix}$$

$$= -(0-3)$$

$$= \underline{\underline{3}}$$

$$\alpha_{31} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$= (3-4)$$

$$= \underline{\underline{-1}}$$

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$$\alpha_{32} = (-1)^5 \begin{vmatrix} 0 & 2 \\ 1 & 3 \end{vmatrix}$$
$$= -(0-2)$$
$$= \underline{\underline{2}}$$

$$\alpha_{33} = (-1)^6 \begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$$
$$= (0-1)$$
$$= -1$$

$$\text{Adj } A = \text{transpose of } \begin{bmatrix} -1 & 8 & -5 \\ 1 & -6 & 3 \\ -1 & 2 & -1 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Inverse of a Matrix

For a square matrix A if there exist a square matrix B such that $AB = BA = I$ then B is said to be the inverse of A or reciprocal of A. The inverse of A is denoted by A^{-1}.

$$A^{-1} = \frac{1}{|A|} (\text{adj } A)$$

Ques Find the inverse of A where $A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 5 & 7 \\ 2 & -3 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$|A| = 3 \begin{vmatrix} -3 & 1 \\ 1 & 2 \end{vmatrix} - 5 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 7 \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix}$$

$$= 3(-6 - 1) - 5(4 - 1) + 7(2 + 3)$$

$$= -21 - 15 + 35$$

$$= -21 + 20$$

$$= -1$$

$$\begin{array}{r} 35 \\ 15 \\ \hline 20 \end{array}$$

$$\alpha_{11} = (-1)^2 \begin{vmatrix} -3 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= (-6 - 1)$$

$$= -7$$

$$\alpha_{12} = (-1)^3 \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= -(4 - 1)$$

$$= -3$$

$$\alpha_{13} = (-1)^4 \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = 2 + 3$$

$$= 5$$

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$\alpha_{21} = (-1)^3 \begin{vmatrix} 5 & 7 \\ 1 & 2 \end{vmatrix}$

$= -(10 - 7)$

$= -3$

$\alpha_{22} = (-1)^4 \begin{vmatrix} 3 & 7 \\ 1 & 2 \end{vmatrix}$

$= +(6 - 7)$

$= -1$

$\alpha_{23} = (-1)^5 \begin{vmatrix} 3 & 5 \\ 1 & 1 \end{vmatrix}$

$= -(3 - 5)$

$= +2$

$\alpha_{31} = (-1)^4 \begin{vmatrix} 5 & 7 \\ -3 & 1 \end{vmatrix}$

$= (5 + 21)$

$= 26$

$\alpha_{32} = (-1)^5 \begin{vmatrix} 3 & 7 \\ 2 & 1 \end{vmatrix}$

$= -(3 - 14)$

$= 11$

$\alpha_{33} = (-1)^6 \begin{vmatrix} 3 & 5 \\ 2 & -3 \end{vmatrix} = (-9 - 10)$

$= -19$

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adj A = transpose of $\begin{bmatrix} -7 & -3 & 5 \\ -3 & -1 & 2 \\ 26 & 11 & -19 \end{bmatrix}$

$$\text{adj } A = \begin{bmatrix} -7 & -3 & 26 \\ -3 & -1 & 11 \\ 5 & 2 & -19 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 7 & 3 & -26 \\ 3 & 1 & -11 \\ -5 & -2 & 19 \end{bmatrix}$$

Ques Solve the following eqns:-

$$5x - 6y + 4z = 15$$
$$7x + 4y - 3z = 19$$
$$2x + y + 6z = 46$$

$$A = \begin{bmatrix} 5 & -6 & 4 \\ 7 & 4 & -3 \\ 2 & 1 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 15 \\ 19 \\ 46 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$AX = B$$
$$X = A^{-1}B$$

$$A^{-1} = \frac{(\text{adj } A)}{|A|}$$

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$$|A| = 5\begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} + 6\begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} + 4\begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix}$$

$= 5(24+3) + 6(42+6) + 4(7-8)$

$= 5 \times 27 + 6 \times 48 + 4 \times \bar{1}$

$= 135 + 288 \cancel{+} \,^{+}4$

$= \cancel{427}\ \ \underline{\underline{419}}$ 49

27 ×5

135 288

$$\alpha_{11} = (-1)^2 \begin{vmatrix} 4 & -3 \\ 1 & 6 \end{vmatrix} = 24 + 3 = \underline{\underline{27}}$$

$$\alpha_{12} = (-1)^3 \begin{vmatrix} 7 & -3 \\ 2 & 6 \end{vmatrix} = -(42+6) = \underline{\underline{-48}}$$

$$\alpha_{13} = (-1)^4 \begin{vmatrix} 7 & 4 \\ 2 & 1 \end{vmatrix} = 7 - 8 = \underline{\underline{-1}}$$

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$$\alpha_{21} = (-1)^3 \begin{vmatrix} -6 & 4 \\ 1 & 6 \end{vmatrix} = -(\bar{3}6 - 4) = +40$$

$$\alpha_{22} = (-1)^4 \begin{vmatrix} 5 & 4 \\ 2 & 6 \end{vmatrix} = (30-8) = \underline{\underline{22}}$$

18
16
34

$$\alpha_{23} = (-1)^5 \begin{vmatrix} 5 & -6 \\ 2 & 1 \end{vmatrix} = -(5+12) = \underline{\underline{-17}}$$

1
28
15
43

$$\alpha_{31} = (-1)^4 \begin{vmatrix} -6 & 4 \\ 4 & -3 \end{vmatrix} = (18 \cancel{+} 16) = \underline{\underline{\cancel{34}\,2}}$$

$$\alpha_{32} = (-1)^5 \begin{vmatrix} 5 & 4 \\ 7 & -3 \end{vmatrix} = -(-15 + \bar{2}8) = \underline{43}$$

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$$\alpha_{33} = (-1)^6 \begin{vmatrix} 5 & -6 \\ 7 & 4 \end{vmatrix}$$

$$= (20 + 42)$$

$$\alpha_{33} = 62$$

$$\text{adj } A = \text{transpose of } \begin{bmatrix} 27 & -48 & -1 \\ 40 & 22 & -17 \\ \cancel{34}\,2 & 43 & 62 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 27 & 40 & \cancel{34}\,2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}$$

$$A^{-1} = \frac{1}{419} \begin{bmatrix} 27 & 40 & \cancel{34}\,2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix}$$

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Wednesday

$$X = A^{-1} B$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{419} \begin{bmatrix} 27 & 40 & \cancel{34}\,2 \\ -48 & 22 & 43 \\ -1 & -17 & 62 \end{bmatrix} \times \begin{pmatrix} 15 \\ 19 \\ 46 \end{pmatrix}$$

$3 \times \cancel{3}$ $\quad \cancel{3} \times 1$

$$= \frac{1}{419} \begin{bmatrix} 27 \cdot 15 + 40 \cdot 19 + 2 \cdot 46 \\ -48 \cdot 15 + 22 \cdot 19 + 43 \cdot 46 \\ -1 \cdot 15 + -17 \cdot 19 + 62 \cdot 46 \end{bmatrix}$$

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$$= \frac{1}{419}\begin{bmatrix} 405 + 760 + 92 \\ -720 + 418 + 1978 \\ -15 + -323 + 2852 \end{bmatrix}$$

$$= \frac{1}{419}\begin{bmatrix} 1257 \\ 1676 \\ 2514 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}$$

$x = 3, \; y = 4, \; z = 6$

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Cayley Hamilton Theorem

This theorem gives another way to find the inverse of a square matrix

Every square matrix satisfies its own characteristic equation $\boxed{\text{or}}$

Let A be a square matrix of order n with characteristic equation

$a_n\lambda^n + a_{n-1}\lambda^{n-1} + \cdots a_1\lambda + a_0 = 0$

where $a_i, 1 \leq i < n$ are coefficients

then

$a_n A^n + a_{n-1}A^{n-1} + \cdots a_1 A + a_0 I = 0$

Remark 1

If A is an non-singular matrix by Cayley Hamilton theorem we can write

$$a_n A^n + a_{n-1} A^{n-1} + \cdots a_1 A + a_0 I = 0$$

multiply both sides with A^{-1} we get,

$$a_n A^{n-1} + a_{n-1} A^{n-2} + \cdots a_1 I + a_0 A^{-1} = 0$$

$$\therefore A^{-1} = \frac{-1}{a_0}\left[a_n A^{n-1} + a_{n-1} A^{n-2} + \cdots a_1 I\right]$$

Ques Verify Cayley - Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ and find the value of A^{-1}

$$|A - \lambda I| = 0$$

$$= \left| \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right| = 0$$

$$= \begin{vmatrix} 1-\lambda & 3 \\ 2 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(4-\lambda) - 6 = 0$$

$$4 - \lambda - 4\lambda + \lambda^2 - 6 = 0$$

$$\lambda^2 - 5\lambda - 2 = 0$$

$$A^2 - 5A - 2I = 0$$

$$A^2 = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times1+3\times2 & 1\times3+3\times4 \\ 2\times1+4\times2 & 2\times3+4\times4 \end{pmatrix}$$

$$= \begin{pmatrix} 1+6 & 3+12 \\ 2+8 & 6+16 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 15 \\ 10 & 22 \end{pmatrix}$$

$$A^2 - 5A - 2I = \begin{pmatrix} 7 & 15 \\ 10 & 22 \end{pmatrix} - \begin{pmatrix} 5 & 15 \\ 10 & 20 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 7 & 15 \\ 10 & 22 \end{pmatrix} - \begin{pmatrix} 7 & 15 \\ 10 & 22 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = \underline{\underline{0}}$$

This satisfies the Cayley Hamilton theorem

$$A^2 \cdot A^{-1} - 5AA^{-1} - 2IA^{-1} = 0$$

$$= A - 5I - 2A^{-1} = 0$$

$$A - 5I = 2A^{-1}$$

$$A^{-1} = \frac{1}{2}(A - 5I)$$

$$A^{-1} = \frac{1}{2}\left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}\right)$$

$$A^{-1} = \frac{1}{2}\begin{pmatrix} -4 & 3 \\ 2 & -1 \end{pmatrix}$$

Ques verify cayley Hamelton theorem and find A^{-1} and A^4

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$|A - \lambda I| = 0$$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$$= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix} - \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} = 0$$

$$= \cancel{\begin{pmatrix} 18 & -22 \\ & \end{pmatrix}}$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 5 \\ 3 & 5 & 6-\lambda \end{vmatrix} = 0$$

$\frac{\begin{matrix}12 \\ 3\end{matrix}}{36}$

$$(1-\lambda)\begin{vmatrix} 4-\lambda & 5 \\ 5 & 6-\lambda \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 3 & 6-\lambda \end{vmatrix} + 3\begin{vmatrix} 2 & 4-\lambda \\ 3 & 5 \end{vmatrix} = 0$$

$$= (1-\lambda)\left((4-\lambda)(6-\lambda) - 25\right) - 2\left(12 - 2\lambda - 15\right) +$$
$$3\left(10 - 12 + 3\lambda\right)$$

$$= (1-\lambda)(24 - 4\lambda - 6\lambda + \lambda^2 - 25) - (24 - 4\lambda - 30)$$
$$+ (30 - 36 + 9\lambda)$$

$$= \cancel{24}\,(1-\lambda)(-1 - 10\lambda + \lambda^2) + 6 + 4\lambda - 6 + 9\lambda = 0$$

$$= (-1 - 10\lambda + \lambda^2 + \lambda + 10\lambda^2 - \lambda^3) + 13\lambda = 0$$

$-9-5$

$$= -\lambda^3 + \lambda^2 + 10\lambda^2 - 10\lambda + \lambda + 13\lambda - 1 = 0$$

$$= -\lambda^3 + 11\lambda^2 - 9 + 13\lambda - 1 = 0$$
$$-\lambda^3 + 11\lambda^2 + 4\lambda - 1 = 0$$

$$\lambda^3 - 11\lambda^2 - 4\lambda + 1 = 0$$

$$A^3 - 11A^2 - 4A + 1I = 0$$

$$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+8+15 & 3+10+18 \\ 2+8+15 & 4+16+25 & 6+20+30 \\ 3+10+18 & 6+20+30 & 9+25+36 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{bmatrix}$$

$$A3 = \begin{bmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{bmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

$$= \begin{bmatrix} 14+50+93 & 28+100+155 & 42+125+186 \\ 25+90+168 & 50+180+280 & 75+225+336 \\ 31+112+210 & 62+224+350 & 93+280+420 \end{bmatrix}$$

$$A3 = \begin{bmatrix} 157 & 283 & 353 \\ 283 & 510 & 636 \\ 353 & 636 & 793 \end{bmatrix}$$

$$11A^2 = \begin{bmatrix} 11\times14 & 11\times25 & 11\times31 \\ 11\times25 & 11\times45 & 11\times56 \\ 11\times31 & 11\times56 & 11\times70 \end{bmatrix}$$

$$11A2 = \begin{bmatrix} 154 & 275 & 341 \\ 275 & 495 & 616 \\ 341 & 616 & 770 \end{bmatrix}$$

$$4A = \begin{bmatrix} 4 & 8 & 12 \\ 8 & 16 & 20 \\ 12 & 20 & 24 \end{bmatrix}$$

$$A^3 - 11A^2 - 4A + 1I = 0$$

$$= \begin{bmatrix} 157 & 283 & 353 \\ 283 & 510 & 636 \\ 353 & 636 & 793 \end{bmatrix} - \begin{bmatrix} 154 & 275 & 341 \\ 275 & 495 & 616 \\ 341 & 616 & 770 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 12 \\ 8 & 16 & 20 \\ 12 & 20 & 24 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$3 - {-3}$

$$= \begin{bmatrix} 3 & 8 & 12 \\ 8 & 15 & 20 \\ 12 & 20 & 23 \end{bmatrix} + \begin{bmatrix} -3 & -8 & -12 \\ -8 & -15 & -20 \\ -12 & -20 & -23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This satisfies Cayley Hamilton Theorem.

$$A^3 \cdot A^{-1} - 11A^2 \cdot A^{-1} - 4A \cdot A^{-1} + 1IA^{-1} = 0$$

$$A^2 - 11A - 4I + 1A^{-1} = 0$$

$$A^2 - 11A - 4I = -A^{-1}$$

$$A^{-1} = -A^2 + 11A + 4I$$

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$$A^{-1} = -\begin{pmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{pmatrix} + \begin{pmatrix} 11 & 22 & 33 \\ 22 & 44 & 55 \\ 33 & 55 & 66 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

$$A^{-1} = -\begin{pmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{pmatrix} + \begin{pmatrix} 15 & 22 & 33 \\ 22 & 48 & 55 \\ 33 & 55 & 70 \end{pmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

$$A^4 = A^3 \cdot A$$

$$= \begin{pmatrix} 157 & 283 & 353 \\ 283 & 510 & 636 \\ 353 & 636 & 793 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$$

$$= \begin{bmatrix} 157+566+1059 & 314+1132+1765 & 471+1415+2118 \\ 283+1020+1908 & 566+2040+3180 & 849+2550+3816 \\ 353+1272+2379 & 706+2544+3965 & 1059+3180+4758 \end{bmatrix}$$

$$A^4 = \begin{pmatrix} 1782 & 3211 & 4004 \\ 3211 & 5786 & 7215 \\ 4004 & 7215 & 8997 \end{pmatrix}$$