Complementary Course to BCA Programme

Semester III–Course II

ST3CMT02 - ADVANCED STATISTICAL METHODS

Hours per week -4

Number of credits -4

Module I

Theoretical distributions. Discrete distribution(Uniform, Bernoulli, binomial and Poisson), mean, variance, moment generating functions and fitting of data. Continuous distribution- Uniform and normal distribution-important properties (without proof) of the distribution (mean, variance, moments, mgf, M.D. and Q.D Area under the normal curve-related problems............................ **(16Hours)**

Module II

Sampling Distributions. definition, Statistic, Parameter, Standard Error, Sampling Distributions of Mean of the sample from Normal population and distribution of Variance(form alone), statement of the form of the distributions χ_2, t and F (without derivation), properties, Inter relationships................**(16 Hours)**

Module III

Estimation of parameters- Point Estimation and Interval estimation, properties of Point Estimation- Unbiasedness, Efficiency; Consistency; Sufficiency, Methods of estimation-method of moments and method of maximum likelihood. Interval Estimation for Mean, Variance of normal population and Proportion of binomial population.. **(20Hours)**.

Module IV

Testing of hypotheses- Statistical hypotheses, Simple and composite hypotheses. Null and Alternate hypothesis, Two types of errors, Critical Region, Size of the test, Significance level P value, Power, Large Sample test Z test-,t test Chi-Square test-goodness of fit, test of independence.......................................(**20 Hours**)

References:

1. S.C. Gupta and V.K. Kapoor: *Fundamentals of Mathematical Statistics*, Sultan Chand and Sons

2. S.C Gupta: *Fundamentals of Mathematical Statistics*, Sultan Chand and Sons.

3. V.K. Rohatgi: *An Introduction to Probability Theory and Mathematical Statistics*, Wiley Eastern

SCHEME OF QUESTION PAPER

(The number of questions from the 4 modules to be included in the 3 parts of the question paper)

Use of non-programmable calculator and statistical tables allowed.

Part	Marks of each Question	No. of Questions					Total Marks	To be answered	
		Module				Total		No. of Questions	Total Marks
		1	2	3	4				
A	2	3	3	3	3	12	24	10	20
B	5	3	2	2	2	9	45	6	30
C	15	0	1	1	2	4	60	2	30
Total Questions		7	6	6	6	25	129	18	80
Total Mark		36	31	31	31	129			

Module-1

Theoritical Distribution

Random Experiment

An experiment is said to be a random experiment if it has 2 or more outcomes which vary in an unpredictable manner from trial to trial.

eg:- tossing a Coin
Throwing a die

Random Variable.

A variable whose value is determined by the outcome of a random experiment is called random ~~ex~~ variable

eg:- Consider the random experiment of tossing 2 coins the possible outcomes are

HH, TT, HT, TH

Consider random variable x as no. of heads, then the values of x are 0, 1, 2.

Discrete and Continuos Random Variables

~~Ra~~ If the random variable assumes on integer values such as 0, 1, 2, 3 etc, then it is called a discrete random variable.

eg: No. of heads while tossing 2 coins

If the random variable assumes on any values within a certain interval, then the random variable is called a continuous random variable.

eg:- Any variable concerned with the measurement of weight, height, volume etc.

The frequency distributions can be broadly classified into two types namely

(a) Observed frequency distributions and

(b) Theoretical or expected frequency distributions or probability distributions.

The Observed frequency distributions are based on actual observations and experimentations. For instance, consider a study of weights of students of a class represented in the form of a Table as well as a Histogram and Frequency curve.

Weights of 85 students of a class:

Weights (in lbs) :	100-105	105-110	110-115	115-120	120-125	125-130	Total
No. of students :	8	10	23	20	18	6	85

The probability distribution of a random variable is a listing of various values of the random variable with the corresponding probabilities associated with each value of the random variable

eg:-

x	$P(x)$
1	0.1
2	0.2
3	0.3
4	0.2
5	0.2

Some of the important probability distributions are given below:

1. Bernoulli Distribution
2. Binomial Distribution (B.D)
3. Poisson Distribution (P.D)
4. Uniform Distribution (U.D)
5. Normal Distribution (N.D)

Among these, Bernoulli, Binomial and Poisson are discrete probability distributions where as Uniform and Normal distributions are continuous probability distributions.

Bernoulli Distribution

Bernoulli trial

A trial is said to be Bernoulli in nature if it satisfies the following conditions:

1. The trial must result either in a success or in a failure.
2. The probability of success should remain constant for any trial.

Examples:

1. An unbiased coin is tossed once. Let X take value 1 if the throw results in Head or 0 if the throw results in Tail respectively. Here X is a Bernoulli variate with parameter $p = \frac{1}{2}$. Here $x = 0, 1$

2. A fair die is rolled once. Let X take value 1 if the throw results in $(1, 2, 4, 5)$ or 0 if the throw results in $(3, 6)$. Here X is a Bernoulli variate with parameter $p = \frac{4}{6}$. Here $x = 0, 1$

Definition

A discrete random variable X is said to follow a Bernoulli distribution, if its probability mass function is given by.

$P(X = x) = p^x q^{1-x}$ where $x = 0, 1$, with parameter $= p$.

Here, p = probability of success, $p > 0$

q = probability of failure, ~~$q = p + 1$~~ $q = 1 - p$

x = number of success.

Mean & Variance of Bernoulli Distribution

$$\text{Mean} = E(x)$$

$$= \sum_{x=0}^{1} x f(x)$$

$$= \sum_{x=0}^{1} x \cdot p^{x} q^{1-x} \qquad \left[\because P(X=x) = f(x) = p^{x} q^{1-x}\right]$$

$$= 0 \times p^{0} \times q^{1-0} + 1 \times p^{1} \times q^{1-1}$$

$$= 0 + p q^{0} = 0 + p \times 1$$

$$= 0 + p = \underline{\underline{p}}$$

$$\therefore \underline{\underline{\text{Mean}}} = E(x) = p$$

$$\text{Variance} = V(x)$$

$$= E\left(x - E(x)\right)^2$$

$$= E\left(x - p\right)^2 \quad \left[\because E(x) = p\right]$$

$$= \sum_{x=0}^{1} (x-p)^2 f(x) \quad \left[\because E(x) = \sum_{x} x f(x)\right]$$

$$= \sum_{x=0}^{1} (x-p)^2\, p^x\, q^{1-x}$$

$$= (0-p)^2 \times p^0 \times q^{1-0} + (1-p)^2 \times p^1 \times q^{1-1}$$

$$= (-p)^2 \times 1 \times q + q^2 \times p \times q^0 \quad \left[\because q = 1-p\right]$$

$$= p^2 q + q^2 p \times 1$$

$$= p^2 q + q^2 p$$

$$= pq\,(p+q)$$

$$= pq \times 1 \quad \left[\because p+q = 1\right]$$

$$= pq$$

$$\therefore \text{Variance} = V(x) = pq$$

$$SD = \sqrt{V(x)} = \sqrt{pq}$$

Relationship between Mean & Variance of Bernoulli Distribution

We know that $p > pq$ $[\because q < 1]$

ie, Mean > Variance

Properties of Bernoulli Distribution

1. It is a discrete distribution
2. It's probability mass function is,
   $P(X = x) = f(x) = p^x q^{1-x}$, $x = 0, 1$, $p > 0$ & $q = 1 - p$
3. Mean $= p$
4. Variance $= pq$ & $SD = \sqrt{pq}$
5. Mean > Variance
6. Parameter is p

1. If X is a Bernoulli variate taking values 1 or 0 with probabilities 0·6 & 0·4 respectively then find mean & variance.

Ans p = probability of success $= 0.6$

q = probability of failure $= 0.4$

Mean $= p = 0.6$

Variance $= pq = 0.6 \times 0.4 = 0.24$

2. X follows Bernoullian distribution with parameter 0·8. Find mean & variance of x. Also, find mean & variance of $2X+3$.

Ans Parameter $= p = 0.8 \Rightarrow q = 1 - p = 1 - 0.8 = 0.2$

Mean $= p = 0.8$

Variance $= pq = 0.8 \times 0.2 = 0.16$

Mean of $2X+3 = E(2X+3)$

$= E(2X) + E(3)$ $[E(X+Y) = E(X) + E(Y)]$

$= 2E(X) + 3$ $[\because E(aX) = aE(X)$, $E(a) = a]$

$= 2 \times 0.8 + 3$ $[\because E(X) = p = 0.8]$

$= 1.6 + 3 = 4.6$

Variance of $2X+3 = V(2X+3)$

$= V(2X) + V(3)$ $[\because V(X+Y) = V(X) + V(Y)]$

$= 2^2 V(X) + 0$ $[\because V(aX) = a^2 V(X)$ & $V(a) = 0]$

$= 4 \times 0.16$

$= 0.64$

3. Write down the probability function with its range of a Bernoulli's distribution with parameter 0.35

Ans $p = 0.35 \Rightarrow q = 1 - p = 1 - 0.35 = 0.65$

Probability function is,

$$P(X = x) = f(x) = p^x q^{1-x}, \quad x = 0, 1$$

$$= (0.35)^x \times (0.65)^{1-x}$$

HW

4. If $p = 0.4$, find mean & SD

5. If $p = 0.6$, write probability function

Binomial Distribution

A discrete random variable X is said to follow Binomial Distribution with parameters n & p if its probability mass function is,

$$P(X=x) = f(x) = {}^{n}C_{x}\, p^{x} q^{n-x}, \text{ where}$$

$x = 0, 1, 2, \ldots, n$

p = probability of success, $p > 0$

q = probability of failure, $q = 1-p$

x = No. of successes

Mean of Binomial Distribution

$$\therefore \text{Mean} = E(x)$$

$$= \sum_{x=0}^{n} x\, f(x) \qquad \left[\because E(x) = \sum_{x} x f(x)\right]$$

$$= \sum_{x=0}^{n} x\; {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!\, x!} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{(n-1)!\, n}{(n-x)!\,(x-1)!\, x} p^{x} q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{(n-1)!\, n}{(n-x)!\,(x-1)!}\, p \times p^{x-1} \cdot q^{n-x}$$

$$= np \sum_{x=0}^{n} \frac{(n-1)!}{(n-x)!\,(x-1)!} p^{x-1} q^{n-x}$$

$$= np \sum_{x=0}^{n} {}^{n-1}C_{x-1}\, p^{x-1} q^{n-x}$$

$$= np \sum_{x=0}^{n} {}^{n-1}C_{x-1}\, q^{n-x} p^{x-1}$$

$= np(q+p)^{n-1}$

$$\left[\because (q+p)^n = \sum_{x=0}^{n} {}^nC_x\, q^{n-x} p^x\right.$$

$$(q+p)^{n-1} = \sum_{x=0}^{n} {}^{n-1}C_{x-1}\, q^{n-1-(x-1)} p^{x-1}$$

$$(q+p)^{n-1} = \sum_{x=0}^{n} {}^{n-1}C_{x-1}\, q^{n-1-x+1} p^{x-1}$$

$$\left.(q+p)^{n-1} = \sum_{x=0}^{n} {}^{n+1}C_{x-1}\, q^{n-x} p^{x-1}\right]$$

$= np \times (1)^{n-1}$ $\quad [\because q+p=1]$

$= np \times 1$

$= \underline{\underline{np}}$

$\therefore$ $\underline{\underline{\text{Mean}}} = np$

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Variance of Binomial Distribution

$$V(X) = E(X^2) - [E(X)]^2 \quad \text{---(1)}$$

$$E(X^2) = \sum_{x=0}^{n} x^2 f(x) \quad \left[\because E(X^2) = \sum_{x} x^2 f(x)\right]$$

$$= \sum_{x=0}^{n} [x(x-1)+x] f(x)$$

$$\left[\because x(x-1)+x = x^2 - x + x = x^2\right]$$

$$= \sum_{x=0}^{n} x(x-1) f(x) + \sum_{x=0}^{n} x f(x)$$

$$= \sum_{x=0}^{n} x(x-1) f(x) + np \quad \left[\because \sum_{x=0}^{n} x f(x) = np\right]$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^{n}C_{x}\, p^{x} q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{n!}{(n-x)!\,x!} p^{x} q^{n-x} + np$$

$$= \sum_{x=0}^{n} x(x-1) \frac{(n-2)!\,(n-1)\,n}{(n-x)!\,(x-2)!\,(x-1)\,x} \times p^2 \times p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=0}^{n} \frac{(n-2)!}{(n-x)!\,(x-2)!} p^{x-2} q^{n-x} + np$$

$$= n(n-1)p^2 \sum_{x=0}^{n} \frac{(n-2)!}{(n-x)!\,(x-2)!} q^{n-x} p^{x-2} + np$$

$$= n(n-1)p^2 \sum_{x=0}^{n} {}^{n-2}C_{x-2}\, q^{n-x} p^{x-2} + np$$

$$= n(n-1)p^2 \times (q+p)^{n-2} + np$$

$$\left[\because (q+p)^{n-2} = \sum_{x=0}^{n} {}^{n-2}C_{x-2}\, q^{n-x} p^{x-2}\right]$$

$$= n(n-1)p^2 \times (1)^{n-2} + np \qquad [\because q+p=1]$$

$= n(n-1)p^2 \times 1 + np$

$= n(n-1)p^2 + np$ — (2)

$[E(x)]^2 = (np)^2 = n^2p^2$ — (3)

Substituting (2) & (3) in (1)

$V(x) = E(x^2) - [E(x)]^2$

$= n(n-1)p^2 + np - n^2p^2$

$= (n^2 - n)p^2 + np - n^2p^2$

$= n^2p^2 - np^2 + np - n^2p^2$

$= -np^2 + np$

$= np(-p+1) = np(1-p)$

$= npq$ $[\because q = 1-p]$

$\therefore$ Variance $= npq \Rightarrow SD = \sqrt{V(x)}$

$= \sqrt{npq}$

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Moment Generating Function of Binomial Distribution

Moment generating function, $m \cdot g \cdot f = M_x(t)$

$$= E(e^{tx})$$

$$= \sum_{x=0}^{n} e^{tx} f(x) \quad \left[\because E(x) = \sum_x x f(x)\right]$$

$$= \sum_{x=0}^{n} e^{tx} \, {}^nC_x \, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x \, p^x e^{tx} q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x \, (pe^t)^x q^{n-x}$$

$$= \sum_{x=0}^{n} {}^nC_x \, q^{n-x} (pe^t)^x$$

$$= (q+pe^t)^n$$

$$\left[\because (a+b)^n = \sum_x {}^nC_x \, a^{n-x} b^x\right]$$

$$\therefore m \cdot g \cdot f = (q+pe^t)^n$$

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Importance of Binomial distribution.

The Binomial distribution is often very useful in decision making situations in business. In quality control it is very widely applied. In acceptance sampling plans, inspection is carried out on the articles drawn in a sample. The Binomial Distribution is used in such a sampling. The Binomial Distribution describes an enormous varieties of real life events. The distribution can be used to judge whether a coin or a die is unbiased or not by comparing the observed frequencies and expected frequencies.

Relationship between Mean & Variance of Binomial Distribution

We know that $np > npq$ $\left[\because q < 1\right]$

ie, Mean $>$ Variance

Properties of Binomial Distribution

1. It is a discrete probability distribution
2. Its probability mass function is
$$P(X=x) = f(x) = {}^nC_x \, p^x q^{n-x}, \quad x = 0, 1, \ldots, n$$
3. Mean $= np$
4. Variance $= npq$, $SD = \sqrt{npq}$
5. Mean $>$ Variance
6. Mean of binomial distribution increases as n increases with p remaining constant

1. Four coins are tossed simultaneously. What is the probability of getting 2 heads?

Ans n = No. of coins tossed = 4

$p = P$ (getting head in a single trial) $= \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

x = No. of heads to be obtained = 2

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x} = {}^{4}C_{x} \times \left(\frac{1}{2}\right)^{x} \times \left(\frac{1}{2}\right)^{4-x}$$

P(getting 2 heads) $= P(X=2) = {}^{4}C_{2} \times \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{4-2}$

$$= \frac{4!}{(4-2)! \times 2!} \times \frac{1}{4} \times \left(\frac{1}{2}\right)^{2}$$

$$= \frac{4!}{2! \times 2!} \times \frac{1}{4} \times \frac{1}{4}$$

$$= \frac{2! \times 3 \times 4}{2! \times 2!} \times \frac{1}{16}$$

$$= \frac{3 \times 4}{2!} \times \frac{1}{16}$$

$$= \frac{12}{1\times 2} \times \frac{1}{16} = \frac{3}{8}$$

2. Eight unbiased coins were tossed simultaneously. Find the probability of getting

(i) exactly 4 heads

(ii) no heads at all

(iii) 6 or more heads

(iv) almost 2 heads

(v) no. of heads ranging from 3 to 5

Ans n = No. of coins tossed $= 8$

$p = P(\text{getting head in a single trial}) = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

x = No. of heads obtained

$$P(X=x) = {}^nC_x\, p^x q^{n-x} = {}^8C_x \times \left(\frac{1}{2}\right)^x \times \left(\frac{1}{2}\right)^{8-x}$$

(i) $P(\text{getting exactly 4 heads}) = P(X=4)$

$$P(X=4) = {}^8C_4 \times \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{8-4}$$

$$= \frac{8!}{(8-4)! \times 4!} \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^4$$

$$= \frac{4! \times 5 \times 6 \times 7 \times 8}{4! \times 4!} \times \left(\frac{1}{2}\right)^8$$

$$= \frac{5 \times 6 \times 7 \times 8}{4!} \times \frac{1}{256}$$

$$= \frac{5 \times 6 \times 7 \times 8}{1 \times 2 \times 3 \times 4} \times \frac{1}{256}$$

$$= \frac{70}{256}$$

(ii) $P(\text{getting no heads}) = P(x=0)$

$$P(x=0) = {}^8C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^{8-0}$$

$$= \frac{8!}{(8-0)! \times 0!} \times 1 \times \left(\frac{1}{2}\right)^8$$

$$= \frac{8!}{8! \times 0!} \times \frac{1}{256}$$

$$= \frac{1}{1} \times \frac{1}{256} \quad \left[\because 0! = 1\right]$$

$$= \frac{1}{256}$$

(iii)

(iii) $P(\text{6 or more heads}) = P(X \geq 6)$

$= P(X = 6 \text{ or } 7 \text{ or } 8)$

$= P(X=6) + P(X=7) + P(X=8)$

$P(X=6) = {}^8C_6 \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^{8-6}$

$= \frac{8!}{(8-6)! \times 6!} \times \frac{1}{64} \times \left(\frac{1}{2}\right)^2$

$= \frac{8!}{2! \times 6!} \times \frac{1}{64} \times \frac{1}{4}$

$= 28 \times \frac{1}{256} = \frac{28}{256}$

$P(X=7) = {}^8C_7 \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^{8-7}$

$= \frac{8!}{(8-7)! \times 7!} \times \frac{1}{128} \times \frac{1}{2}$

$= \frac{8!}{1! \times 7!} \times \frac{1}{256}$

$= \frac{8}{256}$

$$P(x=8) = {}^8C_8 \times \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^{8-8}$$

$$= \frac{8!}{(8-8)! \times 8!} \times \frac{1}{256} \times \left(\frac{1}{2}\right)^0$$

$$= \frac{8!}{0! \times 8!} \times \frac{1}{256} \times 1$$

$$= 1 \times \frac{1}{256} = \frac{1}{256}$$

$\therefore$ P (getting 6 or more heads)

$$= P(x=6) + P(x=7) + P(x=8)$$

$$= \frac{28}{256} + \frac{8}{256} + \frac{1}{256}$$

$$= \frac{37}{256}$$

(iv) P (getting atmost 2 heads) $= P(x \leq 2)$

$$= P(x = 0 \text{ or } 1 \text{ or } 2)$$

$$= P(x=0) + P(x=1) + P(x=2)$$

$$P(X=0) = {}^{8}C_{0} \times \left(\frac{1}{2}\right)^{0} \times \left(\frac{1}{2}\right)^{8-0}$$

$$= \frac{8!}{(8-0)! \times 0!} \times 1 \times \left(\frac{1}{2}\right)^{8}$$

$$= 1 \times 1 \times \frac{1}{256} = \underline{\underline{\frac{1}{256}}}$$

$$P(X=1) = {}^{8}C_{1} \left(\frac{1}{2}\right)^{1} \times \left(\frac{1}{2}\right)^{8-1}$$

$$= \frac{8!}{(8-1)! \times 1!} \times \frac{1}{2} \times \left(\frac{1}{2}\right)^{7}$$

$$= 8 \times \frac{1}{2} \times \frac{1}{128} = \underline{\underline{\frac{8}{256}}}$$

$$P(X=2) = {}^{8}C_{2} \times \left(\frac{1}{2}\right)^{2} \times \left(\frac{1}{2}\right)^{8-2}$$

$$= \frac{8!}{(8-2)! \times 2!} \times \frac{1}{4} \times \left(\frac{1}{2}\right)^{6}$$

$$= 28 \times \frac{1}{4} \times \frac{1}{64} = \underline{\underline{\frac{28}{256}}}$$

$$P(\text{getting atmost 2 heads}) = P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{1}{256} + \frac{8}{256} + \frac{28}{256} = \underline{\underline{\frac{37}{256}}}$$

(v) P(getting number of heads ranging from 3 to 5) = P(X = 3 or 4 or 5)

$$= P(X=3) + P(X=4) + P(X=5)$$

$$P(X=3) = {}^{8}C_{3} \times \left(\frac{1}{2}\right)^{3} \times \left(\frac{1}{2}\right)^{8-3}$$

$$= \frac{8!}{(8-3)! \times 3!} \times \frac{1}{8} \times \left(\frac{1}{2}\right)^{5}$$

$$= 56 \times \frac{1}{8} \times \frac{1}{32} = \frac{56}{256}$$

$$P(X=4) = {}^{8}C_{4} \times \left(\frac{1}{2}\right)^{4} \times \left(\frac{1}{2}\right)^{8-4}$$

$$= \frac{8!}{(8-4)! \times 4!} \times \frac{1}{16} \times \left(\frac{1}{2}\right)^{4}$$

$$= 70 \times \frac{1}{16} \times \frac{1}{16}$$

$$= \frac{70}{256}$$

$$P(X=5) = {}^8C_5 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^{8-5}$$

$$= \frac{8!}{(8-5)! \times 5!} \times \frac{1}{32} \times \left(\frac{1}{2}\right)^3$$

$$= 56 \times \frac{1}{32} \times \frac{1}{8} = \frac{56}{256}$$

$\therefore$ P(getting no. of heads ranging from 3 to 5)

$$= P(X=3) + P(X=4) + P(X=5)$$

$$= \frac{56}{256} + \frac{70}{256} + \frac{56}{256} = \frac{182}{256}$$

3. For a Binomial distribution mean $= 4$ & variance $= \frac{12}{9}$. Write all the terms of the distribution

Ans Mean $= np = 4$ — ①

Variance $= npq = \frac{12}{9}$ — ②

Dividing ② by ①,

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$$\frac{npq}{np} = \frac{\frac{12}{9}}{4}$$

$$q = \frac{12}{9} \times \frac{1}{4} = \frac{1}{3}$$

$$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$$

$$n = \frac{np}{p} = \frac{4}{\frac{2}{3}} = 4 \times \frac{3}{2} = 6$$

Since $n = 6$, $x = 0, 1, 2, 3, 4, 5, 6$.

$$P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x} = {}^{6}C_{x} \times \left(\frac{2}{3}\right)^{x} \times \left(\frac{1}{3}\right)^{6-x}$$

$$P(X = 0) = {}^{6}C_{0} \times \left(\frac{2}{3}\right)^{0} \times \left(\frac{1}{3}\right)^{6-0}$$

$$= \frac{6!}{(6-0)! \times 0!} \times 1 \times \left(\frac{1}{3}\right)^{6}$$

$$= 1 \times 1 \times \frac{1}{729} = \frac{1}{729}$$

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$P(X=1) = {}^6C_1 \times \left(\frac{2}{3}\right)^1 \times \left(\frac{1}{3}\right)^{6-1}$

$= \frac{6!}{(6-1)! \times 1!} \times \frac{2}{3} \times \left(\frac{1}{3}\right)^5$

$= 6 \times \frac{2}{3} \times \frac{1}{243} = \frac{12}{729}$

$P(X=2) = {}^6C_2 \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^{6-2}$

$= \frac{6!}{(6-2)! \times 2!} \times \frac{4}{9} \times \left(\frac{1}{3}\right)^4$

$= 15 \times \frac{4}{9} \times \frac{1}{81} = \frac{60}{729}$

$P(X=3) = {}^6C_3 \times \left(\frac{2}{3}\right)^3 \times \left(\frac{1}{3}\right)^{6-3}$

$= \frac{6!}{(6-3)! \times 3!} \times \frac{8}{27} \times \left(\frac{1}{3}\right)^3$

$= 20 \times \frac{8}{27} \times \frac{1}{27} = \frac{160}{729}$

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$$P(x=4) = {}^6C_4 \times \left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^{6-4}$$

$$= \frac{6!}{(6-4)! \times 4!} \times \frac{16}{81} \times \left(\frac{1}{3}\right)^2$$

$$= 15 \times \frac{16}{81} \times \frac{1}{9} = \frac{240}{729}$$

$$P(x=5) = {}^6C_5 \times \left(\frac{2}{3}\right)^5 \times \left(\frac{1}{3}\right)^{6-5}$$

$$= \frac{6!}{(6-5)! \times 5!} \times \frac{32}{243} \times \frac{1}{3}$$

$$= 6 \times \frac{32}{243} \times \frac{1}{3} = \frac{192}{729}$$

$$P(x=6) = {}^6C_6 \times \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{3}\right)^{6-6}$$

$$= \frac{6!}{(6-6)! \times 6!} \times \frac{64}{729} \times \left(\frac{1}{3}\right)^0$$

$$= 1 \times \frac{64}{729} \times 1 = \frac{64}{729}$$

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4. Consider families with 4 children each. What percentage of families would you expect to have

a) 2 boys & 2 girls

b) atleast one boy

c) no girls

d) at most 2 girls

Ans. $P(\text{getting a boy}) = p = \frac{1}{2}$

$P(\text{getting a girl}) = q = 1-p = 1-\frac{1}{2} = \frac{1}{2}$

$n = 4$, x = No. of boys

$P(X=x) = {}^nC_x\, p^x q^{n-x}$

$= {}^4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

a) $P(\text{getting 2 boys \& 2 girls}) = P(\text{getting 2 boys}) = P(X=2)$

$P(X=2) = {}^4C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{4-2}$

$= \frac{4!}{(4-2)! \times 2!} \times \frac{1}{4} \times \left(\frac{1}{2}\right)^2$

$6 \times \frac{1}{4} \times \frac{1}{4} = \frac{3}{8}$

$\therefore$ Percentage of families with 2 boys & 2 girls $= \frac{3}{8} \times 100 = 0.375 \times 100$

$= \underline{37.5\%}$

b) $P(\text{atleast 1 boy}) = P(X \geq 1)$

$= P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$P(X=1) = {}^4C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^{4-1}$

$= \frac{4!}{(4-1)! \times 1!} \times \frac{1}{2} \times \left(\frac{1}{2}\right)^3$

$= 4 \times \frac{1}{2} \times \frac{1}{8} = \underline{\frac{4}{16}}$

$P(X=2) = {}^4C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{4-2}$

$= \frac{4!}{(4-2)! \times 2!} \times \frac{1}{4} \times \left(\frac{1}{2}\right)^2$

$= \frac{4!}{2! \times 2!} \times \frac{1}{4} \times \frac{1}{4}$

$= 6 \times \frac{1}{16} = \underline{\frac{6}{16}}$

$$P(X=3) = {}^4C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^{4-3}$$

$$= 4 \times \frac{1}{16} = \frac{4}{16}$$

$$P(X=4) = {}^4C_4 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^{4-4}$$

$$= \frac{1}{16}$$

$$P(\text{atleast 1 boy}) = \frac{4}{16} + \frac{6}{16} + \frac{4}{16} + \frac{1}{16} = \frac{15}{16}$$

∴ Percentage of families with atleast 1 boy

$$= \frac{15}{16} \times 100 = 0.9375 \times 100$$

$$= 93.75\ \%$$

c) $P(\text{getting no girls}) = P(\text{getting 4 boys})$

$$= P(X=4)$$

$$= {}^4C_4 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^{4-4}$$

$$= 1 \times \frac{1}{16} \times 1 = \frac{1}{16}$$

∴ Percentage of families with no-girls

$$= \frac{1}{16} \times 100 = 0.0625 \times 100 = 6.25\%$$

d) P (getting at most 2 girls)

$= P$ (getting 0 or 1 or 2 girls)

$= P$ (getting 4 or 3 or 2 boys)

$= P(X=4) + P(X=3) + P(X=2)$

$= \frac{1}{16} + \frac{4}{16} + \frac{6}{16} = \underline{\underline{\frac{11}{16}}}$

Percentage of families with atmost 2 girls $= \frac{11}{16} \times 100 = 0.6875 \times 100$

$= \underline{\underline{68.75\ \%}}$

5. ~~Q~~ 4 dice are thrown 162 times. The occurrence of 2 or 3 is considered a success. In how many throws do you expect

(i) exactly 2 success

(ii) atleast 1 success

<u>Ans</u> $n = 4$, $N = 162$, $x =$ No. of success

$p = P$ (getting 2 or 3) $= \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

$= \underline{\underline{\frac{1}{3}}}$

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

Frequency = Probability × N

$P(X = x) = {}^{n}C_{x}\, p^{x} q^{n-x}$

$= {}^{4}C_{x} \times \left(\frac{1}{3}\right)^{x} \times \left(\frac{2}{3}\right)^{4-x}$

(i) P(getting exactly 2 success) $= P(X = 2)$

$= {}^{4}C_{2} \times \left(\frac{1}{3}\right)^{2} \times \left(\frac{2}{3}\right)^{4-2}$

$= 6 \times \frac{1}{9} \times \left(\frac{2}{3}\right)^{2}$

$= 6 \times \frac{1}{9} \times \frac{4}{9} = \frac{24}{81}$

No. of throws in which exactly 2 success occur = Probability × N $= \frac{24}{81} \times 162 = 48$

(ii) P(atleast one success) = 1 − P(no success)

P(no success) = $P(X=0)$

$= {}^4C_0 \times \left(\frac{1}{3}\right)^0 \times \left(\frac{2}{3}\right)^{4-0}$

$= 1 \times 1 \times \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

∴ P(atleast one success) = 1 − P(no success)

$= 1 - \frac{16}{81}$

$= \frac{65}{81}$

∴ No. of throws in which atleast 1 success occur $= \frac{65}{81} \times 162 = 130$

6. In a Binomial distribution consisting of 5 independent trials, 1^{st} & 2^{nd} terms are 0.4096 & 0.2048 respectively. Find p

Ans $n = 5$

$$P(X=x) = {}^nC_x\, p^x\, q^{n-x} = {}^5C_x\, p^x\, q^{5-x}$$

1^{st} term $= P(X=0) = 0.4096$

$${}^5C_0 \times p^0 \times q^{5-0} = 0.4096$$

$$1 \times 1 \times q^5 = 0.4096 \Rightarrow \underline{q^5 = 0.4096} \quad -(1)$$

2^{nd} term $= P(X=1) = 0.2048$

$${}^5C_1 \times p^1 \times q^{5-1} = 0.2048$$

$$\underline{5 \times p \times q^4 = 0.2048} \quad -(2)$$

Dividing (1) by (2),

$$\frac{q^5}{5 \times p \times q^4} = \frac{0.4096}{0.2048}$$

$$\frac{q}{5 \times p} = 2$$

$\Rightarrow q = 2 \times 5 \times p \Rightarrow q = \underline{\underline{10p}}$

We know, $q = 1 - p$

$\therefore 1 - p = 10p$

$\Rightarrow 1 = 10p + p \Rightarrow 1 = 11p$

$\Rightarrow 11p = 1 \Rightarrow p = \underline{\underline{\frac{1}{11}}}$

For a Binomial distribution with $n=6$, the 3rd term is 9 times the 5th term. Find p

Ans $n = 6$

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x} = {}^{6}C_{x}\, p^{x} q^{6-x}$$

$$3^{rd} \text{ term} = P(X=2) = {}^{6}C_{2} \times p^{2} \times q^{6-2}$$

$$= 15 \times p^{2} \times q^{4}$$

$$5^{th} \text{ term} = P(X=4) = {}^{6}C_{4} \times p^{4} \times q^{6-4}$$

$$= 15 \times p^{4} \times q^{2}$$

$$3^{rd} \text{ term} = 9 \times 5^{th} \text{ term}$$

$$15 \times p^2 \times q^4 = 9 \times 15 \times p^4 \times q^2$$

$$\frac{15 \times q^4}{15 \times q^2} = \frac{9 \times p^4}{p^2}$$

$$q^2 = 9p^2$$

$$\Rightarrow (1-p)^2 = 9p^2 \quad [\because q = 1-p]$$

$$\Rightarrow 1 - 2p + p^2 = 9p^2$$

$$\Rightarrow 9p^2 - p^2 + 2p - 1 = 0$$

$$\Rightarrow 8p^2 + 2p - 1 = 0$$

$$p = \frac{-2 \pm \sqrt{4 - 4 \times 8 \times -1}}{2 \times 8}$$

$$p = \frac{2 \pm \sqrt{4+32}}{16} = \frac{2 \pm \sqrt{36}}{16}$$

$$p = \frac{-2 \pm 6}{16}$$

$$p = \frac{-2+6}{16} = \frac{4}{16} = \frac{1}{4}$$

$$p = \frac{-2-6}{16} = \frac{-8}{16} = \frac{-1}{2}$$

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Since p cannot be -ve,

$$p = \frac{1}{4}$$

9. Find the Binomial distribution with mean 3 & variance 2

Ans Mean $= np = 3$ — ①

Variance $= npq = 2$ — ②

Dividing ② by ①,

$$\frac{npq}{np} = \frac{2}{3}$$

$$\Rightarrow q = \frac{2}{3}$$

$$p = 1 - q = 1 - \frac{2}{3} = \frac{3-2}{3} = \frac{1}{3}$$

$$np = 3 \Rightarrow n \times \frac{1}{3} = 3$$

$$\Rightarrow n = 3 \times 3 = 9$$

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

$$= {}^{9}C_{x} \left(\frac{1}{3}\right)^{x} \left(\frac{2}{3}\right)^{9-x}, \quad x = 0, 1, 2, \ldots, 9$$

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8. Bring out the fallacy in the following "The mean of a Binomial distribution is 5 & the SD is 3

Ans Mean $= np = 5$ — (1)

$SD = \sqrt{npq} = 3$

$\Rightarrow npq = \underline{\underline{(3)^2}} = 9$

$\Rightarrow npq = 9$

$\Rightarrow 5 \times q = 9$ $\boxed{\text{From (1)}}$

$\Rightarrow q = \frac{9}{5} = 1.8 > 1$

But, q cannot be greater than 1

∴ The $\underline{\underline{\text{statement}}}$ is wrong

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$\therefore \quad p \quad 4, \quad 2$

Fitting a Binomial Distribution

Theoretical

1. Determine the values of p and q and n and substitute them in the function $nC_x \, p^x \, q^{n-x}$, we get the probability function of the Binomial Distribution.
2. Put $x = 0, 1, 2.....$ in the function $nC_x \, p^x \, q^{n-x}$, we get $n + 1$ terms.
3. Multiply each such term by N (total frequency), to obtain the expected frequency.

Q1. Eight coins were tossed together 256 times. Find the expected frequencies of heads. Find mean & SD

Ans $n = 8,\ N = 256$

$p = P(\text{getting head in a toss}) = \frac{1}{2}$

$q = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

$P(X = x) = {}^nC_x\, p^x\, q^{n-x}$

$= {}^8C_x \left(\frac{1}{2}\right)^x \times \left(\frac{1}{2}\right)^{8-x},\quad x = 0, 1, 2, \dots, 8$

Expected frequency $= P(X) \times N$

No. of heads X	$P(X) = {}^8C_x \times \left(\frac{1}{2}\right)^x \times \left(\frac{1}{2}\right)^{8-x}$	Expected frequency $P(X) \times N$
0	${}^8C_0 \times \left(\frac{1}{2}\right)^0 \times \left(\frac{1}{2}\right)^{8-0} = \frac{1}{256}$	$\frac{1}{256} \times 256 = 1$
1	${}^8C_1 \times \left(\frac{1}{2}\right)^1 \times \left(\frac{1}{2}\right)^{8-1} = \frac{8}{256}$	$\frac{8}{256} \times 256 = 8$
2	${}^8C_2 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)^{8-2} = \frac{28}{256}$	28
3	${}^8C_3 \times \left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^{8-3} = \frac{56}{256}$	56
4	${}^8C_4 \times \left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^{8-4} = \frac{70}{256}$	70
5	${}^8C_5 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^{8-5} = \frac{56}{256}$	56
6	${}^8C_6 \times \left(\frac{1}{2}\right)^6 \times \left(\frac{1}{2}\right)^{8-6} = \frac{28}{256}$	28
7	${}^8C_7 \times \left(\frac{1}{2}\right)^7 \times \left(\frac{1}{2}\right)^{8-7} = \frac{8}{256}$	8
8	${}^8C_8 \times \left(\frac{1}{2}\right)^8 \times \left(\frac{1}{2}\right)^{8-8} = \frac{1}{256}$	1

$$\text{Mean} = np = 8 \times \frac{1}{2} = 4$$

$$SD = \sqrt{npq} = \sqrt{8 \times \frac{1}{2} \times \frac{1}{2}} = \sqrt{2} = \underline{\underline{1.414}}$$

Ex. 13: The following data show the number of seeds germinating out of 5 ib damp filter for 80 sets of seeds. Fit a binomial distribution of this data and find the expected frequencies.

x	**:**	**0**	**1**	**2**	**3**	**4**	**5**
f	**:**	**6**	**20**	**28**	**12**	**8**	**6**

Ans :

Since 'p' is not given find mean of the given data

x	f	fx
0	6	0
1	20	20
2	28	56
3	12	36
4	8	32
5	6	30
	80	174

2. (Continuation) $\rightarrow$ $\Sigma f = 80$, $\Sigma fx = 174$, $n = 5$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{174}{80} = 2.175, \quad N = \Sigma f = 80$$

But, Mean $= np$

$\Rightarrow 2.175 = np$

$\Rightarrow 2.175 = 5 \times p$

$$\Rightarrow p = \frac{2.175}{5} = 0.435$$

$$q = 1 - p = 1 - 0.435 = 0.565$$

$$P(X = x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$$

$$= {}^{5}C_{x} \times (0.435)^{x} \times (0.565)^{5-x}, \quad x = 0, 1, 2, 3, 4, 5$$

Expected frequency $= P(X=x) \times N$

x	$P(x) = {}^5C_x (0.435)^x \times (0.565)^{10-x}$	Expected frequency $P(x) \times 80$
0	${}^5C_0 \times (0.435)^0 \times (0.565)^{10-0} = 0.0625$	5
1	${}^5C_1 \times (0.435)^1 \times (0.565)^{10-1} = 0.225$	18
2	${}^5C_2 \times (0.435)^2 \times (0.565)^{10-2} = 0.3375$	27
3	${}^5C_3 \times (0.435)^3 \times (0.565)^{10-3} = 0.2625$	21
4	${}^5C_4 \times (0.435)^4 \times (0.565)^{10-4} = 0.075$	6
5	${}^5C_5 \times (0.435)^5 \times (0.565)^{10-5} = 0.0875$	7

Situations when binomial distributions can be applied:-

- The experiment is repeated finite no of times.
- The random experiment has two outcomes Success and failure.
- Trials are independent
- Probability for success in a single trial remains constant from trial to trial.

Q)

Poisson Distribution

A discrete random variable X is said to follow Poisson distribution if its probability mass function is,

$$P(X=x) = f(x) = \frac{e^{-m} \times m^{x}}{x!}, \text{ where}$$

$x = 0, 1, 2, \ldots$ with parameter m, where

$m = \text{mean} = np$

Poisson Distribution as a limiting case of Binomial Distribution

Poisson distribution may be obtained as a limiting case of Binomial distribution under the following conditions:

1. the number of trials is very large (ie, $n \to \infty$)
2. the probability of success for each trial is very small (ie, $p \to 0$)
3. $m = np$ is finite

Uses or Importance of Poisson Distribution

1. Poisson distribution can be used to count the number of road accidents taking place in a year

2. It is used to count the number of defects per unit of a manufactured product

3. It is used to count the number of customers arriving at a super market

4. It is used to count the number of telephone calls arriving at a telephone switch board in unit time.

5. It is used to count the number of casualties due to a rare disease in a year.

Mean of Poisson Distribution

$$\text{Mean} = E(x) = \sum x f(x)$$

$$= \sum_{x=0}^{\infty} x \times \frac{e^{-m} \times m^{x}}{x!}$$

$$= \sum_{x=0}^{\infty} x \times \frac{e^{-m} \times m^{x}}{(x-1)! \times x}$$

$$= \sum_{x=0}^{\infty} \cancel{x} \times \frac{e^{-m} \times m^{x}}{(x-1)! \times \cancel{x}}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} \times m^{x}}{(x-1)!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} \times m \times m^{x-1}}{(x-1)!}$$

$$= e^{-m} \times m \sum_{x=0}^{\infty} \frac{m^{x-1}}{(x-1)!}$$

$$= e^{-m} \times m \times e^{m} \qquad \left[\because e^{m} = \sum_{x=0}^{\infty} \frac{m^{x}}{x!}\right]$$

$$= m \times e^{-m} \times e^{m}$$

$$= m \times e^{-m+m}$$

$$= m \times e^{0}$$

$$= m \times 1$$

$$= m$$

$\therefore$ Mean $= m$

Variance of Poisson Distribution

$$Var(x) = E(x^2) - [E(x)]^2 \quad \text{— (1)}$$

$$E(x^2) = \sum x^2 f(x)$$

$$= \sum_{x=0}^{\infty} x^2 f(x)$$

$$= \sum_{x=0}^{\infty} [x(x-1)+x] f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) f(x) + \sum_{x=0}^{\infty} x f(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) f(x) + E(x)$$

$$= \sum_{x=0}^{\infty} x(x-1) f(x) + m \qquad [\because E(x) = m]$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m} \times m^x}{x!} + m$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-m} \times m^x}{(x-2)! \times (x-1) \times x} + m$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} \times m^x}{(x-2)!} + m$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} \times m^2 \times m^{x-2}}{(x-2)!} + m$$

$$= e^{-m} \times m^2 \sum_{x=0}^{\infty} \frac{m^{x-2}}{(x-2)!} + m$$

$= e^{-m} \times m^2 \times e^m + m$ $\left[\because \sum_{x=0}^{\infty} \frac{m^x}{x!} = e^m\right]$

$= e^{-m} \times e^m \times m^2 + m$

$= e^{-m+m} \times m^2 + m$

$= e^0 \times m^2 + m$

$= 1 \times m^2 + m = \underline{\underline{m^2 + m}}$

$\therefore E(x^2) = \underline{\underline{m^2 + m}}$

Substituting in (1),

$Var(x) = m^2 + m - (m)^2$ $[\because E(x) = m]$

$= m^2 + m - m^2$

$= \underline{\underline{m}}$

$\therefore \underline{\underline{Var(x) = m}}$

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Moment Generating Function of Poisson Distribution

$$M_x(t) = E(e^{tx})$$

$$= \sum e^{tx} f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-m} \times m^x}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} \times (e^{tx} \times m^x)}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-m} \times (e^t m)^x}{x!}$$

$$= e^{-m} \sum_{x=0}^{\infty} \frac{(e^t m)^x}{x!}$$

$$= e^{-m} \times e^{e^t m} \quad \left[\because \sum_{x=0}^{\infty} \frac{m^x}{x!} = e^m\right]$$

$$= e^{-m+e^t m}$$

$$= e^{e^t m - m} = e^{m(e^t - 1)}$$

Mode of poisson distribution

when m is an integer, m and $m-1$ are the mode. Now m not an integer, mode is a integeal part of m.

eg: when $m = 2$

mode = 2 and 1

when $m = 1.2$

mode = 1

Properties of Poisson Distribution

1. It is a discrete probability distribution
2. Its probability mass function is,

$$P(X=x) = f(x) = \frac{e^{-m} \times m^{x}}{x!},$$ where

$m = np$ & $x = 0, 1, 2, \ldots$

3. Mean $= m = np$
4. Variance $= m$ & $SD = \sqrt{m}$
5. Mean = Variance
6. It has only one parameter, m

1. If mean of Poisson distribution is 1.5. Find mode & SD

Ans

Mean $= m = 1.5$

Mode = Integer part of m

$= \underline{\underline{1}}$

$SD = \sqrt{m} = \sqrt{1.5} = \underline{\underline{1.22}}$

2. Comment on the following

For a Poisson distribution mean = 8 & variance = 7

Ans For a Poisson distribution,

Mean = Variance

But, here Mean ≠ Variance

∴ The given distribution is not Poisson

Note

$$e^{-m} = \frac{1}{\text{antilog}\,(m \times \log e)}$$

$$e^{-m} = \frac{1}{\text{antilog}\,(m \times 0.4343)}$$

3. If $n = 10$, $p = 0.1$, find $P(X=2)$ by both Binomial & Poisson distributions & S.T PD is an approximation of BD

Ans $n = 10$, $p = 0.1$

$q = 1 - p = 1 - 0.1 = \underline{0.9}$

For BD, $P(X=x) = {}^{n}C_{x}\, p^{x}\, q^{n-x}$

$$P(X=2) = {}^{10}C_{2} \times (0.1)^{2} \times (0.9)^{10-2}$$

$$= 45 \times 0.01 \times 0.4305$$

$$= \underline{0.19}$$

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For PD, $P(X=x) = \dfrac{e^{-m} \times m^x}{x!}$

$m = np = 10 \times 0.1 = \underline{1}$

$P(X=2) = \dfrac{e^{-1} \times (1)^2}{2!}$.

$e^{-m} = \dfrac{1}{\text{antilog}(m \times 0.4343)} = \dfrac{1}{\text{antilog}(1 \times 0.4343)}$

$= \dfrac{1}{\text{antilog}(0.4343)} = \dfrac{1}{2.718}$

$= \underline{\underline{0.3679}}$

$P(X=2) = \dfrac{0.3679 \times 1}{2} = \underline{\underline{0.183}}$

So, PD is an <u>approximation</u> of BD

Antilog (0·4343)

Consider decimal part ·4343

Consider ·43 in the left column of the antilog table & move horizontally to the right to the column headed by 4 & we get 2716. Again move to right to the mean difference column headed by 3 & we get 2.

Now, $2716 + 2 = 2718$

$\therefore$ Antilog $(0{\cdot}4343) = \underline{\underline{2{\cdot}718}}$

4. If a random variable X follows PD such that $P(X=1) = P(X=2)$. Find $P(X=0)$

Ans $P(x=x) = \dfrac{e^{-m} \times m^{x}}{x!}$

$P(x=1) = \dfrac{e^{-m} \times m^{1}}{1!} = e^{-m} \times m$

$P(x=2) = \dfrac{e^{-m} \times m^{2}}{2!} = \dfrac{e^{-m} \times m^{2}}{2}$

$P(x=1) = P(x=2)$

$\Rightarrow e^{-m} \times m = \dfrac{e^{-m} \times m^{2}}{2}$

$\Rightarrow 2 = \dfrac{e^{-m} \times m^{2}}{e^{-m} \times m}$

$\Rightarrow 2 = m$

$\therefore m = 2$

$P(x=0) = \dfrac{e^{-m} \times m^{0}}{0!} = \dfrac{e^{-2} \times 1}{1}$ $[\because m=2]$

e^{-2}

$$= \frac{1}{\text{antilog}\,(m \times 0.4343)} = \frac{1}{\text{antilog}\,(2 \times 0.4343)}$$

$$= \frac{1}{\text{antilog}\,(0.8686)} = \frac{1}{7.389} = 0.1355$$

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Differences 1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	2
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	3	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	6	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

Antilog 2.4645 = 291.4

Antilog 1.4645 = 29.14

Antilog 0.4645 = 2.914

Anti log $\bar{1}.4645$ = .2914

Anti log $\bar{2}.4645$ = .02914

Antilog $\bar{3}.4645$ = .002914

ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
											MEAN DIFFERENCES								
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	6	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	7	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	10	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	11	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	6	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	12	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	12	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	13	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	14	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	21

5. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly 5 bulbs are defective

Ans

$n = 100$.

p = probability of a defective bulb

$= 3\% = \frac{3}{100} = \underline{\underline{0.03}}$

Since p is small & n is large, we use Poisson distribution.

$m = np = 100 \times 0.03 = \underline{\underline{3}}$

$$P(X=x) = \frac{e^{-m} \times m^{x}}{x!} = \frac{e^{-3} \times 3^{x}}{x!}$$

$$P(X=5) = \frac{e^{-3} \times 3^{5}}{5!}$$

$$e^{-3} = \frac{1}{\text{antilog}(3 \times 0.4343)} = \frac{1}{\text{antilog}(1.3029)}$$

$$= \frac{1}{20.08} = \underline{\underline{0.0498}}$$

$$P(X=5) = \frac{0.0498 \times 243}{120} = \underline{\underline{0.1008}}$$

$P(5 \text{ bulbs are defective}) = P(x=5) = P_5 = \frac{\quad}{5!} = .0498 \times \frac{\quad}{120} = \underline{\underline{0.1008}}$

Ex. 2: It is known from past experience that in a certain plant there are on the average 4 industrial accidents per month. Find the probability that in a given year there will be less than 3 accidents. Assume poisson distribution.

Ans: $m = 4$ [Here n and p are not seperately given. But 'm' is di-

6. Ans $m = \text{Mean} = 4$

$$P(X=x) = \frac{e^{-m} \times m^{x}}{x!} = \frac{e^{-4} \times 4^{x}}{x!}$$

Probability that there will be less than 3 accidents $= P(X<3)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= \frac{e^{-4} \times 4^{0}}{0!} + \frac{e^{-4} \times 4^{1}}{1!} + \frac{e^{-4} \times 4^{2}}{2!}$$

$$= \frac{e^{-4} \times 1}{1} + \frac{e^{-4} \times 4}{1!} + \frac{e^{-4} \times 16}{2}$$

$$= e^{-4} + 4e^{-4} + 8e^{-4}$$

$$= e^{-4}(1+4+8) = e^{-4} \times 13$$

Now, $e^{-4} = \dfrac{1}{\text{antilog}(4 \times 0.4343)} = \dfrac{1}{\text{antilog}(1.7372)}$

$$= \frac{1}{54.61} = 0.01832$$

$\therefore P(X<3) = e^{-4} \times 13 = 0.01832 \times 13$

$= 0.2382$

$$= e^{-4} \times 13 = 0.01832 \times 13 = \underline{\underline{0.2382}}$$

Ex. 3: In a town 10 accidents took place in a span of 100 days. Assuming that the number of accidents follows Poisson, find the probability that there will be 3 or more accidents in a day.

7. Ans Average no. of accidents $= \frac{10}{100} = 0.1$

$m = \text{Mean} = 0.1$

$$P(X=x) = \frac{e^{-m} \times m^{x}}{x!} = \frac{e^{-0.1} \times (0.1)^{x}}{x!}$$

$P(\text{3 or more accidents}) = P(X \geq 3)$

$= 1 - P(X=0,1,2)$

$= 1 - \left[P(X=0) + P(X=1) + P(X=2)\right]$

$$= 1 - \left[\frac{e^{-0.1} \times (0.1)^{0}}{0!} + \frac{e^{-0.1} \times (0.1)^{1}}{1!} + \frac{e^{-0.1} \times (0.1)^{2}}{2!}\right]$$

$$= 1 - \left[\frac{e^{-0.1} \times 1}{1} + \frac{e^{-0.1} \times 0.1}{1} + \frac{e^{-0.1} \times 0.01}{2}\right]$$

$$= 1 - \left[e^{-0.1} + e^{-0.1} \times 0.1 + e^{-0.1} \times 0.005\right]$$

$$= 1 - e^{-0.1} \times \left[1 + 0.1 + 0.005\right]$$

$$= 1 - e^{-0.1} \times \left[1.105\right]$$

$$e^{-0.1} = \frac{1}{\text{antilog}(0.1 \times 0.4343)} = \frac{1}{\text{antilog}(0.04343)}$$

$$= \frac{1}{1.105} = \underline{\underline{0.9048}}$$

$$P(X \geq 3) = 1 - 0.9048 \times 1.105$$

$$= 1 - 0.9998$$

$$= \underline{\underline{0.0002}}$$

Ex. 5: A manufacturer of blades knows that 5% of his product is defective. If he sells blades in boxes of 100, and guarantees that no more than 10 blades will be defective, what is the probability (approximately) that a box will fail to meet the guaranteed quality ?

Ans: p = Probability of a defective blade = 5% = 0.05

8. Ans $n = 100$

p = Probability of a defective blade

$= 5\% = \frac{5}{100} = \underline{\underline{0.05}}$

Since the probability of a defective blade is small, we use Poisson distribution

$m = np = 100 \times 0.05 = \underline{\underline{5}}$

$$P(X=x) = \frac{e^{-m} \times m^{x}}{x!} = \frac{e^{-5} \times 5^{x}}{x!}$$

A box will fail to meet the guaranteed quality if the no. of defective blades in it is more than 10. So, the required probability is,

$$P(X > 10) = 1 - P(X \leq 10)$$

$$= 1 - [P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4) + P(x=5) + P(x=6) + P(x=7) + P(x=8) + P(x=9) + P(x=10)]$$

$$= 1 - \left[\frac{e^{-5} \times 5^0}{0!} + \frac{e^{-5} \times 5^1}{1!} + \frac{e^{-5} \times 5^2}{2!} + \frac{e^{-5} \times 5^3}{3!} + \cdots + \frac{e^{-5} \times 5^{10}}{10!}\right]$$

$$= 1 - e^{-5}\left[1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!} + \frac{5^4}{4!} + \frac{5^5}{5!} + \frac{5^6}{6!} + \frac{5^7}{7!} + \frac{5^8}{8!} + \frac{5^9}{9!} + \frac{5^{10}}{10!}\right]$$

$$= 1 - e^{-5} \times \underline{\underline{146.37}}$$

$$e^{-5} = \frac{1}{\text{antilog}(5 \times 0.4343)} = \frac{1}{\text{antilog}(2.1715)}$$

$$= \frac{1}{148.5} = \underline{\underline{0.0067}}$$

$$P(x > 10) = 1 - 0.0067 \times 146.37$$

$$= \underline{\underline{0.019}}$$

SHOT ON REDMI NOTE 7
MI DUAL CAMERA

$$\boxed{\textbf{Note: } f = N \times \text{Probability}}$$

Ex. 7: Out of 500 items selected for inspection .2% are found to be defective. Find how many lots will contain exactly no defective if there are 1000 lots (use Poisson distribution)

Fitting Poisson Distribution

1. Find the value of mean, m
2. Put $x = 0, 1, 2, \ldots$ in $P(X=x) = \dfrac{e^{-m} \times m^{x}}{x!}$

   & find corresponding probabilities
3. The expected (or theoretical) frequencies is obtained by $N \times P(x)$

(5)

x!

Ex. 10: Fit a poisson distribution to the following data and calculate the theoretical frequencies

x	**:**	**0**	**1**	**2**	**3**	**4**
f	**:**	**123**	**59**	**14**	**3**	**1**

Ans:

x	f	fx
0	123	0
1	59	59
2	14	28
3	3	9
4	1	4
	200	100

$N = \Sigma f = 200$

$\therefore \quad \bar{x} = \frac{\Sigma fx}{N} = \frac{100}{200} = 0.5$ Thus m = 0.5

$\therefore$ The poisson distribution, $P(x) = \frac{e^{-0.5}(0.5)^x}{x!}$

Theoretical frequencies are

x	P (x)	Theoretical frequency = N × P (x)
0	$e^{-0.5}\frac{(0.5)^0}{0!} = 0.6065$	$200 \times .6065 = 121$
1	$e^{-0.5}\frac{(0.5)^1}{1!} = 0.30325$	$200 \times .303 = 61$
2	$e^{-0.5}\frac{(0.5)^2}{2!} = 0.0766$	$200 \times .0766 = 15$
3	$e^{-0.5}\frac{(0.5)^3}{3!} = 0.0127$	$200 \times .0127 = 3$
4	$e^{-0.5}\frac{(0.5)^4}{4!} = 0.0016$	$200 \times .0016 = 0$

$$e^{-0.5} = \frac{1}{\text{antilog}(0.5 \times 0.4343)} = \frac{1}{\text{antilog}(0.21715)}$$

$$= \frac{1}{\text{antilog}(0.2172)} = \frac{1}{1.649} = \underline{\underline{0.6065}}$$

Ex. 11: A systematic sample of 100 pages was taken from the Concise Oxford Dictionary and the observed frequency distribution of foreign words per page was found to be as follows. Calculate the expected frequencies using Poisson Distribution. Also compute variance of fitted distribution.

No. of foreign words per page (x)	**:**	**0**	**1**	**2**	**3**	**4**	**5**	**6**
Frequency (f)	**:**	**48**	**27**	**12**	**7**	**4**	**1**	**1**

Ans:

x	f	fx
0	48	0
1	27	27
2	12	24
3	7	21
4	4	16
5	1	5
6	1	6
	100	99

$N = \Sigma f = 100$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{99}{100} = .99$$

$\therefore$ m = .99

$$\therefore \quad \text{Poisson distribution: } P(x) = \frac{e^{-0.99}(0.99)^x}{x!}$$

St.

Calculation of theoretical frequencies

x	P (x)	Theoretical frequency N × P (x)
0	$e^{-0.99}\dfrac{(0.99)^0}{0!} = 0.3716$	$100 \times .3716 = 37.2$
1	$e^{-0.99}\dfrac{(0.99)^1}{1!} = 0.3679$	$100 \times .3679 = 36.8$
2	$e^{-0.99}\dfrac{(0.99)^2}{2!} = 0.1821$	$100 \times .1821 = 18.2$
3	$e^{-0.99}\dfrac{(0.99)^3}{3!} = 0.0601$	$100 \times .0601 = 6$
4	$e^{-0.99}\dfrac{(0.99)^4}{4!} = 0.0149$	$100 \times ..0149 = 1.5$
5	$e^{-0.99}\dfrac{(0.99)^5}{5!} = 0.0029$	$100 \times .0029 = 0.3$
6	$e^{-0.99}\dfrac{(0.99)^6}{6!} = 0.0005$	$100 \times .0005 = 0.1$

Hence the theoretical (expected) frequencies of the poisson distribution are:

x	0	1	2	3	4	5	6
Expected frequency (Rounded)	37	37	18	6	2	0	0

For poisson distribution, mean and variance are equal to m.

$\therefore$ Variance $= m = 0.99$

$$e^{-0.99} = \frac{1}{\text{antilog}(0.99 \times 0.4343)} = \frac{1}{\text{antilog}(0.4299)}$$

$$= \frac{1}{2.691} = \underline{\underline{0.3716}}$$

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6.2

Definition Normal Distribution

A continous random variable X is said to follow Normal distribution if its probability function is $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$ where μ and σ are constants (μ being the mean σ being the standard deviation of x). The variable X varies between $-\infty$ and $+\infty$.

Mean of Normal Distribution

$$\text{Mean} = E(x)$$

$$= \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \times \frac{1}{\sqrt{2\pi}\,\sigma} \times e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} dx$$

$$= \mu$$

Variance of Normal Distribution

$$\text{Variance} = E\left(X - E(X)\right)^2$$

$$= E\left(X - \mu\right)^2$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \times \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} dx$$

$$= \underline{\underline{\sigma^2}}$$

$$SD = \sqrt{\text{Variance}} = \sqrt{\sigma^2} = \underline{\underline{\sigma}}$$

Moment Generating Function of Normal Distribution

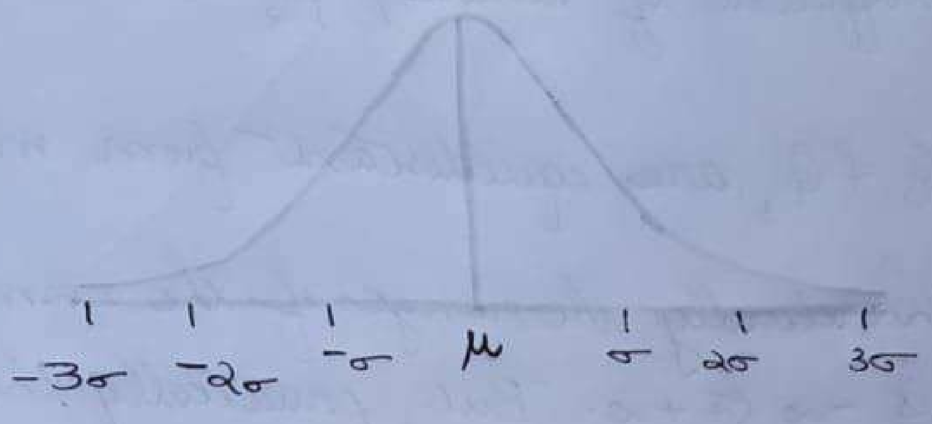
$$M_x(t) = E\left(e^{tx}\right)$$

$$= \int_{-\infty}^{\infty} e^{tx} \times f(x)\, dx$$

$$= \int_{-\infty}^{\infty} e^{tx} \times \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} dx$$

$$= e^{\mu t + \frac{1}{2}t^2\sigma^2}$$

Properties of Normal Distribution



1. The normal curve is a continous curve
2. The normal curve is bell shaped
3. Normal curve is symmetric about the mean
4. Mean, Median & Mode are equal for a normal distribution
5. The height of Normal curve is maximum at the mean.
6. No portion of the curve lies below the X-axis
7. The normal curve is unimodel
8. All odd moments of the normal distribution are zero & even moments are given by

$$\mu_{2n} = (2n-1) \times \sigma^2 \times \mu_{2n-2}$$

9. Coefficient of skewness, $\beta_1 = 0$

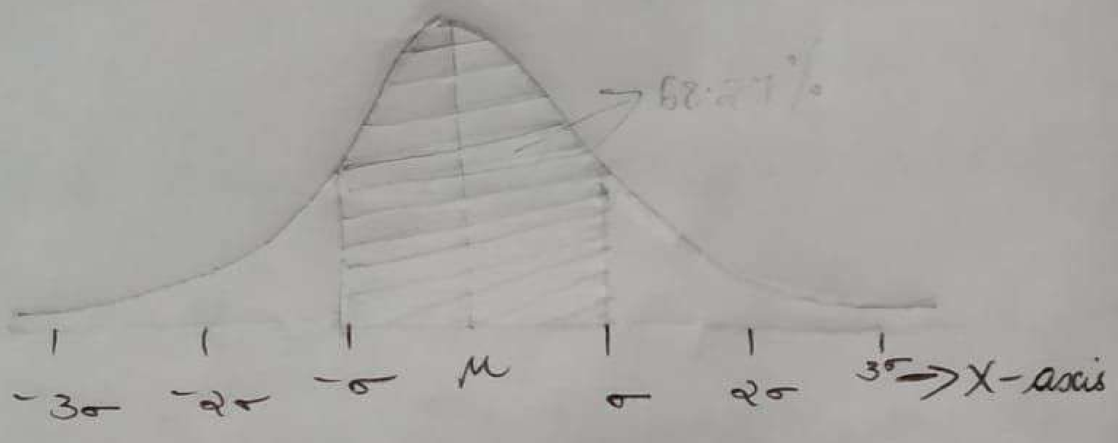
10. Coefficient of kurtosis, $\beta_2 = 3$

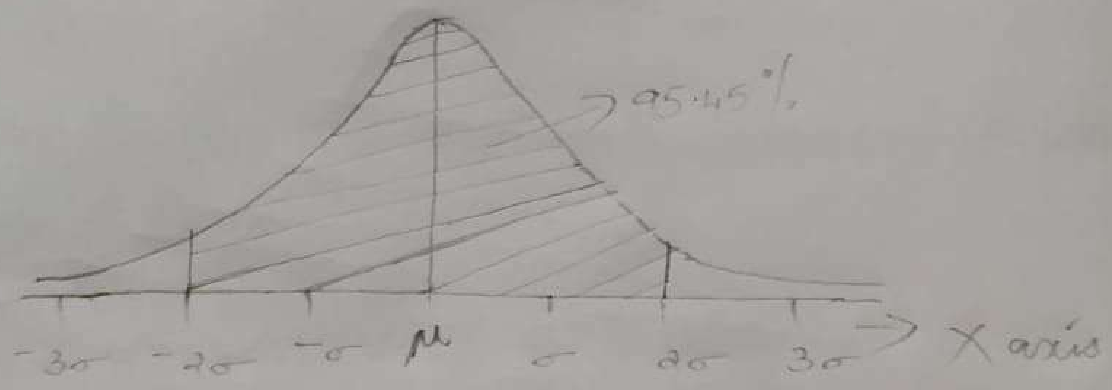
11. Q_1 & Q_3 are equidistant from median (Q_2)

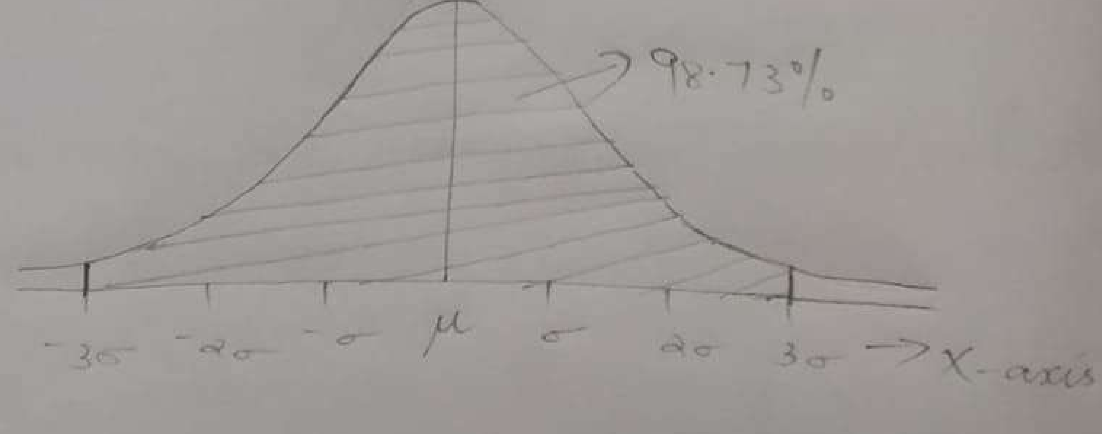
12. Theoretically, the range of the normal curve is $-\infty$ to $+\infty$. But, practically the range is $x - 3\sigma$ to $x + 3\sigma$

13. Area under the normal curve is distributed as follows.

$$\left.\begin{array}{l} \mu \pm \sigma \text{ covers } 68.27\% \text{ area} \\ \mu \pm 2\sigma \text{ covers } 95.45\% \text{ area} \\ \mu \pm 3\sigma \text{ covers } 98.73\% \text{ area} \end{array}\right\} \text{area property}$$


68·27%
-3σ
-2σ
-σ
μ
σ
2σ
3σ
X-axis


95·45%
-3σ
-2σ
-σ
μ
σ
2σ
3σ
X axis


98·73%
-3σ
-2σ
-σ
μ
σ
2σ
3σ
X-axis

14. There is only one maximum point which occurs at the mean.

15. The normal curve approches nearer to the base but it never touches

Normal distribution as a lmiting case of Binominal distribution

Binominal Distribution is an important theoretical distribution for dicrete variables. Binominal distribution tends to Normal Distribution under the following conditions.

1. Number of trials (n) is very large
2. p and q (ie probability for success in a single trail and the probability for its failure) are almost equal.

Then the Binominal distribution can be approximated to normal.

Importance (or use) of Normal Distribution

The study of the normal distribution is of central importance in statistical analysis because of the following reasons.

1. Most of the discrete probabilty distributions (eg: Binominal distribution, Poisson distribution etc) tend to normal distribution as 'n' becomes large.

2. Almost all sampling distributions such as student's t - distribution, F -distribution, Z - distribution, χ^2- distribution etc. conform to the normal distribution for large values of n.

3. The various tests of significance like t -test, F -test etc are based on the assumption that the parent population from which the samples have been drawn follows Normal Distribution.

4. It is extensively used in large sampling theory to find the estimates of parameters from statistics, confidance limits etc. uud is centr

5. Normal distribution has the remarkable property stated in the central limit theorem. As per the theorem, when the sample size is increased, the smple means will tend to be normally distributed. Central

...of normal type.

Merits and Demerits of Normal distribution

Merits

1. Normal distribution is the mostly used distribution in Inferential Statistics.
2. Most of error of measurements and a large variety of physical observations have approximately Normal distributions.
3. The measurements of linear dimensions of large number of articles produced may show individual variations. These follow Normal distribution.
4. The standard normal distribution table shows exhaustively areas for the different intervals of the values of the variable.
5. The Normal distribution has a number of mathematical properties.
6. Most of the distributions in nature are either normal or that can be approximated to normal.

Demerits

1. The variables which are not continous cannot be normally distributed. Therefore many distributions in Economics like distribution of number of children per family, cannot be studied under Normal Distribution.
2. The Normal distribution cannot be applied to situations where the distribution is highly skewed. For example: distribution on income is very much skewed. Therefore here Normal distribution will not be appropriate.

be appropriate.

Standard normal variate (Unit normal variate)

If X is a random varible following Normal distribution with mean μ and standard deviation σ then the variable $Z=\frac{x-\mu}{\sigma}$ is known as standard normal variate.

This 'Z' follows normal disrtibution with mean '0' and S.D = 1.

The distribution of Z is known as Standard Normal Distribution.

The probability function of Z is $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ for $-\infty < z < \infty$

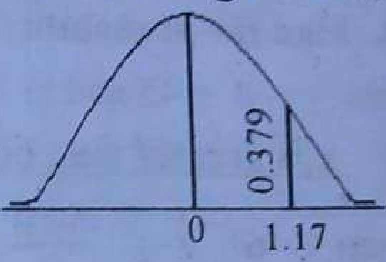
The standard normal distribution table

This is a table showing the probabilty for 'z' taking valuse between 0 and a given value. The probability thus obtained is the area of the standard normal curve between the ordinates at $z = 0$ and at the given value.

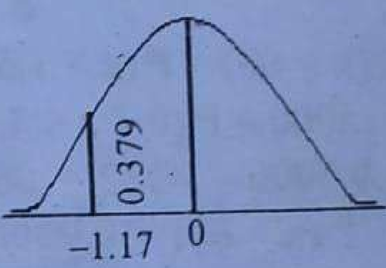
For example, when $z = 1.17$

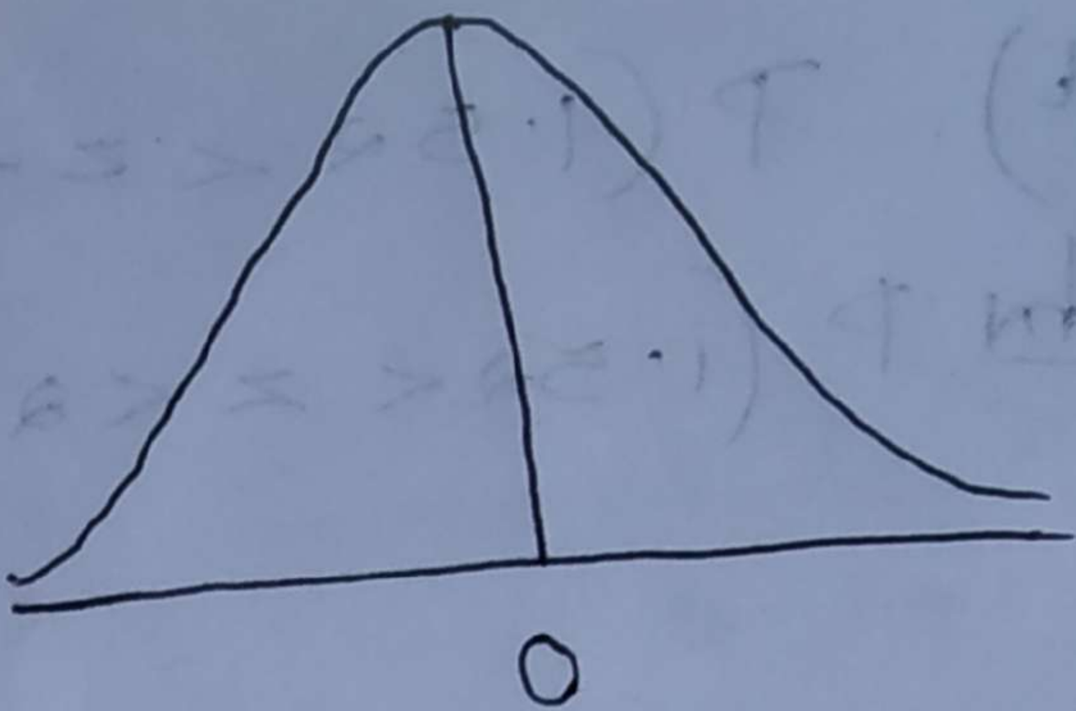
table value = 0.3790

This table value is the area between 0 and 1.17 which can be written as $P(0 < z < 1.17) = 0.3790$



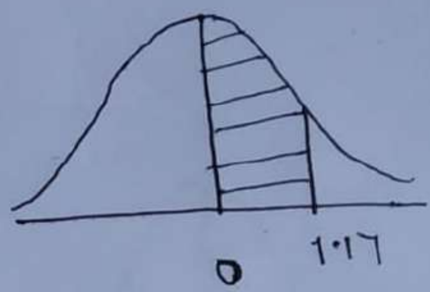
It may be noted that for both negative and positive values of z, area is the same. That is, $P(0 < z < 1.17)$ $= P(-1.17 < z < 0) = .3790$




0

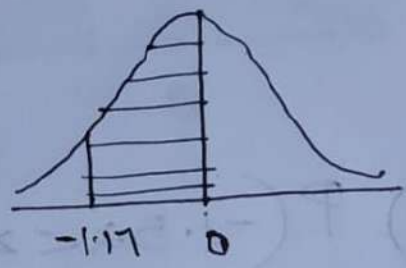
1. Find

a) $P(0 < z < 1.17)$



Ans $P(0 < z < 1.17) = 0.3790$

b) $P(-1.17 < z < 0)$



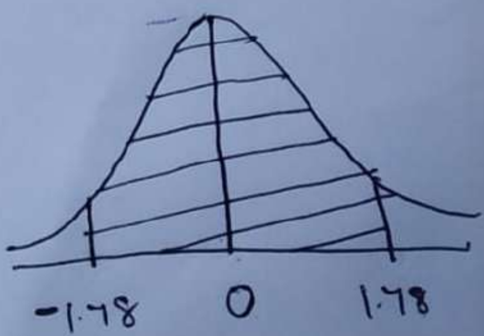
Ans $P(-1.17 < z < 0) = 0.3790$

c) $P(-1.78 < z < 1.78)$

Ans $P(-1.78 < z < 1.78) = P(-1.78 < z < 0) + P(0 < z < 1.78)$

$= 0.4625 + 0.4625$

$= \underline{\underline{0.925}}$

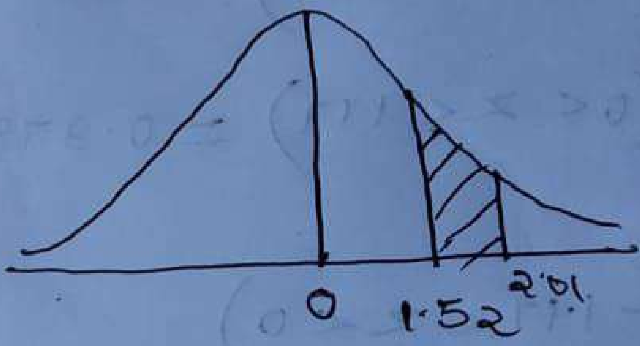


d) $P(1.52 < z < 2.01)$

Ans $P(1.52 < z < 2.01) = P(0 < z < 2.01) - P(0 < z < 1.52)$

$= 0.4778 - 0.4357$

$= \underline{0.0421}$

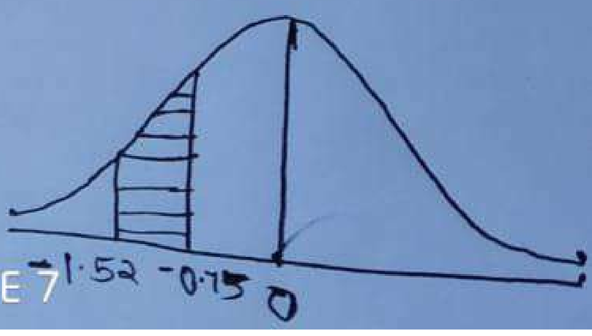


e) $P(-1.52 < z < -0.75)$

Ans $P(-1.52 < z < -0.75) = P(-1.52 < z < 0) - P(-0.75 < z < 0)$

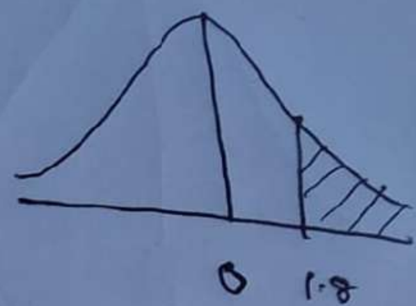
$= 0.4357 - 0.2734$

$= \underline{0.1623}$



f) $P(z > 1.8)$

Ans $P(z > 1.8) = 0.5 - P(0 < z < 1.8)$

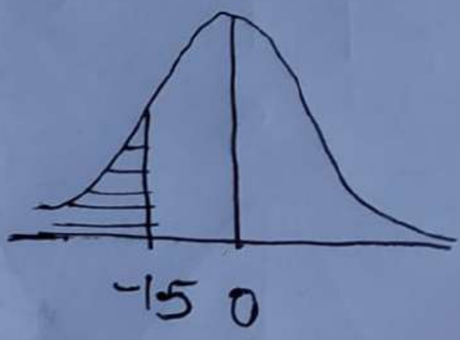


$= 0.5 - 0.4641$

$= 0.0359$

g) $P(z < -1.5)$

Ans $P(z < -1.5) = 0.5 - P(-1.5 < z < 0)$



$= 0.5 - 0.4332$

$= 0.0668$

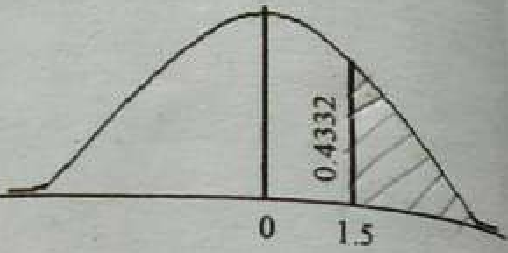
PROBLEMS

Ex.1: The variable X follows a Normal distribution with mean 45 and S.D. = 10. Find the probability that (i) x > 60 (ii) 40 < x < 56

Ans: $\mu = 45$ and $\sigma = 10$

(i) <u>x is greater than 60 (ie x > 60)</u>

when x = 60, $z = \frac{x-\mu}{\sigma} = \frac{60-45}{10} = 1.5$

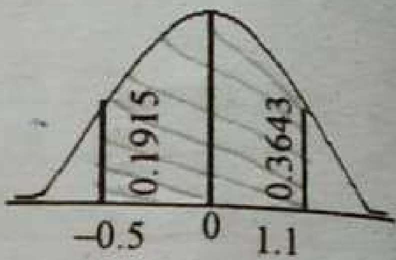


$P(x > 60) = P(z > 1.5)$
$= .5000 - P(0 < z < 1.5)$
$= 0.5000 - .4332 = .0668$
$\therefore P(x > 60) = \underline{\underline{0.0668}}$

(ii) <u>x lies between 40 and 56 (ie 40 < x < 56)</u>

when x = 40, $z = \frac{x-\mu}{\sigma} = \frac{40-45}{10} = -0.5$

when x = 56, $z = \frac{x-\mu}{\sigma} = \frac{56-45}{10} = 1.1$



$P(40 < x < 56) = P(-.5 < z < 1.1)$
$= P(-.5 < z < 0) + P(0 < z < 1.1)$
$= .1915 + .3643 = .5558$
$P(40 < x < 56) = \underline{\underline{0.5558}}$

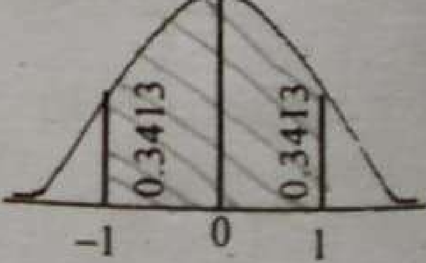
[Note: For every x value, there corresponds an z value obtained by the formula, $z = \frac{x-\mu}{\sigma}$ *similarly for every z value there is a table value]*

$\boxed{\text{Note: Required number} = \text{Probability} \times \text{N}}$

Ex.2: The scores of students in a test follow Normal Distribution with mean = 80 and SD = 15. A sample of 1000 students has been drawn from the population. Find (1) appropriate number of students scoring between 65 and 95 (2) the probability that a randomly chosen student has scores greater than 100.

Ans: $\mu = 80$ and $\sigma = 15$, N = 1000

(1) <u>between 65 and 95 (ie 65 < x < 95)</u>



when x = 65, $z = \frac{x-\mu}{\sigma} = \frac{65-80}{10} = -1$

when $x = 95$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{95-80}{15} = 1$

$\therefore P(65 < x < 95) = P(-1 < z < 1)$

$= P(-1 < z < 0) + P(0 < z < 1)$

$= 0.3413 + 0.3413 = 0.6826$

$P(1{\cdot}33 < z < 2{\cdot}1)$

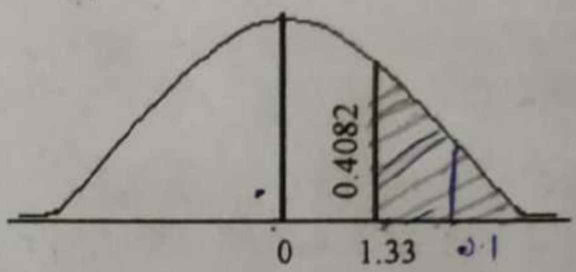
No.of students scoring between 65 and 95

$= \text{Probability} \times N = 0.6826 \times 1000 = 682.6 = \underline{\underline{683}}$

(2) <u>Greater than 100 (ie x > 100)</u>

when x =100

$z = \dfrac{x-\mu}{\sigma} = \dfrac{100-80}{15} = 1.33$



$P(x > 100) = P(z > 1.33)$

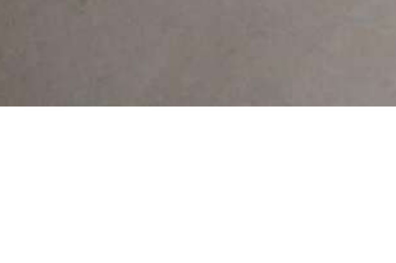
$= 0.5000 - .4082 = .0918$

Number of students scoring more than $100 = 0.0918 \times 1000 = 91.8 = \underline{\underline{92}}$

Ex. 4: The per acre yield of crop in a particular area is observed to follow a normal distribution with mean 15 quintals and S.D of 5 quintals. Find (i) the proportion of the area yielding at least 25 quintals (ii) what extent of the land under the crop can yield between 10 and 20 quintals if the total land under crop is 782 acres ?

Ans: $\mu = 15$ and $\sigma = 5$

(i) <u>at least 25 quintels $(x > 25)$</u>



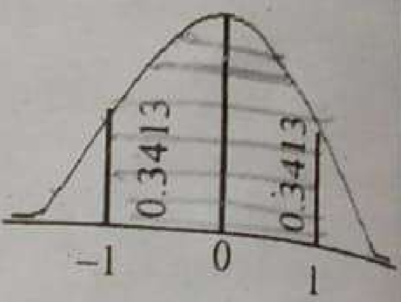
When $x = 25$, $Z = \dfrac{x - \mu}{\sigma} = \dfrac{25 - 15}{5} = 2$

P (x is at least 25)

$= P(x \geq 25) = P(z \geq 2)$
$= 0.5000 - P(0 < z \leq 2)$
$= .5000 - .4772 = 0.0228$

$\therefore$ Proportion of the area yielding at least 25 quintals,
$= 0.0228 \times 100 =$ 2.28%

(ii) Between 10 and 20 quintals (ie $10 < x < 20$)



when $x = 10$, $z = \frac{x-\mu}{\sigma} = \frac{10-15}{5} = -1$

when $x = 20$, $z = \frac{x-\mu}{\sigma} = \frac{20-15}{5} = 1$

$P(10 < x < 20) = P(-1 < z < 1)$
$= P(-1 < z < 0) + P(0 < z < 1) = 0.3413 + 0.3413 = .6826$

Extent of land under the crop yielding between 10 and 20 quintals
$= 0.6826 \times 782 =$ 533.8 acres

Ex. 5: Find the probability that the number of heads lie in the range 185 and 220 when a fair coin is tossed 400 times.

Ans: $n = 400; p = \frac{1}{2}, q = \frac{1}{2}$

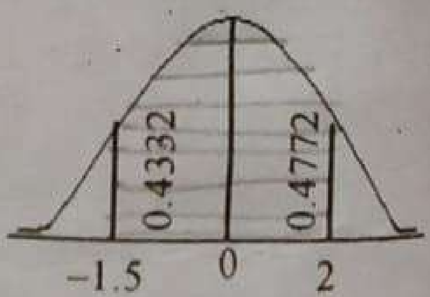
$\mu = np = 400 \times \frac{1}{2} = 200$

$\sigma = \sqrt{npq} = \sqrt{400 \times \frac{1}{2} \times \frac{1}{2}} = 10$

between 185 and 220 (ie $185 < x < 220$)

when $x = 185$, $z = \frac{x-\mu}{\sigma} = \frac{185-200}{10} = -1.5$

when $x = 220$, $z = \frac{220-200}{10} = 2$



$P(185 < x < 220) = P(-1.5 < z < 2)$
$= P(-1.5 < z < 0) + P(0 < z < 2)$
$= 0.4332 + 0.4772 = 0.9104$

$\therefore$ Probabilty that the number of heads range between 185 and 200
$=$ 0.9104

To find the value of z when the area is known

Let a be the unknown value of z

1. Let $P(0 < z < a) = 0.4332$

We have to find the value of a

For that, first find the value 0.4332 or the value just greater than this value in the table. Then take the row & column value corresponding to this value as a

2. Let $P(z > a) = 0.35$

We have to find the value of a

We know that,

$$P(z > a) = 0.5 - P(0 < z < a)$$

$$\Rightarrow P(0 < z < a) = 0.5 - P(z > a)$$
$$= 0.5 - 0.35$$
$$= \underline{\underline{0.15}}$$

Then, from the table, find the value 0.15 or the value just greater than 0.15. Now, take the row & column value corresponding to this value as a

3. Let $P(z<a) = 0.35$. We have to find a

$$P(z<a) = 0.5 - P(a<z<0)$$

$$P(a<z<0) = 0.5 - P(z<a)$$

$$= 0.5 - 0.35$$

$$= \underline{\underline{0.15}}$$

Then, from the table, find the value 0.15 or the value just greater than 0.15. Now, take the row & column value corresponding to this value as a. Since $z<a$, we take -ve value of a

$\therefore$ The value of x that has 20% of the area to its right = $\underline{\underline{48.5}}$

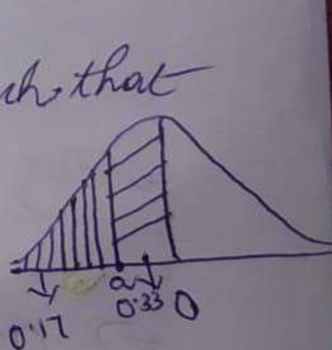
Ex. 9: in a Normal distribution 17% of the items are below 30 and 17% of the area above 60. Find the mean and standard deviation.

Ans: Given $P(x < 30) = .17$

1. Given $P(x<30) = 17\% = \frac{17}{100} = 0.17$

Let a be the corresponding value of z such that

$$P(z<a) = 0.17$$



Now, $P(a<z<0) = 0.5 - P(z<a)$

$$= 0.5 - 0.17 = \underline{\underline{0.33}}$$

Now, when $a = -0.95$, table value $= 0.33$

ie, $\underline{\underline{z = -0.95}}$

Now, $z = \frac{x-\mu}{\sigma}$ $\Rightarrow -0.95 = \frac{30-\mu}{\sigma}$ $[\because x=3$

$$-0.96\sigma = 30-\mu$$

$$\Rightarrow \mu - 0.96\sigma = 30 \quad - (1)$$

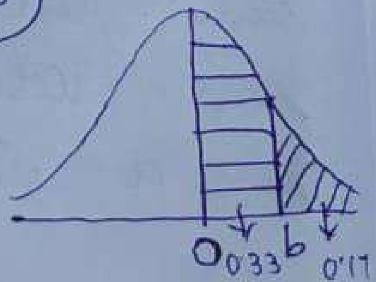
Given $P(x>60) = 17\% = 0.17$

Let b be the corresponding value of z such that $P(z>b) = 0.17$

Now, $P(0<z<b) = 0.5 - P(z>b)$

$$= 0.5 - 0.17$$

$$= 0.33$$



Now, when $b = 0.96$, table value $= 0.33$

ie, $z = 0.96$

Now, $z = \dfrac{x-\mu}{\sigma} \Rightarrow 0.96 = \dfrac{60-\mu}{\sigma}$ $[\because x=60]$

$$\Rightarrow 0.96\sigma = 60-\mu$$

$$\Rightarrow \mu + 0.96\sigma = 60 \quad - (2)$$

From (1) & (2),

$$\begin{aligned} \mu - 0.96\sigma &= 30 \quad + \\ \mu + 0.96\sigma &= 60 \\ \hline 2\mu &= 90 \end{aligned}$$

$$\Rightarrow \mu = \frac{90}{2} = \underline{45}$$

Substituting in (1),

$$45 - 0.96\sigma = 30$$

$$-0.96\sigma = 30 - 45$$

$$-0.96\sigma = -15$$

$$\sigma = \frac{-15}{-0.96} = \underline{\underline{15.8}}$$

$\therefore$ Mean, $\mu = 45$

SD, $\sigma = 15.8$

Hence the lowest weekly wages of the highest paid workers is 76.40 Rs

Ex. 8: Given a Normal distribution with mean = 40 and SD = 10. Find the value of x that has (a) 15% of the area to its left (b) 20% of to area to its right

2.

$\mu = 40, \ \sigma = 10$

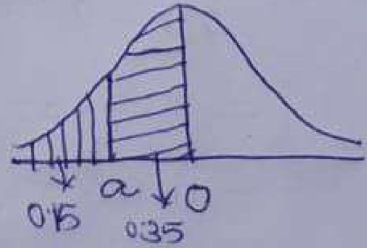
a) 15% of the area to the left

Given area = 15%

Let $x = a$

Given, $P(x < a) = 15\%$

$= 0.15$



$P(a < x < 0) = 0.5 - P(x < a)$

$= 0.5 - 0.15$

$= 0.35$

When $z = -1.04$, table value $= 0.35$

$\therefore z = -1.04$

$z = \frac{x - \mu}{\sigma}$

$\Rightarrow -1.04 = \frac{a - 40}{10}$ $\quad [\because x = a]$

$\Rightarrow -1.04 \times 10 = a - 40$

$\Rightarrow -10.4 = a - 40$

$\Rightarrow a = 40 - 10.4 = 29.6$

$\therefore$ The val[illegible] x that 15[illegible] area to [illegible] left = 29[illegible]

b)

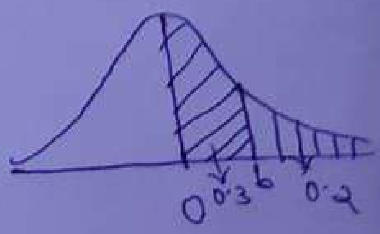
b) 20% of the area on the right

Given area = 20%

Let $x = b$

Given, $P(x>b) = 20\% = 0.2$

$$P(0<x<b) = 0.5 - P(x>b)$$
$$= 0.5 - 0.2$$
$$= \underline{\underline{0.3}}$$



when $z = 0.85$, table value $= 0.3$

$$\therefore z = \underline{\underline{0.85}}$$

$$z = \frac{x-\mu}{\sigma}$$

$$\Rightarrow 0.85 = \frac{b-40}{10} \quad [\because x = b]$$

$$\Rightarrow 0.85 \times 10 = b - 40$$

$$\Rightarrow 8.5 = b - 40$$

$$\Rightarrow b = 8.5 + 40 = \underline{48.5}$$

$\therefore$ The value of x that has 20% of the area to its right $= \underline{\underline{48.5}}$

Fitting Normal Curve

Main problem in fitting the normal distribution is estimating the parameters of the distribution. Parameters of the distribution are μ and σ. So find mean and standard deviation of the given frequency distribution. Take them as the estimates of the parameters μ and σ. Then we get the density function, $f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$. By substituting values of μ and σ in this equation we get the equation of the best fitting normal distribution.

To get the theoretical frequencies following steps are applied.

1. Calculate mean and S.D of the given distribution.
2. Calculate z value for all class limits, using the formula, $z = \frac{x-\mu}{\sigma}$
3. Find the area of each z value from the standard normal table.
4. Find the area of each class (by subtracting or adding , as the case may be, with the areas obtained for 'z' values.)
5. Multiply the area of each class by the total frequency to get the class frequency.

The new frequency distribution with theoritical frequencies, will be a normal approximation to the given frequency distribution.

O. 16

Ex. 14: Fit a normal distribution to the following data:

Marks	:	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of students	:	4	22	48	66	40	16	4

To find the mean and S.D

1.

Class	No. of students (f)	x	fx
10-20	4	15	60
20-30	22	25	550
30-40	48	35	1680
40-50	66	45	2970
50-60	40	55	2200
60-70	16	65	1040
70-80	4	75	300
	$\Sigma f = 200$		$\Sigma fx = 8800$

Mean, $\mu = \frac{\Sigma fx}{\Sigma f} = \frac{8800}{200} = \underline{44}$

x	$(x-\mu)$	$(x-\mu)^2$	f	$f(x-\mu)^2$
15	-29	841	4	3364
25	-19	361	22	7942
35	-9	81	48	3888
45	1	1	66	66
55	11	121	40	4840
65	21	441	16	7056
75	31	961	4	3844
			$\Sigma f = 200$	$\Sigma f(x-\mu)^2 = 31000$

$$SD, \sigma = \sqrt{\frac{\Sigma f(x-\mu)^2}{\Sigma f}} = \sqrt{\frac{31000}{200}} = \sqrt{155}$$

$$= \underline{12.45}$$

$$P(X=x) = \frac{1}{\sqrt{2\pi}\,\sigma} \times e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$$

$$= \frac{1}{\sqrt{2\pi} \times 12.45} \times e^{-\frac{1}{2}\left[\frac{x-44}{12.45}\right]^2}$$

Class limits	$z = \frac{x-\mu}{\sigma} = \frac{x-44}{12.45}$	Area from table	Area for interval	Theoretical frequency
(1)	(2)	(3)	(4)	(5) = (4) x 200
10	$-\infty$	0.5	0.5 − 0.4726 = 0.0274	5
20	−1.92	0.4726	0.1040	21
30	−1.12	0.3686	0.2431	49
40	−0.32	0.1255	0.1255 + 0.1844 = 0.3099	62
50	0.48	0.1844	0.2153	43
60	1.28	0.3997	0.0815	16
70	2.08	0.4812	0.0188	2
80	$+\infty$	0.5		

Various Steps

1. First find mean and S.D of the given distribution ie μ and σ
2. Then for each 'x' value (the class limit), get z value using the formula, $z = \frac{x - \mu}{\sigma}$ [redacted]. These z values form column 5.

3. Column 4 is the difference between two adjacent values of column 3 except two values where z changes from negative to positive.
4. Column 5 is obtained by multiplying all the values in column 4 with total frequency [illegible].

Note: a) First and last 'z' values are taken as $-\infty$ and $+\infty$.

b) In column 4, third value is obtained by adding two values of the previous column, instead of taking the difference (When z value changes from negative to positive we add).

Formulas

1. Mean = Median = Mode = μ

2. $SD = \sigma$

3. $\mu_1 = 0, \mu_3 = 0, \ldots$

4. $\mu_2 = \sigma^2, \mu_4 = 3\sigma^4, \ldots$

5. Coefficient of skewness, $\beta_1 = 0$

6. Measure of kurtosis, $\beta_2 = \frac{\mu_4}{\mu_2^2} = 3$

7. $QD = \frac{2}{3} \times \sigma$

8. $MD = \frac{4}{5} \times \sigma$

Ex. 17: For a normal distribution, mean = 45, SD = 12. Find median, mode, QD, MD, Coefficient of skewness and a measure of kurtosis

Ans: Mean = 45

$\therefore$ Median = 45 and Mode = 45

SD = 12 $\quad \therefore \sigma = 12 \quad \therefore \sigma^2 = 144$

$$QD = \frac{2}{3}\sigma = \frac{2}{3} \times 12 = 8$$

$$MD = \frac{4}{5}\sigma = \frac{4}{5} \times 12 = 9.6$$

Coefficient of skewness = 0 Ans

$\mu_2 = \sigma^2$ and $\mu_4 = 3\sigma^4$

$$\beta_2 = \text{Measure of kurtosis} = \frac{\mu_4}{\mu_2^2} = \frac{3\sigma^4}{(\sigma^2)^2} = \frac{3\sigma^4}{\sigma^4} = 3$$

Uniform Distribution

A continuous random variable X is said to follow uniform distribution if its p.d.f is

$$P(X=x) = f(x) = \frac{1}{b-a}, \quad a \leq x \leq b$$

Note : Mean of Uniform Distribution

$$\text{Mean} = E(X) = \int_x x\, f(x)\, dx$$

$$= \int_a^b x \times \frac{1}{b-a}\, dx = \frac{1}{b-a}\left[\int_a^b x\, dx\right]$$

$$= \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{b-a} \times \left(\frac{b^2-a^2}{2}\right)$$

$$= \frac{1}{b-a} \times \frac{(b+a) \times (b-a)}{2}$$

$$= \frac{a+b}{2}$$

Variance of Uniform Distribution

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= E(x^2) - \left(\frac{a+b}{2}\right)^2$$

$$Var(x) = E(x^2) - \frac{(a+b)^2}{4} \quad — ①$$

$$E(x^2) = \int_a^b x^2 f(x)\, dx$$

$$= \int_a^b x^2 \times \frac{1}{b-a} dx = \frac{1}{b-a} \times \int_a^b x^2 dx$$

$$= \frac{1}{b-a} \times \left[\frac{x^3}{3}\right]_a^b = \frac{1}{b-a} \times \left(\frac{b^3 - a^3}{3}\right)$$

$$= \frac{1}{b-a} \times (b-a)\left(\frac{b^2 + ba + a^2}{3}\right)$$

$$= \frac{b^2 + ba + a^2}{3}$$

Substituting in ①,

$$Var(x) = \frac{b^2 + ba + a^2}{3} - \frac{(a+b)^2}{4}$$

$$= \frac{b^2+ba+a^2}{3} - \frac{(a^2+2ab+b^2)}{4}$$

$$= \frac{4b^2+4ba+4a^2-(3a^2+6ab+3b^2)}{12}$$

$$= \frac{4b^2+4ba+4a^2-3a^2-6ab-3b^2}{12}$$

$$= \frac{a^2-2ab+b^2}{12} = \frac{(a-b)^2}{12}$$