30/01/2020
Thursday

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Module - I

Graph

A graph $G = (V, E)$ consists of V, a non-empty set of vertices or nodes. and E, a set of edges.

Each edge has either one or two vertices associated with it, called its end points. An edge is said to connect its endpoint.

Infinite graph

A graph with an infinite veritex set is called an infinite graph.

Finite graph

A graph with finite vertices is called a finite graph.

Simple graph

A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a simple graph.

Multigraph

Graphs that may have multiple edges connecting the same vertices are called multigraph.

Loops

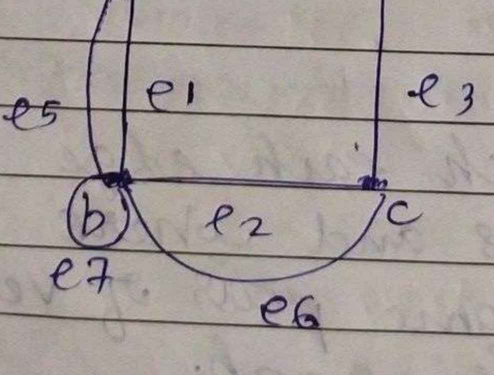
Edges that connects a vertex to itself are called loops.

Pseudograph

Graph with multiple edges and loops connecting same pair of vertices

OR

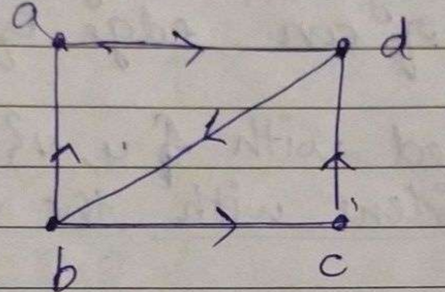
Graphs that may include loops and possibly multiple edges connecting the same pair of vertices are called pseudograph.



Directed graph / Digraph

A directed graph or digraph, (V, E) consists

of a non-empty set of vertices V and a set of directed edges E. Each directed edge is associated with an ordered pair of vertices.
The directed edge associated with the ordered pair (u, v) is said to start at u and end at v.



(a, d)
(b, a)
(b, c)
(d, b)
(c, d)

Simple directed graph

when a directed graph has no loops and has no multiple edges it is called a simple directed graph.

Directed multigraph

The directed graph that may have multiple directed edges ~~is called~~ from a vertex to a second vertex are said to be directed multigraphs

Mixed graph

A graph with both directed and undirected edges is called a mixed graph.

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Graph Terminology and Some special types of graph

Two vertices u and v in an undirected graph G are called <u>adjacent</u> in G if u and v are endpoints of an edge of G. Ques

If E is associated with {u, v} the edge E is called <u>incident with</u> the vertices u and v. (1)

The edge E is also said to <u>connect</u> u and v.

The vertices u and v are called endpoints of an edge associated with {u, v}

<u>Degree of a vertex</u>

The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex. (2)

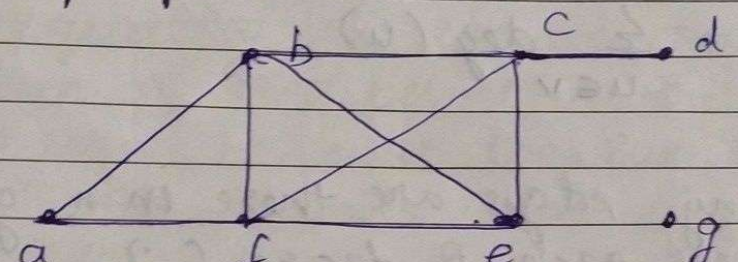
The degree of the vertex is denoted by deg ( ).

A vertex of degree 0 is called <u>isolated</u> i.e. the isolated vertex is not adjacent to any vertex.

A vertex is <u>pentent</u> if and Only if it has degree 1. ie a pentent vertex is adjacent to exactly one vertex

Ques What are the degrees of the vertices in the below given graphs

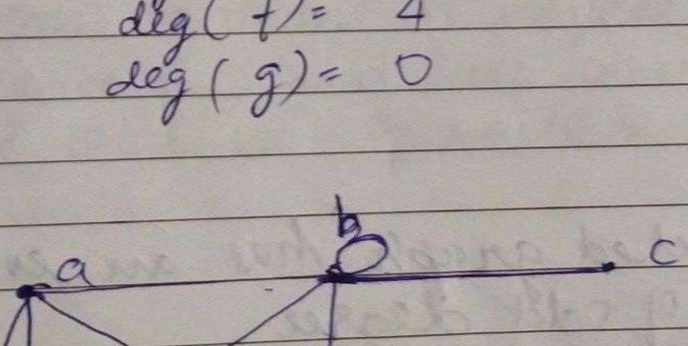
①



degree of a = 2
deg (b) = 4
deg (C) = 4
deg (d) = 1
deg (e) = 3
deg (f) = 4
deg (g) = 0

9 edges
sum = 18

②



degree of
a = 4
b = 6
C = 1
d = 5
e = 6

sum = 22
edge = 11

$$\boxed{2e = \Sigma \text{ degree}(n)}$$

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Hand Shaking Theorem (Theorem 1)

$(2e = \Sigma \deg(u))$

Let $G = (V, E)$ be an undirected graph with e edges then

$$2e = \sum_{u \in V} \deg(u)$$

Ques How many edges are there in a graph with 10 vertices each of degree 6?

$$2 \cdot e = 6 \times 10$$
$$2e = 60$$
$$e = \frac{60}{2}$$
$$e = 30$$

(Theorem 2)

An undirected graph has an even number of vertices of odd degree

Proof

Let V_1 and V_2 be the set of vertices of even degree and the set of vertices of odd degree respectively. in an undirected graph

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$G = (V, E)$ then . $2e = \sum_{u \in V} deg(u) = \sum_{u \in V_1} deg(u) + \sum_{u \in V_2} deg(u)$

Because the degree of u (deg(u)) is even for (every element) $\in V_1$, the first term in the right hand side of the last equality) is even. Furthermore the sum of two terms in the right hand side is even because the sum is 2e. Hence the second term in the sum is also even because the sum of all the terms in the odd degree vertices must be an even number. Thus there are an even number of vertices of odd degree.

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Ques A graph contains 21 edges and 3 vertices of degree 4 and all other vertices of degree 2 Find total no. of vertices

$\frac{42}{2} = 21$

$e = 21$

$2e = \sum_{u \in V_1} deg(u) + \sum_{u \in V_2} deg(u)$

$2 \times 21 = 3 \times 4 \times 3 + 2 \times V$

$42 = 12 + 2V$

$2V = 42 - 12$

$= 30$

$V = 30/2 = \underline{15}$

42
12
30

42 - 21
21
21

$E = 21$

$$2e = \sum_{u \in V_1} deg(u) + \sum_{u \in V_2} deg(u)$$

$$21 \times 2 = 4 \times 3 + (n-3)2$$
$$= 12 + 2n - 6$$
$$42 = 6 + 2n$$
$$36 = 2n$$
$$n = \frac{36}{2}$$
$$n = 18$$

Ques A simple graph G has 24 edges and degree of each vertex is 4. Find the no. of vertices.

$E = 24$

$$2e = \sum deg(u)$$
$$24 \times 2 = 4 \times n$$
$$n = \frac{24 \times 2}{4}$$
$$n = 12$$

Ques A simple graph with 35 edges is having 4 vertices of degree 5, 5 vertices of degree 4 and 4 vertices of degree 3. Find the no. of vertex with degree 2.

$$2e = \sum_{u \in V_1} deg(u) + \sum_{u \in V_2} deg(u)$$

$$35 \times 2 = 4 \times 5 + 5 \times 4 + 4 \times 3 + v \times 2$$

$$35 \times 2 = 20 + 20 + 12 + 2v$$
$$70 = 52 + 2v$$

$$70 - 52 = 2v$$
$$28 = 2v$$
$$v = \frac{18}{2}$$

$$v = 9$$

$$\begin{array}{r} 35 \\ \times 2 \\ \hline 70 \\ -52 \\ \hline 18 \end{array}$$

<u>Degree in Directed Graph</u>

When (u, v) is an edge of the graph G with directed edges, u is said to be adjacent to v and v is said to be adjacent from u. The vertex u is called initial vertex and v is called the terminal or end vertex.

The initial vertex and the terminal vertex of a loop are the same.

In a graph with directed edges, the <u>In degree of a vertex v</u> is denoted by <u>$deg^-(v)$</u> is the number of edges with v as their terminal vertex.

The <u>out degree</u> of v denoted by <u>$deg^+(v)$</u> is the number of edges with v as their

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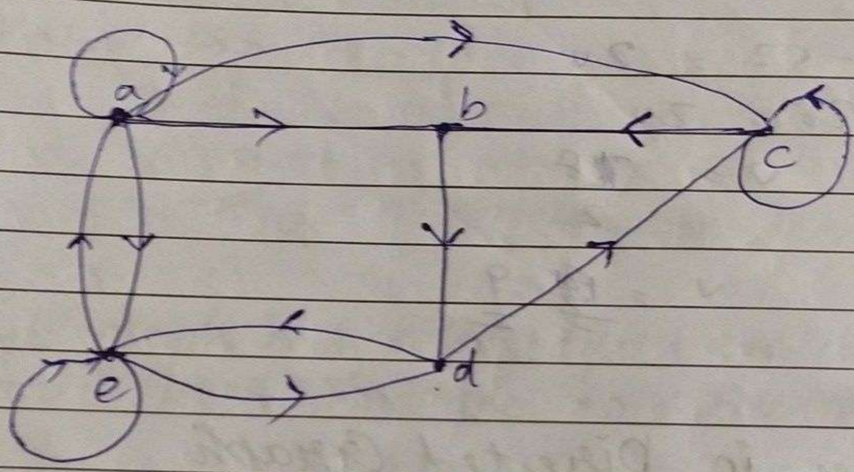
initial vertex.

A loop at a vertex contributes 1 to both the indegree and outdegree of the vertex

Ques Find the in-degree and outdegree of each vertex



a ⇒ indegree - 2
outdegree - 4

b ⇒ indegree - 2
outdegree - 1

c ⇒ indegree - 3
outdegree - 2

d ⇒ integree 2
out degree 2

e ⇒ $deg^-(e)$ - 3
$deg^+(e)$ 3

sum of indegree - 12
sum of outdegree 12

no of edges - 12

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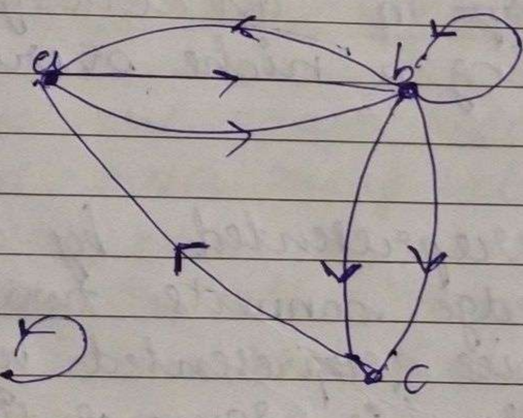
Theorem 3

Let $G = (V, E)$ be a graph with directed edges then

$$\sum_{v \in V} \deg^-(v) = \sum_{v \in V} \deg^+(v) = |E|$$

Sum of indegree = sum of outdegree = no. of edges

Ques Find the indegree and outdegree



$\deg^-(a) = 2$ $\quad\quad$ $\deg^+(a) = 2$
$\deg^-(b) = 3$ $\quad\quad$ $\deg^+(b)$ 4
$\deg^-(c) = 2$ $\quad\quad$ $\deg^+(c)$ 1
$\deg^-(d) = 1$ $\quad\quad$ $\deg^+(d)$ 1

Total - 8 $\quad\quad$ Total 8

No. of edges 8

Graph Models

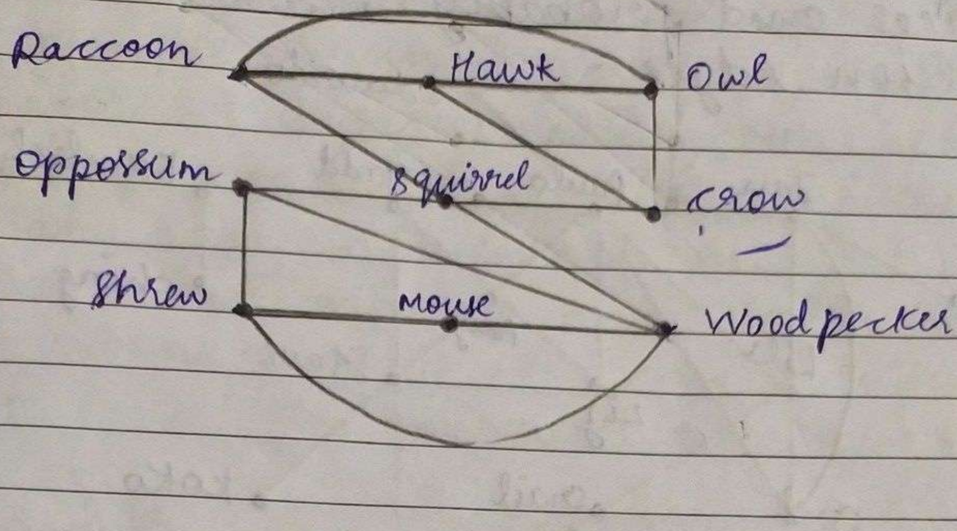
Graphs are used in a wide variety of models. We will present a few graph models from diverse fields here. Others will be introduced in subsequent sections.

1. Niche Overlap Graphs in Ecology

Graphs are used in many models involving the interaction of different species of animals.
For instance, the competition between species ~~between sp~~ in an ecosystem can be modeled using a niche overlap graph.

Each species is represented by a vertex. An undirected edge connects two vertices if the two species represented by these vertices complete (ie, some of the food resources they use are the same)

A niche overlap graph is simple graph because no loops or multiple edges are needed in this model. The graph models the ~~ecosystem~~ ecosystem of a forest. We see from this graph that squirrel and raccoons compete but crows and shrews donot.

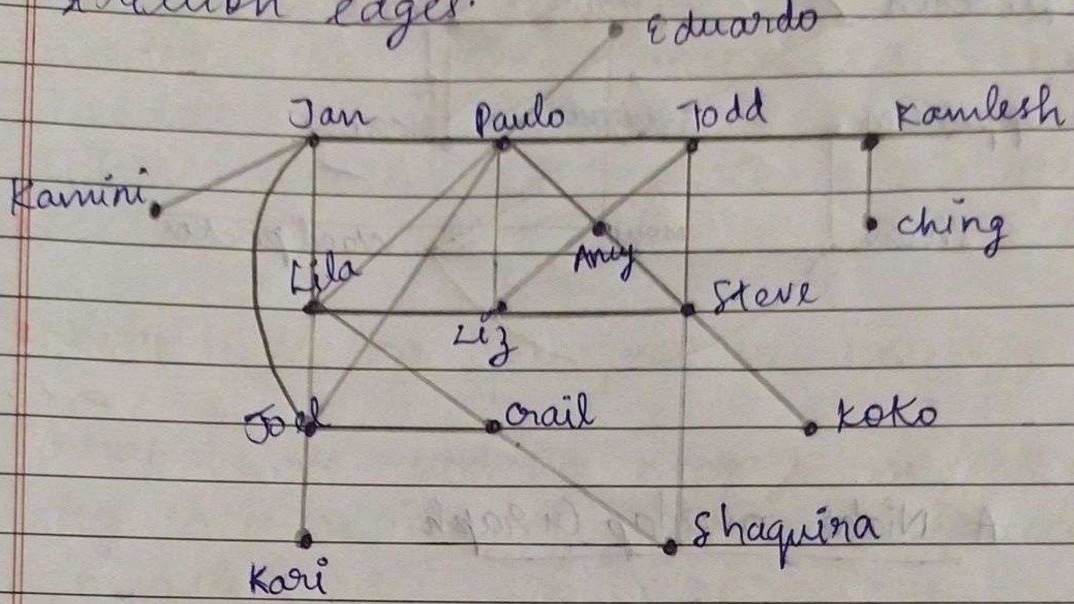


A Niche overlap Graph

20 ## Acquaintanceship Graphs

We can use graph models to represent various relationship between people. For eg:- we can use a simple graph to represent whether two people know each other ie. whether they are acquainted

Each person in a particular group of people is represented by a vertex. An undirected edge is used to connect two people when these people know each other. No multiple edges and usually no loops are used (if we want to include the notion of self-knowledge we would include loops). A small acquaintanceship graph is shown below. The acquaintanceship graph of all people in the world has more than six billion vertices and probably more than one trillion edges.



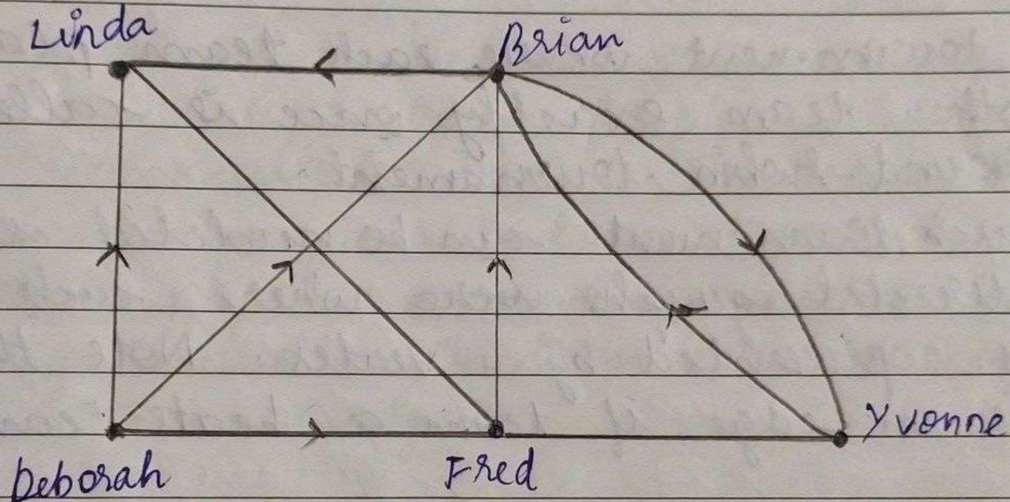
An acquaintanceship Graph

3. Influence Graphs

In studies of group behaviour it is observed that certain people can influence the thinking of others. A directed graph called the influence graph can be used to model this behaviour.

Each person of the group is represented by a vertex. There is a directed edge from vertex a to vertex b, when the person represented by vertex a influence the person represented by vertex b.

This graph doesnot contain loops and it doesnot contain multiple directed edges. An example of an influence graph for members of a group is shown below. In the group modeled by this influence graph, Deborah can influence Brian, Fred and Linda, but no one can influence her. Also Yvonne and Brian can influence each other.



An influence graph

4. The Hollywood Graph

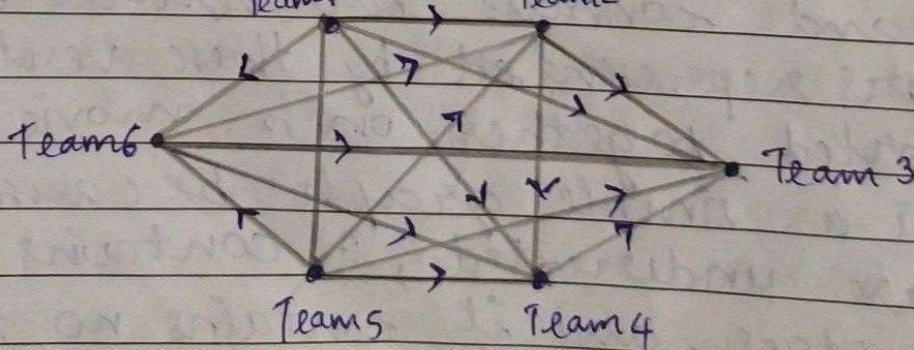
The Hollywood graph represents actors by vertices and connects two vertices when the actors represented by these vertices have worked together on a movie. This graph is a simple graph because its edges are undirected, it contains no multiple edges, and it contains no loops.

According to the Internet Movie Database, in January 2006 the Hollywood graph has 637,099 vertices representing actors who have appeared in 339,896 films and had more than 20 million edges.

5. <u>Round - Robin Tournaments</u>

A tournament where each team plays each other team exactly once is called a round-robin tournament.
Such tournaments can be modeled using directed graphs ~~were~~ where each term is represented by a vertex. Note that (a,b) is an edge if team a beats team b.

This graph is a simple directed graph, containing no loops or multiple directed edges (because no two teams play each other more than once). Such a directed graph model is presented below, In this, Team 1 is undefeated in this tournament and Team 3 is winless.



A Graph Model of Round-Robin Tournament

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6. Collaboration Graphs

A graph called a collaboration graph can be used to model joint authorship of academic papers. In a collaboration graph, vertices represent people (perhaps restricted to members of a certain academic community) and edges link two people if they have jointly written a paper.

This graph is a simple graph because it contains undirected edges and has no loops or multiple edges. The collaboration graph for people working together on research papers in mathematics has been found to have more than 400,000 vertices and 675,000 edges

7. Call graphs

Graphs can be used to model telephone calls made in a network, such as a long-distance telephone network. In particular, a directed multigraph can be used to model calls where each telephone number is represented by a vertex and each telephone call is represented by a directed edge.

The edge representing a call starts at the telephone number from which the call was

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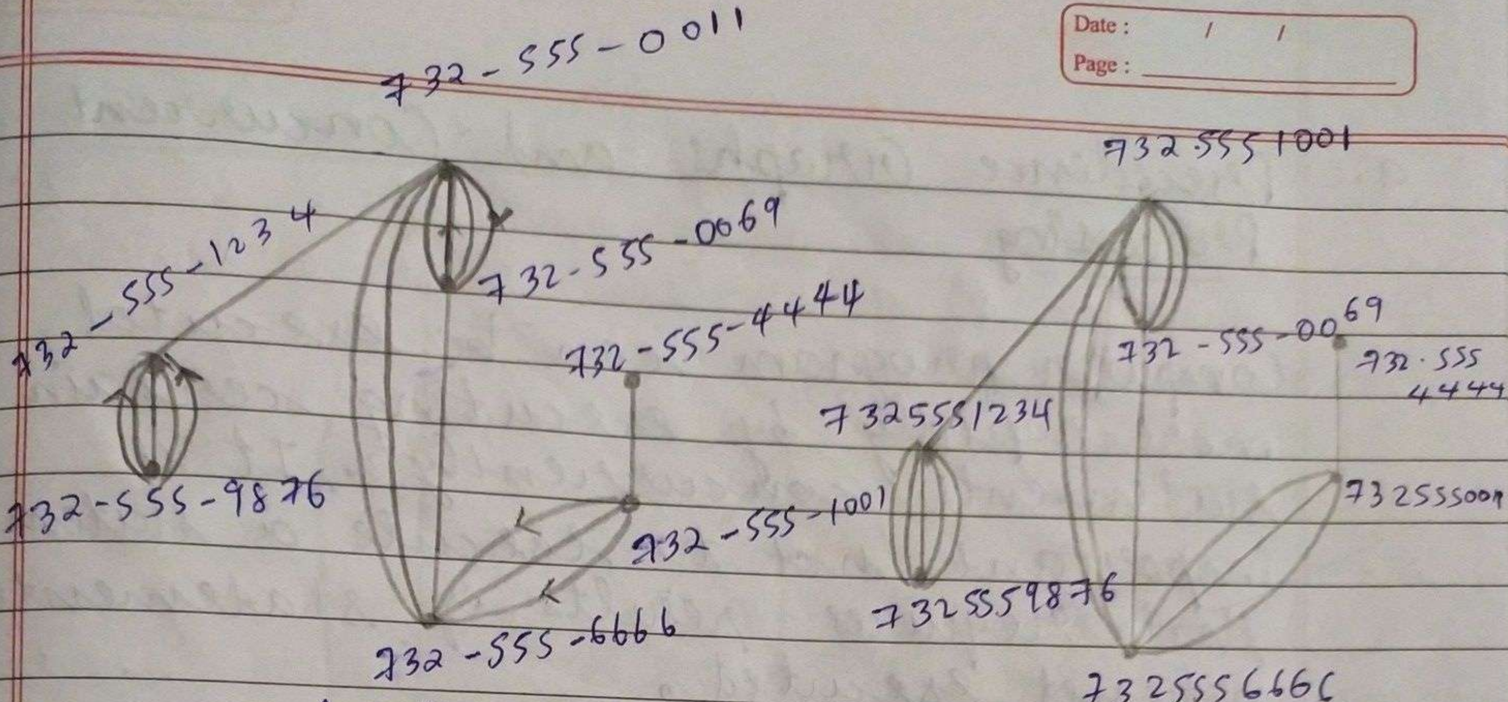
made and ends at the telephone number to which the call was made. We need directed edges because the direction in which the call is made matters. We need multiple directed edges because we ~~ca~~ want to represent each call made from a particular telephone number to a second number.

A small telephone call graph is displayed below, representing seven telephone numbers. This graph shows for instance, that 3 calls have been made from 732-555-1234 to 732-555-9876 and two in the other direction, but no calls have been made 732-555-4444 to any of the other six numbers except 732-555-0011. When we care only whether there has been a call connecting 2 telephone numbers, we use an ~~d~~ undirected graph with an edge connecting telephone numbers when there has been a call between these numbers. This version of the call graph is displayed below.

Call graphs that model actual calling activities can be huge. For example, one call graph studied at A and T, which models calls during 20 days, has about 290 million vertices and 4 billion edges.



A Call graph

8. The web Graph

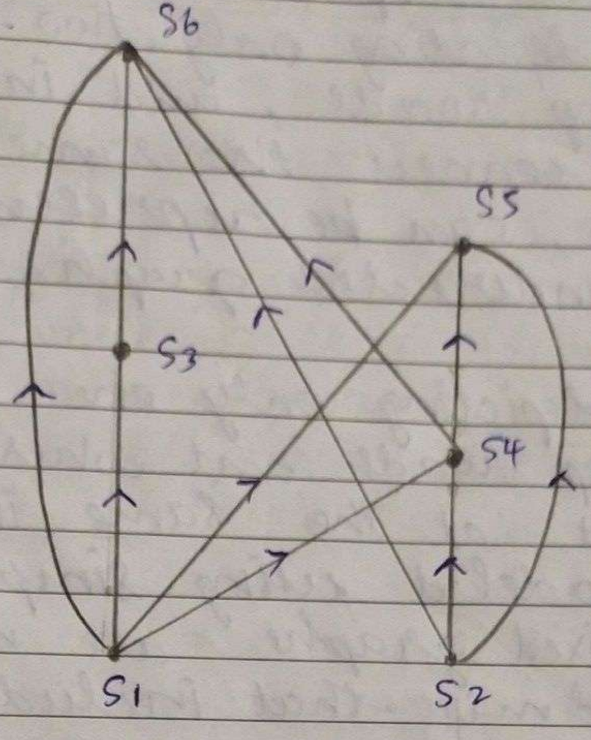
The www can be modelled as a directed graph where each the web page 'b' if there is a link on 'a' pointing to 'b'. Because new web pages are created and others removed somewhere on the web almost every second, the web graph changes on an almost continual basis. Currently the web graph has more than three billion vertices and 20 billion edges. Many people are studying the properties of the web graph to better understand the nature of the web.

9. Precedence Graphs and Concurrent Processing

Computer program can be executed more rapidly by executing certain statements concurrently. It is important not to execute a statement that requires results of statements not yet executed.

The dependence of statements on previous statements can be represented by a directed graph. Each statement is represented by a vertex, and there is an edge from one vertex to second vertex. If the statement represented by the second vertex, and there is an edge from one vertex to second vertex. If the statement represented by the first vertex has been executed.

This graph is called a precedence graph. A computer program and its graph are displayed below. For instance, the graph shows that statement S_5 cannot be executed before statements S_1, S_2 and S_4 are executed.



S1 a := 0
S2 b := 1
S3 c := a + 1
S4 d := b + a
S5 e := d + 1
S6 f := c + d

10) Roadmaps

Graphs can be used to model roadmaps. In such models, vertices represent intersections and edges represent roads.

Undirected edges represent two-way roads and directed edges represent one-way roads.

Multiple undirected edges represent multiple 2-way roads connecting the same two intersections.
Multiple directed edges represent multiple one-way roads that start at one intersection and end at a second intersection.

Loops represent loop roads. Consequently roadmaps depicting only two-way roads and no loop roads, and in which no two roads connect the same pair of intersections, can be represented using a simple undirected graph.

Roadmaps depicting only one-way roads and no loop roads and where no two roads start at the same intersection, can be modeled using simple directed graphs. Mixed graphs are needed to depict roadmaps that include both one-way and two-way roads.

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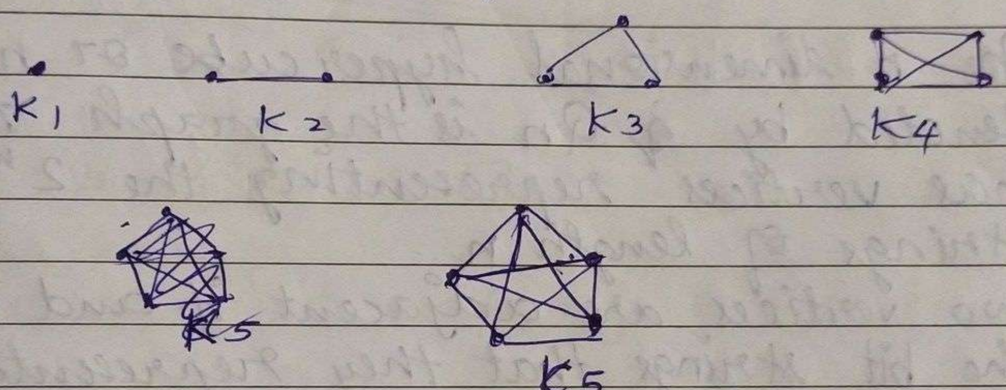
Special Simple Graphs

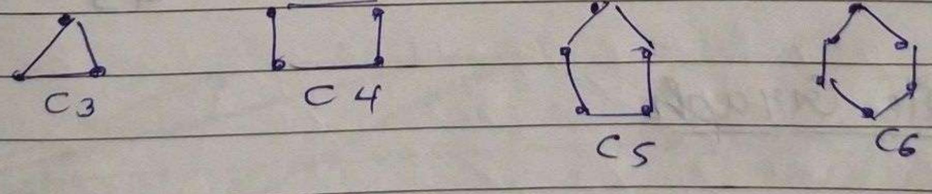
① Complete graph

The complete graph on n vertices denoted by K_n where $n = 1, 2, 3 \ldots\ldots$ is the simple graph that contains exactly one edge between each pair of distinct vertices



② Cycles

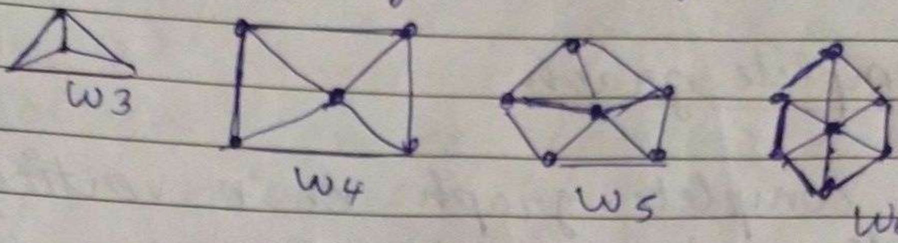
The cycle C_n, $n \geq 3$ consists of n vertices 1, 2, 3 .. n and edges $\{(1,2), (2,3) \ldots [(n-1), n], (n, 1)\}$



③ Wheels

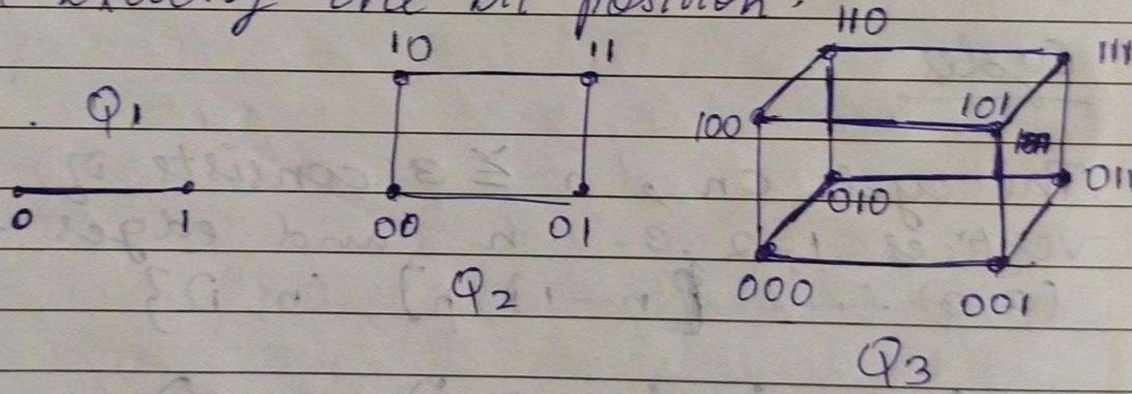
We obtain the wheel W_n when we add an additional vertex to the cycle C_n for $n \geq 3$

and connect this new vertex to each of the n vertices in C_n by new edges



④ n cube

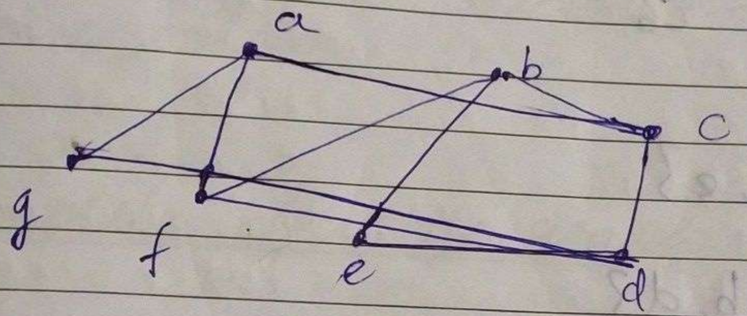
The n dimensional hypercube or n cube denoted by Q_n is the graph that has vertices representing the 2^n bit strings of length n.
Two vertices are adjacent if and only if the bit strings that they represent differ in exactly one bit position.



⑤ Bipartite Graph

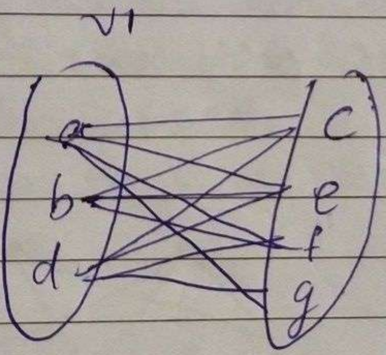
A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph connects a vertex in V_1 and a vertex in V_2 so that no edge in G connects either two vertices in V_1 or two vertices in V_2. when this condition ~~calls~~ holds we call the pair V_1, V_2 a biparition of the vertex set V ~~orf~~ of G.

eg:-

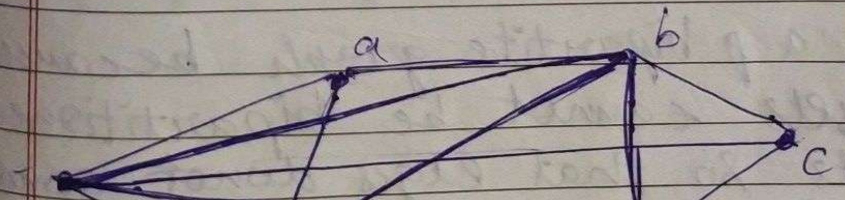


$V_1 = \{a, b, d\}$

$V_2 = \{c, e, f, g\}$



eg:-



It is not a bipartite graph because we ~~can't partition the~~ its vertex set cannot be partitioned into 2 subsets so that edges donot connect two vertices from the same subset

$V_1 = \{a, c\}$

$V_2 = \{b, e, f, d\}$

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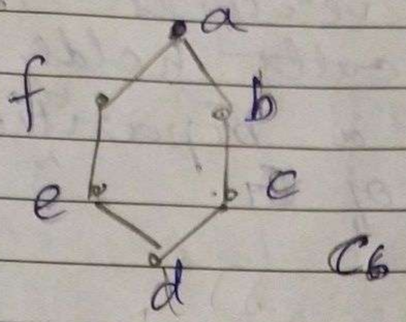
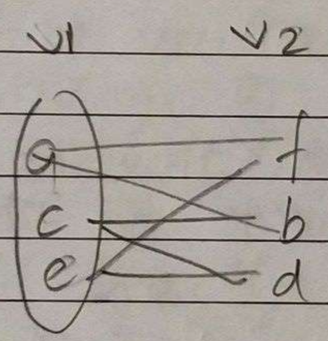
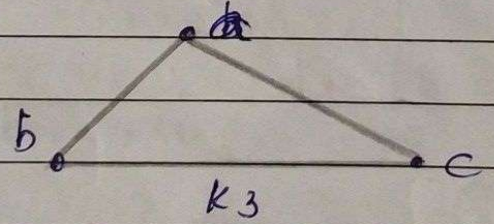
Ques - Check whether C_6 and K_3 are bipartite?



$V_1 = \{a, c, e\}$

$V_2 = \{f, b, d\}$





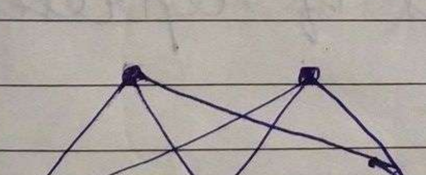
No K_3 is not a bipartite graph because its vertex sets cannot be bipartitioned into 2 subsets so that edges donot connect two vertices from the same subset

Theorem 4

A simple graph is bipartite if and only if it is possible to assign one of two different colours to each vertex of the graph so that no two adjacent vertices are assigned the same colour.

⑥ Complete bipartite graph

The complete bipartite graph denoted by $K_{m,n}$ is the graph that has its vertex set partitioned into two subsets of m and n vertices respectively. There is an edge between two vertices if and only if one vertex is in the first subset and the other vertex is ~~th~~ in the second subset

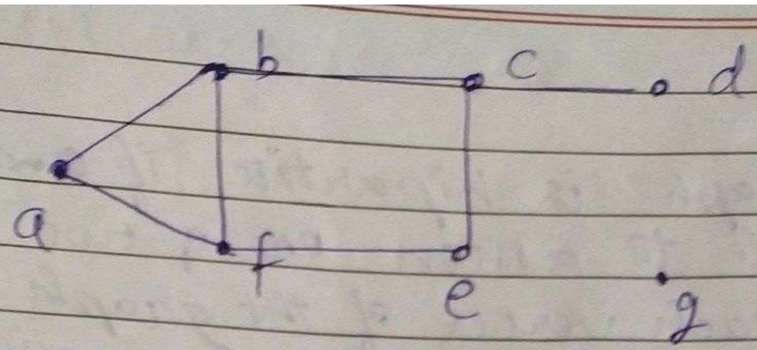


$K_{2,3}$

$K_{3,3}$

Degree Sequence

Degree sequence of a graph is the sequence of degree of the vertices of the graph in ~~Increasing~~ non increasing order (decreasing)



deg(a) = 2
deg(b) = 3
deg(c) = 3
deg(d) = 1
deg(e) = 2
deg(f) = 3
deg(g) = 0

3, 3, 3, 2, 2, 1, 0

5/02/2020
wednesday

Representing Graphs / Representation of Graph

There are three ways of representing graphs.

1. Adjacency ~~Lists~~ Lists
2. Adjacency Matrices
3. Incidence Matrices

1. Adjacency Lists

One way to represent a graph with no multiple edges - is adjacency lists which specify the vertices that are adjacent to

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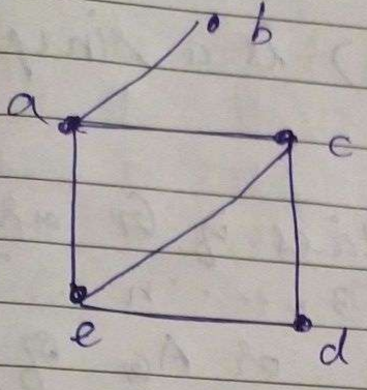
each vertex of the graph.

eg:-

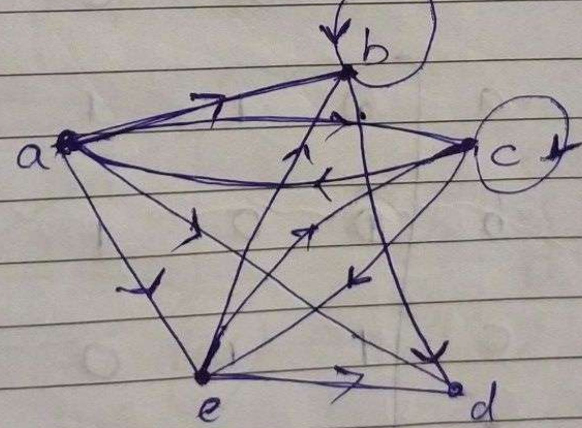


undirected graph

Vertex	Adjacency vertex
a	b, c, e
b	a
c	a, d, e
d	c, e
e	a, c, d

NOTE:- In a directed graph the adjacent vertices are replaced by the terminal vertices in the adjacency lists.

eg:-



Initial vertex	Terminal vertex
a	b, c, d, e
b	b, d
c	a, c, e
d	—
e	b, c, d

2. Adjacency Matrices

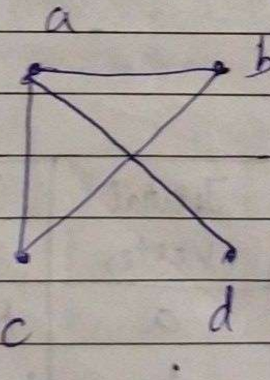
Suppose that $G = (v, e)$ is a simple graph where ~~no~~ $|V| = n$
(no. of v)

Suppose that the vertices of G are listed arbitararly as 1, 2, 3 . . . . n. The adjacency Matrix A or A_G of G with respect to the listing of vertices is the $n \times n$ ~~0 1~~ zero-one matrix with 1 as its (i, j)th entry where i and j are adjacent and 0 as its (i, j)th entry when they are not adjacent.

In other words, if its adjacency matrix $A = [a_{ij}]$ then,

$$a_{ij} = \begin{cases} 1, & \text{where } (i,j) \text{ is an edge of } G \\ 0, & \text{otherwise} \end{cases}$$

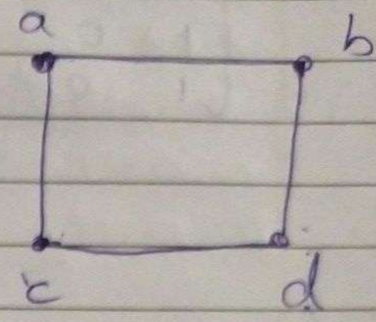
eg:-



$$\begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

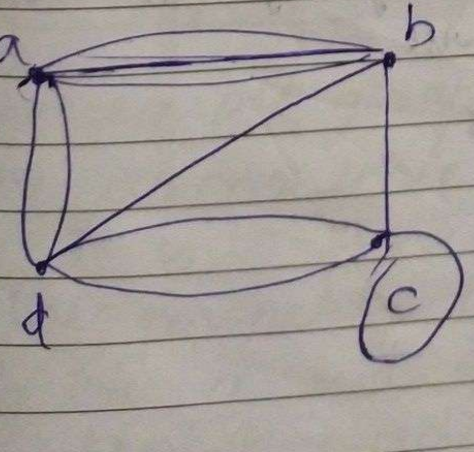
Q: Draw a graph with the adjacency matrix

$$\begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \end{array}$$



OTE An adjacency matrix of a graph is based on the ordering chosen for the vertices. Hence there are n factorial different adjacency matrices for a graph with n vertices because there are n factorial (n!) different ~~oders~~ orderings of n vertices

ues Write the adjacency matrix for the pseudograph given below

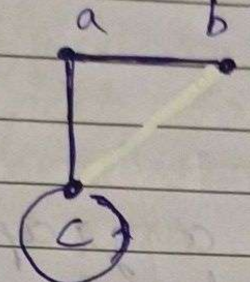


$$\begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

Ques Draw a graph with adjacency matrix

$$\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

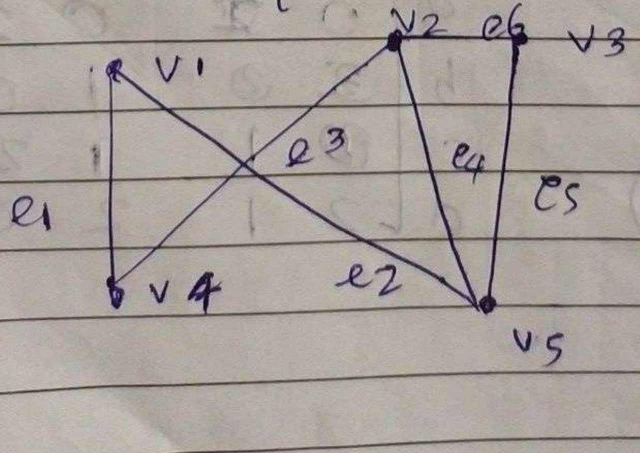
$$\begin{array}{c|ccc} & a & b & c \\ \hline a & & & \\ b & & & \\ c & & & \end{array} \quad \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$



3. Incidence Matrices

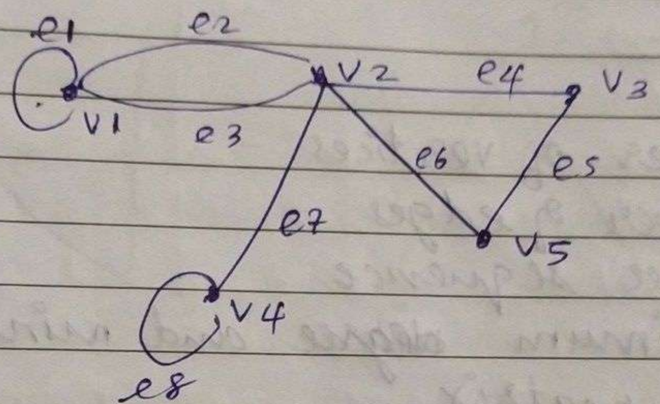
Let $G = (V, e)$ be an undirected graph. Suppose $1, 2, 3 \cdots n$ are the vertices and $e_1, e_2 \ldots e_m$ are the edges of G then the incidence matrix with respect to this ordering of V and e is the $n \times m$ matrix ~~where~~ $M = [m_{ij}]$ where

$$m_{ij} = \begin{cases} 1, & \text{when edge } e_j \text{ is incident with } i \\ 0, & \text{otherwise} \end{cases}$$



$$
\begin{array}{c}
 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5
\end{array}
\begin{array}{c}
\begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$

eg:-



$$
\begin{array}{c|cccccccc}
 & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \\
\hline
v_1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
v_2 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\
v_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
v_4 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
v_5 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{array}
$$

Isomorphism of Graphs

The simple graph $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic if there is a one-to-one and onto function from V_1 to V_2 a with the property that a and b are adjacent in G_1 if and only if f (a) and f(b) are ~~adjoint~~ adjacent in G_2 for all a and b in V_1 such a function f is called ~~an~~ isomorphism.

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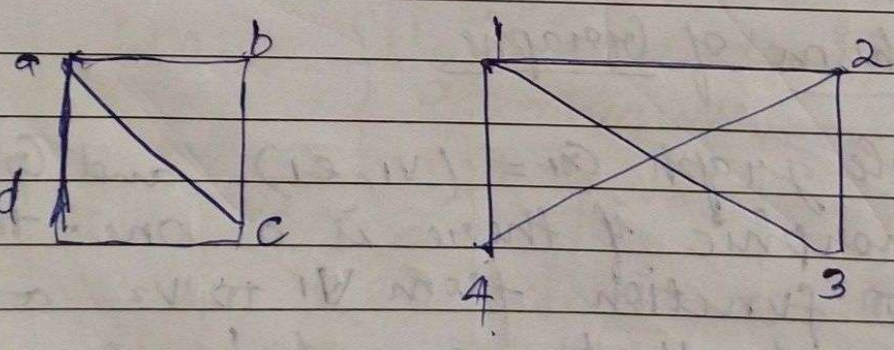
In other words, two graphs are said to be isomorphic if they perphaps the same graph just drawn differently with different names i.e. they have identical behavior for any graph theoretic property

Properties:-

(a) The number of vertices
(b) The number of edges
(c) The degree sequence
(d) The maximum ~~degree~~ degree and minimum degree
(e) Adjacency matrix
(f) Subgraph

10/02/2020
Monday

Ques Check whether these two are isomorphic



a) No of vertices	4	4
b) No of edges	5	5
c) No of deg sequence	<3,3,2,2>	<3,3,2,2>
d) max degree	3	3

Date : / /
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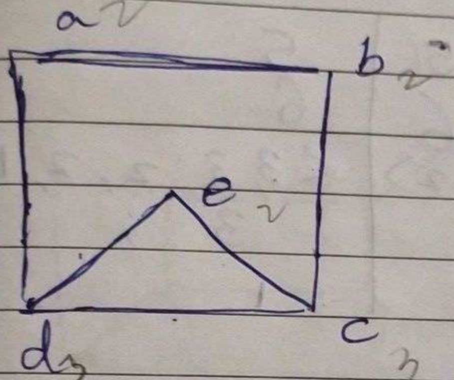
d) Min degree : 2 | 2

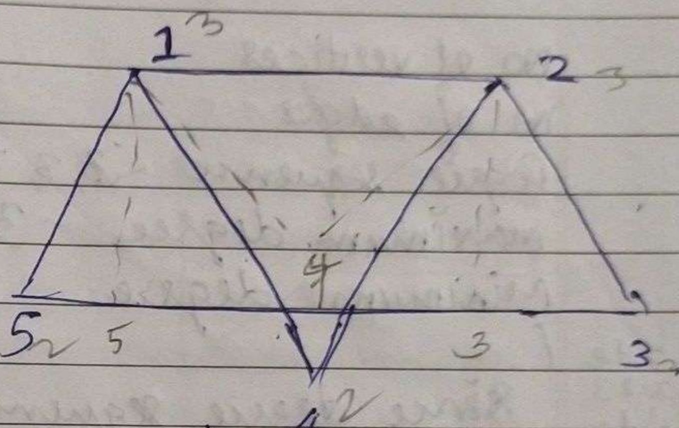
e)

$$\begin{array}{c|c} & a \quad b \quad c \quad d \\ a & \\ b & \\ c & \\ d & \end{array} \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{c} \\ 1 \\ 3 \\ 2 \\ 4 \end{array} \begin{array}{c} 1 \quad 3 \quad 2 \quad 4 \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Ques:-





1) No of vertices : 5 | 5
2) No of edges 6 | 6
3) Degree of sequence : ⟨3, 3, 2, 2, 2⟩ | ⟨3, 3, 2, 2, 2⟩
4) Maximum ~~edge~~: 3 | 3
Minimum degree: 2 | 2
5) ~~Adjacen~~

a → 3
4 → e
b → 5
1 → 2
c → 1

degree a - 2 ; b - 2 ; c - 3 ; d - 3 ; e - 2 ;
degree 1 - 3 ; 2 - 3 ; 3 - 2 ; 4 - 2 ; 5 - 2 ;

$$\begin{array}{c|c} & a \quad b \quad c \quad d \quad e \\ a & \\ b & \\ c & \\ d & \\ e & \end{array} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

$$\begin{array}{c|c} & 3 \quad 5 \quad 1 \quad 2 \quad 4 \\ 3 & \\ 5 & \\ 1 & \\ 2 & \\ 4 & \end{array} \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

These two graphs are isomorphic.

Date : / /
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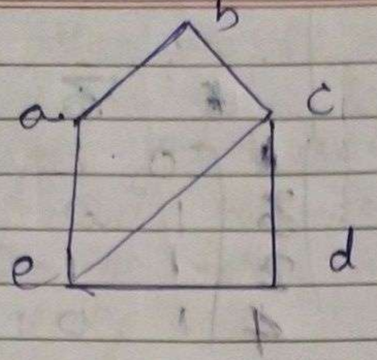
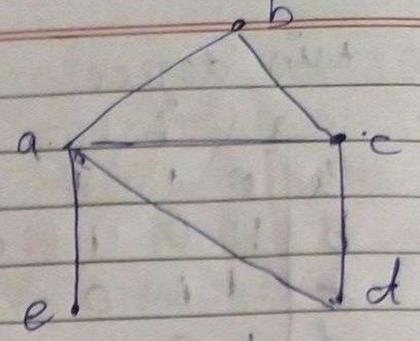
Ques



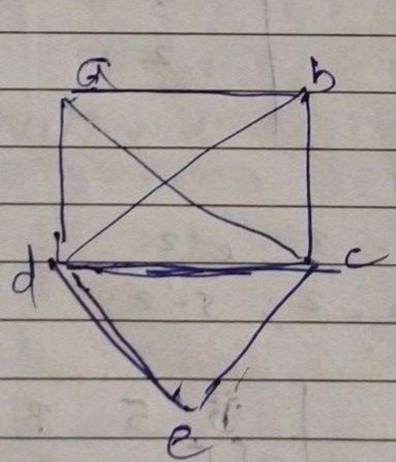


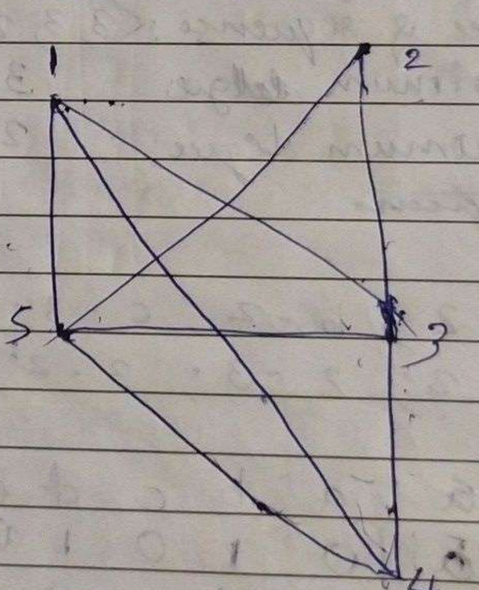
No of vertices	5	5
No of edges	6	6
Degree sequence	<3, 3, 3, 2, 2>	<3, 3, 2, 2, 1>
Maximum degree	3	3
Minimum degree	2	1

a	b	c	d	e
2	2	3	2	3
3	2	3	2	1

Since degree sequence and minimum degree is not same, it is not isomorphic.

Ques



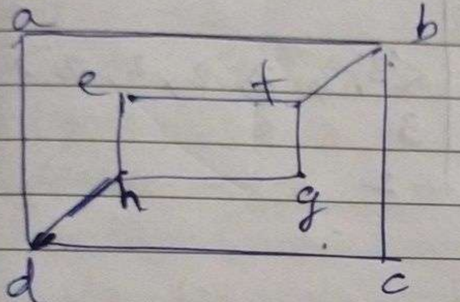


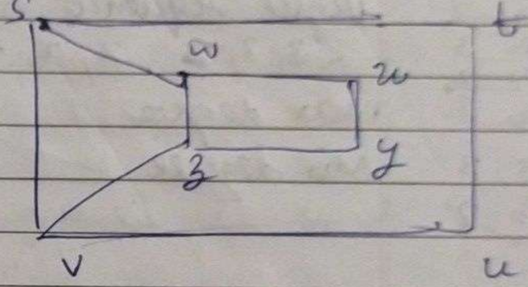
a	b	c	d	e
3	3	4	4	2

1	2	3	4	5
3	2	4	3	4

No of vertices	5	5
No of edges	8	8
Degree sequence	<4, 4, 3, 3, 2>	<4, 4, 3, 3, 2>
Min degree	2	2
max degree	4	4

$$
\begin{array}{c|c}
& a \quad b \quad c \quad d \quad e \\
\hline
\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} &
\begin{pmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0
\end{pmatrix}
\end{array}
\qquad
\begin{array}{c|c}
& 1 \quad 4 \quad 5 \quad 3 \quad 2 \\
\hline
\begin{array}{c} 1 \\ 4 \\ 5 \\ 3 \\ 2 \end{array} &
\begin{pmatrix}
0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 0
\end{pmatrix}
\end{array}
$$





No of vertices	8	8
No of edges	10	10
Degree sequence	$\langle 3,3,3,3,2,2,2,2 \rangle$	$\langle 3,3,3,3,2,2,3,2 \rangle$
Max degree	3	3
Min degree	2	2

$$
\begin{array}{c|c}
& a \quad b \quad c \quad d \quad e \quad f \quad g \quad h \\
\hline
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \\ h \end{array} &
\begin{pmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 1 & 0
\end{pmatrix}
\end{array}
$$

0 1 0

This is not isomorphic

U1 U2 U3 U4 U5 U6

V6 V3 V4 V5 V4 V1

Date : / /

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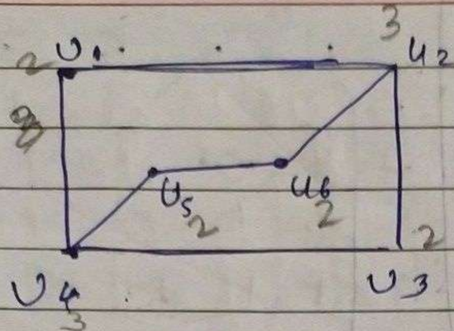
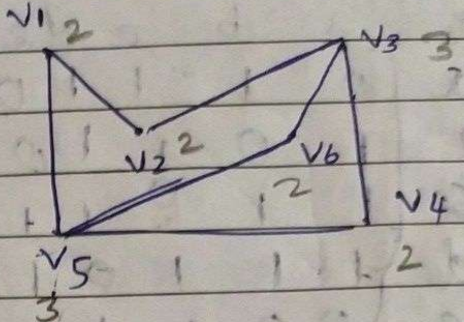
12/02/2020

Wednesday

Ques





Vertices -	6	6
Edges -	7	7
Degree Sequence		
⟨3, 3, 2, 2, 2, 2⟩		⟨3, 3, 2, 2, 2, 2⟩
max degree	3	3
Min degree	2	2

U1 2

U2 3

U3 2

U4 3

U5 2

U6 2.

V1 2

V2 2

V3 3

V4 2

V5 3

V6 2

$$\begin{array}{c|cccccc} & U_1 & U_2 & U_3 & U_4 & U_5 & U_6 \\ \hline U_1 & 0 & 1 & 0 & 1 & 0 & 0 \\ U_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ U_3 & 0 & 1 & 0 & 1 & 0 & 0 \\ U_4 & 1 & 0 & 1 & 0 & 1 & 0 \\ U_5 & 0 & 0 & 0 & 1 & 0 & 1 \\ U_6 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

$$\begin{array}{c|cccccc} & V_6 & V_5 & V_4 & V_3 & V_2 & V_1 \\ \hline V_6 & 0 & 1 & 0 & 1 & 0 & 0 \\ V_5 & & & & & & 1 \\ V_4 & & & & & & \\ V_3 & & & & & & \\ V_2 & & & & & & \\ V_1 & & & & & & \end{array}$$

U4 - V3

U2 V5

~~U5 - V4~~

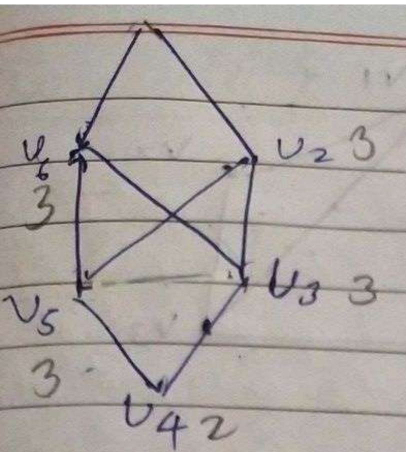
U1 - V2

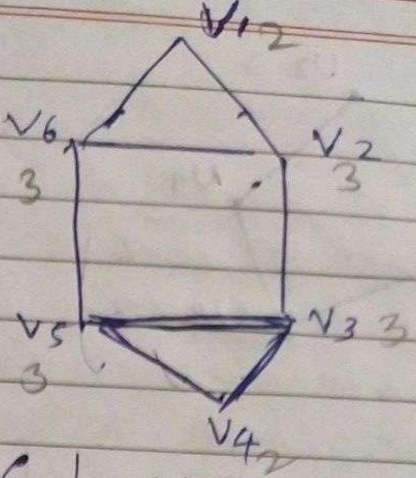
U3 V6

U5 - V1

U6 - V3

Ques





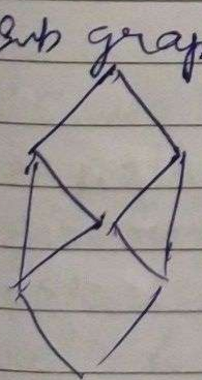
vertices -	6	6
edges -	8	8
dg sequence -	$\langle 3,3,3,3,2,2 \rangle$	$\langle 3,3,3,3,2,2 \rangle$
max degree -	3	3
min degree -	2	2

	u_1	u_2	u_3	u_4	u_5	u_6
u_1	0	1	0	0	0	1
u_2	1	0	1	0	1	0
u_3	0	1	0	1	0	1
u_4	0	0	1	0	1	0
u_5	0	1	0	1	0	1
u_6	1	0	1	0	1	0

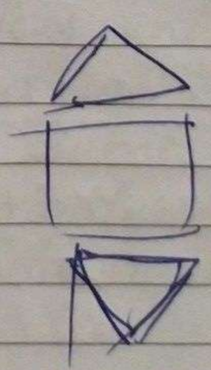
	v_1	v_2	v_3	v_4	v_5	v_6
v_1						
v_2						
v_3	1	1				
v_4		1				
v_5	1		1	1		
v_6		1		1		
			1	1		

It is not isomorphic

sub graph

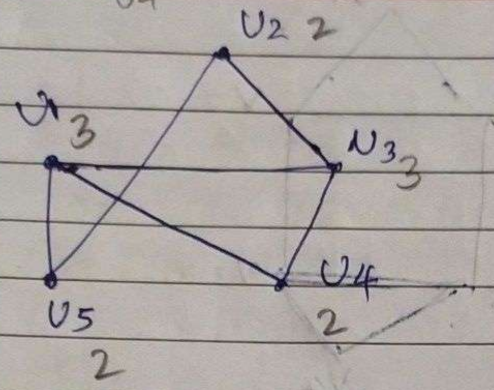


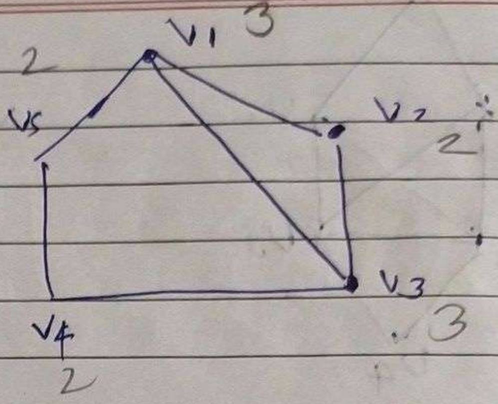
sub graph



$u_1 \to v_1$
$u_2 \to v_4$
$u_3 \to v_3$
$u_4 \to v_2$
$u_5 \to v_5$

Ques





Vertices	5	5
edges	6	6
degree sequence	$\langle 3, 3, 2, 2, 2 \rangle$	degree sequence $(3, 3, 2, 2, 2)$
max degree	3	3
min degree	2	2

	U_1	U_2	U_3	U_4	U_5
U_1	0	0	1	1	1
U_2	0	0	1	0	1
U_3	1	1	0	1	0
U_4	1	0	1	0	0
U_5	1	1	0	0	0

	V_1	V_4	V_3	V_2	V_3
V_1	0	0	1	1	1
V_4	0	0	1	0	1
V_3	1	1	0	1	0
V_2	1	0	1	0	0
V_5	1	1	0	0	0

13/02/2020 Thursday

Connectivity

Path

Informally a path is a sequence of edges that begin at a vertex of a graph and travels from vertex to vertex along edges of a graph.

The formal definition of a path is :-

Let n be a non-negative integer and G an undirected graph. A path of length n from u to v in G is ~~the~~ a sequence of n edges $e_1, e_2 \ldots e_n$ of G such that e_1 is associated with $\{x_0, x_1\}$, e_2 is associated with $\{x_1, x_2\}$ and so on with e_n associated with $\{x_{n-1}, x_n\}$ where $x_0 = u$ and $x_n = v$.

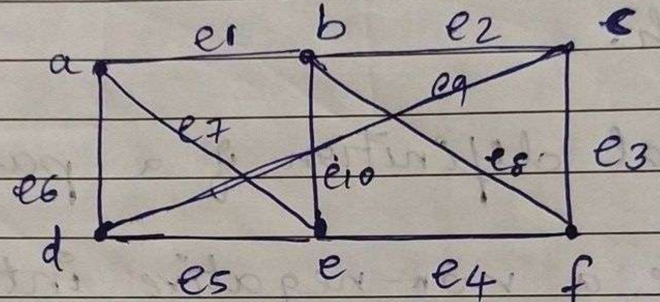
When the graph is simple we denote its path by its vertex sequence $x_0, x_1, \ldots x_n$.

(Because listing these vertices uniquely determines the path.) The path is a circuit if it begins and ends at the same vertex i.e. if $u = v$ and has length greater than zero. A circuit in a graph is also called cycle in a graph.

Walk

A walk is defined to be an alternating sequence of vertices and edges of a graph, $v_0, e_1, v_1, e_2 \ldots\ldots v_{n-1}, e_n, v_n$ where v_{i-1} and v_i are the endpoints of e_i for $i = 1, 2, 3 \ldots n$

Eg:



path - a, d, c, f, e

walk - $a e_6 d e_9 c e_3 f e_4 e$

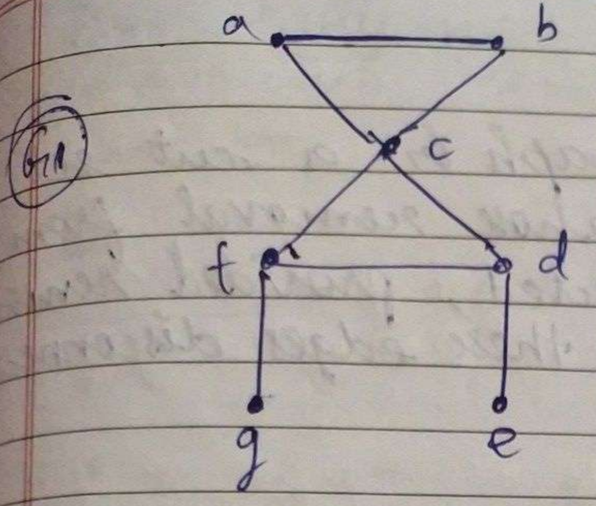
Circuit - a b e d a

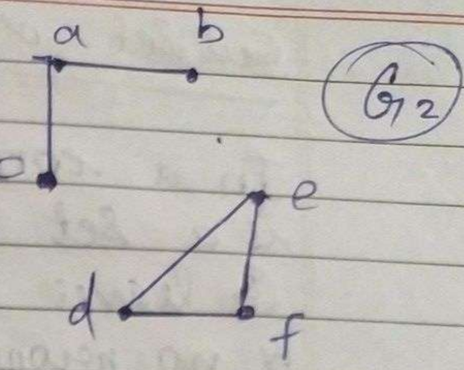
Closed walk - $a e_1 b e_{10} e e_5 d e_6 a$

Trail - $a e_1 b e_2 c e_3 f$

Connectedness in Undirected Graph

A undirected graph is called connected if there is a path between every pair of distinct vertices of the graph





G1 is a connected graph

G2 is a unconnected graph.

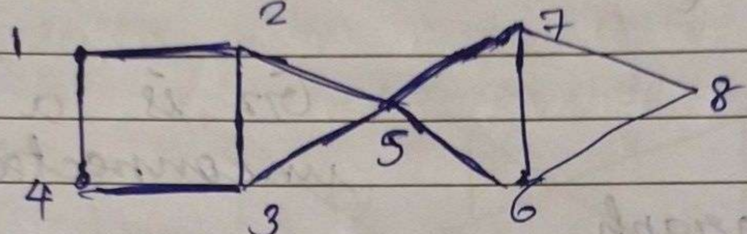
Cut Vertex and Cut Edge

In a graph, or the removal of a vertex and all edges incident with it produces a sub graph with more connected components than in the original graph. Such vertices are called cut vertices or articulation points. ~~The remo~~

The removal of a cut vertex from a connected graph produces a subgraph that is not connected. Similarly nearly an edge whose removal produces a graph with more connected components than in the original graph is called a cut edge or bridge

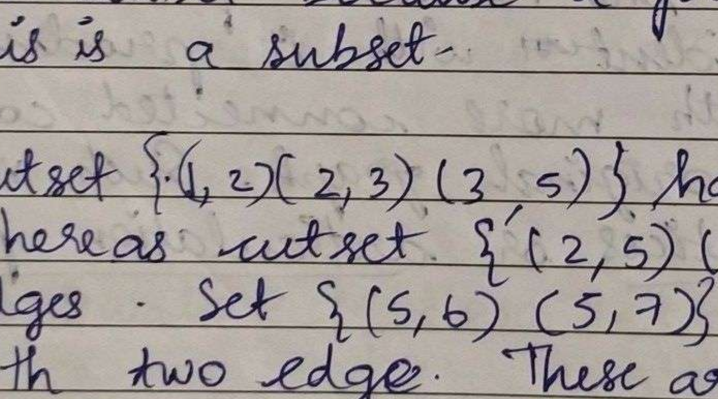
Cut Set or Co-cycle

In a connected graph G a cut set is a set of edges whose removal from G leaves G disconnected, provided removal of no proper subset of these edges disconnects G.



In the above graph set {(2,5) ~~and~~ (3,5)} is a cut set. {(1,2) (2,3) (3,5)} is also a cutset. The set {(1,2) (2,3) (3,5) (2,5)} is not a cutset because a proper subset of this is a subset.

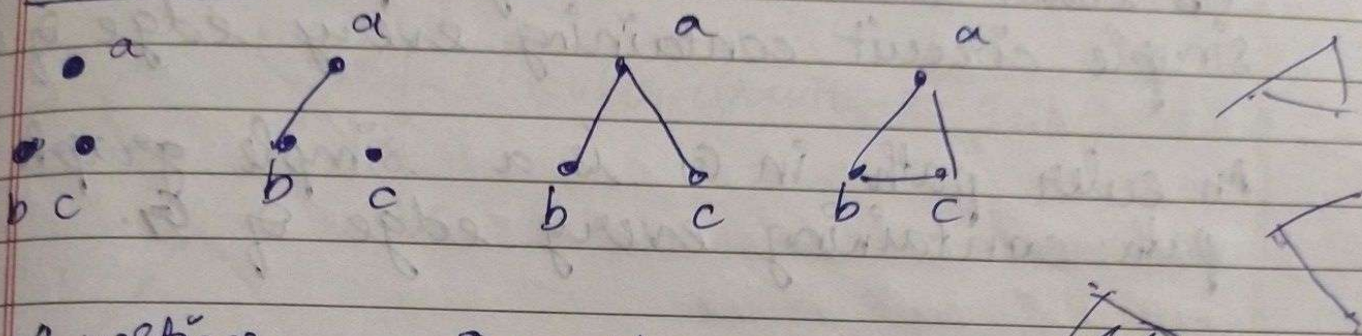
Cutset {(1,2) (2,3) (3,5)} has three edges whereas cutset {(2,5) (3,5)} has two edges. Set {(5,6) (5,7)} is also a cutset with two edge. These are called minimal cutsets or simple cutset



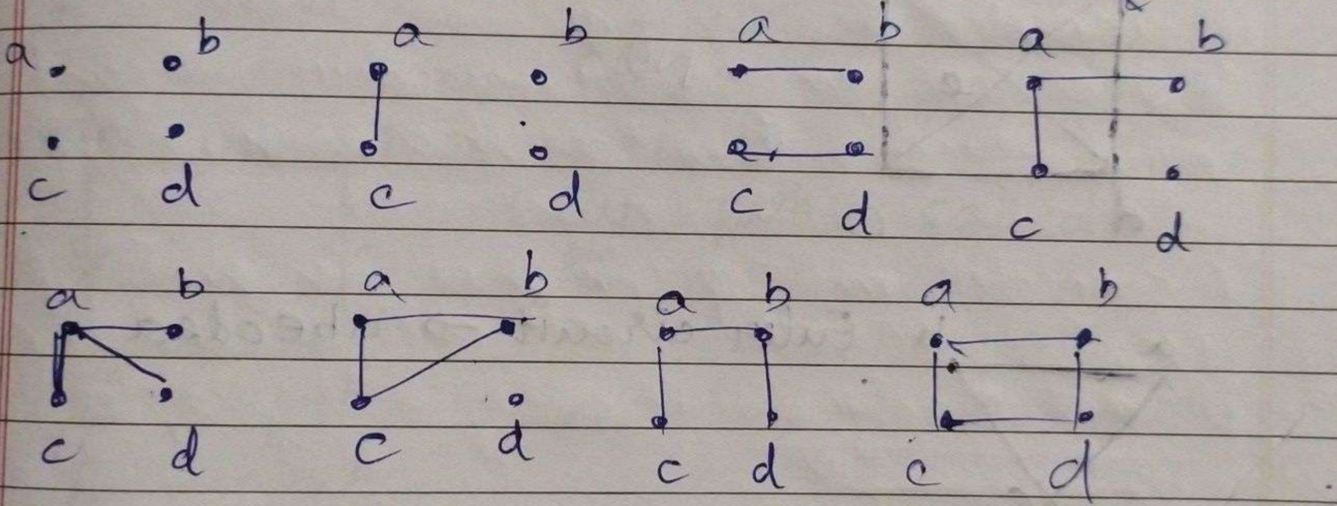
(c,e) (f,e)
(d,c) (c,e) (f,e)
(d,c) (c,e) (f,e)

Ques How many non-isomorphic graphs can be drawn with 3 vertices? / 4 vertices.
~~How mo~~

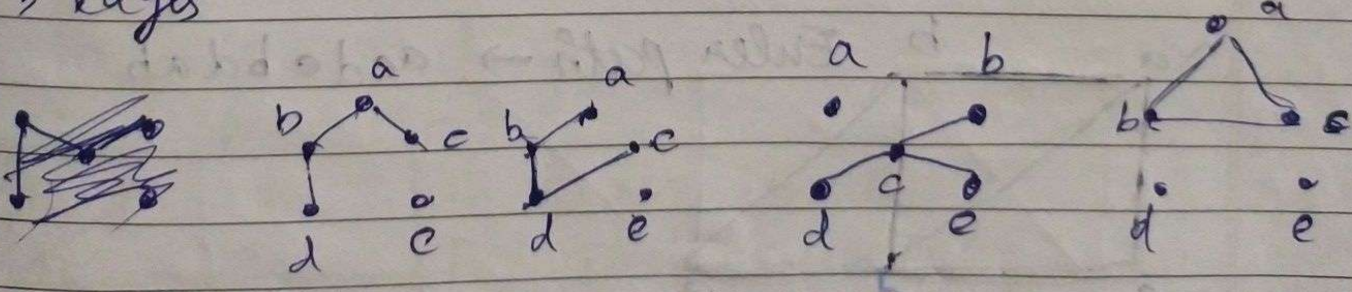
3 vertices → 4 graphs



4 vertices → 8 graphs



Ques How many non-isomorphic simple graphs ~~can be dr~~ are there with 5 vertices and 3 edges



To check whether euler → all edges should be travelled/traversed and edges should not be repeated, but the vertex can be repeated.

(oiler)

24/02/2020

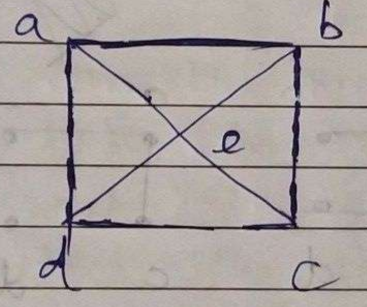
Monday

Euler and Hamilton paths

Euler paths and circuits

An euler circuit in a graph G is a simple circuit containing every edge of G

An euler path in G is a simple ~~graph~~ path containing every edge of G.



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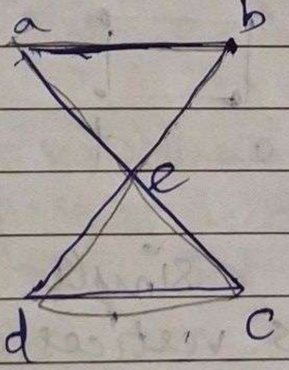
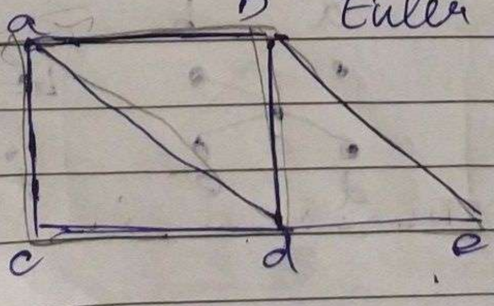
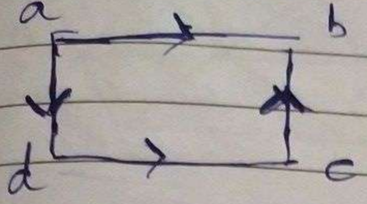


Euler circuit → abecdea

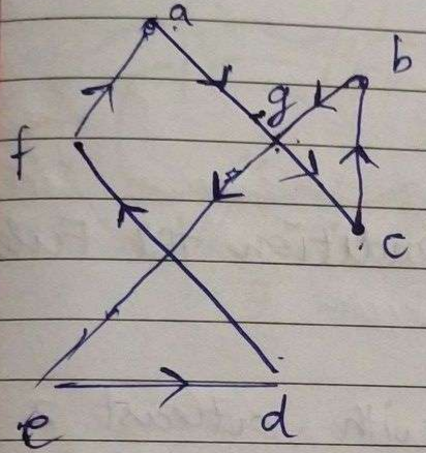


Euler path → acdebdab

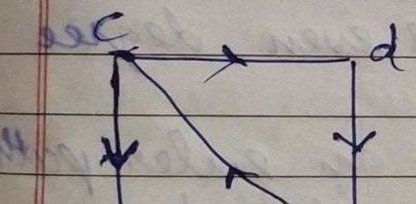
Euler path → exactly 2 vertices has odd degree
Euler circuit → all the degree must be even



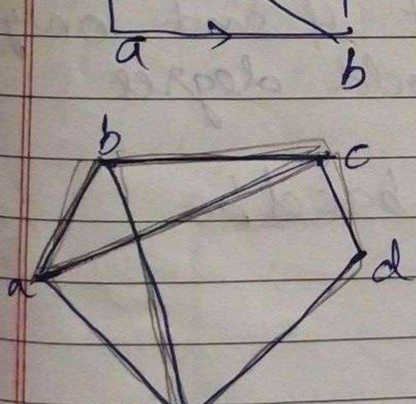
NO X

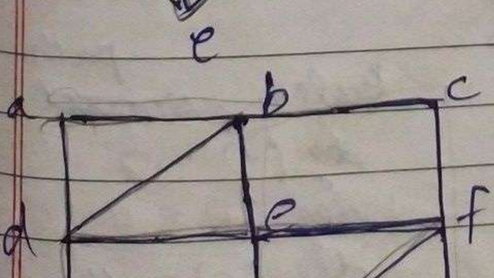


Euler circuit → agcbgedfa

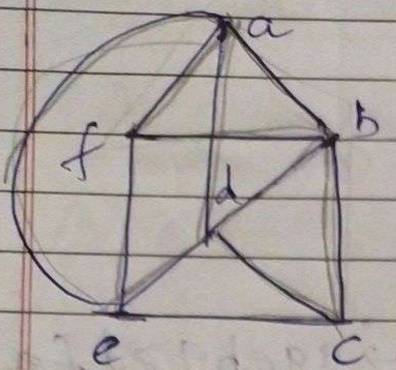


Euler path → cabcdb





Euler circuit →
abdebcfehfihgda



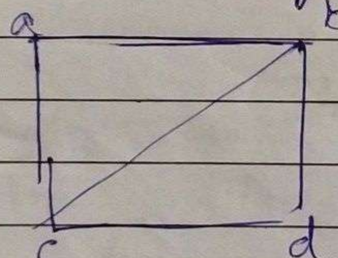
26/02/2020
wednesday

Necessary and Sufficient condition for Euler circuits and paths

A connected multigraph with atleast 2 vertices has an euler circuit if and only if each of its vertices has even degree

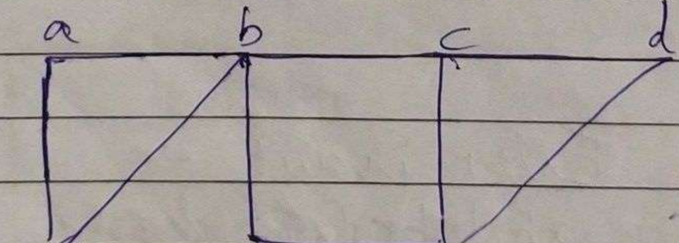
A connected multigraph has an euler path but not an euler circuit if and only if it has exactly 2 vertices of odd degree

G1



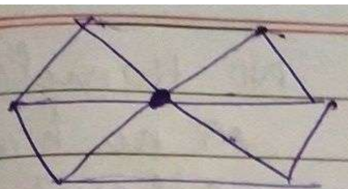
euler path → bacdbc

G2

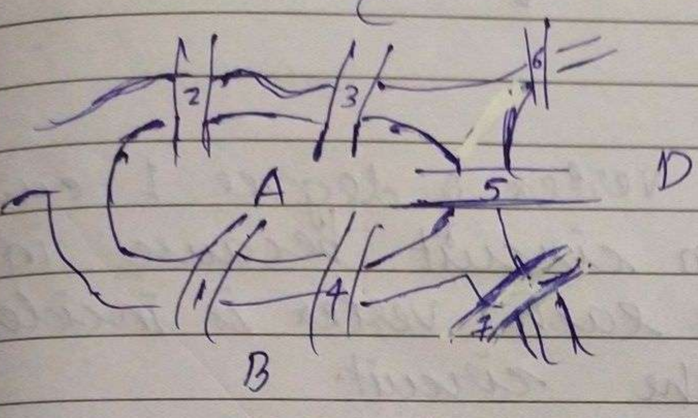


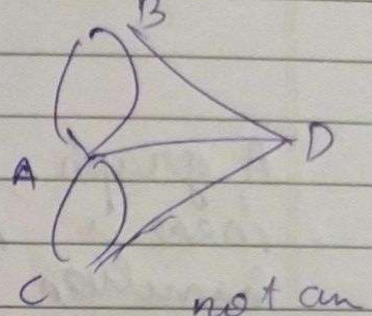
Euler ~~circuit~~ path
deg (a) = 2
deg (b) = 4
deg (c) = 4
deg (d) = 2
deg (e) = 3
deg (f) = 4
deg (g) = 3

gabcdefgbfce



as all the vertices has degree 3 it does not have euler path or circuit



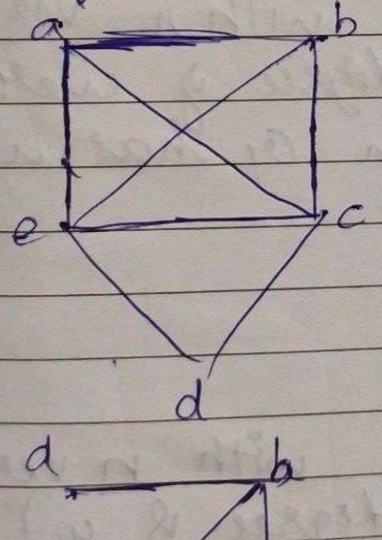


not an euler circuit

Hamilton Paths and Circuits

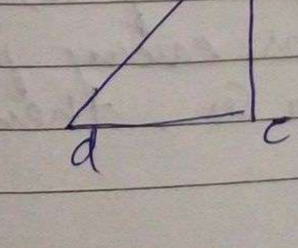
A simple path in a graph G that passes through every vertex exactly once is called a hamilton path. and a simple circuit in a graph G that passes through every vertex exactly once is called a halimlton circuit.

all vertex only one time should be traversed through all vertex once and vertex cannot be repeated & can go through or not go through all edges

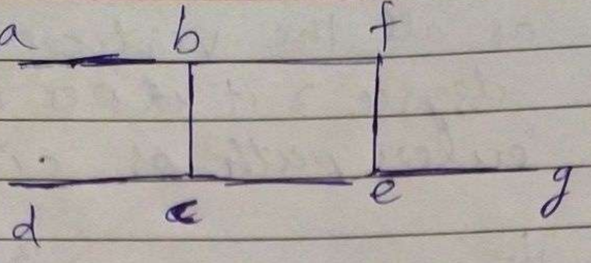


Hamilton circuit
abcdea
Hamilton path
abcde

If vertex is having only 1 degree it will not be hamilton circuit but may or maybe hamilton path



Hamilton path
abcd



No Hamilton path or no hamilton circuit.

A graph with a vertex of degree 1 cannot have a hamilton circuit because in a hamilton circuit each vertex is incident with 2 edges in the circuit.

~~Morevor~~ Moreover, if a vertex in a graph has degree 2 then both edges that are incident with this vertex must be a path of any hamilton circuit.

Dirac's Theorem ~~and~~

If G is a simple graph with n vertices where $n \geq 3$ such that the degree of every vertex in G is atleast $n/2$ then G has a hamilton circuit.

Ore's Theorem

If G is a simple graph with n vertices with $n \geq 3$ such that (degree of u) $\deg(u) + 1 \deg(v) \geq n$ for every pair of non-adjacent vertices u and v in G, then G has a hamilton circuit.