BCA – Fourth Semester

Operations Research

Syllabus

Module I: Basics of O.R (10 hours)

The nature and uses of O.R, Main concepts and approaches of O R, Models in O R, Advantages of a model, Phases of O R

Module II: Linear Programming Problems (25 hours)

Mathematical formulation of a LPP, General LPP problems, Solution of a LPP, Graphical method for solving a LPP, Simplex method, slack and surplus variables, Reduction of any feasible solution to a basic feasible solution, Unbounded solution, Optimality conditions, Artificial variable techniques, Big M method

Module III: Transportation and Assignment problems

Transportation model, solution by simplex method, north west corner rule, lowest cost entry method, vogel's method, MODI method, degeneracy, assignment problems

Module IV: Game Theory

Two persons zero sum games, pure and mixed strategy with saddle point, solution of pure strategy games, solution of mixed strategy problems by arithmetic method, principle of dominance

Module 1

Basics of Operations Research

Meaning of OR

The word operation may be defined as some action that we apply to some problems or hypothesis. The word research is an organized process of seeking out facts about the same. Operations research is the systematic and method oriented study of the basic structure, functions and relationships of an organization. OR provides quantitative measure for decision making. It provides techniques for taking wise decisions and arriving at optimal solutions. It is a study of optimization techniques.

Definition

OR is an applied decision theory which involves a team approach of experts from various disciplines such as scientists, engineers, business executives, psychologists etc. The team tries to analyse the cause and effect relationship for a given problem, between various parameters and provides solutions known as optimal solutions which best suit the purpose of the organization as a whole.

Features or Characteristics of OR (Nature of OR)

1. **System orientation.** OR studies the problem as a whole. It emphasis an overall approach to the system.
2. **Interdisciplinary team approach**. OR is inter-disciplinary in nature. It is performed by a team of experts drawn from different disciplines such as Mathematics, Statistics, Economics, engineering, Management, etc.
3. **Scientific Approach.** OR uses scientific methods to solve complex problems.
4. **Decision making.** OR is a decision science which helps management to take better decision.
5. **Optimization Objective.** OR attempts to find the best and optimal solution to a problem using OR techniques. It tries to optimize a well defined function subject to given constraints.
6. **Mathematical model and quantitative solution**. OR uses models built by quantitative measurement of variables concerning a given problem and derives a quantitative solution from the model.

7. **Use of computers.** OR often requires a computer to solve the complex mathematical method.

Importance of OR (Functions of OR)

OR has gained increasing importance in everyday life. OR is emerging as an inter-disciplinary area of knowledge that can make contribution to the solution of the problems in diverse d areas of our interest. It greatly helps in tackling the intricate and complex problems of modern business and industry. The various functions of OR are;

1. **OR provides a tool for scientific analyser.** OR provides the executives with a more precise description of the cause and effect relationship and risk underlying the business operations in measurable terms. OR replaces the conventional and subjective approach of decision making by an analytical and objective approach.
2. **OR provides solution for various business problems.** OR techniques are used in the field of production, procurement, marketing, finance and other allied fields. For allocating available resources to various products so that in a given time, the profits are maximum or the cost is minimum, OR techniques can be applied. To arrange the time and quantity of orders of stocks to earn maximum profit for the given resources and for determining the optimum number of men and machines to be employed, OR techniques are useful.
3. **Enables proper development of resources.** OR renders valuable help in proper deployment of resources. The OR technique namely PERT (Programme Evaluation and Review Technique) enables us to determine the earliest and the latest times for each of the events and activities and helps in identification of critical path. This enables the project completion on time. Also provides for determining the probability of completing an event or project by a specified date.
4. **Helps in minimizing waiting and servicing costs.** The waiting line of queuing theory is another OR technique which helps in minimizing total waiting and service coasts. OR enables the management to decide when to buy and how much to buy. The technique of inventory planning enables the management to decide when to buy.
5. **OR assists in choosing an optimum strategy.** Game theory is specially used to determine the optimum strategy. In competitive situation game theory enables the businessman to minimize loss by adopting the optimum strategy. OR renders great help in optimum resource allocation. Technique LP is used to allocate resources in an optimum manner in problems of scheduling, product mix and so on.
6. **Facilitates the process of decision making.** Decision theory enables the businessmen to select the best course of action when information is given in probabilistic form. Simulation is another important technique used to imitate an operation or process prior to actual performance.

Scope of OR (Applications or uses of OR)

OR has wide scope and it is very important in every day life. It has applications in many different areas like, defence, engineering, economics, management, public administration and other social and behavioural scervices. Indian organizations like Railways, Indian Airlines etc. are using OR techniques. Defence Organizations, TATA, STC etc. are using OR techniques. Kirloskar Co. Uses assignment models. DCM, Calico, Benny's are using Linear Programming. Many organizations are using PERT/ CPM techniques.

1. **In Defence operations.** Defence operations are carried out by a number of components namely, Air Force, Army and Navy. The activities of these components can be further subdivided as administration, intelligence operation, training and supply. In defence operation it is necessary to coordinate the activities of various components and their subdivisions,

which gives maximum benefit to the organization as a whole. The final strategy is formulated by a team of scientists drawn from various disciplines. The one who is able to select the best strategy wins the battle. The techniques of OR much help to select optimum strategy.

2. **In Industry.** There is a need to integrate the different components of one industry and to frame a policy for effective use of the components so as to serve the best interest of the organization as a whole. In Industrial organizations there is division of management functions, each division has its own objectives and its own contributions. Production department minimizes cost of production but maximizes output, Marketing department maximizes output, but minimizes cost of unit sales. Finance department tries to optimize capital investment. Personnel department appoints good people at minimum cost. The objectives of various departments or components come in conflict with each other. OR techniques help in overcoming this difficulty by integrating the diversified activities of various components.
3. **In Planning.** In modern times, for the government it is necessary to have careful planning for economic development of the country. In the field of planning the basic problem is to obtain maximum growth of per capita income in the shortest possible time by considering the existing restrictions. OR techniques can be effectively applied to maximize per capita income with minimum sacrifice and time. Government can use OR for framing future economic and social policies.
4. **In Agriculture.** With the increase in population there is a need to increase agricultural output. But there are a number of restrictions like climatic conditions, available facilities, distribution of water etc. under which agricultural production is to be studied. The problem of optimum allocation of land for various crops can be solved by the application of OR techniques.
5. **In Transportation.** OR techniques like Monte Carlo technique, queuing theory and linear programming are of great use in transportation activities. Monte Carlo methods can be applied to regulate the train arrivals and processing times. Queuing theory can be applied to minimize the passenger's waiting time and reduce the congestion. Linear programming technique can be used to formulate suitable transportation policy reducing the cost and time of transshipment.

OR and Modern Business Mangement

OR renders valuable service in the field of business management. OR is a problem solving and decision making science. It ensures improvement in the quality of managerial decisions in all functional areas of management.

1. **Allocation and Distribution.** OR is useful to the directing authority in deciding optimum allocation of various limited resources such as men, machines, materials, time, money etc. for achieving the optimum goal.
2. **Production Management.** OR is useful to production management in (i) selecting the building site for a plant, scheduling and controlling it, development and designing its lay out (selection, location and design of production plant) (ii) project scheduling and allocation of resources (iii) Calculation of the number and size of items to be produced (iv) Scheduling and sequencing the production by proper allocation of machines (v) Calculating the optimum product mix.
3. **Marketing Management.** OR is useful in Marketing management in determining; (i) where distribution points and warehousing should be located, their size, quantity to be stocked and the choice of customers (ii) Selection of advertising media (iii) the optimum allocation of sale budget to promotional expenses (iv) the customer preference relating to size, colour packaging etc. for various products.

4. **Personnel Management.** OR is useful to the personnel management in finding out; (i) skilled persons at minimum cost (ii) the number of persons to be maintained (iii) the optimum manner of sequencing personnel to a variety of jobs (iv) in studying personnel procedures, accident rates and labour turn-over (v) recruiting policies and assignment of job.
5. **Finanace Management.** OR is very useful to the finance management in (i) finding out capital requirement policies (ii) working out a profit plan for the firm (iii) developing capital investment plans (iv) estimating credit and investment risks (v) determination of optimum replacement policies.
6. **Research and Development.** OR is useful in research and development in (i) determination of areas for research and development (ii) reliability and control of development projects (iii) selection of projects and preparation of their budgets.

Role of OR in Engineering

In the field of Engineering OR is useful in

(i) Production, planning, scheduling and control

(ii) Inventory control

(iii) Allocation of resources of services

(iv) Planning of maintenance and replacement of equipments

(v) Optimal design of water resources systems, electrical networks, material handling and control systems

(vi) Optimal design of structures and machines

Phases of OR (process of OR or Methodology of OR)

Various steps in the application of OR technique are;

1. Formulating the problem.
2. Constructing a model to represent the system under study
3. Deriving solution from the model
4. Testing the model and the solution derived from it
5. Establishing control over the solution
6. Implementing the results

1. Formulating the problem

It is very essential that the problem at hand be clearly defined. Formulating a problem consists in identifying, defining and specifying the measures of the components of a decision model. This should yield a statement of the problem's element that include the controllable variables, uncontrollable parameters, the restrictions or constraints of the variables and the objectives for defining an improved solution.

2. Constructing the model

In OR a model is usually a mathematical model. Mathematical model consists of a set of equations which describe the problem. It requires the representation of interrelationship among the elements in terms of mathematical formulae. One or more equations or inequalities is required to represent the system. The model include an objective function which defines the effectiveness of the system. The objective function and the constraints together constitute a model of the problem that we want to solve.

3. Deriving the solution

This phase deals with mathematical calculation for obtaining solution to the model. A solution to the model means those values of the decision variables that optimize the measures of effectiveness in a model. There are various methods available for obtaining the solution like analytical method, numerical method and simulation method.

4. **Testing the validity**

A model is said to be valid if it can give a reliable prediction of the system's performance. A model must have a longer life and must be a good representation of the system and must correspond to reality.

5. **Controlling the solution**

The solution derived from a model is depending on the variables. So a change in the values of the variables may affect the solution. So controls must be established to indicate the limits within which the model and its solution can be considered as reliable. When one or more variables change significantly, the solution goes out of control. In such situations the model is to be modified accordingly.

6. **Implementing the results**

The results of OR must be implemented to improve the system performance. So the solution obtained should be translated into operating procedures. This phase of OR is executed through the cooperation of both OR experts and those who are responsible for managing and operating the system.

OR Models

The objective of model is to provide a means for analysisng the behavior of the system for the purpose of improving its performance. Models provide descriptions and explanations of the operations of the system that they represent. A model in OR may be defined as an idealized representation of a real life system.

Properties (characteristics) of a good model

A model in OR should possess the following characteristics;

1. It should be simple
2. It should be capable of adjustments with new formulations without having any significant change in its frame
3. It should contain very few variables
4. A model should not take much time in its construction

Advantages of model

1. It describes problem more concisely
2. It provide some logical and systematic approach to the problem
3. It indicates limitation and scope of the problem
4. It tends to make the overall structure of the problem more comprehensible
5. It facilitate dealing with the problem in its entirety
6. It enables the use of high-powered mathematical techniques to analyse the problem
7. It helps in finding avenues for new research and improvements in a system

Disadvantages of a model

1. Models are only an attempt to understand an operation and should never be considered as absolute in any sense
2. The validity of any model can only be verified by carrying on experiment and relevant data characteristics

Types of models

There are three types of models that are commonly used in OR.

1. Iconic models
2. Analogue models
3. Symbolic models

Iconic models and analogue models are also known as Physical models and symbolic models are called Mathematical models.

1. Iconic models

Iconic models represent the system as it is, but in different size. Iconic models are obtained by enlarging or reducing the size of the system. They are images. That is, Iconic models are pictorial representation of the system and have the appearance of the real thing. These models are usually scaled up or down. A toy airplane is an iconic model of a real one. In a globe the diameter of the earth is scaled down. But its shape, relative size of continents, ocean etc are approximately correct. Iconic models of atoms and molecules used in Chemistry are scaled up, so as to make it visible to the naked eye. Iconic models may be two dimensional or three dimensional.

Examples: photographs, drawing model airplanes, ships, engines, globes, maps etc.

Advantages. (i) They are specified and concrete (ii) They are easy to construct (iii) They can be studied more easily than the system itself.

Disadvantages. (i) They are difficult to manipulate for experimental purposes (ii) They cannot be used to study the changes in operation of a system (iii) It is not easy to make any modification or improvement in these models (iv) adjustments with changing situations cannot be done in these models.

2. Analogue models

In analogue models one set of properties is used to represent another set of properties. After the problem is solved, the solution is re-interpreted in terms of the original system. For example;
(i) contour lines on a map are analogues of elevation as they represent the rise and fall or heights. (ii) graphs are very simple analogues. They represent properties like force, speed, age time etc. in terms of distance. (iii) In a map a blue colour generally represents water.

Advantages. Analogue models are easier to manipulate than iconic models

Disadvantages. Analogue models are less specific and less concrete.

3. Symbolic models (mathematical models)

In symbolic models, letters, numbers and other types of mathematical symbols are used to represent variables and the relationship between them. Symbolic models are some kind of mathematical equations or inequalities reflecting the structure of the system they represent. Inventory models, queuing models, allocation models, sequencing models, replacement models, routing models, etc. are examples of symbolic models.

Advantages. (i) Symbolic models are most abstract and most general (ii) They are usually the easiest to manipulate experimentally.(iii) They usually yield more accurate results under manipulation

Deterministic and Probabilistic models

Models may be deterministic or probabilistic. In deterministic models variables are completely defined and the outcomes are certain. In such models all the parameters, constraints and functional relationships are assumed to be known with certainty when the decision is made. Linear programming models are examples of deterministic models.

In probabilistic models (stochastic models) there is a risk and uncertainty. Therefore input and output variables assume probability distributions. Models in which at least one parameter or decision variable is a random variable, are called probabilistic models. If we run the model repeatedly with the same initial conditions we may get different results. Eg. Insurance against risk of fire, accidents etc. In a game theory if the strategy is not pure we apply probabilistic model. We can only express the outcome in terms of chance.

Static and Dynamic model

Models can also be classified as static and dynamic.

Static model. Static models are those which do not take time into account. It assumes that the values of the variables do not change with time during a particular period. Ex. Assignment techniques use static model. Inventory model another example.

Dynamic model. A dynamic model considers time as one of the important variables. Change in time affects the model. So a sequence of interrelated decisions over a period of time are made to select the optimal solution. Eg. Replacement model, Dynamic programming.

General methods of solution for OR models

OR models are generally solved by the following methods.

1. **Analytic or deductive method**

In this method classical mathematical tools such as differential calculus and finite difference and graphs are used for solving an OR model. Eg, Inventory model.

2. **Numerical or iterative method**

Whenever the analytic methods fail, we use iterative procedure. The classical method may fail because of the complexity of the constraints or of the number of variables. In this procedure we start with a trial solution and a set of rules for improving it. The trial solution is improved by the given rules and is then replaced by this improved solution. This process is repeated until either no further improvement is possible or the cost of further calculation cannot be justified.

3. **Monte Carlo technique of simulation**

The basis of Monte Carlo method is random sampling of a variable's possible values. It is a simulation technique in which we try to find the best and optimal solution.

The various steps associated are as follows.

For appropriate model of the system, make sample observatins and determine the probability distribution for the variables.

- Convert the probability distribution to cumulative distribution
- Select the sequence of random numbers with the help of random variables
- Determine the sequence of values of variables with the sequence of random numbers obtained in the above step
- Fit an appropriate standard mathematical function to the values obtained above.

OR techniques (tools of OR or Important OR models)

Some of the important OR techniques is given below.

1. **Allocation models**

Allocation models involve the allocation of resources to activities in such a manner that some measure of effectiveness is optimized. Allocation problems can be solved by linear and non- linear programming techniques. Linear programming technique is used to finding solution for optimizing a

given objective such as maximizing profit or minimizing cost under certain constraints. The technique includes an objective function, several alternative constraints. Assignment models and Transportation models are special cases of Linear programming.

2. **Sequencing**

These are concerned with placing items in a certain sequence or order for service.

3. **Waiting or Queuing theory**

These are models that involve waiting for services. In business world several types of interruptions occur. Facilities may break down and therefore repairs may be required. Power failures occur. Workers or the needed materials do not show up where and when expected. These cause waiting line problems. The waiting line problems can be solved by waiting line theory or queuing theory. Waiting line theory aims in minimizing the costs of both servicing and waiting.

4. **Inventory models**

These are models with regard to holding or storing resources. The decisions required generally entail the determination of how much of resources are to be acquired or when to acquire them. Inventory control aims at optimum inventory levels. Inventory planning is meant to take optimum decisions about how much to buy and when to buy. Inventory theory technique is used for solving inventory problems. The technique helps to minimize costs associated with holding inventories, procurement of inventories and the shortage of inventories.

5. **Competitive strategy models (game theory)**

These are models which arise when two or more people are competing for a certain resource. Game model is used to determine the optimum strategy in a competitive situation.

6. **Decision theory**

Decision theory concerns with making sound decisions under conditions of certainty, risk and uncertainty. Decision theory explains how to select a suitable strategy to achieve some object or goal under different conditions.

7. **Network analysis**

Network models involve the determination of an optimum sequence of performing certain operations concerning some jobs in order to minimize over all time or cost. PERT, CPM and other network techniques such as Gant chart come under network models.

8. **Simulation**

Simulation is a technique of testing a model which resembles a real life situation. This technique is used to imitate an operation prior to actual performance. There are two methods of simulation. Monte Carlo method and System simulation method.

9. **Search models**

This model concerns itself with search problems. A search problem is characterized by the need for designing a procedure to collect information on the basis of which one or more decisions are made. Example for search problems are (1) advertising agencies search for customers (2) personnel departments search for good executives.

10. **Replacement theory**

These are models concerned with situation that arise with some items such as machines, electric light bulbs etc. need for replacement because the same may deteriorate with time or may break down

completely or may become out of date due to new developments. This model is concerned with the prediction of replacement costs and determination of the most economic replacement policy.

Role of OR in decision making

Decision making is an important responsibility of today's management. The effective decision making becomes necessary and this can be achieved through scientific approach to the problem. OR provide a quantitative basis for decision making since OR itself is a scientific method. By employing a systematic study of a problem, building mathematical model, experimenting with the model and predicting future operations, OR helps a business management in taking wise decisions.

OR techniques are useful in decision making situations such as suggesting the best location for factories and warehouses, suggesting most economic means for transportation etc. In marketing OR may help in indicating most profitable type and use of advertisement campaigns. Thus with the help of OR, suitable decision can be taken.

Phases of OR (process of OR or methodology of OR)

Various phase (stage or steps) in the application of OR technique are;

1. **Formulating the problem**

Formulating a problem consists in identifying, defining and specifying the measures of the components of a decision model. This should yields a statement of the problem's element that include the controllable variables, the uncontrollable parameters, the restrictions or constraints of the variables and the objectives for defining an improved solution.

2. **Constructing the model**

This phase is concerned with the choice of proper data inputs and design of the appropriate information output. It requires the representation of interrelationship among the elements in terms of mathematical formulae. One or more equations or inequalities (termed as constraints) is required to express the controlled variables. The model also include an objective function which defined the measure of effectiveness of the systems. The objective function and the constraints together constitute a model of the problem.

3. **Deriving the solution**

This phase deals with mathematical calculation for obtaining solution to the model. A solution to the model means those values of the decision variables that optimize the measures of effectiveness in a model. There are various methods available for obtaining the solution like analytical method, numerical method and simulation method.

4. **Testing the validity**

A model is said to be valid if it can give a reliable prediction of the system's performance. A good practitioner of OR realizes that his model must have a longer life and must be a good representation of the system and must correspond to reality.

5. **Controlling the solution**

This phase of the study establishes control over the solution of proper feedback of the information variables which deviated significantly. When one or more variables change significantly, the solution goes out of control. In such situations the model is to be modified accordingly.

6. **Implementing the results**

This phase would basically involve a careful examination of the solution to be adopted and its realities. This phase of OR is primarily executed through the cooperation of both OR experts and those who are responsible for managing and operating the system.

Limitations of OR

1. OR tries to find out the optimal solution, taking all the factors into account. When there are large number of factors involved, a study of all of them becomes difficult or impossible
2. The solution of a problem can be obtained by OR techniques, only if the problem can be quantified. It is not easy or possible to quantify all elements particularly when they are intangible.
3. OR is a specialist's job. It requires the effort of mathematicians and managers put together. When mathematicians fail to understand the business problem or when the manage fails to understand the working of OR, there is a gap between who provides solution and who uses the solution.
4. When the basic data are subjected to frequent changes, incorporating them in the OR models is a costly affair. Greatest difficulty is created by time factor. A solution at a right time will be more useful than a perfect solution arrived ate. Therefore there is the problem of choosing between best solution and timely solution.