Numerisk Analys
FMNF05

Emil Wihlander
dat15ewi@student.lu.se

20 januari 2018

Kapitel 0: Fundamentals

0.1 Evaluating a Polynomial

0.1.1 a)

$$P(x) = 6x^4 + x^3 + 5x^2 + x + 1 = 1 + x(1 + x(5 + x(1 + 6x)))$$

With nested:

$$6 \cdot 1/3 + 1 = 3$$

$$3 \cdot 1/3 + 5 = 6$$

$$6 \cdot 1/3 + 1 = 3$$

$$3 \cdot 1/3 + 1 = 2$$

Without nested:

$$6 \cdot (1/3)^4 + (1/3)^3 + 5 \cdot (1/3)^2 + 1/3 + 1$$

$$6/81 + 1/27 + 5/9 + 1/3 + 1$$

$$6/81 + 3/81 + 45/81 + 27/81 + 81/81$$

$$162/81 = 2$$

b)

$$P(x) = -3x^4 + 4x^3 + 5x^2 - 5x + 1 = 1 + x(-5 + x(5 + x(4 - 3x)))$$

With nested:

$$-3 \cdot 1/3 + 4 = 3$$

$$3 \cdot 1/3 + 5 = 6$$

$$6 \cdot 1/3 - 5 = -3$$

$$-3 \cdot 1/3 + 1 = 0$$

Without nested:

$$-3 \cdot (1/3)^4 + 4 \cdot (1/3)^3 + 5 \cdot (1/3)^2 - 5 \cdot (1/3) + 1$$

$$-3/81 + 4/27 + 5/9 - 5/3 + 1$$

$$-3/81 + 12/81 + 45/81 - 135/81 + 81/81$$

$$0/81 = 0$$

c)

$$P(x) = 2x^4 + x^3 - x^2 + 1 = 1 + x(0 + x(-1 + x(1 + 2x)))$$

With nested:

$$2 \cdot 1/3 + 1 = 5/3$$

$$5/3 \cdot 1/3 - 1 = -4/9$$

$$-4/9 \cdot 1/3 = -4/27$$

$$-4/27 \cdot 1/3 + 1 = 77/81$$

Without nested:

$$2 \cdot (1/3)^4 + (1/3)^3 - (1/3)^2 + 1$$

$$2/81 + 1/27 - 1/9 + 1$$

$$2/81 + 3/81 - 9/81 + 81/81$$

$$77/81$$

0.1.2 a)

$$P(x) = 6x^3 - 2x^2 - 3x + 7 = 7 + x(-3 + x(-2 + 6x))$$

With nested:

$$6 \cdot (-1/2) - 2 = -5$$

$$-5 \cdot (-1/2) - 3 = -1/2$$

$$-1/2 \cdot (-1/2) + 7 = 29/4$$

b)

$$P(x) = 8x^5 - x^4 - 3x^3 + x^2 - 3x + 1 = 1 + x(-3 + x(1 + x(-3 + x(-1 + 8x))))$$

With nested:

$$8 \cdot (-1/2) - 1 = -5$$

$$-5 \cdot (-1/2) - 3 = -1/2$$

$$-1/2 \cdot (-1/2) + 1 = 5/4$$

$$5/4 \cdot (-1/2) - 3 = -29/8$$

$$-29/8 \cdot (-1/2) + 1 = 45/16$$

c)

$$P(x) = 4x^6 - 2x^4 - 2x + 4 = 4 + x(-2 + x(0 + x(0 + x(-2 + x(0 + 4x)))))$$

With nested:

$$4 \cdot (-1/2) = -2$$

$$-2 \cdot (-1/2) - 2 = -1$$

$$-1 \cdot (-1/2) = 1/2$$

$$1/2 \cdot (-1/2) = -1/4$$

$$-1/4 \cdot (-1/2) - 2 = -15/8$$

$$-15/8 \cdot (-1/2) + 4 = 79/16$$

0.1.3

$$P(x) = x^6 - 4x^4 + 2x^2 + 1 = 1 + x^2(2 + x^2(-4 + x^2))$$

With nested:

$$(1/2)^2 - 4 = -15/4$$

$$-15/4 \cdot (1/2)^2 + 2 = 17/16$$

$$17/16 \cdot (1/2)^2 + 1 = 81/64$$

0.1.4 a)

$$P(x) = 1 + x(1/2 + (x - 2)(1/2 + (x - 3)(-1/2)))$$

With nested:

$$-1/2 \cdot (5 - 3) + 1/2 = -1/2$$

$$-1/2 \cdot (5 - 2) + 1/2 = -1$$

$$-1 \cdot 5 + 1 = -4$$

b)

$$P(x) = 1 + x(1/2 + (x-2)(1/2 + (x-3)(-1/2)))$$

With nested:

$$-1/2 \cdot (-1-3) + 1/2 = 5/2$$

$$5/2 \cdot (-1-2) + 1/2 = -7$$

$$-7 \cdot (-1) + 1 = 8$$

0.1.5 a)

$$P(x) = 4 + x(4 + (x-1)(1 + (x-2)(3 + 2(x-3))))$$

With nested:

$$2 \cdot (1/2 - 3) + 3 = -2$$

$$-2 \cdot (1/2 - 2) + 1 = 4$$

$$4 \cdot (1/2 - 1) + 4 = 2$$

$$2 \cdot (1/2) + 4 = 5$$

b)

$$P(x) = 4 + x(4 + (x-1)(1 + (x-2)(3 + 2(x-3))))$$

With nested:

$$2 \cdot (-1/2 - 3) + 3 = -4$$

$$-4 \cdot (-1/2 - 2) + 1 = 11$$

$$11 \cdot (-1/2 - 1) + 4 = -25/2$$

$$-25/2 \cdot (-1/2) + 4 = 41/4$$

0.1.6 a)

$P(x) = a_0 + a_5x^5 + a_{10}x^{10} + a_{15}x^{15} = a_0 + x^5(a_5 + x^5(a_{10} + x^5(a_{15})))$

$a_{15}x^5 + a_{10} = b_1$ 5 multiplications and 1 addition

$b_1x^5 + a_5 = b_2$ (since x^5 is calculated) 1 multiplications and 1 addition

$b_2x^5 + a_0 = b_3$ 1 multiplications and 1 addition

$5 + 1 + 1 = 7$ multiplications, $1 + 1 + 1 = 3$ addition.

b)

$P(x) = a_7x^7 + a_{12}x^{12} + a_{17}x^{17} + a_{22}x^{22} + a_{27}x^{27} = x^7(a_7 + x^5(a_{12} + x^5(a_{17} + x^5(a_{22} + x^5(a_{27})))))$

$a_{27}x^5 + a_{22} = b_1$ 5 multiplications and 1 addition

$b_1x^5 + a_{17} = b_2$ (since x^5 is calculated) 1 multiplications and 1 addition

$b_2x^5 + a_{12} = b_3$ 1 multiplications and 1 addition

$b_3x^5 + a_7 = b_4$ 1 multiplications and 1 addition

$b_4x^7 = b_4$ 2 multiplications

$5 + 1 + 1 + 1 + 2 = 10$ multiplications, $1 + 1 + 1 + 1 = 4$ addition.

0.1.7 n multiplications, $2n$ addition.

(c) 0.1.1

```
format long
x = 1.00001;
p = nest(50,ones(1,51),x)
q = (x^51-1)/(x-1)
estError = abs(p-q)
```

Output:

```
p=51.012752082749991
q=51.012752082745230
estError=0.000000000004761
```

(c) 0.1.2

$$P(x) = 1 - x + x^2 - x^3 + \ldots + x^{98} - x^{99} = 1 - x + x^2(1-x) + \ldots + x^{98}(1-x) = \sum_{k=0}^{49} x^{2k}(1-x) = (1-x)\sum_{k=0}^{49}(x^2)^k = (1-x)\frac{1-(x^2)^{50}}{1-x} = 1 - x^{100}$$

```
format long
x = 1.00001;
p = nest(99,(-1).^(0:99),x)
q = (1-x^100)
estError = abs(p-q)
```

Output:

```
p=-0.000500245079648
q=-0.001000495161746
estError=0.000500250082098
```

0.2 Binary Numbers

0.2.1 a)

$64/2 = 32$ R 0

$32/2 = 16$ R 0

$16/2 = 8$ R 0

$8/2 = 4$ R 0

$4/2 = 2$ R 0

$2/2 = 1$ R 0

$1/2 = 0$ R 1

$(64)_{10} = (1000000)_2$

b)

$17/2 = 8$ R 1

$8/2 = 4$ R 0

$4/2 = 2$ R 0

$2/2 = 1$ R 0

$1/2 = 0$ R 1

$(17)_{10} = (10001)_2$

c)

$79/2 = 32$ R 1

$39/2 = 19$ R 1

$19/2 = 9$ R 1

$9/2 = 4$ R 1

$4/2 = 2$ R 0

$2/2 = 1$ R 0

$1/2 = 0$ R 1

$(79)_{10} = (1001111)_2$

d)

$227/2 = 113$ R 1

$113/2 = 56$ R 1

$56/2 = 28$ R 0

$28/2 = 14$ R 0

$14/2 = 7$ R 0

$7/2 = 3$ R 1

$3/2 = 1$ R 1

$1/2 = 0$ R 1

$(227)_{10} = (11100011)_2$

0.2.2 a)

$1/8 \cdot 2 = 1/4$ R 0

$1/4 \cdot 2 = 1/2$ R 0

$1/2 \cdot 2 = 0$ R 1

$(1/8)_{10} = (.001)_2$

b)

$7/8 \cdot 2 = 3/4$ R 1

$3/4 \cdot 2 = 1/2$ R 1

$1/2 \cdot 2 = 0$ R 1

$(7/8)_{10} = (.111)_2$

c) It's larger than 2, factor it out.

Integer part:

$2/2 = 1$ R 0

$1/2 = 56$ R 1

Fractional part:

$3/16 \cdot 2 = 3/8$ R 0

$3/8 \cdot 2 = 3/4$ R 0

$3/4 \cdot 2 = 1/2$ R 1

$1/2 \cdot 2 = 0$ R 1

$(35/16)_{10} = (10.0011)_2$

d)

$31/64 \cdot 2 = 31/32$ R 0

$31/32 \cdot 2 = 15/16$ R 1

$15/16 \cdot 2 = 7/8$ R 1

$7/8 \cdot 2 = 3/4$ R 1

$3/4 \cdot 2 = 1/2$ R 1

$1/2 \cdot 2 = 0$ R 1

$(31/64)_{10} = (.011111)_2$

0.2.3 a) Solve the integer and fractional part separately.

Integer part:

$10/2 = 5$ R 0

$5/2 = 2$ R 1

$2/2 = 1$ R 0

$1/2 = 0$ R 1

Fractional part:

$.5 \cdot 2 = 0$ R 1

Sum:

$(10.5)_{10} = (1010.1)_2$

b)

$1/3 \cdot 2 = 2/3$ R 0

$2/3 \cdot 2 = 1/3$ R 1

$1/3 \cdot 2 = 2/3$ R 0

The period is two.

$(1/3)_{10} = (.\overline{01})_2$

c)

$5/7 \cdot 2 = 3/7$ R 1

$3/7 \cdot 2 = 6/7$ R 0

$6/7 \cdot 2 = 5/7$ R 1

$5/7 \cdot 2 = 3/7$ R 1

The period is three.

$(5/7)_{10} = (.\overline{101})_2$

d) Solve the integer and fractional part separately.

Integer part:

$12/2 = 6$ R 0

$6/2 = 3$ R 0

$3/2 = 1$ R 1

$1/2 = 0$ R 1

Fractional part:

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

The period is four.

Sum:

$$(12.8)_{10} = (1100.\overline{1100})_2$$

e) Solve the integer and fractional part separately.

Integer part:

$55/2 = 27$ R 1

$27/2 = 13$ R 1

$13/2 = 6$ R 1

$6/2 = 3$ R 0

$3/2 = 1$ R 1

$1/2 = 0$ R 1

Fractional part:

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

The period is four.

Sum:

$$(55.4)_{10} = (110111.\overline{0110})_2$$

f)

$.1 \cdot 2 = .2$ R 0

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

The period is four after first bit.

$$(0.1)_{10} = (0.0\overline{0011})_2$$

0.2.4 a) Solve the integer and fractional part separately.

Integer part:

$11/2 = 5$ R 1

$5/2 = 2$ R 1

$2/2 = 1$ R 0

$1/2 = 0$ R 1

Fractional part:

$.25 \cdot 2 = .5$ R 0

$.5 \cdot 2 = 0$ R 1

Sum:

$$(11.25)_{10} = (1101.01)_2$$

b)

$2/3 \cdot 2 = 1/3$ R 1

$1/3 \cdot 2 = 2/3$ R 0

$2/3 \cdot 2 = 1/3$ R 1

The period is two.

$$(2/3)_{10} = (.\overline{10})_2$$

c)

$3/5 = 0.6$

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

The period is four.

$$(3/5)_{10} = (.\overline{1001})_2$$

d) Solve the integer and fractional part separately.

Integer part:

$3/2 = 1$ R 1

$1/2 = 0$ R 1

Fractional part:

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

The period is four.

Sum:

$$(3.2)_{10} = (11.\overline{0011})_2$$

e) Solve the integer and fractional part separately.

Integer part:

$30/2 = 15$ R 0

$15/2 = 7$ R 1

$7/2 = 3$ R 1

$3/2 = 1$ R 1

$1/2 = 0$ R 1

Fractional part:

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

The period is four.

Sum:

$$(30.6)_{10} = (11110.\overline{1001})_2$$

f) Solve the integer and fractional part separately.

Integer part:

$99/2 = 49$ R 1

$49/2 = 24$ R 1

$24/2 = 12$ R 0

$12/2 = 6$ R 0

$6/2 = 3$ R 0

$3/2 = 1$ R 1

$1/2 = 0$ R 1

Fractional part:

$.9 \cdot 2 = .8$ R 1

$.8 \cdot 2 = .6$ R 1

$.6 \cdot 2 = .2$ R 1

$.2 \cdot 2 = .4$ R 0

$.4 \cdot 2 = .8$ R 0

$.8 \cdot 2 = .6$ R 1

The period is four after the first bit.

Sum:

$$(99.9)_{10} = (1100011.1\overline{1100})_2$$

0.2.5 Solve the integer and fractional part separately. At least 4 decimal points (3.1416) will give the correct answer.

Integer part:

$3/2 = 1$ R 1

$1/2 = 0$ R 1

Fractional part:

$.14159265358979 \cdot 2 = .28318530717958$ R 0

$.28318530717958 \cdot 2 = .56637061435916$ R 0

$.56637061435916 \cdot 2 = .13274122871832$ R 1

$.13274122871832 \cdot 2 = .26548245743664$ R 0

$.26548245743664 \cdot 2 = .53096491487328$ R 0

$.53096491487328 \cdot 2 = .06192982974656$ R 1

$.06192982974656 \cdot 2 = .12385965949312$ R 0

$.12385965949312 \cdot 2 = .24771931898624$ R 0

$.24771931898624 \cdot 2 = .49543863797248$ R 0

$.49543863797248 \cdot 2 = .99087727594496$ R 0

$.99087727594496 \cdot 2 = .98175455188992$ R 1

$.98175455188992 \cdot 2 = .96350910377984$ R 1

$.96350910377984 \cdot 2 = .92701820755968$ R 1

Sum:

$$(\pi)_{10} \approx (11.0010010000111)_2$$

0.2.6 Do it the same way as in the last exercise. At least 4 decimal points (2.7183) will give the correct answer.

$$(e)_{10} \approx (10.1011011111100)_2$$

0.2.7 a)

$$2^6 + 2^4 + 2^2 + 1 = 64 + 16 + 4 + 1 = 85$$