Sailing Through Storms:
Volatility Timing for a Norwegian Investor

Master Thesis

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ABSTRACT

We study whether volatility-timing strategies applied to well-known asset pricing factors improve risk-adjusted returns in the Norwegian stock market. We do it by constructing both individual-factor portfolios and mean-variance multifactor portfolios, applying volatility management techniques based on factor and market volatility, and evaluating their out-of-sample performance. We find that individual volatility-managed factors generally fail to deliver statistically significant improvements, whereas a volatility-managed multifactor portfolio based on market volatility achieves a modestly significant Sharpe ratio increase. We conclude that while volatility-timing may enhance performance under specific conditions, the results lack robustness when accounting for time periods, factor dependencies, and real-world frictions, questioning their practical applicability.

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[1]Special thanks to Raman Uppal for his insightful teaching on the topic and for patiently addressing all our questions related to his paper, which serves as the main reference for this thesis.

Disclaimer

This thesis does not include text generated or suggested directly by AI. We used Grammarly and Writefull to check for grammar and spelling mistakes, deciding ourselves whether to accept or reject the suggestions. ChatGPT was used to improve the clarity and structure of the text. For the coding part of the research, we used GitHub Copilot to make the code more efficient and ChatGPT to suggest or improve parts of the code. Additionally, we used Perplexity to help conduct research online, ensuring access to a broad range of resources and information.

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1 Introduction

Understanding the relationship between risk and return has long been a cornerstone of financial theory and portfolio management. However, recent empirical research has questioned the stability of this relationship, particularly in the context of volatility-timing strategies. This thesis investigates the effectiveness of volatility-timing strategies applied to well-known asset pricing factors in the Norwegian stock market, focusing on whether such strategies can improve risk-adjusted returns for investors, we also build on the current literature by introducing two volatility-managed multifactor portfolios.

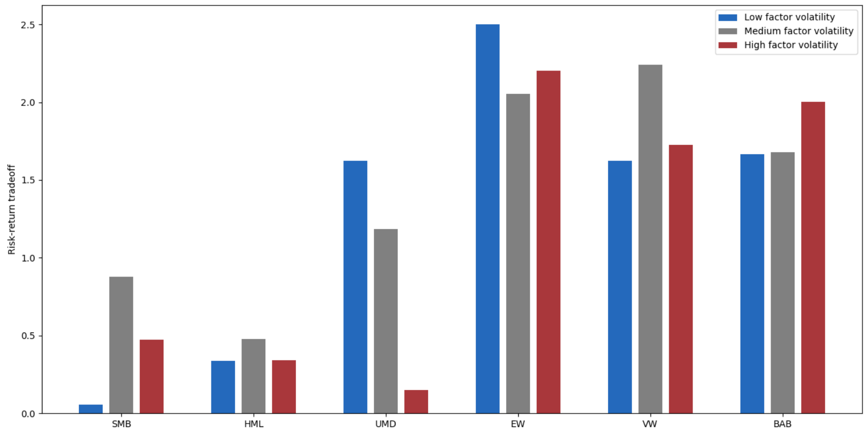
The traditional risk-return trade-off, as established by models like the Capital Asset Pricing Model (CAPM) and multi-factor models, assumes a linear relationship between risk and expected return. However, Moreira and Muir (2017) challenge this notion by proposing that investors can dynamically adjust their exposure–reducing leverage during periods of high volatility and increasing it during periods of low volatility. Their findings suggest that such volatility-managed strategies may yield higher Sharpe ratios by exploiting the non-linear behavior of returns under different volatility regimes. Several studies have replicated their methodology in the Norwegian market, often reporting statistically significant improvements in performance. Nevertheless, these findings have been met with criticism, particularly regarding their lack of robustness in out-of-sample settings and the exclusion of transaction costs, as highlighted by Cederburg et al. (2020) and Barroso and Detzel (2021).

To address these concerns, DeMiguel et al. (2024) introduce a revised framework that emphasizes multifactor portfolios and incorporates considerations such as trading diversification and transaction costs. Their study demonstrates that, while individual factor strategies often fail to maintain significance out-of-sample, a well-constructed volatility-managed multifactor portfolio can achieve superior performance.

Figure 1 illustrates the key relationship explored by Moreira and Muir (2017): the average risk-adjusted return in period $t+1$ categorized by realized volatility in period t. As highlighted by prior literature in Norway, this relationship has been presented as statistically significant, particularly for certain factors. However, as seen in the figure, such significance is predominantly observed for the UMD factor during periods of low volatility as all other factors have a higher risk-adjusted return when volatility is high. This thesis revisits and extends these findings using the improved methodological approach suggested by DeMiguel et al. (2024).

Figure 1: Factor Risk-Return Trade-Off Across Volatility Regimes

This plot illustrates how the risk-return trade-off of the six factors varies across different levels of realized factor volatility. Specifically, we sort the months in our sample into terciles based on the monthly time series of realized factor volatility in time t. For each category, we compute the risk-return trade-off from the factor using the return and volatility in $t+1$ and report the average value in each category.



More specifically, we create volatility-timed multifactor portfolios that allow the weights on each factor to adjust with volatility. This leads us to the following research questions:

- *Can well-known asset pricing factors be volatility-managed to achieve superior returns, on average, out-of-sample in the Norwegian stock market?*

- *Can a volatility-managed multifactor portfolio, adjusting weights inversely to volatility, on average, outperform its unconditional counterpart out-of-sample in the Norwegian stock market?*

The remainder of this thesis is organized as follows. Section 2 reviews relevant literature on volatility-timing and multifactor models. Section 3 details the methodology applied to assess the performance of volatility-managed portfolios. Section 4 describes the dataset used in the analysis. Section 5 presents the empirical results and discussion. Section 6 concludes. Additional material and robustness checks are provided in the Appendix.

2 Literature review

This literature review looks at the extensive research on portfolio management, specifically volatility timing . It will also cover the history behind performance measurements used to understand the effect of these portfolio choices and the models used to create them.

2.1 Theoretical Foundations of Portfolio Choice

Markowitz (1952, 1959) fundamentally transformed portfolio theory by incorporating not only the expected returns of individual assets but also their covariances, recognizing that the interactions among assets play a crucial role in portfolio construction. This groundbreaking framework, which earned Markowitz the Nobel Prize in 1990, established the cornerstone of modern portfolio theory, emphasizing the trade-off between risk and return. The essence of his contribution is captured in the following equation, which identifies the optimal portfolio weights that maximize risk-adjusted returns:

$$w = \frac{1}{\gamma}(\mathbb{V}[R])^{-1}(\mathbb{E}[R] - R_f 1_N),$$

where w denotes the portfolio weights, $\mathbb{V}[R]$ is the $N \times N$ covariance matrix of asset returns, $\mathbb{E}[R]$ represents the $N \times 1$ vector of expected returns, R_f is the risk-free rate, 1_N is a $N \times 1$ vector of ones, and γ is the investor's degree of risk aversion.

Despite its theoretical elegance, mean-variance optimization has been subject to considerable criticism, primarily due to the sensitivity of its inputs to estimation error. Empirical studies consistently show that sample-based estimates of expected returns, $\mathbb{E}[R]$, are highly imprecise, while estimates of the covariance matrix, $\mathbb{V}[R]$, often suffer from being ill-conditioned. Merton (1980) underscores the inherent difficulty of accurately estimating expected returns, pointing out that these estimates remain noisy and do not improve substantially even with longer time-series data. Furthermore, the covariance matrix entails estimating a large number of parameters, which is particularly problematic when the available data is limited, leading to instability in the optimization process.

In response, several methodological advancements have been proposed to mitigate these issues:

- **Shrinkage Estimators**: Ledoit and Wolf (2004b,a) propose shrinking the sample covariance matrix towards a more structured, stable target, thereby reducing estimation error and improving robustness.
- **Constraints on Portfolio Weights**: Studies such as Frost and Savarino (1988), Chopra (1993), and Jagannathan and Ma (2003) recommend imposing constraints–such as prohibiting short sales–to stabilize the optimization and reduce sensitivity to estimation noise.
- **Global Minimum Variance (GMV) Portfolio**: Jorion (1985, 1986) advocate for bypassing expected returns altogether by focusing exclusively on minimizing portfolio variance, effectively removing the need to estimate expected returns.

Further critique is offered by DeMiguel et al. (2009), who demonstrate that, even with shrinkage techniques applied, the mean-variance optimized portfolios tend to underperform the simple equally-weighted, $1/N$, portfolio in out-of-sample tests based on the Sharpe ratio. Additionally, they highlight that such portfolios typically experience significantly higher turnover, resulting in elevated transaction costs. Interestingly, their findings indicate that the GMV portfolio–particularly when the covariance matrix is shrunk through short-sale constraints or the Ledoit-Wolf shrinkage methodology–exhibits relatively improved performance, though, this is the only portfolio that beats the $1/N$ benchmark.

Addressing these limitations from a different angle, Brandt et al. (2009) introduce a parametric portfolio policy, built upon a critical insight. Rather than first modeling asset returns and subsequently deriving optimal portfolio weights, they propose specifying a parametric factor model directly for the portfolio weights. This approach markedly reduces the dimensionality of the optimization problem, particularly in a factor investing context, thereby simplifying the portfolio construction process while maintaining flexibility and tractability.

2.2 Theoretical Foundations of Asset Pricing Models

The CAPM was first introduced by Sharpe (1964) as a foundational model establishing a positive linear relationship between systematic risk (beta) and expected returns. Later, Lintner (1965) and Mossin (1966) formalized and extended the framework. The model is expressed as:

$$\mathbb{E}[R_i] = R_f + \beta_i(\mathbb{E}[R_m] - R_f), \quad \text{where } \beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}.$$

In this equation, $\mathbb{E}[R_i]$ represents the expected return on asset i, R_f is the risk-free rate, β_i measures the asset's systematic risk, and $\mathbb{E}[R_m] - R_f$ is the market risk premium.

Since its introduction, the CAPM has been one of the most widely used models in finance, but it has also faced significant criticism. For instance, Black (1972) shows that the security market line (SML) is flatter than predicted by the CAPM. This means that low-beta stocks earn higher returns than the model predicts, while high-beta stocks earn lower returns than expected.

The literature on this phenomenon is divided into two perspectives. The first perspective attributes it to structural constraints, such as funding limitations.

This view underpins the Betting Against Beta (BAB) factor introduced by Frazzini and Pedersen (2014), which has been further supported by empirical findings from Oliver and Mikhail (2018) and Pelster (2024). This perspective suggests that leverage constraints force investors to overweight high-beta stocks, distorting the relationship between beta and returns.

The second perspective focuses on investor behavior. Brunnermeier and Parker (2007) argue that investors are drawn to lottery-like stocks, which exhibit a high probability of large short-term moves. This behavior is idiosyncratic rather than systematic and has been explored further in studies like Ang et al. (2006, 2009) and Bali et al. (2011), which identify significant factors linked to these preferences.

Building on this, Bali et al. (2017) sort stocks by beta and demonstrate that, after accounting for lottery demand, the abnormal returns associated with low-beta stocks disappear. They also show that lottery demand increases buying pressure on stocks with a high probability of extreme short-term moves, flattening the SML and generating alpha for portfolios that are long low-beta stocks and short high-beta stocks.

Roll (1977) raised another critique, emphasizing that CAPM tests rely on proxies for the market portfolio, which may not fully represent all investable assets, making the model's validity difficult to confirm. Similarly, Banz (1981) identified the "size effect", where smaller firms exhibit higher risk-adjusted returns than predicted by the CAPM, introducing the concept of a size premium that the model fails to incorporate.

Building on these critiques, Fama and French (1992) demonstrated that variables such as size and value have significant explanatory power for asset returns, leading to the development of the Fama-French three-factor model (FF3). This model introduced the Small Minus Big (SMB) factor, capturing the size effect, and the High Minus Low (HML) factor, representing value.

Additionally, Jegadeesh and Titman (1993) documented the momentum effect, showing that stocks with strong past performance tend to continue performing well in the short term, another anomaly not captured by the CAPM. This insight led to the introduction of the Up Minus Down (UMD) factor by Carhart (1997), capturing momentum as an additional explanatory variable.

Complementing these findings, Asness et al. (2013) explore the interplay between value and momentum factors, demonstrating their effectiveness across various asset classes. Their research highlights that value and momentum are negatively correlated, making them highly complementary when combined in a single portfolio. By doing so, investors can achieve better diversification and risk-adjusted returns, further challenging the simplicity of the CAPM and reinforcing the need for multi-factor models.

Studies focusing on the Norwegian market have highlighted key differences in the applicability of asset pricing models. For instance, Fylling and Jacobsen (2023) argues that these factors exhibit lower stability in the Norwegian market compared to their performance in the United States. Similarly, Lønø and Svendsen (2019) demonstrates that the inclusion of additional factors, such as Momentum and Robust Minus Weak, beyond the traditional Fama-French three-factor model, does not significantly enhance the explanatory power for returns in the Norwegian stock market.

2.3 Performance Evaluation Metrics

The Sharpe ratio, introduced by Sharpe (1966), is a widely used metric for evaluating the performance of investment funds relative to the level of risk taken. It is defined as:

$$SR_p = \frac{\mathbb{E}[R_p - R_f]}{\sqrt{\mathbb{V}[R_p - R_f]}} = \frac{\text{excess mean return}}{\text{total portfolio risk}},$$

where $\mathbb{E}[R_p - R_f]$ represents the portfolio's excess return over the risk-free rate, and $\sqrt{\mathbb{V}[R_p - R_f]}$ is the standard deviation of the portfolio's excess return.

Despite its widespread use, it has been subject to several criticisms. For instance, Leland (1999) highlights that the it is unsuitable when return distributions deviate from normality, particularly when skewness or kurtosis is present. Additionally, (Lo, 2002, eq. (20) & Table 2) demonstrates that scaling the SR by $\sqrt{t}$ is valid only under the assumption of Independent and Identically Distributed (IID) returns with no serial correlation. If negative serial correlation exists, the scaling factor can be significantly larger, revealing the limitations of this adjustment in non-IID contexts. Given these criticisms, the Sharpe ratio is still one of the most used ratios to account for portfolio returns.

2.4 Volatility-Timed Factor Strategies

Volatility-timing strategies have garnered substantial attention across academia, the financial press, and among practitioners. BlackRock describes their Managed Volatility V.I. Fund as follows: "In periods of heightened volatility, the portfolio will de-risk into less volatile assets like fixed income and cash and re-risk when market turbulence subsides." This statement captures the essence of volatility-timed strategies. However, academic opinion remains divided regarding their efficacy. While Asness (2016) of AQR expresses skepticism about the feasibility and benefits of factor timing, Moreira and Muir (2017) argue that volatility-timing is both achievable and advantageous.

Moreira and Muir (2017) demonstrate that investors can enhance the Sharpe ratio by reducing exposure to risk factors during periods of elevated volatility, which can also yield larger alphas. Their approach involves scaling down factor exposures following market crashes–such as those experienced in 1929, 1987, and 2008–based on the observation that returns do not rise proportionally with risk during volatile periods. By lowering exposure when volatility is high, investors can reduce risk to a greater extent than returns. Formally, they express the volatility-managed factor return as:

$$f_{t+1}^{\sigma} = \frac{c}{\sigma_t^2(f)} \times f_{t+1},$$

where f_{t+1}^{σ} represents the volatility-adjusted factor excess return, c is a scaling constant ensuring the volatility of f^{σ} matches that of the unadjusted return f, and $\sigma_t^2(f)$ is the prior month's volatility, estimated using daily data.

To evaluate the effectiveness of their strategy, they propose the following regression:

$$f_{t+1}^{\sigma} = \alpha + \beta f_{t+1} + \epsilon_{t+1}.$$

They argue, under theoretical assumptions, α should be zero, implying a proportional relationship between risk and return. However, significant deviations from $\alpha = 0$ suggest that increased risk is not compensated by proportional returns[2]. Their empirical results indicate statistically significant β coefficients and non-zero α values, thereby supporting the argument that volatility-timing can enhance portfolio performance.

Several attempts have been made to replicate and extend this methodology using Norwegian equity data. Johansen and Eckhoff (2016) report that volatility-timing generally leads to higher alphas and improved Sharpe ratios, further showing that volatility forecasts derived from GARCH models are effective. Similarly, Bakken and Horvei (2022) document significant alphas for the HML, PR1YR, and UMD factors when applying the approach of Moreira and Muir (2017). However, they contend that these results are not feasible in real-world settings, primarily due to trading frictions, citing the findings of Barroso and Detzel (2021). Furthermore, Alme and Aarsland (2022) observe that volatility-

[2]Under the assumptions of Markowitz and Black-Litterman, they argue that it holds that $\mu_{t,mkt} = \gamma_{mkt}\sigma_{t,mkt}^2$, supporting the expectation of $\alpha = 0$. We'll revisit this view in Section 2.4.1 as it is critized by Cederburg et al. (2020) that we will cover.

timing improves returns, particularly during financial crises, although the benefits appear diminished in the post-COVID-19 period.

2.4.1 Limitations and Critiques of Volatility-Timing Strategies

The conclusions drawn by Moreira and Muir (2017) have been met with considerable criticism. Cederburg et al. (2020) argue that their reported gains are not replicable in out-of-sample tests, attributing the observed results to a look-ahead bias stemming from alpha estimation conducted over the entire sample period. Their replication and extension of the strategy reveal no consistent evidence that volatility-managed portfolios systematically outperform. Specifically, they find that only 8 out of 103 volatility-managed factors exhibit Sharpe ratios significantly different from their unmanaged counterparts, with these instances concentrated primarily in momentum-based strategies.

Additionally, Barroso and Detzel (2021) demonstrate that once transaction costs are accounted for, the advantages of volatility-timing largely dissipate. They show that such strategies often entail prohibitively high turnover, particularly due to large positions in small-cap stocks, leading to elevated trading costs. The sole exception appears to be the market factor, which retains its significance owing to lower associated transaction costs. However, even for the market factor, improved performance is restricted to periods characterized by elevated investor sentiment, in line with theories suggesting sentiment-driven underreaction to volatility fluctuations.

2.4.2 Multifactor Frameworks for Volatility-Timing

In response to these critiques, DeMiguel et al. (2024) propose an enhanced framework for volatility-timing that addresses the identified limitations. They begin by constructing volatility-managed individual factor portfolios using the methodology of Moreira and Muir (2017), subsequently forming a mean-variance portfolio that combines both a managed and a unmanaged factors. To counter the concerns raised by Cederburg et al. (2020), they employ a bootstrapping approach to assess statistical significance robustly. They also explicitly model transaction costs and account for trading diversification into the optimization problem.

Their findings indicate that, when transaction costs and diversification are properly incorporated, only the UMD, IA, and BAB, 3 out of 9, factors exhibit statistically significant improvements in Sharpe ratios, with IA significant at the 5% level and the others at 10%.

Building on these insights, they introduce a new volatility-timing framework characterized by four key features:

1. The use of multi-factor portfolios instead of isolated factor portfolios, extending the parametric portfolio policy framework.
2. Dynamic adjustment of relative factor weights, as opposed to static weights.
3. Incorporation of trading diversification in transaction cost calculations, following the methodology of DeMiguel et al. (2020).
4. Optimization of factor weights while explicitly integrating transaction costs into the objective function.

Using this refined approach, they demonstrate that volatility-managed multi-factor portfolios consistently outperform both their unconditional counterparts and the earlier strategies proposed by Moreira and Muir (2017) even out-of-sample. Section 3 explains their methodology in detail.

3 Methodology and Testable Hypotheses

This chapter outlines the methodology for constructing volatility-managed *individual* factors, volatility-managed *individual-factor* portfolios, unconditional volatility-managed *multifactor* portfolios, and conditional volatility-managed *multifactor* portfolios.

3.1 Hypothesis for Research Question 1 (RQ-1)

- *Can well-known asset pricing factors be volatility-managed to achieve superior returns, on average, out-of-sample in the Norwegian stock market?*

To address RQ-1, we begin by defining volatility-managed *individual* factors following the methodology of Moreira and Muir (2017). Specifically, the return of the kth volatility-managed individual factor is given by:

$$r^{\sigma_k}_{k,t+1} = \frac{c}{\sigma^2_{k,t}} r_{k,t+1}, \tag{1}$$

where $r_{k,t+1}$ represents the unmanaged return of the kth factor in month $t+1$, $\sigma^2_{k,t}$ is the realized variance of the kth factor for month t, estimated using the sample variance of daily factor returns, and c is a scaling parameter ensuring that the volatility of the managed factor matches that of the unmanaged factor.

Additionally, we construct an alternative set of returns, $r^{\sigma_m}_{k,t+1}$, where factor returns are scaled by market variance instead of their own variance. This follows the same formulation as Equation 1, but replaces $\sigma^2_{k,t}$ with the market's realized variance, $\sigma^2_{m,t}$[3].

Finally, the volatility-managed individual-factor portfolio is constructed as the mean-variance combination of the unmanaged factor and its volatility-managed

[3] The scaling parameter c is adjusted to ensure that the volatility of the managed and unmanaged factors remains identical in both cases.

counterpart, whether scaled by its own volatility or by market volatility. We employ a expanding window approach, using the first 120 months as the starting window, to obtain the portfolio weights using the mean-variance framework. A risk-aversion parameter, γ, of 5 is applied across all mean-variance approaches. Additionally, we introduce a short-selling constraint to prevent extreme weights[4]. To assess the effectiveness of volatility management on *individual factors*, we analyze the statistical significance of differences in Sharpe ratios, thereby addressing RQ-1.

3.2 Hypothesis for Research Question 2 (RQ-2)

- *Can a volatility-managed multifactor portfolio, adjusting weights inversely to volatility, on average, out-of-sample, outperform its counterpart in the Norwegian stock market?*

We define a conditional mean-variance multifactor portfolio that allows the relative weights of different factors to vary with both market volatility or factor volatility[5]. Additionally, we use volatility rather than variance to scale the returns, as Moreira and Muir (2017) and Barroso and Detzel (2021) point out that using volatility reduces transaction costs[6].

We define a conditional multifactor portfolio at time t as:

$$w_t(\theta_t) = \sum_{k=1}^{K} x_{k,t}\theta_{k,t}, \tag{2}$$

where K is the number of factors, $x_{k,t} \in \mathbb{R}^{N_t}$ is the stock portfolio associated with the kth factor at time t, with N_t denoting the number of stocks available

[4]Appendix B.2 shows our results with the relaxation of this constrains and changing the value of γ do not significantly change our results.

[5]DeMiguel et al. (2024) consider only market volatility, arguing that this is a conservative choice, as the effect is even stronger when using factor volatility. In Section 5 we find the opposite. We find that when using factor volatility, our returns turn nonsignificant.

[6]Appendix B.3 show that the results are not signficantly different when using variance over volatility.

at time t. The term $\theta_{k,t}$ represents the portfolio weight on the kth factor at time t, and $\theta_t = (\theta_{1,t}, \theta_{2,t}, \ldots, \theta_{K,t})$ is the factor-weight vector at time t.

Each factor weight, $\theta_{k,t}$, is modeled as an affine function of the inverse of market volatility, defined as

$$\theta_{k,t} = a_k + b_k \frac{1}{\sigma_t}, \tag{3}$$

where σ_t represents realized volatility, either of the market or the factor, estimated as the sample volatility of daily market returns in month t. A positive value of b_k implies that the model reduces exposure to the kth factor when realized volatility is high. Additionally, the model allows each factor's weight to vary independently, as in general, $b_i \neq b_j$ for $i \neq j$.

If $r_{t+1} \in \mathbb{R}^{N_t}$ is the vector of stock returns for month $t+1$ and $r_{k,t+1} \equiv x_{k,t}^\top (r_{t+1} - r_{f,t+1} e_t) \in \mathbb{R}$ is the kth factor return for month $t+1$, where $r_{f,t+1}$ is the return of the risk-free asset at time $t+1$ and e_t is the N_t-dimensional vector of ones, then substituting (3) into (2) gives the return of our conditional multifactor portfolio as:

$$r_{p,t+1}(\theta_t) = \sum_{k=1}^{K} r_{k,t+1} \left(a_k + b_k \frac{1}{\sigma_t} \right) = \sum_{k=1}^{K} r_{k,t+1} \theta_{k,t}. \tag{4}$$

To simplify implementation, we define an "extended" factor portfolio-weight matrix $X_{\text{ext},t}$, factor-return vector $r_{\text{ext},t+1}$, and factor-weight vector η as:

$$X_{\text{ext},t} = \begin{bmatrix} x_{1t}^\top \\ x_{2t}^\top \\ \vdots \\ x_{Kt}^\top \\ x_{1t}^\top \times \frac{1}{\sigma_t} \\ x_{2t}^\top \times \frac{1}{\sigma_t} \\ \vdots \\ x_{Kt}^\top \times \frac{1}{\sigma_t} \end{bmatrix}, \quad r_{\text{ext},t+1} = \begin{bmatrix} r_{1,t+1} \\ r_{2,t+1} \\ \vdots \\ r_{K,t+1} \\ r_{1,t+1} \times \frac{1}{\sigma_t} \\ r_{2,t+1} \times \frac{1}{\sigma_t} \\ \vdots \\ r_{K,t+1} \times \frac{1}{\sigma_t} \end{bmatrix}, \quad \text{and} \quad \eta = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_K \\ b_1 \\ b_2 \\ \vdots \\ b_K \end{bmatrix}. \tag{5}$$

The conditional mean-variance multifactor portfolio is given by the extended factor-weight vector, η, which optimizes the mean-variance utility of an investor with risk-aversion parameter γ:

$$\max_{\eta \geq 0} \ \widehat{\mu}_{\text{ext}}^{\top} \eta - \frac{\gamma}{2} \eta^{\top} \widehat{\Sigma}_{\text{ext}} \eta \tag{6}$$

where $\widehat{\mu}_{\text{ext}}$ and $\widehat{\Sigma}_{\text{ext}}$ represent the sample mean and covariance matrix of the extended factor-return vector, respectively. Here, $\widehat{\mu}_{\text{ext}}^{\top} \eta$ and $\eta^{\top} \widehat{\Sigma}_{\text{ext}} \eta$ denote the sample mean and variance of the conditional multifactor portfolio return[7].

To minimize estimation errors and prevent extreme weights, we impose non-negativity constraints on the factor weight parameters, such that $a_k \geq 0$ and $b_k \geq 0$, ensuring $\eta \geq 0$. In this same framework, we define the unconditional multifactor portfolio in an identical manner, with the additional constraint $b_k = 0$ to problem (6)[8].

We then assess the effectiveness of volatility management by comparing differences in Sharpe ratios and look for significant alphas to answer RQ-2.

To evaluate the statistical significance of our results for RQ-1 and RQ-2, we compute p-values using a bootstrap approach. Specifically, we generate 10,000 bootstrap samples of the volatility-managed and corresponding unmanaged return series, employing the stationary block-bootstrap method with an average block length of five. The return series are bootstrapped jointly to preserve the inherent correlation between them, as the performance of the unmanaged portfolio directly influences the managed portfolio's returns. From these bootstrap samples, we construct the empirical distribution of the differences in Sharpe ratios. The p-value is then calculated as the proportion of instances in which the Sharpe ratio difference is less than zero.

[7] Since our portfolios are zero-cost, we do not impose constraints requiring portfolio weights to sum to one. Appendix B.1 demonstrates that imposing leverage constraints–specifically, limiting the maximum weight assigned to individual factors–results in more statistically significant outcomes.

[8] Appendix B.4 shows that allowing short-selling does not significantly change our results.

4 Data and Preprocessing

This study focuses on equities listed on the Norwegian stock market. Stock return data were collected from the Oslo Børs Information (OBI) database. Additionally, the value-weighted (VW) and equally-weighted (EW) indices of all stocks listed on the Oslo Børs were obtained from Bernt A. Ødegaard's website[9]. The value factor (HML), size factor (SMB), and Carhart momentum factor (UMD) were likewise sourced from Ødegaard's dataset. Risk-free rates were gathered from Norges Bank, Oslo Børs, and NoRe, and accessed through the same source. Our dataset, summarized in Table 1, includes both daily and monthly observations covering the period from January 1980 to December 2023, yielding a sample of 44 years. As the BAB factor requires three years of historical data for construction, it commences in 1983.

4.1 Filters

Prior research, such as Novy-Marx and Velikov (2021), indicates that a substantial portion of the returns generated by the BAB factor can be attributed to an overweight allocation in small and illiquid stocks. Similarly, Barroso and Detzel (2021) argue that such a concentration in small-cap stocks leads to elevated transaction costs, especially when employing volatility-timing strategies. Furthermore, Ødegaard (2021) emphasize that these stocks should be excluded from analyses using OBI data, as their reported returns are often overstated. To address these concerns, we apply data filters consistent with those used by Ødegaard (2021). Specifically, we exclude stocks with a price below 10 NOK if their market capitalization is under 10 million NOK or missing altogether. Additionally, stocks with a market capitalization below 1 million NOK are excluded.

[9] https://ba-odegaard.no/

4.2 Data Cleaning and Manipulation

Regarding the risk-free rate, we encountered 86 missing daily observations, which were imputed using simple linear interpolation. Construction of the BAB factor follows the methodology detailed in Appendix A.4. In the OBI daily and monthly datasets, several observations for the year 2020–particularly between June and December–were missing. To address this, we supplemented the data using Yahoo Finance and Bloomberg to the extent possible; however, we still observe a noticeably lower number of stock return observations during this period.

Table 1: Summary of the data used in the analysis

This table summarizes key variables used in the analysis. "Stocks" refers to unfiltered data, while "Stocks (F)" is the filtered subset. For "Stocks" and "Stocks (F)," N indicates the number of stocks, for the other variables it indicates the number of observations. Periods are based on monthly data. Means and standard deviations (SD) are based on monthly returns and are annualized. Max and Min reflect the highest and lowest monthly returns. All returns, except risk-free (RF) and stocks, are excess returns.

Variable	**Frequency**	**Period**	**N**	**Mean**	**SD**	**Max**	**Min**
Stocks	D & M	1980:01-2023:12	1074	1473%	13305%	249900%	-100%
Stocks (F)	D & M	1980:01-2023:12	995	14.41%	30.02%	11567%	-100%
RF	D & M	1980:01-2023:12	10290 & 492	5.71%	1.33%	2.07%	0.01%
VW	D & M	1980:01-2023:12	10290 & 492	17.51%	20.80%	19.95%	-25.95%
EW	D & M	1980:01-2023:12	10290 & 492	10.41%	24.42%	21.26%	-24.95%
SMB	D & M	1981:02-2023:12	10290 & 492	15.01%	18.81%	62.52%	-11.95%
HML	D & M	1981:07-2023:12	10290 & 492	8.01%	24.58%	91.54%	-29.09%
UMD	D & M	1981:07-2023:12	10290 & 492	17.55%	17.48%	20.54%	-18.33%
BAB	D & M	1983:01-2023:12	10290 & 492	16.70%	20.78%	37.97%	-22.28%

For the empirical analysis, we employ heteroskedasticity-consistent standard errors, specifically those introduced by Newey and West (1987). Unless otherwise specified, a 5% significance level is applied throughout the study. In the out-of-sample analysis, we adopt a expanding-window approach with an starting window of 120 months. To ensure comparability across all analyses, the results presented, except Table 1, are based on the period from February 1993 to December 2023.

5 Empirical Analysis: Volatility Timing Performance

In this section, we examine the economic benefits and statistical significance of volatility management. Section 5.1 assesses the performance of volatility-managed *individual*-factors and volatility-managed *individual*-factor portfolios, which have been a focal point in existing literature. Additionally, Section 3.2 evaluates the out-of-sample performance of the conditional *multifactor* portfolio proposed by DeMiguel et al. (2024), alongside our own portfolio, which integrates methodologies from both approaches and compare them to the unconditional multifactor portfolio.

5.1 Volatility Management of Individual Factors and Portfolios

To begin, we establish the foundation for our analysis by first examining individual volatility-timed factors, following the approach proposed by Moreira and Muir (2017). Next, we construct a mean-variance portfolio that combines the original factor with its volatility-timed counterpart, both in-sample and out-of-sample, as suggested by DeMiguel et al. (2024). Through this approach, we aim to validate the criticisms raised by Barroso and Detzel (2021), that volatility-timing do not work out-of-sample even given prior research finding it significant.

For each factor analyzed, Table 2 presents the annualized Sharpe ratio for the unmanaged factor, $\mathrm{SR}(r_k)$, the volatility-managed individual factor, $\mathrm{SR}(r_k^\sigma)$, and the volatility-managed individual-factor portfolio, $\mathrm{SR}(r_k, r_k^\sigma)$. Additionally, we report the p-value for the difference in Sharpe ratios relative to the original factor.

Panel A reports the performance of each individual factor using factor volatility. Panel B presents the in-sample volatility-managed individual factor port-

Table 2: Performance of Volatility-Managed Individual Factor Portfolios

This table presents the annualized Sharpe ratios for each factor considered in our analysis, including the Sharpe ratios of the unmanaged factor, SR(r_k), the volatility-managed individual factor, SR(r_k^{σ}), and the Sharpe ratio of the mean-variance portfolio that combines the unmanaged factor with its managed counterpart, SR(r_k, r_k^{σ}), as constructed using Equation 1. Panel A, B and C focuses on scaling using factor volatility, and D, E and F focuses on scaling using market volatility. Additionally, the table reports p-values to assess the statistical significance of differences in Sharpe ratios. To ensure consistency with the mean-variance portfolio evaluations, the sample period spans from February 1993 to December 2023.

	VW	EW	SMB	HML	UMD	BAB
Panel A: Factor and volatility-managed counterpart using σ_k						
SR(r_k)	0.974	0.566	0.945	0.244	0.614	0.924
SR($r_k^{\sigma_k}$)	0.948	0.574	0.906	0.160	0.753	0.426
p-value(SR($r_k^{\sigma_k}$) - SR(r_k))	0.601	0.494	0.682	0.832	0.110	0.911
Panel B: In-sample volatility-timed individual-factor portfolio using σ_k						
SR(r_k)	0.974	0.566	0.945	0.244	0.614	0.924
SR(r_k, $r_k^{\sigma_k}$)	1.026	0.614	0.963	0.244	0.757	0.924
p-value(SR(r_k, $r_k^{\sigma_k}$) - SR(r_k))	0.439	0.373	0.551	0.581	0.092	0.589
Panel C: Out-of-sample volatility-timed individual-factor portfolio using σ_k						
SR(r_k)	0.974	0.566	0.945	0.244	0.614	0.924
SR(r_k, $r_k^{\sigma_k}$)	0.962	0.474	0.875	0.234	0.720	0.705
p-value(SR(r_k, $r_k^{\sigma_k}$) - SR(r_k))	0.483	0.500	0.647	0.574	0.120	0.648
Panel D: Factor and volatility-managed counterpart using σ_m						
SR(r_k)	0.974	0.566	0.945	0.244	0.614	0.924
SR($r_k^{\sigma_m}$)	0.948	0.596	0.892	0.262	0.696	1.113
p-value(SR($r_k^{\sigma_m}$) - SR(r_k))	0.511	0.484	0.665	0.533	0.151	0.497
Panel E: In-sample volatility-timed individual-factor portfolio using σ_m						
SR(r_k)	0.974	0.566	0.945	0.244	0.614	0.924
SR(r_k, $r_k^{\sigma_m}$)	1.026	0.618	0.987	0.269	0.697	1.114
p-value(SR(r_k, $r_k^{\sigma_m}$) - SR(r_k))	0.461	0.439	0.597	0.492	0.141	0.404
Panel F: Out-of-sample volatility-timed individual-factor portfolio using σ_m						
SR(r_k)	0.974	0.566	0.945	0.244	0.614	0.924
SR(r_k, $r_k^{\sigma_m}$)	0.962	0.506	0.922	0.232	0.649	0.976
p-value(SR(r_k, $r_k^{\sigma_m}$) - SR(r_k))	0.481	0.492	0.637	0.502	0.146	0.463

folios based on factor volatility, while Panel C shows the out-of-sample results for the same approach. Similarly, Panel D examines the performance of the individual factors using market volatility, Panel E displays the in-sample volatility-managed portfolios based on market volatility, and Panel F presents the corresponding out-of-sample results. The in-sample portfolios in Panels B and E utilize the entire sample period to determine the weights that maximize the Sharpe ratio, thereby introducing look-ahead bias, argued to be consistent with prior literature.

Panel A compares each volatility-managed factor to its unmanaged counterpart. Among the managed factors, only UMD and EW display higher Sharpe ratios, with UMD approaching statistical significance at the 10% level. This finding is particularly noteworthy, as prior academic studies suggest that BAB is one of the factors that benefits most from volatility management. Our results, however, indicate that such advantages may not extend to smaller markets, specifically the Norwegian stock market.

Panel B reports results in line with the methodology of Moreira and Muir (2017), which has been criticized for its in-sample nature. The Sharpe ratio of the volatility-managed individual factor portfolio, denoted as $\mathrm{SR}(r_k, r_k^{\sigma_k})$, is higher for four of the six factors analyzed, with the remaining two factors displaying identical values per construction. Nonetheless, statistical significance is observed only for the UMD factor, and only at the 10% level.

Panel C presents the out-of-sample performance, where Sharpe ratios are predictably lower compared to their in-sample counterparts, and in some instances, even fall below those reported in Panel A. Despite this, none of the differences are statistically significant, although momentum (UMD) is again close to significance at a 10% level. Comparing Panels A, B, and C, we observe that volatility timing using the factor's own volatility, as proposed by Moreira and Muir (2017), generally does not yield statistically significant improvements, except for, arguably, momentum, which aligns with the findings of Barroso and Detzel (2021).

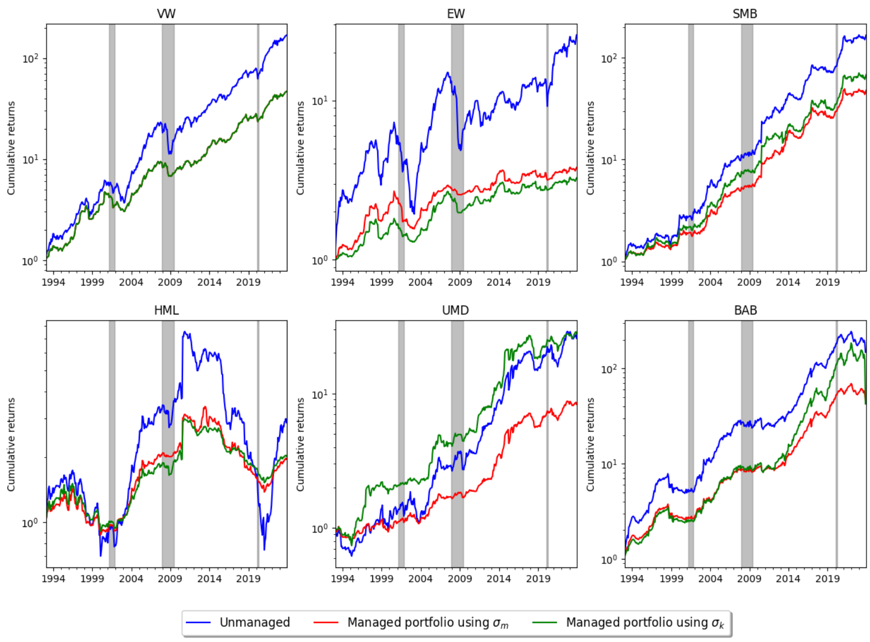
Panels D, E, and F replicate the analysis using market volatility instead of factor volatility. In these cases, Sharpe ratios are generally higher compared to those obtained using factor volatility. However, no statistically significant improvements at the 10% level are observed, even in the in-sample analysis. It is worth noting that the BAB factor shows some improvement, consistent with prior academic suggestions that BAB is conducive to volatility timing, though these results remain statistically insignificant.

Our findings are in line with previous master's theses that identify momentum (UMD) as the factor most responsive to volatility management in the Norwegian stock market. However, whereas prior studies report significant alphas for HML, SMB–in-sample–and occasionally other factors, our results do not corroborate these conclusions. This discrepancy may stem from differences in significance testing methods, as earlier studies typically relied on the approach of Moreira and Muir (2017), or differences in sample periods, since our dataset extends to 2023. Overall, our results suggest that incorporating mean-variance optimization, could in some cases, yields better outcomes than volatility timing alone, indicating that a multifactor approach may offer greater effectiveness.

To visually illustrate these findings, Figure 2 displays the cumulative returns of each unmanaged factor (blue) alongside its corresponding volatility-managed individual factor portfolios, using factor volatility (green) and market volatility (red), over the out-of-sample period from February 1993 to December 2023. Note that the VW factor remains identical across both individual-factor portfolios. Cumulative returns are presented as total returns, representing the growth of a $1 investment in each factor beginning in February 1993. The volatility-managed individual factors are scaled to match the volatility of the unmanaged factor[10].

Figure 2: Cumulative Returns: Volatility-Managed vs. Unmanaged Individual Factor Portfolios (Out-of-Sample)

The six graphs display the cumulative return of the unmanaged factor (blue line) and its corresponding volatility-managed individual factor portfolios using factor volatility (green line) and using market volatility (red line) over the period from February 1993 to December 2023. The volatility-managed individual factor portfolio is scaled to match the variance of the unmanaged portfolio, ensuring comparability in risk-adjusted performance.



From the perspective of cumulative returns illustrated in Figure 2, we do not observe a consistent pattern indicating that volatility timing systematically leads to higher returns, with the exception of a partial effect observed for UMD. This suggests that applying volatility timing to individual factors in isolation does not constitute a viable long-term strategy for achieving superior returns. While there is some evidence that volatility management can mitigate losses during market downturns and crises, this benefit comes at the cost of also missing out on potential upside when volatility is elevated. A potential

[10] First, scaling the factor returns does not affect the Sharpe ratios. Second, since these portfolios are self-financing, they generate payoffs rather than traditional returns; however, for simplicity, we refer to these payoffs as returns.

avenue for future research could involve examining the role of downside volatility specifically, aiming to time factors in a manner that retains exposure to upside volatility while mitigating downside risks.

Based on the presented evidence, we conclude that, consistent with the findings of Cederburg et al. (2020) and DeMiguel et al. (2024), a volatility-managed portfolio constructed from individual factors generally fails to significantly outperform its unmanaged counterpart when evaluated out-of-sample. Although this analysis does not explicitly account for transaction costs, it is reasonable to argue that including such costs would further diminish the Sharpe ratios. However, it is worth noting that DeMiguel et al. (2024) emphasize that trading diversification benefits are more pronounced in volatility-managed portfolios, but the overall trading costs are higher for the volatility-managed portfolios[11]. Therefore, we further conclude that the positive findings reported in previously, which employed the approach of Moreira and Muir (2017), do not hold when evaluated out-of-sample.

5.2 Performance of Mean-Variance Multifactor Portfolios

In the previous section, we analyzed the performance of volatility-managed individual factors and portfolios, an area that has been explored in the context of the Norwegian stock market. In this section, we shift focus to a multifactor perspective by considering an investor who has the ability to invest in, and apply volatility management to, all six factors simultaneously. Specifically, we compare three portfolios: the unconditional mean-variance multifactor portfolio (UMV) and two conditional mean-variance multifactor portfolios–one using market-volatility scaled factors (CMV_m) and another using factor-volatility

[11]The result about trading diversification may seem counterintuitive, but because we are working with factor portfolios, adding more factors can actually reduce overall trading activity. For example, if we are shorting stock X in BAB and simultaneously longing it in UMD by the same amount, these positions offset each other, leading to a net zero position and thus less trading.

scaled factors (CMV_k)[12]. Each portfolio is obtained by solving the optimization problem outlined in equation (6). For the UMV portfolio, we impose an additional constraint by setting $b_k = 0$ for all $k = 1, 2, \ldots, K$. In essence, the UMV portfolio solves the same optimization problem but without access to returns scaled by past volatility.

For each of the portfolios, Table 3 reports the out-of-sample annualized mean return, standard deviation, and Sharpe ratio, along with the p-values for the difference in Sharpe ratios between the conditional and unconditional portfolios. Additionally, the table presents the annualized alpha from a time-series regression of the conditional portfolios' out-of-sample returns on those of the unconditional portfolio, including the Newey-West t-statistic for the alpha.

Table 3: Performance of Mean-Variance Multifactor Portfolios

This table reports the out-of-sample performance of the three multifactor portfolios: the two conditional mean-variance multifactor portfolios (CMV), obtained by solving Equation 6, and the unconditional mean-variance multifactor portfolio (UMV), also derived from Equation 6 but with the additional constraint that $b_k = 0$ for $k = 1, 2, \ldots, K$. This constraint ensures that the weights on the K factors are set to zero for the volatility-managed factor by construction. For each portfolio, the table presents the annualized mean, standard deviation, Sharpe ratio, and p-values for the difference between the Sharpe ratios of the CMV portfolios and the UMV portfolio. Additionally, the table reports the annualized alpha from the time-series regression of the CMV's out-of-sample returns on those of the UMV, along with the Newey-West t-statistic for alpha. The evaluation period spans from February 1993 to December 2023.

	UMV	CMV_k	CMV_m
Mean	0.500	0.576	0.574
Standard deviation	0.306	0.360	0.330
Sharpe ratio	1.633	1.598	1.742
p-value(SR_{CMV} - SR_{UMV})		0.241	0.031
α (%)		1.696	6.797
$t(\alpha)$		0.670	2.059

Table 3 demonstrates that the conditional multifactor portfolio utilizing market volatility (CMV_m) achieves a statistically significant higher Sharpe ratio

[12] DeMiguel et al. (2024) focus exclusively on market volatility; however, given the lack of significant results in the previous section, we include both market and factor volatility for completeness.

compared to its unconditional counterpart (UMV) at the 5% level. Specifically, CMV_m exhibits an improvement in the Sharpe ratio of approximately 7%. However, opposite of what DeMiguel et al. (2024) argues, this enhancement is not observed when using factor volatility; in fact, CMV_k results in a slightly lower Sharpe ratio relative to UMV. The superior performance of CMV_m could stem from the Norwegian stock market's unique structure. With a heavy concentration in the energy sector, market-wide volatility often reflect global commodity price swings and geopolitical events, making it a more predictable signal than factor-specific volatility. Factor volatility may also be noisier due to the smaller number of stocks in each factor, reducing the effectiveness for timing strategies.

In terms of alpha, both conditional portfolios show positive values, though statistical significance is only evident for CMV_m, which delivers an annualized alpha of 6.8%[13]. While both conditional portfolios exhibit higher standard deviations compared to the unconditional portfolio, these are accompanied by correspondingly higher mean returns. It is important to highlight that the elevated mean returns across all portfolios stem from the fact that we do not impose leverage constraints, as all portfolios are constructed to be zero-cost. This choice allows us to better understand the behavior of portfolio weights without restrictions, and importantly, imposing such constraints would only affect the Sharpe ratio positive, making our difference more significant as shown in Appendix B.1.

Nevertheless, DeMiguel et al. (2024) also report that the transaction costs associated with the CMV portfolio are higher than those of the UMV portfolio. This finding aligns with the argument put forth by Cederburg et al. (2020), who suggest that such costs may offset the observed differences in performance as

[13]For the t-statistic, a one-month lag is employed. We have tested multiple lag structures and find that $t(\alpha)$ remains statistically significant at the 5% level for up to six lags for CMV_m.

well as their statistical significance. Furthermore, we observe that the Sharpe ratio difference in our results is smaller than that reported by DeMiguel et al. (2024) prior to accounting for transaction costs. Although we lack transaction cost data, when compared to the existing literature, it is important to have in mind that a smaller market, like Norway, will come with higher transaction costs, which further casting doubt on the robustness of our result.

Figure 3: Cumulative Returns: Conditional vs. Unconditional Mean-Variance Multifactor Portfolios (Out-of-Sample)

This figure illustrates the out-of-sample cumulative returns of the unconditional mean-variance multifactor portfolio (UMV) alongside the two conditional mean-variance multifactor portfolios (CMV_m and CMV_k) over the period from February 1993 to December 2023.

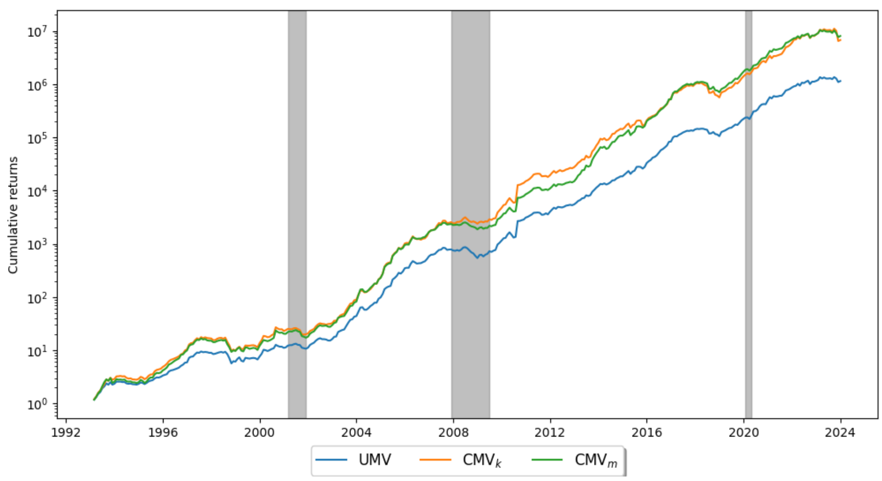


Figure 3 presents the out-of-sample cumulative returns of the unconditional mean-variance multifactor portfolio (UMV) alongside the two conditional mean-variance multifactor portfolios (CMV_m and CMV_k). The figure demonstrates that both conditional portfolios consistently outperform the unconditional portfolio over time, with the two conditional portfolios crossing each other on several occasions.

When comparing the results presented in Section 5.1 with the findings of this section, it becomes evident that constructing a volatility-timed multifactor portfolio can, in certain cases, enhance the Sharpe ratio and yield significant alphas. However, it is important to recognize that these results may lose significance once transaction costs are taken into account, and only holds true when using market volatility. Furthermore, when comparing the multifactor portfolios to the individual-factor portfolios, it is not surprising to observe that the multifactor approach outperforms the individual-factor portfolios across all performance metrics.

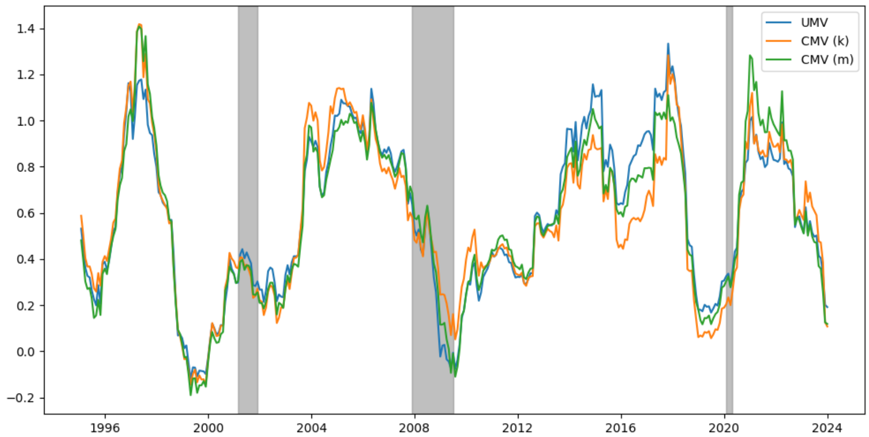
5.3 Understanding the Robustness of Our Portfolio

To assess the robustness of our volatility-managed multifactor portfolio, we conduct several checks detailed in the appendices. Firstly, imposing realistic leverage constraints enhances the statistical significance of Sharpe ratio improvements, as shown in Appendix B.1, suggesting that volatility-timing may perform better under real-world conditions. Appendix B.2 demonstrates that varying the γ parameter still yields significant results. Appendix B.3 shows that substituting variance for volatility does not alter our findings. Lastly, Appendix B.4 finds that relaxing the no-short-selling constraint also preserves significant results.

However, temporal stability poses a challenge. As shown in Appendix B.5, splitting the sample into three decades (1993–2003, 2003–2013, and 2013–2023) reveals that significance holds only in the first decade, with a lower Sharpe ratio and negative alpha emerging in 2003–2013. Economically, the temporal instability may reflect Norway's market conditions during this periods. During the early 2000, for instance, we saw significant volatility in energy markets which likely influenced the Norwegian stock market, making it less fortunate to do volatility-timing due to a change in the risk-return relationship. This inconsistency across periods questions the reliability of volatility-timing, particularly in smaller markets like Norway.

Figure 4: Rolling 2-year Sharpe Ratios Out-of-Sample

This figure illustrates the out-of-sample 2-year Sharpe ratios at a rolling basis for the unconditional mean-variance multifactor portfolio (UMV) alongside the two conditional mean-variance multifactor portfolios (CMV_m and CMV_k) over the period from February 1993 to December 2023.



To further illustrate the time variation in the performance of volatility-managed portfolios, we compute and plot a rolling two-year Sharpe ratio for both the unconditional and conditional multifactor portfolios. As shown in Figure 4, the Sharpe ratio fluctuates considerably over the sample period, with notable peaks around the late 1990s and early 2000s, coinciding with periods of heightened market volatility. However, post-2008, the rolling Sharpe ratios show reduced and inconsistent improvements relative to the unconditional strategy. This builds on the previous finding of the instability of the strategy, and as seen in the figure, it is no clear relationship that the conditional portfolios out-perform the unconditional one in terms of Sharpe ratio. It suggests that while the strategy may deliver strong risk-adjusted returns in select periods, its effectiveness is sensitive to prevailing market conditions and unlikely to persist uniformly over time.

Further analysis in Appendix B.6 explores the portfolio's dependence on individual factors. Systematically removing factors shows that the Sharpe ratio remains significant only when the BAB factor is retained, indicating its outsized contribution to the results. Given the BAB factor's well-documented criticism regarding real-world applicability, this reliance raises doubts about the generalizability of our findings.

To address estimation errors, we explore alternative approaches in Appendix B.7. The Global Minimum-Variance (GMV) portfolio, which minimizes variance without maximizing the Sharpe ratio, yields significant results for the GMV_m portfolio at the 1% level. While this does not directly address our research question, it eliminates estimation error in expected returns. Similarly, applying the Ledoit-Wolf shrinkage method (introduced in Section 2) to the mean-variance framework produces results significant at the 1% level[14]. These findings suggest that reducing reliance on expected return estimates and emphasizing volatility minimization can enhance performance, highlighting the potential impact of estimation errors in the framework.

Collectively, these robustness tests reveal that while volatility-timing can improve risk-adjusted returns under specific conditions, its effectiveness is sensitive to model assumptions, time periods, and factor selection. The dependence on the BAB factor, temporal instability, and susceptibility to estimation errors suggest that practical applicability may be limited in real-world settings, where leverage, transaction costs, and data constraints are critical considerations.

[14]Shrinkage also makes the results significant at a 5% level for the two first decades and significant at a 12% level for the last decade. All decades gets a positive alpha, where only the two last decades have a significant alpha.

6 Conclusion

This thesis set out to investigate whether volatility-managed asset pricing factors could deliver superior risk-adjusted returns in the Norwegian stock market. Addressing RQ-1, our results indicate that volatility-timing applied to individual factors and individual-factor portfolios does not yield statistically significant improvements when evaluated out-of-sample. Although using market volatility instead of factor volatility marginally improves significance, the results remain inconclusive.

For RQ-2, we find that a volatility-managed multifactor portfolio scaled by market volatility achieves a statistically significant improvement in the Sharpe ratio compared to its unconditional counterpart. However, this advantage is modest and potentially offset by higher transaction costs and other frictions.

Our robustness analysis reveals that the effectiveness of volatility-timing strategies is highly sensitive to assumptions regarding leverage, factor composition, and market conditions. We also find that minimizing estimation error gives us more robust results. This underscores the need for cautious interpretation of the results.

We conclude that while volatility management can offer performance enhancements under certain conditions, these benefits are unlikely to be consistently achievable in practice. Future research could focus on incorporating predictive models for volatility, exploring alternative market environments, or assessing the role of downside volatility in enhancing portfolio performance.

APPENDIX

This section provides the technical details and supporting work underlying the analyses conducted in this thesis. While this material may not be of interest to the general reader, it offers a deeper understanding of the methodologies and processes used. Readers focused solely on the conclusions may choose to skip this section, but those seeking a comprehensive exploration of our approach are encouraged to delve into the details provided here.

The full replication code can be found here.

A Data Handling

A.1 Stock Returns

For the stock returns, we begin by identifying missing tickers. Since we have the company names, we can manually fill in these missing tickers. The replacements are as follows:

- Kongsberg Automotive $\rightarrow$ KOA,
- Northern Offshore and Northern Ocean Ltd. $\rightarrow$ NOL,
- EVRY $\rightarrow$ EVRY,
- BW Energy Limited $\rightarrow$ BWE,
- Atlantic Sapphire $\rightarrow$ ASA,
- Pexip Holding $\rightarrow$ PEXIP.

Some tickers remain missing, but these correspond to companies that do not have a full trading month of data and are therefore excluded from the dataset[15].

[15] This does not make a difference for our analysis, as we use thresholds over a month for our factor replication.

Going forward, we need to take into account the companies with different tickers but the same ISIN. Here, we simply replace older tickers with the new ones.

Furthermore, we observe missing returns and prices for the period spanning 2020-07-01 to 2020-11-30. To address this, we retrieve data from Yahoo Finance and S&P CIQ, filling the gaps to the best of our ability. For the missing period, we also require values for shares outstanding to compute market capitalizations. We handle missing values by interpolating between known values if both 2020-06 and 2020-12 data points exist. If only one of these values is available, we use it to populate the missing data.

Regarding data filtering, we restrict the dataset to the period 1980-2023 due to limited availability for 2024. Additionally, we apply the following data filters:

- Exclude all returns where the market capitalization is below 10 million NOK.
- Remove returns where the stock price is under 10 NOK and market capitalization is below 10 million NOK or missing. The rationale behind this is that filtering purely based on price may not be appropriate, as market capitalization better reflects the company's size rather than price alone.

A.2 Factors

We include both the value-weighted (VW) and equally-weighted (EW) indices as factors, which can be considered analogous to "market" factors. In our analysis, we use VW as the primary market factor. Since VW and EW do not represent excess returns, we subtract the risk-free rate accordingly.

A.3 Risk-Free Rate

For the risk-free rate, we address missing daily values through interpolation to ensure consistency with our stock return data.

A.4 Betting Against Beta (BAB) Replication

We extend our gratitude to Masood and Guttulsrød (2024) for providing access to their replication code and robustness tests.

A.4.1 Beta Estimation

We estimate betas using excess stock returns and excess market returns. Daily data is employed for beta estimation, as higher frequency data improves covariance estimates, as noted by Merton (1980). Consequently, volatilities also benefit from higher frequency data.

Once the daily betas are estimated, we convert them into monthly betas by selecting the last available beta at the end of each month. This ensures continuity by carrying over the most recent estimate from the previous period into the subsequent month's analysis. This approach aligns with conventional financial data resampling techniques, maintaining consistency while preserving the latest market information at each period's transition.

The beta estimation follows:

$$\hat{\beta}_i^{ts} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m}, \tag{7}$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ represent the estimated volatilities of the stock and the stock market, respectively, and $\hat{\rho}$ denotes their correlation.

A rolling window approach is applied, using a one-year period for volatility estimation and a five-year period for correlation estimation, as correlations tend to evolve more gradually than volatilities. Daily log returns are used to estimate volatilities, whereas correlations are estimated using overlapping three-day log returns:

$$r_{i,t}^{3d} = \sum_{k=0}^{2} \ln(1 + r_{t+k}^{i}). \tag{8}$$

This approach mitigates the impact of non-synchronous trading. Furthermore, we require a minimum of 120 non-missing data points for volatility estimation and 750 non-missing data points for correlation estimation to ensure robustness.

Finally, we shrink the time-series estimate of beta towards the cross-sectional mean following the methodology of Vasicek (1973) and Elton et al. (2003). Mathematically, this is expressed as:

$$\hat{\beta}_i = w_i \hat{\beta}_i^{TS} + (1 - w_i)\hat{\beta}^{XS}, \tag{9}$$

where the cross-sectional mean beta is set as $\hat{\beta}^{XS} = 1$ and the shrinkage weight is defined as $w_i = 0.6$ for simplicity.

A.4.2 Portfolio Weights

Our objective is to construct a long-short portfolio by taking long positions in low-beta stocks and short positions in high-beta stocks. To achieve this, we design portfolio weights that appropriately adjust to the data.

Each year, we compute an $n \times 1$ vector of beta ranks, defined as:

$$z_i = \text{rank}(\beta_{it}), \tag{10}$$

where β_{it} represents the estimated beta for stock i at time t. We then compute the average rank:

$$\bar{z} = \frac{1_n^\top z}{n}, \tag{11}$$

where n is the number of betas and 1_n is an $n \times 1$ vector of ones. The portfolio weights for the high-beta (w_H) and low-beta (w_L) portfolios are then computed as:

$$w_H = k(z - \bar{z})^+, \quad \text{and} \quad w_L = k(z - \bar{z})^-, \tag{12}$$

where k is a normalizing constant given by:

$$k = \frac{2}{1_n^\top |z - \bar{z}|}. \tag{13}$$

The operators x^+ and x^- indicate the positive and negative elements of a vector x, respectively, and return zero for non-positive/non-negative values. This ensures that stocks with beta estimates above the average receive positive weights in the high-beta portfolio, while those below the average receive positive weights in the low-beta portfolio.

To maintain consistency and ensure that portfolio weights sum to one by construction, we apply this methodology separately to both monthly and daily betas. This separation prevents distortions arising from frequency differences in beta estimation while preserving the intended risk exposures in the long and short positions.

A.4.3 Hedging Procedure

Finally, we compute the BAB factor return using the following formulation:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^L - r_{t+1}^f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r_{t+1}^f), \tag{14}$$

where:

$$r_{t+1}^L = w_L^\top r_{t+1}, \qquad \beta_t^L = \beta_t^\top w_L,$$
$$r_{t+1}^H = w_H^\top r_{t+1}, \qquad \beta_t^H = \beta_t^\top w_H.$$

This formulation ensures that the strategy is market-neutral by scaling the portfolio to achieve a zero market beta. The long and short positions are adjusted to neutralize exposure to systematic risk, allowing the factor returns to reflect the performance of the beta anomaly rather than overall market movements.

Additionally, we introduce an offsetting position in the risk-free asset to make the strategy self-financing. This ensures that the returns are generated purely from the beta spread rather than any excess leverage or capital allocation considerations.

B Robustness Tests: Volatility Timing

In this section we will take a closer look at the assumptions we take during our analysis, and what happens if we change them.

B.1 Imposing Leverage Constraints

We begin by implementing leverage constraints on the model, where we impose a maximum allocation limit on each factor, ranging from 20% to 200%. As shown in Table 4, an interesting pattern emerges: the more stringent the leverage constraint, the more statistically significant the results become. This suggests that volatility management within a portfolio framework can enhance the Sharpe ratio, particularly when applying realistic leverage limitations.

B.2 Sensitivity to Risk Aversion Parameter (γ)

Next, we adjust the value of γ in the optimization. As illustrated in Table 5 and consistent with the findings of DeMiguel et al. (2024), the results remain largely unaffected by variations in γ, indicating that the model's performance is robust to changes in the risk aversion parameter.

B.3 Replacing Volatility with Variance

Table 6 presents the performance of the mean-variance multifactor portfolios when variance, as proposed by Moreira and Muir (2017), is used in place of volatility. The results indicate that employing variance renders the outcomes statistically insignificant, thereby raising concerns regarding the robustness of the findings.

B.4 Relaxing Short-Selling Constraints

Table 7 presents the performance of the mean-variance multifactor portfolios when short-selling is allowed. The table show that the difference in Sharpe ratio stays significant when allowing short-selling. We do see that the t-stat for the alpha becomes insignificant.

B.5 Time-Period Subsample Analysis

When splitting the sample period into three distinct decades, an interesting pattern emerges as seen in Table 8. Between 1993 and 2003, the CMV_m portfolio maintains a statistically significant difference in Sharpe ratios; however, its alpha becomes insignificant. In the subsequent period, 2003 to 2013, the results largely lose significance, with the alpha for CMV_m even turning negative. Finally, during the 2013 to 2023 period, both the difference in Sharpe ratios and the alpha for CMV_m are statistically insignificant. These findings suggest that the effectiveness of volatility-timing is highly sensitive to the specific time period considered and does not consistently yield significant results across different decades.

B.6 Factor Exclusion Analysis

We proceed by systematically removing individual factors from the optimization problem to assess their contribution to the portfolio's performance. The results, shown in Table 9, show that the significance of the CMV_m portfolio remains intact when each factor is removed, with the exception of the BAB factor. This indicates that a substantial portion of the volatility-timing advantage for the CMV_m portfolio is driven by the inclusion of the BAB factor. Additionally, it is noteworthy that the alpha becomes insignificant when either the HML or SMB factors are excluded, highlighting their role in explaining abnormal returns.

B.7 Covariance Shrinkage and Global Minimum Variance Portfolio Analysis

In this section, we extend our analysis by incorporating shrinkage and a global minimum variance (GMV) portfolio into the optimization framework. The shrinkage technique applies the Ledoit-Wolf approach to adjust the covariance matrix, reducing estimation error, while the GMV portfolio focuses solely on minimizing volatility, disregarding expected returns. As shown in Table 10, both approaches yield even more statistically significant results, suggesting that addressing estimation uncertainty in the covariance matrix and emphasizing volatility minimization enhances the performance of the volatility-managed portfolios.

Table 4: Performance of Mean-Variance Multifactor Portfolios under Leverage Constraints

This table replicates the results of Table 3, but imposes leverage constraints by restricting the maximum allocation to each factor, thereby evaluating the effect of limiting factor exposures on the portfolio's performance.

	UMV	CMV_k	CMV_m
Panel A: No leverage constrains on any of the weights			
Sharpe ratio	1.633	1.598	1.742
p-value($\mathrm{SR}_{\mathrm{CMV}}$ - $\mathrm{SR}_{\mathrm{UMV}}$)		0.241	0.031
α (%)		1.696	6.797
$t(\alpha)$		0.670	2.059
Panel B: Maximum weight of 20% per factor			
Sharpe ratio	1.513	1.635	1.748
p-value ($\mathrm{SR}_{\mathrm{CMV}}$ - $\mathrm{SR}_{\mathrm{UMV}}$)		0.004	0.000
alpha (%)		3.557	5.183
t(alpha)		2.540	5.191
Panel C: Maximum weight of 50% per factor			
Sharpe ratio	1.648	1.627	1.782
p-value		0.073	0.000
alpha (%)		2.311	6.525
t(alpha)		0.944	3.576
Panel D: Maximum weight of 100% per factor			
Sharpe ratio	1.630	1.598	1.742
p-value		0.216	0.017
alpha (%)		1.799	6.890
t(alpha)		0.709	2.086
Panel E: Maximum weight of 200% per factor			
Sharpe ratio	1.633	1.598	1.742
p-value		0.238	0.032
alpha (%)		1.696	6.797
t(alpha)		0.670	2.059

Table 5: Sensitivity of Mean-Variance Multifactor Portfolios to Risk Aversion Parameter (γ)

This table presents the same analysis as Table 3, altering the value of γ in the optimization to investigate the sensitivity of the portfolio performance to varying levels of risk aversion.

	UMV	CMV_k	CMV_m
Panel A: Using the original value, $\gamma = 5$			
Sharpe ratio	1.633	1.598	1.742
p-value(SR_{CMV} - SR_{UMV})		0.241	0.031
α (%)		1.696	6.797
$t(\alpha)$		0.670	2.059
Panel B: Not using γ by solving for tangency weights			
Sharpe ratio	1.668	1.635	1.752
p-value (SR_{CMV} - SR_{UMV})		0.241	0.034
alpha (%)		0.532	1.722
t(alpha)		0.763	1.919
Panel C: Using $\gamma = 2$			
Sharpe ratio	1.632	1.597	1.742
p-value		0.229	0.027
alpha (%)		4.254	17.102
t(alpha)		0.670	2.028
Panel D: Using $\gamma = 7$			
Sharpe ratio	1.630	1.592	1.714
p-value		0.227	0.028
alpha (%)		1.143	4.037
t(alpha)		0.630	2.133

Table 6: Performance of Mean-Variance Multifactor Portfolios using Variance Instead of Volatility

This table replicates the results of Table 3, with the distinction that factors are scaled using variance and not volatility.

	UMV	CMV_k	CMV_m
Mean	0.500	0.522	0.523
Standard deviation	0.306	0.324	0.317
Sharpe ratio	1.633	1.611	1.649
p-value(SR_{CMV} - SR_{UMV})		0.125	0.138
α		0.186	1.052
$t(\alpha)$		0.150	1.225

Table 7: Performance of Mean-Variance Multifactor Portfolios with Short-Selling Constraints Relaxed

This table replicates the results of Table 3, with the distinction that short-selling is allowed in the optimization.

	UMV	CMV_k	CMV_m
Mean	1.043	1.212	1.089
Standard deviation	0.593	0.661	0.614
Sharpe ratio	1.759	1.834	1.771
p-value(SR_{CMV} - SR_{UMV})		0.145	0.030
α		12.012	4.232
$t(\alpha)$		1.722	1.104

Table 8: Time-Period Subsample Performance of Mean-Variance Multifactor Portfolios

This table replicates the results of Table 3, but splits the sample period into three distinct ten-year intervals to assess the consistency of the factor performance across different market conditions.

	UMV	CMV_k	CMV_m
Panel A: Full period of February 1993 to December 2023			
Sharpe ratio	1.633	1.598	1.742
p-value(SR_{CMV} - SR_{UMV})		0.241	0.031
α (%)		1.696	6.797
$t(\alpha)$		0.670	2.059
Panel B: Period spans from 1993 to 2003			
Sharpe ratio	1.530	1.542	1.515
p-value (SR_{CMV} - SR_{UMV})		0.276	0.040
alpha (%)		2.994	2.100
t(alpha)		1.106	0.771
Panel C: Period spans from 2003 to 2013			
Sharpe ratio	1.755	1.584	1.558
p-value (SR_{CMV} - SR_{UMV})		0.961	0.956
alpha (%)		6.231	-2.085
t(alpha)		0.895	-0.504
Panel D: Period spans from 2013 to 2023			
Sharpe ratio	2.210	1.931	2.09
p-value (SR_{CMV} - SR_{UMV})		0.170	0.054
alpha (%)		-4.987	5.000
t(alpha)		-0.494	0.765

Table 9: Factor Exclusion Analysis: Performance of Mean-Variance Multifactor Portfolios

This table replicates the results of Table 3, with the distinction that factors are excluded one after one to assess the impact of each factor's removal on the statistical significance of the results.

	UMV	CMV_k	CMV_m
Panel A: Using all six factors as in our original analysis			
Sharpe ratio	1.633	1.598	1.742
p-value(SR_{CMV} - SR_{UMV})		0.241	0.031
α (%)		1.696	6.797
$t(\alpha)$		0.670	2.059
Panel B: Removing the VW factor from the data			
Sharpe ratio	1.261	1.288	1.398
p-value(SR_{CMV} - SR_{UMV})		0.105	0.006
α (%)		3.912	6.510
$t(\alpha)$		1.329	2.452
Panel C: Removing the EW factor from the data			
Sharpe ratio	1.639	1.599	1.744
p-value(SR_{CMV} - SR_{UMV})		0.233	0.026
α (%)		1.579	6.652
$t(\alpha)$		0.630	2.107
Panel D: Removing the SMB factor from the data			
Sharpe ratio	1.451	1.384	1.453
p-value(SR_{CMV} - SR_{UMV})		0.278	0.017
α (%)		0.381	2.188
$t(\alpha)$		0.162	1.302
Panel E: Removing the HML factor from the data			
Sharpe ratio	1.832	1.804	1.813
p-value(SR_{CMV} - SR_{UMV})		0.235	0.032
α (%)		2.052	2.366
$t(\alpha)$		0.924	1.492
Panel F: Removing the UMD factor from the data			
Sharpe ratio	1.516	1.465	1.709
p-value(SR_{CMV} - SR_{UMV})		0.218	0.028
α (%)		0.640	8.546
$t(\alpha)$		0.279	2.906
Panel G: Removing the BAB factor from the data			
Sharpe ratio	1.270	1.283	1.390
p-value(SR_{CMV} - SR_{UMV})		0.182	0.139
α (%)		1.945	6.067
$t(\alpha)$		1.051	2.389

Table 10: Effect of Covariance Shrinkage and GMV Strategy on Mean-Variance Multifactor Portfolio Performance

This table replicates the results of Table 3, with the distinction that we introduce a GMV approach and a shrinkage of the covariance matrix.

	UMV	CMV_k	CMV_m
Panel A: Using regular mean-variance without shrinkage			
Mean	0.500	0.576	0.574
Standard deviation	0.306	0.360	0.330
Sharpe ratio	1.633	1.598	1.742
p-value(SR_{CMV} - SR_{UMV})		0.241	0.031
α (%)		1.696	6.797
$t(\alpha)$		0.670	2.059
Panel B: Minimizing variance (GMV)			
Mean	0.147	0.149	0.156
Standard deviation	0.098	0.100	0.096
Sharpe ratio	1.499	1.496	1.630
p-value(SR_{CMV} - SR_{UMV})		0.340	0.004
α		0.352	1.525
$t(\alpha)$		0.736	3.383
Panel C: Introducing shrinkage			
Mean	0.478	0.579	0.588
Standard deviation	0.295	0.360	0.338
Sharpe ratio	1.619	1.609	1.742
p-value(SR_{CMV} - SR_{UMV})		0.284	0.009
α		2.136	6.502
$t(\alpha)$		0.895	2.557

REFERENCES

Alme, S., and S. L. Aarsland. 2022. *Return Premiums in Volatility-Managed Portfolios*. Master's thesis, University of Agder, Agder, Norway. Supervisor: Steen Koekebakker.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang. 2006. The Cross-Section of Volatility and Expected Returns. *Journal of Finance* 61:259–299. doi:10.1111/j.1540-6261.2006.00836.x.

Ang, A., R. J. Hodrick, Y. Xing, and X. Zhang. 2009. High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence. *Journal of Financial Economics* 91:1–23. doi:10.1016/j.jfineco.2007.12.005.

Asness, C. 2016. The Siren Song of Factor Timing. *Journal of Portfolio Management* 42:1–6. doi:10.3905/jpm.2016.42.5.001.

Asness, C. S., T. J. Moskowitz, and L. H. Pedersen. 2013. Value and Momentum Everywhere. *Journal of Finance* 68:929–985. doi:10.1111/jofi.12021.

Bakken, M., and J. Horvei. 2022. *Does Volatility Timing Enhance Portfolio Performance?* Master of science in business - major in finance, BI Norwegian Buisness School, Oslo, Norway. Supervisor: Geri Hooidal Bjoonnes.

Bali, T., S. J. Brown, Y. Tang, and M. O. Demirtas. 2017. Do Hedge Funds Outperform? Estimating Risk and Skill in Absolute Returns. *Journal of Financial Economics* 126:1–25. doi:10.1016/j.jfineco.2017.06.002.

Bali, T., N. Cakici, X. S. Yan, and Z. Zhang. 2011. Does Idiosyncratic Volatility Really Matter? *Journal of Finance* 66:973–1007. doi:10.1111/j.1540-6261.2011.01655.x.

Banz, R. W. 1981. The Relationship Between Return and Market Value of Common Stocks. *Journal of Financial Economics* 9:3–18. doi:10.1016/0304-405X(81)90018-0.

Barroso, P., and A. L. Detzel. 2021. Do limits to arbitrage explain the benefits of volatility-managed portfolios? *Journal of Financial Economics* 140:744–767.

Black, F. 1972. Capital Market Equilibrium with Restricted Borrowing. *The Journal of Business* 45:444–455. doi:10.1086/295472.

Brandt, M. W., P. Sant-Clara, and R. Valkanov. 2009. Parametric portfolio policies: Exploiting characteristics in the cross section of equity returns. *Review of Financial Studies* 22:3411–3447.

Brunnermeier, M. K., and J. A. Parker. 2007. Optimal Expectations. *American Economic Review* 97:1092–1118. doi:10.1257/aer.97.4.1092.

Carhart, M. M. 1997. On persistence in mutual fund performance. *Journal of Finance* 52:57–82.

Cederburg, S., M. S. O'Doherty, F. Wang, and X. Yan. 2020. On the performance of volatility-managed portfolios. *Journal of Financial Economics* 138:95–117.

Chopra, V. K. 1993. Improving Optimization. *The Journal of Investing* 2:51–59.

DeMiguel, V., L. Garlappi, F. J. Nogales, and R. Uppal. 2009. A generalized approach to portfolio optimization: Improving performance by constraining portfolio norms. *Management Science* 55:798–812.

DeMiguel, V., A. Martin-Utrera, and R. Uppal. 2024. A multifactor perspective on volatility-managed portfolios. *Journal of Finance* Forthcoming, Available at SSRN 3982504.

DeMiguel, V., A. MartÃn-Utrera, F. J. Nogales, and R. Uppal. 2020. A transaction-cost perspective on the multitude of firm characteristics. *Review of Financial Studies* 33:2180–2222.

Elton, E. J., M. J. Gruber, S. J. Brown, and W. N. Goetzmann. 2003. *Modern Portfolio Theory and Investment Analysis*. 6th ed. Hoboken, NJ: John Wiley & Sons.

Fama, E. F., and K. R. French. 1992. The cross-section of expected stock returns. *Journal of Finance* 47:427–465.

Frazzini, A., and L. H. Pedersen. 2014. Betting against beta. *Journal of Financial Economics* 111:1–25.

Frost, P. A., and J. E. Savarino. 1988. For Better Performance: Constrain Portfolio Weights. *The Journal of Portfolio Management* 15:29–34. doi:10.3905/jpm.1988.409183.

Fylling, E. O. V., and E. Jacobsen. 2023. *Stability of Asset Pricing Models at the Oslo Stock Exchange*. Master's thesis in financial economics, Norwegian University of Science and Technology (NTNU), Trondheim, Norway. Supervisor: Snorre Lindset.

Jagannathan, R., and T. Ma. 2003. Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *The Journal of Finance* 58:1651–1684. doi:10.1111/1540-6261.00580.

Jegadeesh, N., and S. Titman. 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. *The Journal of Finance* 48:65–91. doi:10.1111/j.1540-6261.1993.tb04702.x.

Johansen, T. A., and L. K. Eckhoff. 2016. *Managing Volatility: An Empirical Analysis of the Time-series Relation Between Risk and Return*. Master of science in economics and business administration, Norwegian School of Economics, Bergen, Norway. Supervisor: Francisco Santos.

Jorion, P. 1985. International Portfolio Diversification with Estimation Risk. *The Journal of Business* 58:259–278. doi:10.1086/296305.

Jorion, P. 1986. Bayes-Stein Estimation for Portfolio Analysis. *Journal of Financial and Quantitative Analysis* 21:279–292. doi:10.2307/2331041.

Ledoit, O., and M. Wolf. 2004a. Honey, I Shrunk the Sample Covariance Matrix. *The Journal of Portfolio Management* 30:110–119. doi:10.3905/jpm.2004.110.

Ledoit, O., and M. Wolf. 2004b. A Well-Conditioned Estimator for Large-Dimensional Covariance Matrices. *Journal of Multivariate Analysis* 88:365–411. doi:10.1016/S0047-259X(03)00096-4.

Leland, H. E. 1999. Beyond Mean-Variance: Performance Measurement in a Nonsymmetrical World. *Financial Analysts Journal* 55:27–36. doi:10.2469/faj.v55.n1.2246.

Lintner, J. 1965. The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics* 47:13–37. doi:10.2307/1924119.

Lo, A. W. 2002. The Statistics of Sharpe Ratios. *Financial Analysts Journal* 58:36–52. doi:10.2469/faj.v58.n4.2453.

Lønø, B. E., and C. E. Svendsen. 2019. *A Comparison of Asset Pricing Models in the Norwegian Stock Market*. Master of science, BI Norwegian Business School, Oslo, Norway.

Markowitz, H. M. 1952. Portfolio selection. *Journal of Finance* 7:77–91.

Markowitz, H. M. 1959. *Portfolio selection: Efficient diversification of investments*. New York: Wiley.

Masood, S., and C. Guttulsrød. 2024. *The Robustness of Betting Agaist Beta: Implications of Portfolio Weights and Skewness Risk*. Master's thesis, BI Norwegian Business School.

Merton, R. C. 1980. On Estimating the Expected Return on the Market: An Exploratory Investigation. *Journal of Financial Economics* 8:323–361. doi:10.1016/0304-405X(80)90007-0.

Moreira, A., and T. Muir. 2017. Volatility-managed portfolios. *Journal of Finance* 72:1611–1644.

Mossin, J. 1966. Equilibrium in a Capital Asset Market. *Econometrica* 34:768–783. doi:10.2307/1910098.

Newey, W. K., and K. D. West. 1987. A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix. *Econometrica* 55:703–708. doi:10.2307/1913610.

Novy-Marx, R., and M. Velikov. 2021. Betting against betting against beta. *Journal of Financial Economics* 143:80–106.

Oliver, P., and A. Mikhail. 2018. Investor Behavior and the Impact of Market Volatility. *Journal of Behavioral Finance* 19:223–240. doi:10.1080/15427560.2018.1485362.

Ødegaard, B. A. 2021. Empirics of the Oslo Stock Exchange: Basic, descriptive, results 1980-2020. *University of Stavanger* .

Pelster, M. 2024. The Dynamics of Risk-Taking in Financial Markets: Evidence from Behavioral Data. *Journal of Financial Economics* 145:25–47. doi:10.1016/j.jfineco.2023.07.001.

Roll, R. 1977. A critique of the asset pricing theoryâ€™s tests Part I: On past and potential testability of the theory. *Journal of Financial Economics* 4:129–176.

Sharpe, W. 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19:425–442.

Sharpe, W. F. 1966. Mutual Fund Performance. *The Journal of Business* 39:119–138. doi:10.1086/294846.

Vasicek, O. A. 1973. A Note on Using Cross-Sectional Information in Bayesian Estimation of Security Betas. *The Journal of Finance* 28:1233–1239. doi:10.1111/j.1540-6261.1973.tb01452.x.