# Package 'SurrogateParadoxTest'

June 11, 2025

**Description** Provides functions to nonparametrically assess assumptions necessary to prevent the surrogate paradox through hypothesis tests of stochastic dominance, monotonicity of regres-

scribed in: Hsiao E, Tian L, and Parast L (2025). ``Avoiding the surrogate paradox: an empiri-

sion functions, and non-negative residual treatment effects. Details are de-

cal framework for assessing assumptions." Journal of Nonparametric Statis-

Type Package

Version 2.1 Date 2025-06-11

Title Empirical Testing of Surrogate Paradox Assumptions

tics <doi:10.1080 10485252.2025.2498609="">. Additionally, there are functions to assess resilience to the surrogate paradox via calculation of the resilience probability, the resilience bound, and the resilience set. Details will be available in Hsiao E, Tian L, and Parast L, ``Resilience Measures for the Surrogate Paradox" (Under Review). A tutorial for this package can be found at <a href="https://www.laylaparast.com/surrogateparadoxtest">https://www.laylaparast.com/surrogateparadoxtest</a>&gt;.</doi:10.1080>	
License GPL	
Imports stats, MonotonicityTest, MASS, ggplot2	
NeedsCompilation no	
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a\_b\_hat

Helper function

#### **Description**

Helper function for monotonicity test

# Usage

```
a_b_n(r, s, X, Y)
```

# **Arguments**

r	Index to start summation.
S	Index to end summation.
Χ	Vector of X values.
Υ	Vector of Y values.

#### Value

a_hat	Numeric value of a_hat
b_hat	Numeric value of b_hat

#### Author(s)

**Emily Hsiao** 

barrett\_donald\_cutoff Helper function for stochastic dominance test

# Description

Rejection cutoff value for stochastic dominance value based on alpha level.

### Usage

```
barrett_donald_cutoff(alpha)
```

### **Arguments**

alpha

Desired alpha level for stochastic dominance test.

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#### Value

Cutoff value for stochastic dominance test

### Author(s)

Emily Hsiao

barrett\_donald\_p

Helper function for stochastic dominance test

# Description

Calculates p-value of the stochastic dominance test

### Usage

```
barrett_donald_p(statistic)
```

# **Arguments**

statistic

Test statistic calculated in stochastic dominance test

#### Value

p-value of the test statistic

### Author(s)

Emily Hsiao

calculate\_bandwidth

Helper function for kernel smoother

# **Description**

Calculates the appropriate bandwidth for Nadaraya-Watson kernel smoother.

# Usage

```
calculate_bandwidth(s)
```

# Arguments

s

Vector of surrogate values.

### Value

Desired bandwidth for kernel smoother.

### Author(s)

4 fourier\_interval

interval Fourier Resilience Interval
--------------------------------------

# Description

Calculates the resilience probability and resilience bound, assuming that the mean function of Study B data is generated by adding a Fourier basis with random coefficients with specified parameters.

### Usage

# Arguments

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
var_vec	Length-4 vector governing the variance of the random coefficients of the mean function for Study B.
period	Length-3 vector dictating the period of th Fourier basis for Study B. Each item represents a proportion of the range of the surrogate values of Study A.
n.iter	Number of $\Delta_B$ samples to generate.
М	Number of bootstrap iterations to estimate the SE of $\hat{p}_0$ and $\hat{q}_{\alpha}$ .
q_quant	Desired quantile for the resilience bound.
plot	TRUE or FALSE; include plots of randomly generated functions in results. Default is FALSE.
intervals	TRUE or FALSE; return the inner 95% of $\Delta_B$ samples. Default is TRUE.
get_var	TRUE or FALSE; return an estimated variance of the $\hat{p}_0$ and $\hat{q}_\alpha$ parameters. Default is TRUE.

# **Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

#### Value

```
Delta_hats Vector of samples of \Delta_B.

Delta_estimate Mean of samples of \Delta_B.

p_hat Estimated value \hat{p}_0, the probability of the surrogate paradox.

q_hat Estimated value \hat{q}_{\alpha}
, the resilience bound.
```

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p_se	Estimated variance of $\hat{p}_0$ ; only returned if get_var = TRUE.
q_se	Estimated variance of $\hat{q}_{\alpha}$ ; only returned if get_var = TRUE.
control_plot	Plot including data points of control group of Study A and functions generated for Study B; only returned if plot = $TRUE$ .
treatment_plot	Plot including data points of treatment group of Study A and functions generated for Study B; only returned if plot = TRUE.

#### Author(s)

**Emily Hsiao** 

#### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

#### **Examples**

```
s0.A <- rnorm(20, 3, sqrt(3))
y0.A <- sapply(s0.A, function(x) 2 * x - 1)
s1.A <- rnorm(20, 4, sqrt(3))
y1.A <- sapply(s1.A, function(x) x + 3)
s0.B <- rnorm(20, 4.75, sqrt(1))
s1.B <- rnorm(20, 5.25, sqrt(1))
result <- fourier_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
var_vec = c(0.25, 0.25, 0.1, 0.1), period = c(0.5, 0.25, 0.1), n.iter = 20, M = 20)
result$p_hat</pre>
```

fourier\_resilience\_set

Fourier Resilience Set

### **Description**

Creates a plot of the resilience set i.e., the possible variance parameters of the coefficients of the Fourier terms of the mean function of Study B such that the probability of the surrogate paradox is below a threshold  $\alpha$ . Note that this function assumes that the covariance matrix of the random coefficients takes the form  $\Sigma = diag(\sigma_{11}^2, \sigma_{11}^2, \sigma_{22}^2, \sigma_{22}^2)$ .

### Usage

```
fourier_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
sig1_values, sig2_values, alpha = 0.05)
```

### **Arguments**

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.

fourier\_resilience\_set

s1.B	Vector of surrogate values in the treatment group of Study B.
sig1_values	Vector of values of $\sigma_{11}^2$ for which the probability of surrogate paradox should be calculated with the given data.
sig2_values	Vector of values of $\sigma^2_{22}$ for which the probability of surrogate paradox should be calculated with the given data.
alpha	Threshold value of the surrogate paradox.

#### **Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Value

Returns a ggplot object with values of  $\sigma_{11}^2$  on the x-axis and  $\sigma_{22}^2$  on the y-axis, and regions where  $P(\Delta_B) < \alpha$  with given data and parameters highlighted in blue.

### Author(s)

**Emily Hsiao** 

#### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

# Examples

```
n_A=200
n_B=200
s0.A \leftarrow rnorm(n_A, mean = 10, sd = 2)
y0.A < -5 + 0.5 * s0.A + rnorm(n_A, mean = 0, sd = 0.5)
s1.A \leftarrow rnorm(n_A, mean = 12, sd = 2.5)
y1.A \leftarrow 5.5 + 0.6 * s1.A + rnorm(n_A, mean = 0, sd = 0.6)
 s0.B <- rnorm(n_B, 10, 2)
 s1.B <- rnorm(n_B, 11, 3)
 sig1\_vals\_example \leftarrow seq(0.1, 1, length.out = 30) # Sigma squared values
 sig2\_vals\_example \leftarrow seq(0.1, .5, length.out = 30)
 # Run the function with the generated example data
 fourier_resilience_set(
   s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
   sig1_values = sig1_vals_example,
   sig2_values = sig2_vals_example,
   alpha = 0.05
 )
```

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gaussian_kernel	Helper function for kernel smoother
Baassian_kerner	Tresper junction for kernet smoother

# Description

Gaussian kernel used for kernel smoother.

# Usage

```
gaussian_kernel(x)
```

# **Arguments**

X X

### Value

Gaussian kernel applied to x

# Author(s)

Emily Hsiao

```
{\tt gaussian\_process\_interval}
```

Gaussian Process Resilience Interval

# Description

Calculates the resilience probability and resilience bound, assuming that the mean function of Study B data is generated according to a Gaussian Process with specified parameters.

# Usage

```
gaussian_process_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2, theta,
n.iter = 500, M = 100, q_quant = 0.1, plot = FALSE, intervals = TRUE, get_var = TRUE)
```

# Arguments

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
sigma2	Variance parameter of the Gaussian Process.
theta	Smoothness parameter of the Gaussian Process.
n.iter	Number of $\Delta_B$ samples to generate.

M Number of bootstrap iterations to estimate the SE of  $\hat{p}_0$  and  $\hat{q}_{\alpha}$ .

q\_quant Desired quantile for the resilience bound.

plot TRUE or FALSE; include plots of randomly generated functions in results. De-

fault is FALSE.

intervals TRUE or FALSE; return the inner 95% of  $\Delta_B$  samples. Default is TRUE.

get\_var TRUE or FALSE; return an estimated variance of the  $\hat{p}_0$  and  $\hat{q}_{\alpha}$  parameters.

Default is TRUE.

#### **Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

#### Value

Delta\_hats Vector of samples of  $\Delta_B$ . Delta\_estimate Mean of samples of  $\Delta_B$ .

p\_hat Estimated value  $\hat{p}_0$ , the probability of the surrogate paradox.

q\_hat Estimated value  $\hat{q}_{\alpha}$ 

, the resilience bound.

p\_se Estimated variance of  $\hat{p}_0$ ; only returned if get\_var = TRUE. q\_se Estimated variance of  $\hat{q}_{\alpha}$ ; only returned if get\_var = TRUE.

control\_plot Plot including data points of control group of Study A and functions generated

for Study B; only returned if plot = TRUE.

treatment\_plot Plot including data points of treatment group of Study A and functions generated

for Study B; only returned if plot = TRUE.

p\_interval Inner 95% of  $\hat{p}_0$  samples from bootstrap; only returned if intervals = TRUE. q\_interval Inner 95% of  $\hat{q}_{\alpha}$  samples from bootstrap; only returned if intervals = TRUE.

#### Author(s)

Emily Hsiao

### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

# Examples

```
s0.A <- rnorm(20, 3, sqrt(3))
y0.A <- sapply(s0.A, function(x) 2 * x - 1)
s1.A <- rnorm(20, 4, sqrt(3))
y1.A <- sapply(s1.A, function(x) x + 3)
s0.B <- rnorm(20, 4.75, sqrt(1))
s1.B <- rnorm(20, 5.25, sqrt(1))
result <- gaussian_process_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2 = 0.25, theta = 2, n.iter = 20, M = 20)
result$p_hat</pre>
```

gp\_resilience\_set 9

esilience_set Gaussian Process Resilience Set
---

### **Description**

Creates a plot of the resilience set i.e., the possible parameters of the Gaussian Process such that the probability of the surrogate paradox is below a threshold  $\alpha$ .

### Usage

```
gp_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2_vals, theta_vals,
alpha = 0.05)
```

### Arguments

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
sigma2_vals	Vector of values of $\sigma^2$ for which the probability of surrogate paradox should be calculated with the given data.
theta_vals	Vector of values of $\theta$ for which the probability of surrogate paradox should be calculated with the given data.
alpha	Threshold value of the surrogate paradox.

#### **Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

#### Value

Returns a ggplot object with values of  $\theta$  on the x-axis and  $\sigma^2$  on the y-axis, and regions where  $P(\Delta_B) < \alpha$  with given data and parameters highlighted in blue.

### Author(s)

Emily Hsiao

### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

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### **Examples**

```
n_A=200
n_B=200
s0.A <- rnorm(n_A, mean = 10, sd = 2)
y0.A <- 5 + 0.5 * s0.A + rnorm(n_A, mean = 0, sd = 0.5)

s1.A <- rnorm(n_A, mean = 12, sd = 2.5)
y1.A <- 5.5 + 0.6 * s1.A + rnorm(n_A, mean = 0, sd = 0.6)

s0.B <- rnorm(n_B, 10, 2)
s1.B <- rnorm(n_B, 11, 3)

sigma2_values <- seq(0.01, 10, length.out = 30)
theta_values <- seq(0.01, 10, length.out = 30)
gp_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2_values, theta_values, alpha = 0.05)</pre>
```

 $modified\_S\_stat$ 

Helper function for non-negative residual treatment effect test.

# Description

Calculates the value of the test statistic in the NNR test.

### Usage

```
modified_S_stat(mu0_hat, mu1_hat, s0, y0, s1, y1, grid_x, boot = FALSE)
```

### **Arguments**

mu0_hat	Kernel-smoothed estimate of $\mu_0$ function.
mu1_hat	Kernel-smoothed estimate of $\mu_1$ function.
s0	Vector of surrogate values in control group.
у0	Vector of endpoint values in control group.
s1	Vector of surrogate values in treatment group.
y1	Vector of endpoing values in the treatment group.
grid_x	Values of s over which supremum is calculated.
boot	Whether this is a bootstrapped statistic or the test statistic.

#### Value

s_hat	Value of the test statistic
sup	Value of the supremum over grid_x

# Author(s)

monotonicity\_test 11

	monotonicity_test	Monotonicity test
--	-------------------	-------------------

# **Description**

Runs the test of monotonicity for a regression function.

### Usage

```
monotonicity_test(X, Y, h = NA, m = 5, bootstrap_n = 100,
alpha = 0.05)
```

# Arguments

X	Vector of X values.
Υ	Vector of Y values.

h Bandwidth for the kernel smoother.

m Window size to calculate linear regression.

bootstrap\_n Desired number of bootstrap samples.

alpha Desired alpha level of the test.

### Value

T\_m\_value Value of the test statistic.

p\_val p-value of test

reject whether the test rejects the null

T\_m\_samples Vector of bootstrapped values of test statistic

### Author(s)

Emily Hsiao

### References

Hall, Peter, and Nancy E. Heckman. "Testing for monotonicity of a regression mean by calibrating for linear functions." Annals of Statistics (2000): 20-39.

polynomial\_interval

nnr_test Non-negative residual treatment effect function	
--	--

# Description

Runs the test of non-negative residual treatment effect given two sets of surrogate and primary endpoint values.

### Usage

```
nnr_{test}(s0, y0, s1, y1, n_{test}) = 200, alpha = 0.05)
```

### **Arguments**

s0	Vector of surrogate values in control group.
y0	Vector of endpoint values in control group.
s1	Vector of surrogate values in treatment group.
y1	Vector of endpoint values in treatment group.
n_bootstrap	Desired number of bootstrap samples.

alpha Desired alpha level of test.

#### Value

p\_value p-value of test.

reject Whether the test rejects the null hypothesis.

s\_hat Calculated value of test statistic.

s\_vec Vector of bootstrapped values of test statistic.

#### Author(s)

**Emily Hsiao** 

### References

Hsiao et all 2024 (Under review)

# Description

Calculates the resilience probability and resilience bound, assuming that the mean function of Study B data is generated by adding a polynomial basis with random coefficients with specified parameters.

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#### Usage

```
polynomial_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, var_vec, n.iter = 500,
M = 100, q_quant = 0.1, plot = FALSE, intervals = TRUE, get_var = TRUE)
```

#### **Arguments**

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
var_vec	Length-4 vector governing the variance of the random coefficients of the mean function for Study B.
n.iter	Number of $\Delta_B$ samples to generate.
М	Number of bootstrap iterations to estimate the SE of $\hat{p}_0$ and $\hat{q}_{\alpha}$ .
q_quant	Desired quantile for the resilience bound.
plot	TRUE or FALSE; include plots of randomly generated functions in results. Default is FALSE.
intervals	TRUE or FALSE; return the inner 95% of $\Delta_B$ samples. Default is TRUE.
get_var	TRUE or FALSE; return an estimated variance of the $\hat{p}_0$ and $\hat{q}_\alpha$ parameters. Defaults to TRUE.

# **Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Value

Delta\_hats Vector of samples of  $\Delta_B$ . Delta\_estimate Mean of samples of  $\Delta_B$ . p\_hat Estimated value  $\hat{p}_0$ , the probability of the surrogate paradox. q\_hat Estimated value  $\hat{q}_{\alpha}$ , the resilience bound. Estimated variance of  $\hat{p}_0$ ; only returned if get\_var = TRUE. p\_se Estimated variance of  $\hat{q}_{\alpha}$ ; only returned if get\_var = TRUE. q\_se Plot including data points of control group of Study A and functions generated control\_plot for Study B; only returned if plot = TRUE. treatment\_plot Plot including data points of treatment group of Study A and functions generated for Study B; only returned if plot = TRUE.

### Author(s)

#### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

#### **Examples**

```
s0.A <- rnorm(20, 3, sqrt(3))
y0.A <- sapply(s0.A, function(x) 2 * x - 1)
s1.A <- rnorm(20, 4, sqrt(3))
y1.A <- sapply(s1.A, function(x) x + 3)
s0.B <- rnorm(20, 4.75, sqrt(1))
s1.B <- rnorm(20, 5.25, sqrt(1))
result <- polynomial_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, var_vec = c(0.25, 0.25, 0.1, 0.1),n.iter = 20, M = 20)
result$p_hat</pre>
```

```
polynomial_resilience_set
```

Polynomial Resilience Set

#### **Description**

Creates a plot of the resilience set i.e., the possible variance parameters of the coefficients of the polynomial terms of the mean function of Study B such that the probability of the surrogate paradox is below a threshold  $\alpha$ . Note that this function assumes that the covariance matrix of the random coefficients takes the form  $\Sigma = diag(\sigma_{11}^2, \sigma_{11}^2, \sigma_{22}^2, \sigma_{22}^2)$ .

#### Usage

```
polynomial_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sig1_values,
sig2_values, alpha = 0.05)
```

#### **Arguments**

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
sig1_values	Vector of values of $\sigma_{11}^2$ for which the probability of surrogate paradox should be calculated with the given data.
sig2_values	Vector of values of $\sigma_{22}^2$ for which the probability of surrogate paradox should be calculated with the given data.
alpha	Threshold value of the surrogate paradox.

#### **Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

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#### Value

Returns a ggplot object with values of  $\sigma_{11}^2$  on the x-axis and  $\sigma_{22}^2$  on the y-axis, and regions where  $P(\Delta_B) < \alpha$  with given data and parameters highlighted in blue.

#### Author(s)

**Emily Hsiao** 

#### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

# **Examples**

```
n_A=200
n_B=200
s0.A <- rnorm(n_A, mean = 10, sd = 2)
  y0.A <- 5 + 0.5 * s0.A + rnorm(n_A, mean = 0, sd = 0.5)

s1.A <- rnorm(n_A, mean = 12, sd = 2.5)
  y1.A <- 5.5 + 0.6 * s1.A + rnorm(n_A, mean = 0, sd = 0.6)

s0.B <- rnorm(n_B, 10, 2)
s1.B <- rnorm(n_B, 11, 3)

sigma1_values <- seq(0.01, 1.2, length.out = 30)
sigma2_values <- seq(0.01, .4, length.out = 30)

polynomial_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma1_values, sigma2_values, alpha = 0.05)</pre>
```

Helper function for monotonicity test

# Description

Helper function for monotonicity test; should not be called directly by user.

#### Usage

Q

```
Q(r, s, X)
```

#### **Arguments**

r Index to start summation.

s Index to end summation.

X Vector of X values over which to sum.

# Value

Q

16 S

# Author(s)

Emily Hsiao

# References

Hall, Peter, and Nancy E. Heckman. "Testing for monotonicity of a regression mean by calibrating for linear functions." Annals of Statistics (2000): 20-39.

S Helper function

# Description

Helper function for monotonicity test.

# Usage

# Arguments

а	Value of a (regression coefficient)
b	Value of b (regression coefficient)
r	Index to start summation
s	Index to end summation
Χ	Vector of X values
Υ	Vector of Y values

### Value

Mean-squared error

# Author(s)

sd\_test 17

sd_test Stochastic dominance test function	
--	--

# Description

Runs the test of stochastic dominance given two vectors of surrogate values.

#### Usage

```
sd_test(s0, s1, alpha = 0.05)
```

### **Arguments**

s0	Vector of surrogate values in control group.
s1	Vector of surrogate values in treatment group.
alpha	Desired alpha level of hypothesis test.

### Value

s_hat	Value of test statistic
p.value	p-value of test
reject	Rejection decision of test

### Author(s)

Emily Hsiao

### References

Barrett, Garry F., and Stephen G. Donald. "Consistent tests for stochastic dominance." Econometrica 71.1 (2003): 71-104.

# Description

Nadaraya-Watson kernel smoother

# Usage

```
smoother_fitter(X, Y, kernel = gaussian_kernel, h)
```

# Arguments

Χ	Vector of X values
Υ	Vector of Y values

kernel Kernel to use in the kernel smoother; defaults to Gaussian kernel

h Bandwidth

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#### Value

Returns a function which is the smoothed function; input takes in an x value.

#### Author(s)

**Emily Hsiao** 

test\_assumptions

*Test assumptions to prevent surrogate paradox* 

#### **Description**

Tests the assumptions necessary to prevent the surrogate paradox: stochastic dominance of surrogate values in the treatment group over control group, monotonicity of the relationship between surrogate and primary endpoint in both treatment and control group, and non-negative residual treatment effect of the treatment group over the control group. For computational efficiency, Version 2.0 of this package uses the monotonicity\_test function from the MonotonicityTest package.

### Usage

```
test_assumptions(s0 = NULL, y0 = NULL, s1 = NULL, y1 = NULL, trim = 0.95, alpha = 0.05, type = "all", all_results = TRUE, direction = "positive", monotonicity_bootstrap_n = 100, nnr_bootstrap_n = 200)
```

#### **Arguments**

s0	Vector of surrogate values in control group.	
y0	Vector of primary endpoint values in control group.	
s1	Vector of surrogate values in treatment group.	
y1	Vector of primary endpoint values in treatment group.	
trim	Proportion of data to keep after trimming the outliers. Defaults to 95%. Trims data by sorting by surrogate value and removing (1 - trim)/2 % of the lowest and highest surrogate values with their corresponding primary endpoint values.	
alpha	Desired alpha level of tests.	
type	Type of test to run. Defaults to "all"; possible inputs are "sd" (stochastic dominance), "monotonicity" (monotonicity), and "nnr" (non-negative residual treatment effect).	
all_results	TRUE or FALSE; return all outputs from hypothesis tests. Defaults to TRUE.	
direction	Direction of the test. Defaults to "positive", which tests that the treatment group stochastically dominates the control group, that $\mu_0$ and $\mu_1$ are monotonically increasing, and that $\mu_0 \leq \mu_1 \forall s$ . Parameter "negative" tests that the control group stochastically dominates the treatment group, that $\mu_0$ and $\mu_1$ are monotonically decreasing, and that $\mu_1 \leq \mu_0 \forall s$ .	
monotonicity_bootstrap_n		

nnr\_bootstrap\_n

Number of bootstrap samples for nnr test.

Number of bootstrap samples for monotonicity test.

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#### Value

result Table or string of results of the tests

sd\_result Detailed results of stochastic dominance test; only returned if all\_results is TRUE

monotonicity0\_result Detailed results of monotonicity test in control group; only returned if all\_results is TRUE

monotonicity1\_result Detailed results of monotonicity test in treatment group; only returned if all\_results is TRUE

nnr\_result Detailed results of nnr test; only returned if all\_results is TRUE

#### Author(s)

**Emily Hsiao** 

#### References

Barrett, Garry F., and Stephen G. Donald. "Consistent tests for stochastic dominance." Econometrica 71.1 (2003): 71-104.

Hall, Peter, and Nancy E. Heckman. "Testing for monotonicity of a regression mean by calibrating for linear functions." Annals of Statistics (2000): 20-39.

Hsiao, Tian, Parast. "Avoiding the Surrogate Paradox: An Empirical Framework for Assessing Assumptions." 2025 (Under Review)

### **Examples**

```
m_c <- function(s) 1 + 2 * s
m_t <- function(s) 1 + 2 * s

s_c <- rnorm(100, 3, 1)
y_c <- sapply(s_c, function(s) rnorm(1, m_c(s), 1))
s_t <- rnorm(100, 3, 1)
y_t <- sapply(s_t, function(s) rnorm(1, m_t(s), 1))

test_assumptions(
s0 = s_c, y0 = y_t, s1 = s_t, y1 = y_t, type = "sd"
)

test_assumptions(
s0 = s_c, y0 = y_t, s1 = s_t, y1 = y_t, type = "all")</pre>
```

Helper function for monotonicity test

T\_m

#### **Description**

Calculates the test statistic

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# Usage

 $T_m(m, X, Y)$ 

# Arguments

m m window sizeX Vector of X valuesY Vector of Y values

# Value

stat Value of the test statistic

stat\_vals Vector of statistics before taking maximum

 $\begin{array}{ll} b\_vals & Values \ of \ b \\ Q\_vals & Values \ of \ Q \end{array}$ 

# Author(s)

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