

# Package ‘SurrogateParadoxTest’

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**Type** Package

**Title** Empirical Testing of Surrogate Paradox Assumptions

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**Description** Provides functions to nonparametrically assess assumptions necessary to prevent the surrogate paradox through hypothesis tests of stochastic dominance, monotonicity of regression functions, and non-negative residual treatment effects. Details are described in: Hsiao E, Tian L, and Parast L (2025). ``Avoiding the surrogate paradox: an empirical framework for assessing assumptions." Journal of Nonparametric Statistics <doi:10.1080/10485252.2025.2498609>. Additionally, there are functions to assess resilience to the surrogate paradox via calculation of the resilience probability, the resilience bound, and the resilience set. Details will be available in Hsiao E, Tian L, and Parast L, ``Resilience Measures for the Surrogate Paradox" (Under Review). A tutorial for this package can be found at <<https://www.laylaparast.com/surrogateparadoxtest>>.

**License** GPL

**Imports** stats, MonotonicityTest, MASS, ggplot2

**NeedsCompilation** no

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a_b_hat	<i>Helper function</i>
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### Description

Helper function for monotonicity test

### Usage

a\_b\_hat(r, s, X, Y)

### Arguments

r	Index to start summation.
s	Index to end summation.
X	Vector of X values.
Y	Vector of Y values.

### Value

a_hat	Numeric value of a_hat
b_hat	Numeric value of b_hat

### Author(s)

Emily Hsiao

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barrett_donald_cutoff	<i>Helper function for stochastic dominance test</i>
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### Description

Rejection cutoff value for stochastic dominance value based on alpha level.

### Usage

barrett\_donald\_cutoff(alpha)

### Arguments

alpha	Desired alpha level for stochastic dominance test.
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**Value**

Cutoff value for stochastic dominance test

**Author(s)**

Emily Hsiao

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barrett\_donald\_p

*Helper function for stochastic dominance test*

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**Description**

Calculates p-value of the stochastic dominance test

**Usage**

barrett\_donald\_p(statistic)

**Arguments**

statistic      Test statistic calculated in stochastic dominance test

**Value**

p-value of the test statistic

**Author(s)**

Emily Hsiao

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calculate\_bandwidth

*Helper function for kernel smoother*

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**Description**

Calculates the appropriate bandwidth for Nadaraya-Watson kernel smoother.

**Usage**

calculate\_bandwidth(s)

**Arguments**

s      Vector of surrogate values.

**Value**

Desired bandwidth for kernel smoother.

**Author(s)**

Emily Hsiao

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fourier_interval	<i>Fourier Resilience Interval</i>
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### Description

Calculates the resilience probability and resilience bound, assuming that the mean function of Study B data is generated by adding a Fourier basis with random coefficients with specified parameters.

### Usage

```
fourier_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, var_vec, period,
n.iter = 500, M = 100, q_quant = 0.1,      plot = FALSE, intervals
= TRUE, get_var = TRUE)
```

### Arguments

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
var_vec	Length-4 vector governing the variance of the random coefficients of the mean function for Study B.
period	Length-3 vector dictating the period of the Fourier basis for Study B. Each item represents a proportion of the range of the surrogate values of Study A.
n.iter	Number of $\Delta_B$ samples to generate.
M	Number of bootstrap iterations to estimate the SE of $\hat{p}_0$ and $\hat{q}_\alpha$ .
q_quant	Desired quantile for the resilience bound.
plot	TRUE or FALSE; include plots of randomly generated functions in results. Default is FALSE.
intervals	TRUE or FALSE; return the inner 95% of $\Delta_B$ samples. Default is TRUE.
get_var	TRUE or FALSE; return an estimated variance of the $\hat{p}_0$ and $\hat{q}_\alpha$ parameters. Default is TRUE.

### Details

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Value

Delta_hats	Vector of samples of $\Delta_B$ .
Delta_estimate	Mean of samples of $\Delta_B$ .
p_hat	Estimated value $\hat{p}_0$ , the probability of the surrogate paradox.
q_hat	Estimated value $\hat{q}_\alpha$
	, the resilience bound.

p_se	Estimated variance of $\hat{p}_0$ ; only returned if get_var = TRUE.
q_se	Estimated variance of $\hat{q}_\alpha$ ; only returned if get_var = TRUE.
control_plot	Plot including data points of control group of Study A and functions generated for Study B; only returned if plot = TRUE.
treatment_plot	Plot including data points of treatment group of Study A and functions generated for Study B; only returned if plot = TRUE.

**Author(s)**

Emily Hsiao

**References**

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

**Examples**

```

s0.A <- rnorm(20, 3, sqrt(3))
y0.A <- sapply(s0.A, function(x) 2 * x - 1)
s1.A <- rnorm(20, 4, sqrt(3))
y1.A <- sapply(s1.A, function(x) x + 3)
s0.B <- rnorm(20, 4.75, sqrt(1))
s1.B <- rnorm(20, 5.25, sqrt(1))
result <- fourier_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
var_vec = c(0.25, 0.25, 0.1, 0.1), period = c(0.5, 0.25, 0.1), n.iter = 20, M = 20)
result$p_hat

```

fourier\_resilience\_set

*Fourier Resilience Set***Description**

Creates a plot of the resilience set i.e., the possible variance parameters of the coefficients of the Fourier terms of the mean function of Study B such that the probability of the surrogate paradox is below a threshold  $\alpha$ . Note that this function assumes that the covariance matrix of the random coefficients takes the form  $\Sigma = \text{diag}(\sigma_{11}^2, \sigma_{11}^2, \sigma_{22}^2, \sigma_{22}^2)$ .

**Usage**

```

fourier_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
sig1_values, sig2_values, alpha = 0.05)

```

**Arguments**

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.

s1.B	Vector of surrogate values in the treatment group of Study B.
sig1_values	Vector of values of $\sigma_{11}^2$ for which the probability of surrogate paradox should be calculated with the given data.
sig2_values	Vector of values of $\sigma_{22}^2$ for which the probability of surrogate paradox should be calculated with the given data.
alpha	Threshold value of the surrogate paradox.

### Details

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Value

Returns a ggplot object with values of  $\sigma_{11}^2$  on the x-axis and  $\sigma_{22}^2$  on the y-axis, and regions where  $P(\Delta_B) < \alpha$  with given data and parameters highlighted in blue.

### Author(s)

Emily Hsiao

### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Examples

```
n_A=200
n_B=200
s0.A <- rnorm(n_A, mean = 10, sd = 2)
y0.A <- 5 + 0.5 * s0.A + rnorm(n_A, mean = 0, sd = 0.5)

s1.A <- rnorm(n_A, mean = 12, sd = 2.5)
y1.A <- 5.5 + 0.6 * s1.A + rnorm(n_A, mean = 0, sd = 0.6)

s0.B <- rnorm(n_B, 10, 2)
s1.B <- rnorm(n_B, 11, 3)
sig1_vals_example <- seq(0.1, 1, length.out = 30) # Sigma squared values
sig2_vals_example <- seq(0.1, .5, length.out = 30)

# Run the function with the generated example data

fourier_resilience_set(
  s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
  sig1_values = sig1_vals_example,
  sig2_values = sig2_vals_example,
  alpha = 0.05
)
```

---

gaussian_kernel	<i>Helper function for kernel smoother</i>
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**Description**

Gaussian kernel used for kernel smoother.

**Usage**

```
gaussian_kernel(x)
```

**Arguments**

x	x
---	---

**Value**

Gaussian kernel applied to x

**Author(s)**

Emily Hsiao

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gaussian_process_interval	<i>Gaussian Process Resilience Interval</i>
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**Description**

Calculates the resilience probability and resilience bound, assuming that the mean function of Study B data is generated according to a Gaussian Process with specified parameters.

**Usage**

```
gaussian_process_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2, theta,
  n.iter = 500, M = 100, q_quant = 0.1, plot = FALSE, intervals = TRUE, get_var = TRUE)
```

**Arguments**

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
sigma2	Variance parameter of the Gaussian Process.
theta	Smoothness parameter of the Gaussian Process.
n.iter	Number of $\Delta_B$ samples to generate.

M	Number of bootstrap iterations to estimate the SE of $\hat{p}_0$ and $\hat{q}_\alpha$ .
q_quant	Desired quantile for the resilience bound.
plot	TRUE or FALSE; include plots of randomly generated functions in results. Default is FALSE.
intervals	TRUE or FALSE; return the inner 95% of $\Delta_B$ samples. Default is TRUE.
get_var	TRUE or FALSE; return an estimated variance of the $\hat{p}_0$ and $\hat{q}_\alpha$ parameters. Default is TRUE.

### Details

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Value

Delta_hats	Vector of samples of $\Delta_B$ .
Delta_estimate	Mean of samples of $\Delta_B$ .
p_hat	Estimated value $\hat{p}_0$ , the probability of the surrogate paradox.
q_hat	Estimated value $\hat{q}_\alpha$ , the resilience bound.
p_se	Estimated variance of $\hat{p}_0$ ; only returned if get_var = TRUE.
q_se	Estimated variance of $\hat{q}_\alpha$ ; only returned if get_var = TRUE.
control_plot	Plot including data points of control group of Study A and functions generated for Study B; only returned if plot = TRUE.
treatment_plot	Plot including data points of treatment group of Study A and functions generated for Study B; only returned if plot = TRUE.
p_interval	Inner 95% of $\hat{p}_0$ samples from bootstrap; only returned if intervals = TRUE.
q_interval	Inner 95% of $\hat{q}_\alpha$ samples from bootstrap; only returned if intervals = TRUE.

### Author(s)

Emily Hsiao

### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Examples

```
s0.A <- rnorm(20, 3, sqrt(3))
y0.A <- sapply(s0.A, function(x) 2 * x - 1)
s1.A <- rnorm(20, 4, sqrt(3))
y1.A <- sapply(s1.A, function(x) x + 3)
s0.B <- rnorm(20, 4.75, sqrt(1))
s1.B <- rnorm(20, 5.25, sqrt(1))
result <- gaussian_process_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2 = 0.25,
  theta = 2, n.iter = 20, M = 20)
result$p_hat
```



---

gp_resilience_set	<i>Gaussian Process Resilience Set</i>
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### Description

Creates a plot of the resilience set i.e., the possible parameters of the Gaussian Process such that the probability of the surrogate paradox is below a threshold  $\alpha$ .

### Usage

```
gp_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2_vals, theta_vals,
  alpha = 0.05)
```

### Arguments

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
sigma2_vals	Vector of values of $\sigma^2$ for which the probability of surrogate paradox should be calculated with the given data.
theta_vals	Vector of values of $\theta$ for which the probability of surrogate paradox should be calculated with the given data.
alpha	Threshold value of the surrogate paradox.

### Details

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

### Value

Returns a ggplot object with values of  $\theta$  on the x-axis and  $\sigma^2$  on the y-axis, and regions where  $P(\Delta_B) < \alpha$  with given data and parameters highlighted in blue.

### Author(s)

Emily Hsiao

### References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

**Examples**

```

n_A=200
n_B=200
s0.A <- rnorm(n_A, mean = 10, sd = 2)
y0.A <- 5 + 0.5 * s0.A + rnorm(n_A, mean = 0, sd = 0.5)

s1.A <- rnorm(n_A, mean = 12, sd = 2.5)
y1.A <- 5.5 + 0.6 * s1.A + rnorm(n_A, mean = 0, sd = 0.6)

s0.B <- rnorm(n_B, 10, 2)
s1.B <- rnorm(n_B, 11, 3)

sigma2_values <- seq(0.01, 10, length.out = 30)
theta_values <- seq(0.01, 10, length.out = 30)

gp_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma2_values, theta_values,
alpha = 0.05)

```

---

modified\_S\_stat

*Helper function for non-negative residual treatment effect test.*


---

**Description**

Calculates the value of the test statistic in the NNR test.

**Usage**

```
modified_S_stat(mu0_hat, mu1_hat, s0, y0, s1, y1, grid_x, boot = FALSE)
```

**Arguments**

mu0_hat	Kernel-smoothed estimate of $\mu_0$ function.
mu1_hat	Kernel-smoothed estimate of $\mu_1$ function.
s0	Vector of surrogate values in control group.
y0	Vector of endpoint values in control group.
s1	Vector of surrogate values in treatment group.
y1	Vector of endpoint values in the treatment group.
grid_x	Values of s over which supremum is calculated.
boot	Whether this is a bootstrapped statistic or the test statistic.

**Value**

s_hat	Value of the test statistic
sup	Value of the supremum over grid_x

**Author(s)**

Emily Hsiao

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monotonicity_test	<i>Monotonicity test</i>
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**Description**

Runs the test of monotonicity for a regression function.

**Usage**

```
monotonicity_test(X, Y, h = NA, m = 5, bootstrap_n = 100,  
alpha = 0.05)
```

**Arguments**

X	Vector of X values.
Y	Vector of Y values.
h	Bandwidth for the kernel smoother.
m	Window size to calculate linear regression.
bootstrap_n	Desired number of bootstrap samples.
alpha	Desired alpha level of the test.

**Value**

T_m_value	Value of the test statistic.
p_val	p-value of test
reject	whether the test rejects the null
T_m_samples	Vector of bootstrapped values of test statistic

**Author(s)**

Emily Hsiao

**References**

Hall, Peter, and Nancy E. Heckman. "Testing for monotonicity of a regression mean by calibrating for linear functions." *Annals of Statistics* (2000): 20-39.

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nnr_test	<i>Non-negative residual treatment effect function</i>
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---

**Description**

Runs the test of non-negative residual treatment effect given two sets of surrogate and primary endpoint values.

**Usage**

```
nnr_test(s0, y0, s1, y1, n_bootstrap = 200, alpha = 0.05)
```

**Arguments**

s0	Vector of surrogate values in control group.
y0	Vector of endpoint values in control group.
s1	Vector of surrogate values in treatment group.
y1	Vector of endpoint values in treatment group.
n_bootstrap	Desired number of bootstrap samples.
alpha	Desired alpha level of test.

**Value**

p_value	p-value of test.
reject	Whether the test rejects the null hypothesis.
s_hat	Calculated value of test statistic.
s_vec	Vector of bootstrapped values of test statistic.

**Author(s)**

Emily Hsiao

**References**

Hsiao et all 2024 (Under review)

---

polynomial_interval	<i>Polynomial Resilience Interval</i>
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---

**Description**

Calculates the resilience probability and resilience bound, assuming that the mean function of Study B data is generated by adding a polynomial basis with random coefficients with specified parameters.

**Usage**

```
polynomial_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, var_vec, n.iter = 500,
  M = 100, q_quant = 0.1, plot = FALSE, intervals = TRUE, get_var = TRUE)
```

**Arguments**

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
var_vec	Length-4 vector governing the variance of the random coefficients of the mean function for Study B.
n.iter	Number of $\Delta_B$ samples to generate.
M	Number of bootstrap iterations to estimate the SE of $\hat{p}_0$ and $\hat{q}_\alpha$ .
q_quant	Desired quantile for the resilience bound.
plot	TRUE or FALSE; include plots of randomly generated functions in results. Default is FALSE.
intervals	TRUE or FALSE; return the inner 95% of $\Delta_B$ samples. Default is TRUE.
get_var	TRUE or FALSE; return an estimated variance of the $\hat{p}_0$ and $\hat{q}_\alpha$ parameters. Defaults to TRUE.

**Details**

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

**Value**

Delta_hats	Vector of samples of $\Delta_B$ .
Delta_estimate	Mean of samples of $\Delta_B$ .
p_hat	Estimated value $\hat{p}_0$ , the probability of the surrogate paradox.
q_hat	Estimated value $\hat{q}_\alpha$
	, the resilience bound.
p_se	Estimated variance of $\hat{p}_0$ ; only returned if get_var = TRUE.
q_se	Estimated variance of $\hat{q}_\alpha$ ; only returned if get_var = TRUE.
control_plot	Plot including data points of control group of Study A and functions generated for Study B; only returned if plot = TRUE.
treatment_plot	Plot including data points of treatment group of Study A and functions generated for Study B; only returned if plot = TRUE.

**Author(s)**

Emily Hsiao

## References

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

## Examples

```
s0.A <- rnorm(20, 3, sqrt(3))
y0.A <- sapply(s0.A, function(x) 2 * x - 1)
s1.A <- rnorm(20, 4, sqrt(3))
y1.A <- sapply(s1.A, function(x) x + 3)
s0.B <- rnorm(20, 4.75, sqrt(1))
s1.B <- rnorm(20, 5.25, sqrt(1))
result <- polynomial_interval(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B,
var_vec = c(0.25, 0.25, 0.1, 0.1), n.iter = 20, M = 20)
result$p_hat
```

---

polynomial\_resilience\_set

*Polynomial Resilience Set*

---

## Description

Creates a plot of the resilience set i.e., the possible variance parameters of the coefficients of the polynomial terms of the mean function of Study B such that the probability of the surrogate paradox is below a threshold  $\alpha$ . Note that this function assumes that the covariance matrix of the random coefficients takes the form  $\Sigma = \text{diag}(\sigma_{11}^2, \sigma_{11}^2, \sigma_{22}^2, \sigma_{22}^2)$ .

## Usage

```
polynomial_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sig1_values,
sig2_values, alpha = 0.05)
```

## Arguments

s0.A	Vector of surrogate values in the control group of Study A.
y0.A	Vector of primary outcome values in the control group of Study A.
s1.A	Vector of surrogate values in the treatment group of Study A.
y1.A	Vector of primary outcome values in the treatment group of Study A.
s0.B	Vector of surrogate values in the control group of Study B.
s1.B	Vector of surrogate values in the treatment group of Study B.
sig1_values	Vector of values of $\sigma_{11}^2$ for which the probability of surrogate paradox should be calculated with the given data.
sig2_values	Vector of values of $\sigma_{22}^2$ for which the probability of surrogate paradox should be calculated with the given data.
alpha	Threshold value of the surrogate paradox.

## Details

More details can be found in Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

**Value**

Returns a ggplot object with values of  $\sigma_{11}^2$  on the x-axis and  $\sigma_{22}^2$  on the y-axis, and regions where  $P(\Delta_B) < \alpha$  with given data and parameters highlighted in blue.

**Author(s)**

Emily Hsiao

**References**

Hsiao, Tian, Parast. "Resilience Measures for the Surrogate Paradox." (2025) Under Review.

**Examples**

```
n_A=200
n_B=200
s0.A <- rnorm(n_A, mean = 10, sd = 2)
y0.A <- 5 + 0.5 * s0.A + rnorm(n_A, mean = 0, sd = 0.5)

s1.A <- rnorm(n_A, mean = 12, sd = 2.5)
y1.A <- 5.5 + 0.6 * s1.A + rnorm(n_A, mean = 0, sd = 0.6)

s0.B <- rnorm(n_B, 10, 2)
s1.B <- rnorm(n_B, 11, 3)

sigma1_values <- seq(0.01, 1.2, length.out = 30)
sigma2_values <- seq(0.01, .4, length.out = 30)

polynomial_resilience_set(s0.A, y0.A, s1.A, y1.A, s0.B, s1.B, sigma1_values,
sigma2_values, alpha = 0.05)
```

**Description**

Helper function for monotonicity test; should not be called directly by user.

**Usage**

```
Q(r, s, X)
```

**Arguments**

r	Index to start summation.
s	Index to end summation.
X	Vector of X values over which to sum.

**Value**

Q

**Author(s)**

Emily Hsiao

**References**

Hall, Peter, and Nancy E. Heckman. "Testing for monotonicity of a regression mean by calibrating for linear functions." *Annals of Statistics* (2000): 20-39.

---

S	<i>Helper function</i>
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---

**Description**

Helper function for monotonicity test.

**Usage**

S(a, b, r, s, X, Y)

**Arguments**

- |   |                                     |
|---|-------------------------------------|
| a | Value of a (regression coefficient) |
| b | Value of b (regression coefficient) |
| r | Index to start summation            |
| s | Index to end summation              |
| X | Vector of X values                  |
| Y | Vector of Y values                  |

**Value**

Mean-squared error

**Author(s)**

Emily Hsiao



---

sd_test	<i>Stochastic dominance test function</i>
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---

**Description**

Runs the test of stochastic dominance given two vectors of surrogate values.

**Usage**

```
sd_test(s0, s1, alpha = 0.05)
```

**Arguments**

s0	Vector of surrogate values in control group.
s1	Vector of surrogate values in treatment group.
alpha	Desired alpha level of hypothesis test.

**Value**

s_hat	Value of test statistic
p.value	p-value of test
reject	Rejection decision of test

**Author(s)**

Emily Hsiao

**References**

Barrett, Garry F., and Stephen G. Donald. "Consistent tests for stochastic dominance." *Econometrica* 71.1 (2003): 71-104.

---

smoother_fitter	<i>Helper function</i>
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---

**Description**

Nadaraya-Watson kernel smoother

**Usage**

```
smoother_fitter(X, Y, kernel = gaussian_kernel, h)
```

**Arguments**

X	Vector of X values
Y	Vector of Y values
kernel	Kernel to use in the kernel smoother; defaults to Gaussian kernel
h	Bandwidth

**Value**

Returns a function which is the smoothed function; input takes in an x value.

**Author(s)**

Emily Hsiao

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test_assumptions	<i>Test assumptions to prevent surrogate paradox</i>
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**Description**

Tests the assumptions necessary to prevent the surrogate paradox: stochastic dominance of surrogate values in the treatment group over control group, monotonicity of the relationship between surrogate and primary endpoint in both treatment and control group, and non-negative residual treatment effect of the treatment group over the control group. For computational efficiency, Version 2.0 of this package uses the monotonicity\_test function from the MonotonicityTest package.

**Usage**

```
test_assumptions(s0 = NULL, y0 = NULL, s1 = NULL, y1 = NULL, trim = 0.95,
alpha = 0.05, type = "all", all_results = TRUE, direction = "positive",
monotonicity_bootstrap_n = 100, nnr_bootstrap_n = 200)
```

**Arguments**

s0	Vector of surrogate values in control group.
y0	Vector of primary endpoint values in control group.
s1	Vector of surrogate values in treatment group.
y1	Vector of primary endpoint values in treatment group.
trim	Proportion of data to keep after trimming the outliers. Defaults to 95%. Trims data by sorting by surrogate value and removing (1 - trim)/2 % of the lowest and highest surrogate values with their corresponding primary endpoint values.
alpha	Desired alpha level of tests.
type	Type of test to run. Defaults to "all"; possible inputs are "sd" (stochastic dominance), "monotonicity" (monotonicity), and "nnr" (non-negative residual treatment effect).
all_results	TRUE or FALSE; return all outputs from hypothesis tests. Defaults to TRUE.
direction	Direction of the test. Defaults to "positive", which tests that the treatment group stochastically dominates the control group, that $\mu_0$ and $\mu_1$ are monotonically increasing, and that $\mu_0 \leq \mu_1 \forall s$ . Parameter "negative" tests that the control group stochastically dominates the treatment group, that $\mu_0$ and $\mu_1$ are monotonically decreasing, and that $\mu_1 \leq \mu_0 \forall s$ .
monotonicity_bootstrap_n	Number of bootstrap samples for monotonicity test.
nnr_bootstrap_n	Number of bootstrap samples for nnr test.

**Value**

result	Table or string of results of the tests
sd_result	Detailed results of stochastic dominance test; only returned if all_results is TRUE
monotonicity0_result	Detailed results of monotonicity test in control group; only returned if all_results is TRUE
monotonicity1_result	Detailed results of monotonicity test in treatment group; only returned if all_results is TRUE
nnr_result	Detailed results of nnr test; only returned if all_results is TRUE

**Author(s)**

Emily Hsiao

**References**

Barrett, Garry F., and Stephen G. Donald. "Consistent tests for stochastic dominance." *Econometrica* 71.1 (2003): 71-104.

Hall, Peter, and Nancy E. Heckman. "Testing for monotonicity of a regression mean by calibrating for linear functions." *Annals of Statistics* (2000): 20-39.

Hsiao, Tian, Parast. "Avoiding the Surrogate Paradox: An Empirical Framework for Assessing Assumptions." 2025 (Under Review)

**Examples**

```
m_c <- function(s) 1 + 2 * s
m_t <- function(s) 1 + 2 * s

s_c <- rnorm(100, 3, 1)
y_c <- sapply(s_c, function(s) rnorm(1, m_c(s), 1))
s_t <- rnorm(100, 3, 1)
y_t <- sapply(s_t, function(s) rnorm(1, m_t(s), 1))

test_assumptions(
  s0 = s_c, y0 = y_t, s1 = s_t, y1 = y_t, type = "sd"
)

test_assumptions(
  s0 = s_c, y0 = y_t, s1 = s_t, y1 = y_t, type = "all")
```

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T\_m

*Helper function for monotonicity test*


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**Description**

Calculates the test statistic

**Usage** $T_m(m, X, Y)$ **Arguments**

$m$	$m$ window size
$X$	Vector of $X$ values
$Y$	Vector of $Y$ values

**Value**

$stat$	Value of the test statistic
$stat\_vals$	Vector of statistics before taking maximum
$b\_vals$	Values of $b$
$Q\_vals$	Values of $Q$

**Author(s)**

Emily Hsiao

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