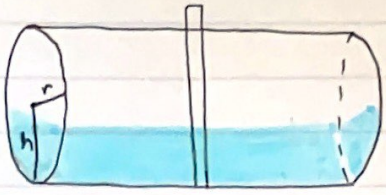
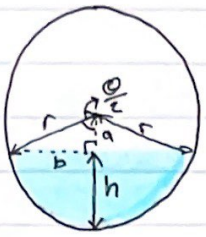
Cylindrical Model:



Side view



in this right angled triangle, by trigonometry

$$a = r\cos\left(\frac{\theta}{2}\right)$$

$$b = r\sin\left(\frac{\theta}{2}\right)$$

the area of the triangle is:

$$A_{triangle} = \frac{1}{2}ab$$

$$= \frac{1}{2}r^2\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)$$

$$= \frac{1}{4}r^2\sin\theta$$

Identity $\sin(2\theta) = 2\sin\theta\cos\theta$

$\sin(\theta) = 2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

$\frac{1}{2}\sin(\theta) = \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$

The area of both triangles is therefore : $\frac{1}{2}r^2\sin\theta$

The area of the sector is:

$$A_{sector} = \frac{1}{2}r^2\theta$$

So $A_L = A_{sect} - 2A_{triangle}$

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$= \frac{1}{2}r^2(\theta - \sin\theta)$$

For a Given volume, V_0 we solve $V_0 = \frac{1}{2}lr^2(\theta - \sin\theta)$ to obtain a value of θ where $\theta = \theta_0$ and $0 \leq \theta_0 \leq \pi$. To ~~convert this~~ to calibrate a dipstick the volume, V_0 will be marked h_0 distance from the bottom, where $h_0 = r - r\cos\left(\frac{\theta_0}{2}\right)$