proofs

April 3, 2025

1 Proofs

1.1 Question 13

1.1.1 Question

1. $(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$
2. $(x)(y)(Kxy \supset Lyx)$

$\therefore (x)Lxx$

1.1.2 S/I rules strictly

3. ASM: $\sim (x)Lxx$
4. $\therefore (\exists x) \sim Lxx$ {from 3}
5. $\therefore \sim Laa$ {from 4}
6. $\therefore (y)(z)((Lay \cdot Lyz) \supset Laz)$ {from 1}
7. $\therefore (z)((Laa \cdot Laz) \supset Laz)$ {from 6}
8. $\therefore ((Laa \cdot Laa) \supset Laa)$ {from 7}
9. $\therefore (y)(Kay \supset Lya)$ {from 2}
10. $\therefore (Kaa \supset Laa)$ {from 9}
11. $\therefore \sim Kaa$ {from 5 and 10}

REFUTE

1.1.3 Resolution proof

1. $(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$
2. $(x)(y)(Kxy \supset Lyx)$

$\therefore (x)Lxx$

1.2 Question 15

1.2.1 Question

1. $(x)(y)(Lxy \supset (Fx \cdot \sim Fy))$

$\therefore (x)(y)(Lxy \supset \sim Lyx)$

1.2.2 S/I rules strictly

2. ASM: $\sim (x)(y)(Lxy \supset \sim Lyx)$

3. $\therefore(\exists x) \sim (y)(Lxy \supset\sim Lyx)$ {from 2}
4. $\therefore \sim (y)(Lay \supset\sim Lya)$ {from 3}
5. $\therefore(\exists y) \sim (Lay \supset\sim Lya)$ {from 4}
6. $\therefore \sim (Lab \supset\sim Lba)$ {from 5}
7. $\therefore Lab$ {from 6}
8. $\therefore Lba$ {from 6}
9. $\therefore(y)(Lay \supset (Fa\cdot \sim Fy))$ {from 1}
10. $\therefore(y)(Lby \supset (Fb\cdot \sim Fy))$ {from 1}
11. $\therefore(Lab \supset (Fa\cdot \sim Fb))$ {from 9}
12. $\therefore(Lba \supset (Fb\cdot \sim Fa))$ {from 10}
13. $\therefore(Fa\cdot \sim Fb)$ {from 7 and 11}
14. $\therefore(Fb\cdot \sim Fa)$ {from 12}
15. $\therefore Fa$ {from 13}
16. $\therefore \sim Fb$ {from 13}
17. $\therefore Fb$ {from 14}
18. $\therefore(x)(y)(Lxy \supset\sim Lyx)$ {16 contradicts 17}