Proofs

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1 Proofs

1.1 Question 13

1.1.1 Question

1. $(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$
2. $(x)(y)(Kxy \supset Lyx)$

$\therefore (x)Lxx$

1.1.2 S/I rules strictly

1. $(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$
2. $(x)(y)(Kxy \supset Lyx)$

$\therefore (x)Lxx$ 3. ASM: $\sim (x)Lxx$ 4. $\therefore (\exists x) \sim Lxx$ {from 3} 5. $\therefore \sim Laa$ {from 4} 6. $\therefore (y)(z)((Lay \cdot Lyz) \supset Laz)$ {from 1} 7. $\therefore (z)((Laa \cdot Laz) \supset Laz)$ {from 6} 8. $\therefore ((Laa \cdot Laa) \supset Laa)$ {from 7} 9. $\therefore (y)(Kay \supset Lya)$ {from 2} 10. $\therefore (Kaa \supset Laa)$ {from 9} 11. $\therefore \sim Kaa$ {from 5 and 10}

REFUTE

1.1.3 Resolution proof

1. $(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$

- $\sim Lxy \vee \sim Lyz \vee Lxz$
- Clauses:
  - $\{\sim Lxy, \sim Lyz, Lxz\}$

2. $(x)(y)(Kxy \supset Lyx)$

- $\sim Kxy \vee Lyx$
- Clauses:
  - $\{\sim Kxy, Lyx\}$

3. ASM: $\sim (x)Lxx$

- Clauses:
  - $\{\sim Lxx\}$
- Substitute $x = a$:
  - $\{\sim Lay, \sim Lyz, Laz\}$ {from 1}
  - $\{\sim Kay, Lya\}$ {from 2}
  - $\sim Laa$ {from 3}

- Substitute $y = a$:
  - $\{\sim Laa, \sim Laz, Laz\}$
  - $\{\sim Kaa, Laa\}$
  - $\sim Laa$
- Resolving $\sim Laa$ with $\{\sim Kaa, Laa\}$ gives $\sim Kaa$
- Substitute $z = a$:
  - $\{\sim Laa, \sim Laa, Laa\} = \{\sim Laa, Laa\}$
  - $\sim Kaa$
- Resolving $\sim Laa$ with $\{\sim Laa, Laa\}$ gives $\sim Laa$ or tautology
- Can't resolve $\sim Kaa$ so REFUTE

1.2 Question 15

1.2.1 Question

1. $(x)(y)(Lxy \supset (Fx \cdot \sim Fy))$

$\therefore (x)(y)(Lxy \supset \sim Lyx)$

1.2.2 S/I rules strictly

1. $(x)(y)(Lxy \supset (Fx \cdot \sim Fy))$

$\therefore (x)(y)(Lxy \supset \sim Lyx)$ 2. ASM: $\sim (x)(y)(Lxy \supset \sim Lyx)$ 3. $\therefore (\exists x) \sim (y)(Lxy \supset \sim Lyx)$ {from 2} 4. $\therefore \sim (y)(Lay \supset \sim Lya)$ {from 3} 5. $\therefore (\exists y) \sim (Lay \supset \sim Lya)$ {from 4} 6. $\therefore \sim (Lab \supset \sim Lba)$ {from 5} 7. $\therefore Lab$ {from 6} 8. $\therefore Lba$ {from 6} 9. $\therefore (y)(Lay \supset (Fa \cdot \sim Fy))$ {from 1} 10. $\therefore (y)(Lby \supset (Fb \cdot \sim Fy))$ {from 1} 11. $\therefore (Lab \supset (Fa \cdot \sim Fb))$ {from 9} 12. $\therefore (Lba \supset (Fb \cdot \sim Fa))$ {from 10} 13. $\therefore (Fa \cdot \sim Fb)$ {from 7 and 11} 14. $\therefore (Fb \cdot \sim Fa)$ {from 12} 15. $\therefore Fa$ {from 13} 16. $\therefore \sim Fb$ {from 13} 17. $\therefore Fb$ {from 14} 3. $\therefore (x)(y)(Lxy \supset \sim Lyx)$ {16 contradicts 17}

1.2.3 Resolution proof

1. $(x)(y)(Lxy \supset (Fx \cdot \sim Fy))$

- $(\sim Lxy \vee Fx)$
- $(\sim Lxy \vee \sim Fy)$
- Clauses:
  - $\{\sim Lxy, Fx\}$
  - $\{\sim Lxy, \sim Fy\}$

2. ASM: $\sim (x)(y)(Lxy \supset \sim Lyx)$

- $(\exists x)(\exists y) \sim (Lxy \supset \sim Lyx) = Lxy \cdot Lyx$
- Clauses:
  - Lxy
  - Lyx
- Substituting $x = a$:
  - $\{\sim Lay, Fa\}$ {from 1}
  - $\{\sim Lay, \sim Fy\}$ {from 1}
  - Lay {from 2}
  - Lya {from 2}
- Substituting $y = a$:

  - $\{\sim Laa, Fa\}$
  - $\{\sim Laa, \sim Fa\}$
  - Laa
- Resolving Laa with $\{\sim Laa, Fa\}$ gives Fa
- Resolving Laa with $\{\sim Laa, \sim Fa\}$ gives $\sim Fa$
- Resolving Fa with $\sim Fa$ gives empty clause