1.

p	q	p NAND q	p NOR q	p ≡ q
0	0	1	1	1
0	1	1	0	0
1	0	1	0	0
1	1	0	0	1

2, a)

p	q	$p \Rightarrow q$	NOT p OR q	E
0	0	1	1	1
0	1	1	1	1
1	0	0	0	1
1	1	1	1	1

2b.

p q r	$\overbrace{\text{r OR NOT p}}^{a}$	$\overbrace{q \to a}^{b}$	$p \to b$
0 0 0	1	1	1
0 0 1	1	1	1
0 1 0	1	1	1
0 1 1	1	1	1
1 0 0	0	1	1
1 0 1	1	1	1
1 1 0	0	0	0
1 1 1	1	1	1

2c.

p q	p OR q	p AND q	(p OR q) → (p AND q)
0 0	0	0	1
0 1	1	0	0
1 0	1	0	0
1 1	1	1	1

3. Exclusion set operator

4. (p NAND q) NAND r   0 0 1 = 0
   p NAND (q NAND r)   0 0 1 = 1

   $(p \to q) \to r$ 0 0 0 = 0
   $p \to (q \to r)$ 0 0 0 = 1

   (p NOR q) NOR r   0 0 1 = 0
   p NOR (q NOR r)   0 0 1 = 1

5. 4: FALSE, TRUE, NOT p, NOT q

6.

p q	False	AND	NOT implies	p	NOT($q \to p$)	q	XOR	OR	NOR	≡	NOT q	$q \to p$	NOT p	$p \to q$	NAND	True
0 0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0 1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1 0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1 1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

1/31/20

7.

p q	p XOR q
0 0	0
0 1	1
1 0	1
1 1	0

Yes, it is commutative and associative.

8. $a = \bar{p}\bar{q}r + p\bar{q}\bar{r} + p\bar{q}r + pq\bar{r} + pqr$

$b = \bar{p}\bar{q}\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r}$

9. a) $a = (p+q+r)(p+\bar{q}+r)(p+\bar{q}+\bar{r})$

b) $b = (p+\bar{q}+\bar{r})(\bar{p}+q+r)(\bar{p}+q+\bar{r})(\bar{p}+\bar{q}+r)(\bar{p}+\bar{q}+\bar{r}$

c) $z = (x+y+c)(x+\bar{y}+\bar{c})(\bar{x}+y+\bar{c})(\bar{x}+\bar{y}+c)$

10. a)

pq \ rs	00	01	11	10
00	0	1	1	1
01	1	1	1	1
11	1	1	0	1
10	1	1	1	1

b)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	0	1
11	1	0	0	0
10	1	1	0	1

c)

pq \ rs	00	01	11	10
00	0	1	0	1
01	1	0	1	0
11	0	1	1	1
10	1	0	1	0

d)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	0
10	1	1	1	1

e)

pq \ rs	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	0	0	0	0
10	1	1	0	0

11. a) $\bar{p}\bar{q}s + r + q\bar{r}\bar{s} + \bar{p}qs + p\bar{r}s + p\bar{q}s$

b) $\bar{r}\bar{s} + \bar{p}\bar{q} + \bar{p}\bar{r}s + p\bar{q}\bar{r} + \bar{p}\bar{r}s + p\bar{q}\bar{s}$

c) $\bar{p}\bar{q}r\bar{s} + \bar{p}\bar{q}\bar{r}s + \bar{p}q\bar{r}\bar{s} + p\bar{q}\bar{r}\bar{s} + pqs +$
$pqr + qrs + prs$

d) $\bar{r}\bar{s} + \bar{r}s + rs + pr\bar{s} + p\bar{q}$

e) $\bar{p}\bar{q} + \bar{p}q + p\bar{q}\bar{r}$

No, you do not need to use all of the prime implicants.

12.

		a = 0	a = 1
p	0	0	1
	1	1	1

P OR q

p	q	p+q
0	0	0
0	1	1
1	0	1
1	1	1

$p\bar{q} + \bar{p}q + pq$

$\bar{p}q + p\bar{q} + pq$

13. a) $pqrs + \bar{p}\bar{q}\bar{r}\bar{s}$

b) $\bar{p}\bar{q}\bar{s} + \bar{p}\bar{q}\bar{r} + \bar{r}\bar{s}\bar{q} + \bar{r}\bar{s}\bar{p}$

c) $pqrs + p\bar{q}r\bar{s} + p\bar{q}\bar{r}s + \bar{p}\bar{q}rs + p\bar{q}\bar{r}s +$
$\bar{p}q\bar{r}s$

d) $\bar{p}\bar{q}\bar{r}s$

e) $\bar{p}\bar{q} + \bar{p}q\bar{r}$

2/1/20

14. a) pq XOR rs

b) $\bar{p}q + \bar{r}s + \bar{p}q\bar{r}s + \bar{p}qr\bar{s} + p\bar{q}\bar{r}s + p\bar{q}r\bar{s}$

c) (p XOR q XOR r XOR s) + ($pqrs$) + (pqr) + (pqs)

d) $p + q + r + \bar{s}$

e) $\bar{p}\bar{q} + \bar{p}q + p\bar{q}\bar{r}$

15 a) 24

b) 8

c) 8

d) 6

pq \ rs	00	01	11	10
00	1	1	0	1
01	1	0	0	0
11	0	0	0	0
10	1	0	0	0

16. a) $pqr \to p + q$

Yes, it is a tautology

pqr	pqr	p+q	E
000	0	0	1
001	0	0	1
010	0	1	1
011	0		

b) $((p \to q)(q \to r)) \to (p \to r)$

Not a tautology

pqr	$p \to q$	$q \to r$	$p \to r$	E
000	1	1	1	1
001	1	1	1	1
010	1	0	1	0

c) $(p \to q) \to p$

Not a tautology

p q	$p \to q$	$(p \to q) \to p$
00	1	0
01		
10		
11		

d) $(p \equiv (q+r)) \rightarrow (q \rightarrow pr)$

Yes, it is a tautology

pqr	q+r	q→pr	E
000	0	1	1
001	1	1	1
010	1	0	1
011			
100			

17. a) $(pqr \rightarrow p+q) \equiv (p+q)+\bar{p}$

pqr	pqr	p+q	(pqr)→p+q	(p+q)+$\bar{p}$	E
000	0	0	1	1	1
001	0	0	1	1	1
010	0	1	1	1	1
011	0	1	1	1	1
			⋮	⋮	⋮

b) If an expression is a tautology, it will always be true, thus it solves the satisfiability problem

18. 1.

p	p≡p
0	0
1	1

2.

pq	p≡q	q≡p	E
00	1	1	1
01	0	0	1
10	0	0	1
11	1	1	1

3.

pqr	p≡q (a)	q≡r (b)	a AND b	p≡r	E
000	1	1	1	1	1
001	1	0	0	0	1
010	0	0	0	1	1
011	0	1	0	0	1
100	0	1	0	0	1
101	0	0	0	1	1
110	1	0	0	0	1
111	1	1	1	1	1

4.

p q	$\underbrace{p \equiv q}_{a}$	$\underbrace{\bar{p} \equiv \bar{q}}_{b}$	$a \equiv b$
0 0	1	1	1
0 1	0	0	1
1 0	0	0	1
1 1	1	1	1

5.

p q	pq	qp	$pq \equiv qp$
0 0	0	0	1
0 1	0	0	1
1 0	0	0	1
1 1	1	1	1

6.

p q r	qr	p(qr)	(pq)	(pq)r	$p(qr) \equiv (pq)r$
0 0 0	0	0	0	0	1
0 0 1	0	0	0	0	1
0 1 0	0	0	0	0	1
0 1 1	1	0	0	0	1
1 0 0	0	0	0	0	1
1 0 1	0	0	0	0	1
1 1 0	0	0	1	0	1
1 1 1	1	1	1	1	1

7.

p q	p+q	q+p	E
0 0	0	0	1
0 1	1	1	1
1 0	1	1	1
1 1	1	1	1

8.

8.

p q r	(q+r)	p+(q+r)	(p+q)	(p+q)+r	E
000	0	0	0	0	1
001	1	1	0	1	1
010	1	1	1	1	1
011	1	1	1	1	1
100	0	1	1	1	1
101	1	1	1	1	1
110	1	1	1	1	1
111	1	1	1	1	1

9.

p q r	q+r	p(q+r)	pq	pr	(pq+pr)	E
000	0	0	0	0	0	1
001	1	0	0	0	0	1
010	1	0	0	0	0	1
011	1	0	0	0	0	1
100	0	0	0	0	0	1
101	1	1	0	1	1	1
110	1	1	1	0	1	1
111	1	1	1	1	1	1

10.

P	P AND 1	(P AND 1) ≡ P
0	0	1
1	1	1

11.

P	P OR 0	(P OR 0) ≡ P
0	0	1
1	1	1

12.

P	P AND 0	(P AND 0) ≡ 0
0	0	1
1	0	1

13.

P	NOT NOT P	(NOT NOT P ≡
0	0	1
1	1	1

18. 14.

p q r	q r	p + (q r)	(p + q)	(p + r)	(p + q)(p + r)	E
0 0 0	0	0	0	0	0	1
0 0 1	0	0	0	1	0	1
0 1 0	0	0	1	0	0	1
0 1 1	1	1	1	1	1	1
1 0 0	0	1	1	1	1	1
1 0 1	0	1	1	1	1	1
1 1 0	0	1	1	1	1	1
1 1 1	1	1	1	1	1	1

15.

p	1 OR p	(1 OR p) ≡ 1
0	1	1
1	1	1

16.

p	p.p	pp ≡ p
0	0	1
1	1	1

17.

p	p + p	(p + p) ≡ p
0	0	1
1	1	1

18.

p q	p q	p + p q	E
0 0	0	0	1
0 1	0	0	1
1 0	0	1	1
1 1	1	1	1

19.

p q	$\bar{p}+q$	$p(\bar{p}+q)$	pq	E
0 0	1	0	0	1
0 1	1	0	0	1
1 0	0	0	0	1
1 1	1	1	1	1

20.

p q	NOT (pq)	$\bar{p}+\bar{q}$	E
0 0	1	1	1
0 1	1	1	1
1 0	1	1	1
1 1	0	0	1

21.

p q	$p \to q$	$q \to p$	$(p \to q)$ AND $(q \to p)$	$p \equiv q$	E
0 0	1	1	1	1	1
0 1	1	0	0	0	1
1 0	0	1	0	0	1
1 1	1	1	1	1	1

22.

p q	$p \equiv q$	$p \to q)$	$(p \equiv q) \to (p \to q)$
0 0	1	1	1
0 1	0	1	1
1 0	0	0	1
1 1	1	1	1

23.

p q r	$p \to q$	$q \to r$	$(p \to q)$ AND $(q \to r)$	$(p \to r)$	E
000	1	1	1	1	1
001	1	1	1	1	1
010	1	0	0	1	1
011	1	1	1	1	1
100	0	1	0	0	1
101	0	1	0	1	1
110	1	0	0	0	1
111	1	1	1	1	1

24.

p q	$p \to q$	$\bar{p} + q$	$(p \to q) \equiv (\bar{p} + q)$
0 0	1	1	1
0 1	1	1	1
1 0	0	0	1
1 1	1	1	1

19. 1.

x y	$(x+y)$	$(x+y) \equiv (x+y)$
00	0	1
01	1	1
10	1	1
11	1	1

2.

x y z	$(x+y) \equiv yz$	$yz \equiv (x+y)$	E
000	1	1	1
001	1	1	1
010	0	0	1
011	1	1	1
100	0	0	1
101	0	0	1
110	0	0	1
111	1	1	1

3.

x y z	$\overbrace{(x+y) \equiv yz}^{a}$	$\overbrace{yz \equiv x}^{b}$	a AND b	$(x+y) \equiv x$	E
000	1	1	1	1	1
001	1	1	1	1	1
010	0	1	0	0	1
011	1	0	0	0	1
100	0	0	0	1	1
101	0	0	0	1	1
110	0	0	0	1	1
111	1	1	1	1	1

20. a) $pq+rs$ to $(p+r)(p+s)(q+r)(q+s)$

By using the distributive law for

$pq+rs \equiv (p+r)(p+s)(q+r)(q+s)$

b) $pq+p\bar{q}r$ into $p(q+r)$

Elimination of certain negations → $pq+pr$

Distributive Law $pq+pr \equiv p(q+r)$

21. $p + pq \equiv p \qquad p(p+q) \equiv p$

$$p + pq \equiv (p+p)(p+q)$$
$$p(p+q)$$
$$pp + pq \equiv p$$

22. 1. $$\text{NOT}(pq + \overline{pr}) \equiv \text{NOT}(pq) + \text{NOT}(\bar{p}r)$$
$$\equiv \text{NOT}(pq) + pr$$
$$\equiv \bar{p} + \bar{q} + pr$$

2. $$\text{NOT}(\text{NOT } p + q(\text{NOT}(r + \bar{s})))$$
$$\equiv \text{NOT}(\text{NOT } p + q(\bar{r} + s))$$
$$\equiv \text{NOT}(\text{NOT } p + q\bar{r} + qs)$$
$$\equiv p + (\bar{q} + r)(\bar{q} + \bar{s})$$

23. 1. $$w\bar{x} + w\bar{x}y + \bar{z}\bar{x}w$$
$$\equiv w(\bar{x} + \bar{x}y + \bar{z}\bar{x})$$
$$\equiv w\bar{x}(1 + \bar{y} + \bar{z})$$
$$\equiv w\bar{x}$$

2. $$(w + \bar{x})(w + y + \bar{z})(\bar{w} + \bar{x} + \bar{y})(\bar{x})$$
$$\equiv \text{NOT}((\bar{w}x) + (\bar{w}\bar{y}z) + (wxy) + (x))$$
$$\equiv \text{NOT}((\cancel{x}) + (\bar{y}z) + (wy) + (\cancel{x}))$$
$$\equiv \text{NOT}((\bar{y}z) + (wy))$$
$$\equiv (y + \bar{z})(\bar{w} + \bar{y})$$