

1. a) CMPT333 \rightarrow variable
- b) cmpt333 \rightarrow constant
- c) 333 \rightarrow constant
- d) "cmpt333" \rightarrow constant
- e) $p(X, x)$ \rightarrow non-ground atomic formula
- f) $p(3, 4, 5)$ \rightarrow ground atomic formula
- g) "p(3, 4, 5)" \rightarrow constant

2/7/20

2. $\text{csg}(\text{"CMPT 333"}, S, G) \text{ AND } \text{snap}(S, \text{"L. Brown"}, A, P)$
 $\rightarrow \text{answer}(G)$

What grade did L. Van Pelt get in "CMPT 220"?

C: "CMPT 220"

N: "L. Van Pelt"

$\text{csg}(\text{"CMPT 220"}, S, G) \text{ AND } \text{snap}(S, \text{"L. Van Pelt"}, A, P)$
 $\rightarrow \text{answer}(G)$

3. a. $(\forall x)((\exists y)(\text{NOT}(p(x) \text{ OR } (p(y) \text{ AND } q(x))))))$

$\hookrightarrow (\forall x)(\exists y) \text{NOT}(p(x) \text{ OR } p(y) \text{ AND } q(x))$

b. $(\exists x)((\text{NOT } p(x)) \text{ AND } ((\exists y)(p(y)) \text{ OR } (\exists x)(q(x, z))))$

$\hookrightarrow (\exists x) \text{NOT } p(x) \text{ AND } ((\exists y) p(y) \text{ OR } (\exists x) q(x, z))$

4. a.

$\forall x$

|

$\exists x$

|

NOT

|

OR

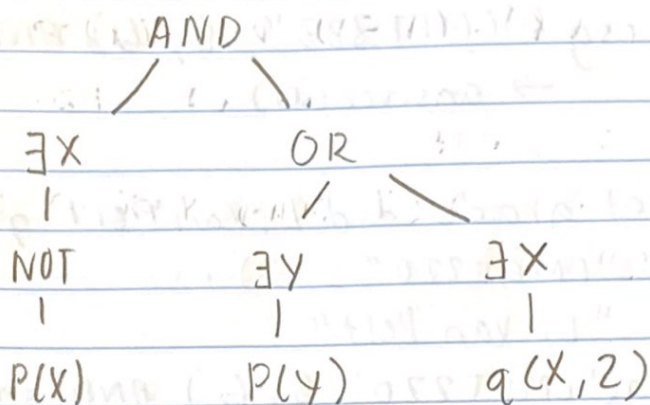
$p(x)$

AND

$p(y)$

$q(x)$

4b.



5. $(\exists x) \text{NOT } P(x) \text{ AND } ((\exists y) P(y) \text{ OR } (\exists x) q(x, z))$

$\hookrightarrow (\exists x) \text{NOT } P(x) \text{ AND } ((\exists y) P(y) \text{ OR } (\exists m) q(m, z))$

6. 1. C. Brown is an A student

N: "C. Brown"

G: "A"

$(\forall C) (sg(C, S, "A") \text{ AND } snap(S, "C. Brown", A, P) \rightarrow \text{answer}("A"))$

2. C. Brown is NOT an A student

$(\exists C) (\text{NOT}(sg(C, S, "A") \text{ AND } snap(S, "C. Brown", A, P)) \rightarrow \text{answer}("A"))$

7. a) $(\forall x)(\exists y)(\text{loves}(x, y))$

True: $\begin{cases} \text{loves}(a, b) \\ \text{loves}(c, b) \end{cases}$ D: a, b, c

False: $\text{loves}(b, a)$

b) $p(x) \rightarrow \text{NOT } p(x)$

True: $P(x)$: False

False: $P(x)$: True

c) $(\exists x) P(x) \rightarrow (\forall x) P(x)$

True: $P(x)$: False

False: $P(x)$: True

$$7d. (p(x, y) \text{ AND } p(y, z)) \rightarrow p(x, z)$$

True: Domain of all real numbers

$$p: x < y$$

$$\text{Thus, } x < y \text{ AND } y < z \rightarrow x < z$$

False: No such domain exists

$$8. a) (p(x) \text{ OR } q(y)) \equiv (q(y) \text{ OR } p(x))$$

True based on commutative law for AND

$$b) (p(x, y) \text{ AND } p(x, y)) \equiv p(x, y)$$

True based on Idempotence of AND

$$c) (p(x) \rightarrow \text{False}) \equiv \text{NOT } p(x)$$

True because when, $p(x) = \text{True}$, both sides will return false and when False, both sides return true

$$9. a) ((\exists x)(\text{NOT } p(x)) \text{ AND } ((\exists y)p(y))) \text{ OR } ((\exists x)q(x, z)))$$

$$\hookrightarrow ((\exists x)(\text{NOT } p(x)) \text{ AND } ((\exists y)p(y))) \text{ OR } ((\exists M)q(M, z)))$$

$$b) (\exists x)(\exists x)p(x) \text{ OR } (x)q(x) \text{ OR } r(x)$$

$$\hookrightarrow (\exists x)p(x) \text{ OR } (y)q(y) \text{ OR } r(M)$$

$$10. a) p(x, y) \text{ AND } (\exists y)q(y)$$

$$\hookrightarrow p(x, y) \text{ AND } (\exists y)q(y) \text{ AND } (\exists x)r(x)$$

$$b) (\exists x)(p(x, y) \text{ OR } (\exists x)p(y, x))$$

$$\hookrightarrow (\exists x)p(x, y) \text{ OR } (\exists M)p(y, M)$$

$$11. p(x, y) \text{ AND } (\exists x)q(x) \equiv (\exists x)(p(x, y) \text{ AND } q(x))$$

Yes, the law $(E \text{ AND } (QX)F) \rightarrow (QX)(E \text{ AND } F)$

implies the two statements because you can reorder the quantifiers outside of the AND

12 a. $(\exists x)(\text{NOT } p(x)) \text{ AND } ((\exists y) p(y)) \text{ OR } ((\exists x) q(x, z))$

$(\exists x)(\exists y)(\text{NOT } p(x) \text{ AND } p(y) \text{ OR } q(x, z))$

b. $(\exists x)(\exists x)p(x) \text{ OR } (x)q(x) \text{ OR } r(x)$

$\hookrightarrow (\exists x)(x)(p(x) \text{ OR } q(x) \text{ OR } r(x))$

13. $((Q_1 x)E) \rightarrow (Q_2 y)F$

$\hookrightarrow (Q_1 x)(Q_2 y)(E \rightarrow F)$

14. 1. $\text{NOT}((\exists x)(\exists y)p(x, y))$

$\hookrightarrow \exists (\forall x)(\forall y)(\text{NOT } p(x, y))$

2. $\text{NOT}((\exists x)p(x) \text{ OR } (\exists y)q(x, y))$

$\hookrightarrow \exists (\forall x)(\forall y)(\text{NOT}(p(x) \text{ OR } q(x, y)))$

15. No, it is not true that E is a tautology whenever $(\exists x)E$ is a tautology because just because 'there exists an x such that E is true' is a tautology, that doesn't mean that E will always be true.