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1. a) CMPT333 → variable
   b) cmpt333 → constant
   c) 333 → constant
   d) "cmpt333" → constant
   e) p(X, x) → non-ground atomic formula
   f) p(3,4,5) → ground atomic formula
   g) "p(3,4,5)" → constant

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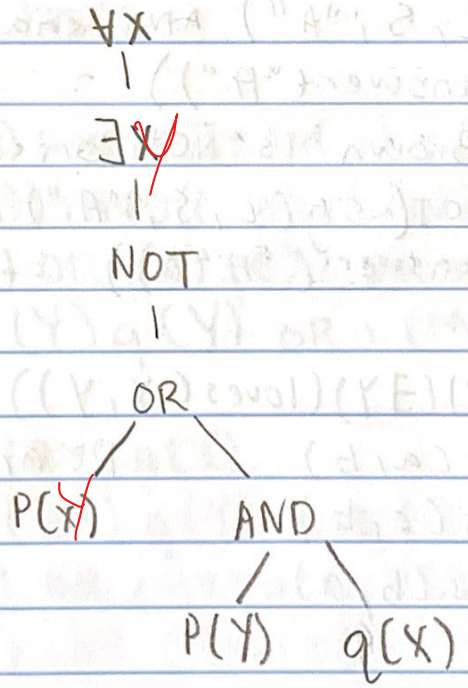
2. csg("CMPT 333", S, G) AND snap(S, "C. Brown", A, P))
→ answer(G)

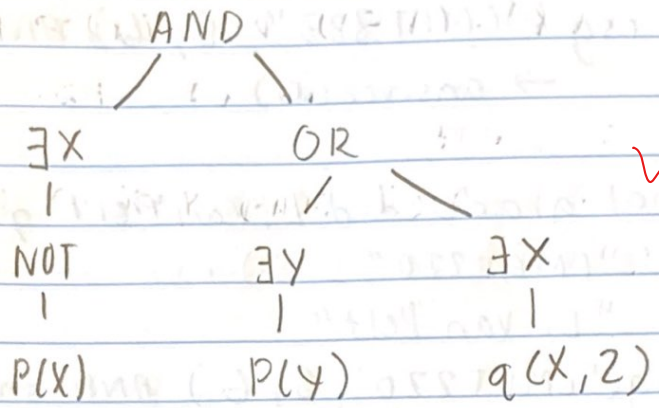
What grade did L. Van Pelt get in "CMPT 220"?
C: "CMPT 220"
N: "L. Van Pelt"
csg("CMPT 220", S, G) AND snap(S, "L. Van Pelt", A, P)
→ answer(G)

3. a. $(\forall x)((\exists Y)(NOT(p(X) \text{ OR } (p(Y) \text{ AND } q(X)))))$
↳ $(\forall X)(\exists Y)\ NOT\ (P(x) \text{ OR } P(Y) \text{ AND } q(X))$
b. $(\exists X)((NOT\ p(X)) \text{ AND } ((\exists Y)(p(Y)) \text{ OR } (\exists X)(q(X,Z))))$
↳ $(\exists X)\ NOT\ p(X) \text{ AND } ((\exists Y)\ p(Y) \text{ OR } (\exists X)\ q(X,Z))$

4. a.



4b.



5. (∃X) NOT P(X) AND ((∃Y) P(Y) OR (∃X) q(X,2))
↳ (∃X) NOT P(X) AND ((∃Y) P(Y) OR (∃M) q(M,2))

6. 1. C. Brown is an A Student
N: "C. Brown"
G: "A"
(∀C)(sg(C, S, "A") AND snap(S, "C. Brown", A, P)
→ answer("A"))

2. C. Brown is NOT an A student
(∃C)(NOT(sg(C, S, "A") AND snap(S, "C. Brown", A, P)
→ answer("A")))

7. a) (∀X)(∃Y)(loves(X, Y))
True [loves(a, b) P: a, b, c
loves(c, b)
False - loves(b, a)

b) p(X) → NOT p(X)
True: P(x): False
False: P(x): True

c) (∃x) P(x) → (∀x) P(x)
True: P(x): False
False: P(x): True

7d. $(p(X,Y) \text{ AND } p(Y,Z)) \rightarrow p(X,Z)$

True: Domain of all real numbers

P: $X < Y$

Thus, $X<Y$ AND $Y<Z \rightarrow X<Z$

False - No such domain exists

8. a) $(p(x) \text{ OR } q(Y)) \equiv (q(Y) \text{ OR } p(X))$

True based on commutative law for AND

b) $(p(X,Y) \text{ AND } p(X,Y)) \equiv p(X,Y)$

True based on Idempotence of AND

c) $(p(X) \rightarrow \text{False}) \equiv \text{NOT } p(X)$

True because when, P(X) = True, both sides will return false and when False, both sides return true.

9. a) $(\exists x)(\text{NOT } p(X)) \text{ AND } ((\exists Y)p(Y))) \text{ OR } ((\exists X) q(X,Z))))$

↳ $(\exists x)(\text{NOT } p(X)) \text{ AND } ((\exists Y) p(Y))) \text{ OR } ((\exists M) q(M,Z)))$

b) $(\exists x)(\exists X) p(X) \text{ OR } (X) q(X) \text{ OR } r(X))$

~~$\exists M$~~ $(\exists X)p(X) \text{ OR } \exists Y) q(Y) \text{ OR } r(M))$ -1

10. a) $p(X,Y) \text{ AND } (\exists Y)\, q(Y)$

↳ $p(X,Y) \text{ AND } (\exists Y)\, q(Y) \text{ AND } (\exists X)\, r(X)$ -3

b) $(\exists X)(p(X,Y) \text{ OR } (\exists X) p(Y,X))$

↳ $(\exists X)\, p(X,Y) \text{ OR } (\exists M)\, p(Y,M)$

11 $p(X,Y) \text{ AND } (\exists X)\, q(X) \equiv (\exists X)(p(X,Y) \text{ AND } q(x))$

Yes, the law $(E \text{ AND } (QX)F) \rightarrow (QX)(E \text{ AND } F)$ implies the two statements because you can reorder the quantifiers outside of the AND -4

12 a. $(\exists X)(\text{NOT } p(x))$ AND $((\exists Y)\, p(Y))$ OR $((\exists X)\, q(X,Z))$

$(\exists X)(\exists Y)(\text{NOT } p(x)$ AND $p(Y)$ OR $q(X,Z))$ ✓

b. $(\exists X)(\exists X)p(X)$ OR $(X)q(X)$ OR $r(X)$

↳ $(\exists X)(\exists X)(p(x)$ OR $q(X)$ OR $r(X))$ ✓

13. $((Q_1 X)E) \rightarrow (Q_2 Y)F)$

↳ $(Q_1 X)(Q_2 Y)(E \rightarrow Y)$ −3

14. 1. NOT $((\exists X)(\exists Y)\, p(X,Y))$

↳ $\equiv (\forall X)(\forall Y)(\text{NOT } p(X,Y))$ ✓

2. NOT $((\exists X)\, p(X)$ OR $(\exists Y)q(X,Y)$

↳ $\equiv (\forall X)(\forall Y)(\text{NOT}(p(x)$ OR $q(X,Y)))$ ✓

15. No, it is not true that E is a tautology whenever $(\exists X)E$ is a tautology because just because 'there exists an X such that E is true' is a tautology, that doesn't mean that E will always be true. ✓