Q1 Eda Assignment

$$m(X) = \frac{1}{N} \sum_{i=1}^{N} X_i$$

$$\mathrm{cov}(X,Y) = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))(y_i - m(Y))$$

$$s^2 = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))^2$$

1. $$m(a+bX) = \frac{1}{N} \sum_{i=1}^{N} (a+bX_i)$$
$$= \frac{1}{N} \sum_{i=1}^{N} a + \frac{b}{N} \sum_{i=1}^{N} X_i$$
$$= a + bm(X)$$

2. $$\mathrm{cov}(X, a+bY) = \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X))[(a+by_i) - (a+bm(Y)]$$
$$= \frac{1}{N} \sum_{i=1}^{N} (X_i - m(X)) b (y_i - m(Y))$$
$$= b\,\mathrm{cov}(X,Y)$$

3. $$\mathrm{cov}(a+bX, a+bX) = \frac{1}{N} \sum_{i=1}^{N} ((a+bX_i) - m(a+bX))^2$$
$$= \frac{1}{N} \sum_{i=1}^{n} (b(X_i - m(X)))^2$$
$$= b^2 \frac{1}{N} \sum_{i=1}^{n} (X_i - m(X))^2$$
$$= b^2 s^2 \quad (\text{since } s^2 = \frac{1}{N} \sum (X_i - m(X))^2 = \mathrm{cov}(X,X)$$

4. If you apply a non-decreasing transformation g, the median of g(X) is g(median(X)). This also works for any quantile. The IQR range doesn't stay the same. It gets distorts unless g is linear (like a + bX with b>0). So, monotone transformations preserve order (and thus quantiles/medians) but not distances (like IQR or range)

5. No, not always.
Here's a counterexample:

$X = \{0, 2\}$, mean=1. With $g(X) = X^2$

$m(g(X)) = \frac{0^2 + 2^2}{2} = 2$, $g(m(X)) = 1^2 = 1$

$2 \neq 1$