Doing our supply/demand contribution graph properly

Which influencer has the most influence?

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Purpose: When I did the original note for the classification of sectors and being the *originator* of a supply or demand shock, I thought we could do a similar graph to Shapiro's inflation graph showing contributions of demand and supply with our classifications directly. Based on our last meeting, it became clear that this was incorrect. If a sector (like oil) is the source of a supply shock *everywhere* we need to have the right approach to capture this.

Here is what I would suggest in order to operationalize this. Note that I will use the same notation of matrices that we have in our other notes – $\boldsymbol{\Omega}$ is the share of intermediates row sources from column, $\underline{\gamma}$ is the intermediate share of sales for each industry.

1. Start by writing a function that produces the graphs. This function should take as inputs the following:

   - π_t – CPI inflation (T x 1 total time series)
   - $\boldsymbol{E}$ – Expenditure weights by sector (P x T) where P is the number of products and T is the number of time periods. Note that $1'_P E = 1'_T$ – the sum of expenditure weights over the product/industry dimension P is 1.
   - A Demand shock matrix $\boldsymbol{D}$ (P x T) where entries are between 0 and 1. If an entry is 0.75 for sector 3 in 2000Q1, then it means that in 2000Q1, sector 3 was 75% affected by a demand shock and 25% affected by a supply shock.
   - A matrix of sectoral inflations $\boldsymbol{\Pi}$ (P x T) where $(1'_P(\boldsymbol{E} \cdot \boldsymbol{\Pi}) \approx \pi_t)$ noting that 1_P is a P x 1 vector of ones and $A \cdot B$ means the elementwise multiplication of A and B.[1]

   Note that there might be more inputs I have forgotten.

   This function will then plot CPI inflation as a line, then calculate $1'_P(\boldsymbol{D} \cdot \boldsymbol{E} \cdot \boldsymbol{\Pi})$ and plot it as a bar and call it the "demand contribution to CPI". For supply bars it will calculate $1'_P((1 - \boldsymbol{D}) \cdot \boldsymbol{E} \cdot \boldsymbol{\Pi})$.

2. Use this function to replicate the Shapiro classification. In this case entries of D will either be 1 or 0.

[1] What I mean by this math is if you take the expenditure shares and scale them by inflation in each product/industry and then sum over products/industries, you should get CPI inflation and it should be close to the first input.

3. Next we will use this function to implement our classification. We will get a initial matrix $\tilde{D}$ from our procedure. This however only characterizes the sectors based on the *initial* source of the shock (supply of demand). What we want is to calculate the influence of these shocks throughout the economy. To do this let's do the following:

   (a) We should already have a classification where if P_{VA} (price of value added) and <u>real</u> quantity (i.e. real production and not sales) both rise/fall in industry i at time t, we say $\tilde{D}_{i,t} = 1$. If instead P_{VA} and quantity move in opposite directions, we say $\tilde{D}_{i,t} = 0$.[2]

   (b) Next we want to calculate the influence of this onto prices.[3] To find that first let us calculate the following $\boldsymbol{\Theta} \equiv (I - diag(\underline{\gamma})\boldsymbol{\Omega})^{-1} = (I - \boldsymbol{\Delta})^{-1}$. Note that $\boldsymbol{\Theta} \neq \boldsymbol{\Xi}$ because $\boldsymbol{\Xi} = (I - \boldsymbol{\Delta}')^{-1}$. $\boldsymbol{\Theta}$ is also a P x P matrix.

   (c) Next we should calculate the influence shares of P_{VA} movements onto output prices (and therefore sectoral inflation). However doing so is a little tricky because even though $\boldsymbol{\Theta}$ is constant, the value added prices vary over time.

   (d) Next calculate $\boldsymbol{\theta}_t \equiv \boldsymbol{\Theta} \cdot (\log(\underline{P}_{VA,t}) \otimes 1'_P)$ where $\underline{P}_{VA,t}$ is a P x 1 matrix when doing this multiplication.[4] Note that we need to do this for every time period $t = [1, \ldots, T]$. Note also that $\boldsymbol{\theta}_t$ is a P x P matrix (and the we have T of these).

   (e) Calculate the column sum of $\underline{\kappa}_t = 1'_P \boldsymbol{\theta}_t$. Note that $\underline{\kappa}_t$ should be 1 by P.

   (f) Influence is calculated by $\boldsymbol{\lambda}_t \equiv diag(1/\underline{\kappa}_t)\boldsymbol{\theta}_t$. This is a P x P matrix and there are T of them – one for each t. Since $\boldsymbol{\Lambda}_t$ is an influence share, the column-sum should be 1: $1'_P \boldsymbol{\lambda} = 1_P$ or summing over rows within a column should be 1.

   (g) Calculate the input $\boldsymbol{D}$ one column at a time. That is if $\underline{D}_t$ is the t^{th} column of $\boldsymbol{D}$ and $\underline{\tilde{D}}_t$ is the t^{th} column of $\tilde{D}$, then it is calculated as: $\underline{D}_t \equiv \boldsymbol{\lambda}_t \underline{\tilde{D}}_t$. Note that since $\boldsymbol{\lambda}_t$ is P x P and $\tilde{D}_t$ is P x 1, each $\underline{D}_t$ is P x 1.

   (h) Finally concatenate the $\underline{D}_t$s to get:

$$\boldsymbol{D} = \begin{bmatrix} \underline{D}_1 & \ldots & \underline{D}_T \end{bmatrix}$$

4. Before using $\boldsymbol{D}$ as an input we should verify that all entries are between 0 and 1. Let me know if any are negative – this might be possible and we need to think about what to do in that case.

[2] Tony can you please check we are doing the classification this way?

[3] Ben, note that if we wanted to do the influence on quantities we would need to do something a little different to this. Let me know if you want the math for that.

[4] $\otimes$ represents the kronecker product but effectively all I want is P repetitions of the vector $\log(\underline{P}_{VA,t})$ so that all rows have the same number in them.