

Two sector examples of calculating CPI

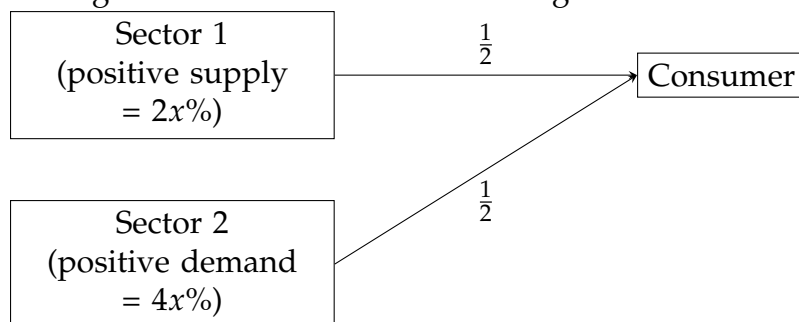
N

February 19, 2026

Purpose: Provide a 2 sector example where we see what the current CPI aggregation method does. I will show that it doesn't work as it should but that switching to weighting up value-added inflation with value-added weights will work without issue. I then outline how to code this up.

1 Horizontal Economy – Shapiro works well

Let's imagine the case as set out in the diagram below.



Here we have two sectors *horizontally* selling a good to the household.

Let's imagine that sector 1 has a positive supply shock and sector 2 has a negative demand shock. Let's imagine that the supply shock *lowers* sector 1 prices by $2x\%$ and the sector 2 demand shock *raises* sector 2 prices by $4x\%$.

We can calculate CPI as:

$$\pi^{CPI} = -\frac{1}{2}2x + \frac{1}{2}4x = x$$

With these numbers, we would have demand contributing $+4x$ to CPI and supply $-2x$ to CPI.

How do we calculate this formally? Well we create a vector (matrix when there are multiple time periods) indicating if the sector has experienced mostly a demand shock or supply shock. Call this \underline{D} and let $D_i = 1$ when it has experienced mostly a demand shock. Then we take the expenditure weights $\omega_{PCE} \equiv \left[\frac{1}{2} \quad \frac{1}{2}\right]'$ and the sectoral CPI changes:

$$\pi_{Demand} = \sum_i D_i \omega_{PCE,i} \pi_i = 0 \times \frac{1}{2} \times (-2x) + 1 \times \frac{1}{2} \times 4x = 2x \checkmark$$

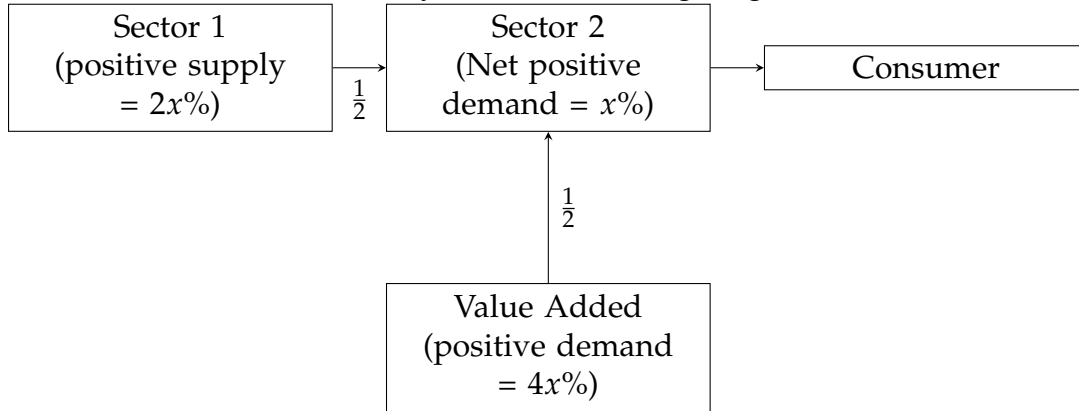
$$\pi_{Supply} = \sum_i (1 - D_i) \omega_{PCE,i} \pi_i = 1 \times \frac{1}{2} \times (-2x) + 0 \times \frac{1}{2} \times 4x = -x \checkmark$$

where we can see that $\pi_{Demand} + \pi_{Supply} = \pi_{CPI}$.

Note that ShapirIO is the same because there is no IO to correct for. Let's now see if we get it for the vertical economy.

2 Vertical Economy – does ShapirIO work well

Let's switch to a vertical economy like the following diagram:



In this vertical economy, note that sector 2's output price is identical to CPI. If we were doing Shapiro we would be getting:

$$\pi_{Demand} = 0 \times 0 \times -2x + 1 \times 1 \times x = x$$

$$\pi_{Supply} = 1 \times 0 \times -2x + 0 \times 1 \times x = 0$$

which is incorrect for the exact reasons we wanted to adjust Shapiro – it misses IO interconnections.

OK, so let's do our IO corrections. There are two approaches: Adjust D and switch to value added prices.

Adjust D: This approach does the same aggregation as Shapiro but uses a D that reflects the breakdown between supply and demand shocks along the IO network. *Crucially, it does not distinguish between sectors with price rises or falls.*

So in this case, we would correct sector 2 based on the fact that its own demand shock is only half of its output price. This would give us:

$$\tilde{D} = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$$

Then if we use this \tilde{D} matrix to calculate supply and demand inflation we would get:

$$\begin{aligned}\pi_{Demand} &= 0 \times 0 \times -2x + \frac{1}{2} \times 1 \times x = \frac{x}{2} \neq 4x \\ \pi_{Supply} &= 1 \times 0 \times -2x + \frac{1}{2} \times 1 \times x = \frac{x}{2} \neq -x\end{aligned}$$

The reason this is incorrect is that we used the wrong \tilde{D} . To see this, let $\tilde{D} = \begin{bmatrix} 0 & 2 \end{bmatrix}'$. Now we have:

$$\begin{aligned}\pi_{Demand} &= 0 \times 0 \times -2x + 2 \times 1 \times x = 2x\checkmark \\ \pi_{Supply} &= 1 \times 0 \times -2x + (1 - 2) \times 1 \times x = -x\checkmark\end{aligned}$$

So one option to fix this is to allow for negative \tilde{D} when the upstream sector has a price fall. But I think there is an easier way:

Switch to Value Added Prices/Inflation: Next, let's revert to a indicator vector for demand shocks \underline{D} but instead of using ω_{PCE} and π_i (output price inflation in each sector), we will use the value added weights for CPI which are:

$$\omega_{VA,i} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

And the inflation we will use will be value added inflation calculated as:

$$\pi_{VA} = \begin{bmatrix} -2x \\ 4x \end{bmatrix}$$

Now the calculation is given by:

$$\begin{aligned}\pi_{Demand} &= 0 \times \frac{1}{2} \times -2x + 1 \times \frac{1}{2} \times 4x = 2x\checkmark \\ \pi_{Supply} &= 1 \times \frac{1}{2} \times -2x + 0 \times \frac{1}{2} \times 4x = -x\checkmark\end{aligned}$$

And we are good!

Note that this approach is much simpler than the previous one so let me outline a set of coding steps to operationalize this.

3 Implementing the new calculations

Let's start at the code where we have $D \in \{0, 1\}$ – our classification of shocks as either mostly demand or supply (and not in-between), $\omega_{VA,i,t}$ and $\{P_{VA,i,t}\}_{i,t}$ – our time series data of value added *prices*. But before we begin, let's note that there is a small problem:

1. CPI inflation in Shapiro is calculated linearly: $\pi_{CPI,t} = \sum_i \omega_{PCE,i,t-1} \times \pi_{i,t}$. I used a difference formula in the note before but let's stick with this one. [Tony, do we use official CPI data at any point in the analysis? If so, does either this formula or the one I gave you earlier match it exactly?]
2. But our relationship between output prices $P_{i,t}$ and value added prices $P_{VA,i,t}$ is *logarithmic*!

$$\log(\underline{P}_t) = \Theta \log(\underline{P}_{VA,t})$$

The correct formula for output price inflation in each sector i is given by:

$$\begin{aligned} \pi_{i,t} &\equiv \frac{P_{i,t}}{P_{i,t-1}} - 1 = \frac{\prod_j P_{VA,j,t}^{\theta_{i,j}}}{\prod_k P_{VA,j,t-1}^{\theta_{i,k}}} - 1 \\ &= \prod_j \left(\frac{P_{VA,j,t}}{P_{VA,j,t-1}} \right)^{\theta_{i,j}} - 1 \\ &= \prod_j \left(\frac{P_{VA,j,t}}{P_{VA,j,t-1}} - 1 \right)^{\theta_{i,j}} \quad \text{because } \sum_j \theta_{i,j} = 1. \text{ [but please confirm!]} \\ &= \prod_j \pi_{VA,j,t}^{\theta_{i,j}} \end{aligned}$$

CPI in terms of $\pi_{VA,j,t}$ is therefore:

$$\pi_{CPI,t} = \sum_i \omega_{PCE,i} \prod_j \pi_{VA,j,t}^{\theta_{i,j}}$$

This is no-longer linear in $\pi_{VA,j,t}$!!!

There are two options:

1. approximate the equation as:

$$\begin{aligned} \pi_{CPI,t} &\approx \sum_i \omega_{PCE,i} \sum_j \theta_{i,j} \pi_{VA,j,t} \\ &= \sum_j \underbrace{\left(\sum_i \omega_{PCE,i} \theta_{i,j} \right)}_{\equiv \omega_{\pi_{VA,j,t}}} \pi_{VA,j,t} \end{aligned}$$

2. Continue with the non-linear version to get an exact match

I recommend we start with the approximation, check it is accurate and if not, do both.

3.1 Steps for the approximated method

1. Verify that $\log(\underline{P}_t) = \Theta \log(\underline{P}_{VA,t})$, $\sum_i \omega_{PCE,i} = 1$ and that $\sum_j \theta_{j,i} = 1$ for all i (simpler to just check $\Theta \underline{1} = \underline{1}$ where $\underline{1}$ is a vector of ones).
2. Calculate $\underline{P}_t^{approx} \equiv \Theta \underline{P}_{VA}$ (note that we are ignoring the logs here).
3. Check the differences between \underline{P}_t and \underline{P}_t^{approx} . Specifically let's calculate the mean absolute difference and largest absolute difference. **If these are very large, we should be worried but still continue to the next step. If they are larger than 100% or -100% we are probably going to be in trouble....**
4. Calculate the (approximated) CPI weights on value-added inflation (matrix method probably quicker):

$$\omega_{VA,i,t} \approx \sum_j \omega_{PCE,j,t-1} \theta_{j,i} = \underline{\omega}'_{PCE,t-1} \Theta$$

5. Check these weights sum to 1 (see proof just after this for why they should):

$$\sum_i \omega_{VA,i,t} = \sum_i \sum_j \omega_{PCE,j,t-1} \theta_{j,i} = \sum_j \omega_{PCE,j,t-1} \underbrace{\sum_i \theta_{j,i}}_{=1} = \sum_j \omega_{PCE,j,t-1} = 1$$

If these don't sum to 1 properly email Nick

6. Calculate VA inflation and output price inflation (BEA price data) as:

$$\pi_{VA,i,t} = \frac{P_{VA,i,t}}{P_{VA,i,t-1}}$$

and

$$\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$$

7. Calculate approximate CPI inflation and compare to Shaprio's CPI inflation calculation:

$$\begin{aligned} \pi_{CPI,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,t} \\ \pi_{CPI,t}^{approx} &= \sum_i \omega_{\pi_{VA},i,t-1} \pi_{VA,i,t} \end{aligned}$$

8. Check the differences between $\pi_{CPI,t}$ and $\pi_{CPI,t}^{approx}$. Specifically let's calculate the mean absolute difference and largest absolute difference. **We want an average error less than 3% and a max error less than 10%.** If these are satisfied, we can skip the exact method.
9. Now we need demand driven and supply driven inflation. These are calculated as:

$$\begin{aligned} \pi_{Demand} &= \sum_i D_i \omega_{\pi_{VA},i,t-1} \pi_{VA,i,t} \\ \pi_{Supply} &= \sum_i (1 - D_i) \omega_{\pi_{VA},i,t-1} \pi_{VA,i,t} \end{aligned}$$

3.2 Steps for the exact method

1. [If not already done] Verify that $\log(\underline{P}_t) = \Theta \log(\underline{P}_{VA,t})$, $\sum_i \omega_{PCE,i} = 1$ and that $\sum_j \theta_{i,j} = 1$ for all i (simpler to just check $\Theta \underline{1} = \underline{1}$ where $\underline{1}$ is a vector of ones).
2. Calculate VA inflation and output price inflation (BEA price data) as:

$$\pi_{VA,i,t} = \frac{P_{VA,i,t}}{P_{VA,i,t-1}}$$

and

$$\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$$

3. Calculate CPI inflation via the Shapiro method for comparison purposes later.

$$\pi_{CPI,t} = \sum_i \omega_{PCE,i,t-1} \pi_{i,t}$$

4. Calculate the following concepts:

$$\pi_{i,t}^{IO} = \prod_j \pi_{VA,j,t}^{\theta_{i,j}}$$

Should equal $\pi_{i,t}$ (output inflation for sector i)

$$\pi_{i,Demand,t}^{IO} = \sum_j \pi_{VA,j,t}^{D_{j,t}\theta_{i,j}}$$

$$\pi_{i,Supply,t}^{IO} = \sum_j \pi_{VA,j,t}^{(1-D_{j,t})\theta_{i,j}}$$

$$\pi_{i,Interactions,t}^{IO} = \pi_{i,t}^{IO} - \pi_{i,Demand,t}^{IO} - \pi_{i,Supply,t}^{IO}$$

The first should be the exact calculation of output price inflation. The second and third are the contributions of demand and supply shocks to output prices in sector i . The fourth is to account for the fact that supply and demand's contribution to inflation in each sector is ****not**** linearly separable. That means that $\pi_{i,t} \neq \pi_{i,Demand,t}^{IO} + \pi_{i,Supply,t}^{IO}$

Note: these are not linearly separable! I will elaborate later.

5. Check the following:

- Sectoral inflation calculated via $\pi_{VA,j,t}$ is equal to output price inflation from BEA
 $\pi_{i,t}^{IO} = \pi_{i,t}$.
- Overall output inflation in each sector equals *the product* of demand and supply driven inflation: $\pi_{i,t}^{IO} = \pi_{i,Demand,t}^{IO} \cdot \pi_{i,Supply,t}^{IO}$

Email Nick if they are not equal

6. Now we can calculate the contribution of demand and supply shocks to inflation:

$$\begin{aligned}\pi_{CPI,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,t}^{IO} \\ \pi_{CPI,Demand,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,Demand,t}^{IO} \\ \pi_{CPI,Supply,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,Supply,t}^{IO} \\ \pi_{CPI,Interactions,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,Interactions,t}^{IO}\end{aligned}$$

7. Confirm that these three contributions all add up to CPI

$$\pi_{CPI,t} = \pi_{CPI,Demand,t} + \pi_{CPI,Supply,t} + \pi_{CPI,Interactions,t}$$

8. plot them and let's see what we get! Hopefully interactions are not super large....

4 Earlier Incorrect Version

1. Verify that $\sum_i \omega_{VA,i,t} P_{VA,i,t} = P_{CPI,t}$ (or that $\sum_i \omega_{i,PCE} P_{i,t} = \sum_i \omega_{VA,i,t} P_{VA,i,t}$)

2. Calculate value added *inflation*:

$$\pi_{VA,i,t} = \frac{P_{VA,i,t}}{P_{VA,i,t-1}} - 1$$

3. Calculate some inflation-based expenditure weights:

$$\omega_{\pi_{VA},i,t} = \frac{\omega_{VA,i,t} P_{VA,i,t-1}}{\sum_j \omega_{VA,j,t-1} P_{VA,j,t-1}}$$

based on the following logic:

$$\pi_{CPI,t} = \frac{P_{CPI,t}}{P_{CPI,t-1}} = \frac{\sum_i \omega_{VA,i,t} P_{VA,i,t}}{\sum_j \omega_{VA,j,t-1} P_{VA,j,t-1}} = \sum_i \left(\frac{\omega_{VA,i,t} P_{VA,i,t-1}}{\sum_j \omega_{VA,j,t-1} P_{VA,j,t-1}} \times \pi_{VA,i,t} \right)$$

4. Verify these weights sum to 1: $\sum_i \omega_{\pi_{VA},i,t} = 1$.

5. Verify that with these weights and value added inflation we recover CPI inflation $\pi_{CPI} = \sum_i \omega_{\pi_{VA},i,t} \pi_{VA,i,t}$ (another check is whether $\sum_i \omega_{\pi_{VA},i,t} \pi_{VA,i,t} = \sum_i \omega_{PCE,i,t} \pi_{i,t}$)

6. Then use these weights with the original D to calculate demand and supply:

$$\begin{aligned}\pi_{Demand} &= \sum_i D_i \omega_{\pi_{VA},i,t} \pi_{VA,i,t} \\ \pi_{Supply} &= \sum_i (1 - D_i) \omega_{\pi_{VA},i,t} \pi_{VA,i,t}\end{aligned}$$