

# Two sector examples of calculating CPI

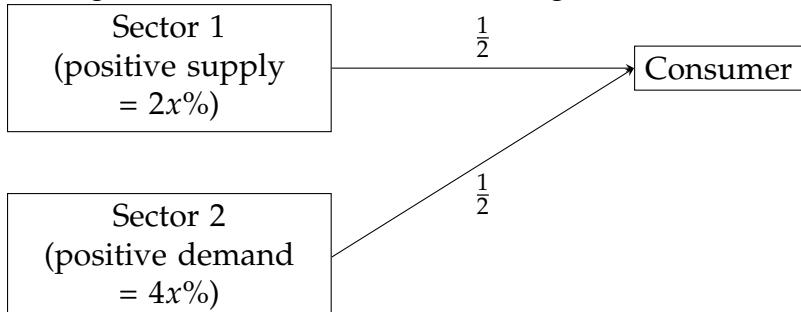
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February 19, 2026

**Purpose:** Provide a 2 sector example where we see what the current CPI aggregation method does. I will show that it doesn't work as it should but that switching to weighting up value-added inflation with value-added weights will work without issue. I then outline how to code this up.

## 1 Horizontal Economy – Shapiro works well

Let's imagine the case as set out in the diagram below.



Here we have two sectors *horizontally* selling a good to the household.

Let's imagine that sector 1 has a positive supply shock and sector 2 has a negative demand shock. Let's imagine that the supply shock *lowers* sector 1 prices by  $2x\%$  and the sector 2 demand shock *raises* sector 2 prices by  $4x\%$ .

We can calculate CPI as:

$$\pi^{CPI} = -\frac{1}{2}2x + \frac{1}{2}4x = x$$

With these numbers, we would have demand contributing  $+4x$  to CPI and supply  $-2x$  to CPI.

How do we calculate this formally? Well we create a vector (matrix when there are multiple time periods) indicating if the sector has experienced mostly a demand shock or supply shock. Call this  $D$  and let  $D_i = 1$  when it has experienced mostly a demand shock. Then we take the expenditure weights  $\omega_{PCE} \equiv [\frac{1}{2} \quad \frac{1}{2}]'$  and the sectoral CPI changes:

$$\pi_{Demand} = \sum_i D_i \omega_{PCE,i} \pi_i = 0 \times \frac{1}{2} \times (-2x) + 1 \times \frac{1}{2} \times 4x = 2x \checkmark$$

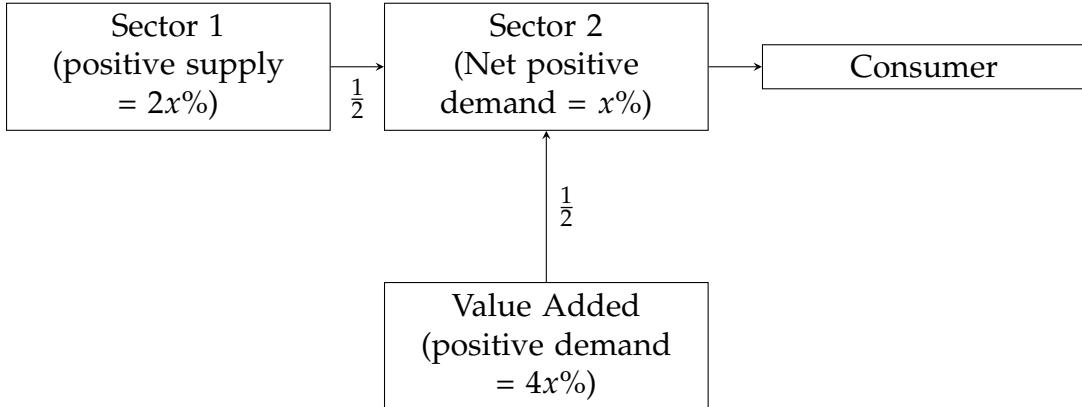
$$\pi_{Supply} = \sum_i (1 - D_i) \omega_{PCE,i} \pi_i = 1 \times \frac{1}{2} \times (-2x) + 0 \times \frac{1}{2} \times 4x = -x \checkmark$$

where we can see that  $\pi_{Demand} + \pi_{Supply} = \pi_{CPI}$ .

Note that ShapirIO is the same because there is no IO to correct for. Let's now see if we get it for the vertical economy.

## 2 Vertical Economy – does ShapirIO work well

Let's switch to a vertical economy like the following diagram:



In this vertical economy, note that sector 2's output price is identical to CPI. If we were doing Shapiro we would be getting:

$$\begin{aligned}\pi_{Demand} &= 0 \times 0 \times -2x + 1 \times 1 \times x = x \\ \pi_{Supply} &= 1 \times 0 \times -2x + 0 \times 1 \times x = 0\end{aligned}$$

which is incorrect for the exact reasons we wanted to adjust Shapiro – it misses IO interconnections.

OK, so let's do our IO corrections. There are two approaches: Adjust  $D$  and switch to value added prices.

**Adjust D:** This approach does the same aggregation as Shapiro but uses a  $D$  that reflects the breakdown between supply and demand shocks along the IO network. *Crucially, it does not distinguish between sectors with price rises or falls.*

So in this case, we would correct sector 2 based on the fact that its own demand shock is only half of its output price. This would give us:

$$\tilde{D} = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$$

Then if we use this  $\tilde{D}$  matrix to calculate supply and demand inflation we would get:

$$\begin{aligned}\pi_{Demand} &= 0 \times 0 \times -2x + \frac{1}{2} \times 1 \times x = \frac{x}{2} \neq 4x \\ \pi_{Supply} &= 1 \times 0 \times -2x + \frac{1}{2} \times 1 \times x = \frac{x}{2} \neq -x\end{aligned}$$

The reason this is incorrect is that we used the wrong  $\tilde{D}$ . To see this, let  $\tilde{D} = [0 \ 2]'$ . Now we have:

$$\begin{aligned}\pi_{Demand} &= 0 \times 0 \times -2x + 2 \times 1 \times x = 2x \checkmark \\ \pi_{Supply} &= 1 \times 0 \times -2x + (1 - 2) \times 1 \times x = -x \checkmark\end{aligned}$$

So one option to fix this is to allow for negative  $\tilde{D}$  when the upstream sector has a price fall. But I think there is an easier way:

**Switch to Value Added Prices/Inflation:** Next, let's revert to a indicator vector for demand shocks  $D$  but instead of using  $\omega_{PCE}$  and  $\pi_i$  (output price inflation in each sector), we will use the value added weights for CPI which are:

$$\omega_{VA,i} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

And the inflation we will use will be value added inflation calculated as:

$$\pi_{VA} = \begin{bmatrix} -2x \\ 4x \end{bmatrix}$$

Now the calculation is given by:

$$\begin{aligned}\pi_{Demand} &= 0 \times \frac{1}{2} \times -2x + 1 \times \frac{1}{2} \times 4x = 2x \checkmark \\ \pi_{Supply} &= 1 \times \frac{1}{2} \times -2x + 0 \times \frac{1}{2} \times 4x = -x \checkmark\end{aligned}$$

And we are good!

Note that this approach is much simpler than the previous one so let me outline a set of coding steps to operationalize this.

### 3 Implementing the new calculations

Let's start at the code where we have  $D \in \{0, 1\}$  – our classification of shocks as either mostly demand or supply (and not in-between),  $\omega_{VA,i,t}$  and  $\{P_{VA,i,t}\}_{i,t}$  – our time series data of value added prices. But before we begin, let's note that there is a small problem:

1. CPI inflation in Shapiro is calculated linearly:  $\pi_{CPI,t} = \sum_i \omega_{PCE,i,t-1} \times \pi_{i,t}$ . I used a difference formula in the note before but let's stick with this one. [Tony, do we use official CPI data at any point in the analysis? If so, does either this formula or the one I gave you earlier match it exactly?]
2. But our relationship between output prices  $P_{i,t}$  and value added prices  $P_{VA,i,t}$  is *logarithmic*!

$$\log(\underline{P}_t) = \Theta \log(\underline{P}_{VA,t})$$

The correct formula for output price inflation in each sector  $i$  is given by:

$$\begin{aligned}\pi_{i,t} &\equiv \frac{P_{i,t}}{P_{i,t-1}} - 1 = \frac{\prod_j P_{VA,j,t}^{\theta_{i,j}}}{\prod_k P_{VA,j,t-1}^{\theta_{i,k}}} - 1 \\ &= \prod_j \left( \frac{P_{VA,j,t}}{P_{VA,j,t-1}} \right)^{\theta_{i,j}} - 1 \\ &= \prod_j \left( \frac{P_{VA,j,t}}{P_{VA,j,t-1}} - 1 \right)^{\theta_{i,j}} \quad \text{because } \sum_j \theta_{i,j} = 1. \text{ [but please confirm!]} \\ &= \prod_j \pi_{VA,j,t}^{\theta_{i,j}}\end{aligned}$$

CPI in terms of  $\pi_{VA,j,t}$  is therefore:

$$\pi_{CPI,t} = \sum_i \omega_{PCE,i} \prod_j \pi_{VA,j,t}^{\theta_{i,j}}$$

This is no-longer linear in  $\pi_{VA,j,t}$ !!!

There are two options:

1. approximate the equation as:

$$\begin{aligned}\pi_{CPI,t} &\approx \sum_i \omega_{PCE,i} \sum_j \theta_{i,j} \pi_{VA,j,t} \\ &= \sum_j \underbrace{\left( \sum_i \omega_{PCE,i} \theta_{i,j} \right)}_{\equiv \omega_{VA,j,t}} \pi_{VA,j,t}\end{aligned}$$

2. Continue with the non-linear version to get an exact match

I recommend we start with the approximation, check it is accurate and if not, do both.

### 3.1 Steps for the approximated method

1. Verify that  $\log(\underline{P}_t) = \Theta \log(\underline{P}_{VA,t})$ ,  $\sum_i \omega_{PCE,i} = 1$  and that  $\sum_j \theta_{i,j} = 1$  for all  $i$  (simpler to just check  $\Theta \underline{1} = \underline{1}$  where  $\underline{1}$  is a vector of ones).
2. Calculate  $\underline{P}_t^{approx} \equiv \Theta \underline{P}_{VA}$  (note that we are ignoring the logs here).
3. Check the differences between  $\underline{P}_t$  and  $\underline{P}_t^{approx}$ . Specifically let's calculate the mean absolute difference and largest absolute difference. **If these are very large, we should be worried but still continue to the next step. If they are larger than 100% or -100% we are probably going to be in trouble....**
4. Calculate the (approximated) CPI weights on value-added inflation (matrix method probably quicker):

$$\omega_{VA,i,t} \approx \sum_j \omega_{PCE,j,t-1} \theta_{j,i} = \underline{\omega}'_{PCE,t-1} \Theta$$

5. Check these weights sum to 1 (see proof just after this for why they should):

$$\sum_i \omega_{VA,i,t} = \sum_i \sum_j \omega_{PCE,j,t-1} \theta_{j,i} = \sum_j \omega_{PCE,j,t-1} \underbrace{\sum_i \theta_{j,i}}_{=1} = \sum_j \omega_{PCE,j,t-1} = 1$$

**If these don't sum to 1 properly email Nick**

6. Calculate VA inflation and output price inflation (BEA price data) as:

$$\pi_{VA,i,t} = \frac{P_{VA,i,t}}{P_{VA,i,t-1}}$$

and

$$\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$$

7. Calculate approximate CPI inflation and compare to Shaprio's CPI inflation calculation:

$$\begin{aligned}\pi_{CPI,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,t} \\ \pi_{CPI,t}^{approx} &= \sum_i \omega_{\pi_{VA,i,t-1}} \pi_{VA,i,t}\end{aligned}$$

8. Check the differences between  $\pi_{CPI,t}$  and  $\pi_{CPI,t}^{approx}$ . Specifically let's calculate the mean absolute difference and largest absolute difference. **We want an average error less than 3% and a max error less than 10%.** If these are satisfied, we can skip the exact method.
9. Now we need demand driven and supply driven inflation. These are calculated as:

$$\begin{aligned}\pi_{Demand} &= \sum_i D_i \omega_{\pi_{VA,i,t-1}} \pi_{VA,i,t} \\ \pi_{Supply} &= \sum_i (1 - D_i) \omega_{\pi_{VA,i,t-1}} \pi_{VA,i,t}\end{aligned}$$

### 3.2 Steps for the exact method

1. [If not already done] Verify that  $\log(\underline{P}_t) = \Theta \log(\underline{P}_{VA,t})$ ,  $\sum_i \omega_{PCE,i} = 1$  and that  $\sum_j \theta_{i,j} = 1$  for all  $i$  (simpler to just check  $\Theta \underline{1} = \underline{1}$  where  $\underline{1}$  is a vector of ones).
2. Calculate VA inflation and output price inflation (BEA price data) as:

$$\pi_{VA,i,t} = \frac{P_{VA,i,t}}{P_{VA,i,t-1}}$$

and

$$\pi_{i,t} = \frac{P_{i,t}}{P_{i,t-1}}$$

3. Calculate CPI inflation via the Shapiro method for comparison purposes later.

$$\pi_{CPI,t} = \sum_i \omega_{PCE,i,t-1} \pi_{i,t}$$

4. Calculate the following concepts:

$$\begin{aligned} \pi_{i,t}^{IO} &= \prod_j \pi_{VA,j,t}^{\theta_{i,j}} && \text{Should equal } \pi_{i,t} \text{ (output inflation for sector } i\text{)} \\ \pi_{i,Demand,t}^{IO} &= \sum_j \pi_{VA,j,t}^{D_{j,t}\theta_{i,j}} \\ \pi_{i,Supply,t}^{IO} &= \sum_j \pi_{VA,j,t}^{(1-D_{j,t})\theta_{i,j}} \\ \pi_{i,Interactions,t}^{IO} &= \pi_{i,t}^{IO} - \pi_{i,Demand,t}^{IO} - \pi_{i,Supply,t}^{IO} \end{aligned}$$

The first should be the exact calculation of output price inflation. The second and third are the contributions of demand and supply shocks to output prices in sector  $i$ . The fourth is to account for the fact that supply and demand's contribution to inflation in each sector is \*\*not\*\* linearly separable. That means that  $\pi_{i,t} \neq \pi_{i,Demand,t}^{IO} + \pi_{i,Supply,t}^{IO}$

**Note: these are not linearly separable! I will elaborate later.**

5. Check the following:

- Sectoral inflation calculated via  $\pi_{VA,j,t}$  is equal to output price inflation from BEA  $\pi_{i,t}^{IO} = \pi_{i,t}$ .
- Overall output inflation in each sector equals *the product* of demand and supply driven inflation:  $\pi_{i,t}^{IO} = \pi_{i,Demand,t}^{IO} \cdot \pi_{i,Supply,t}^{IO}$

Email Nick if they are not equal

6. Now we can calculate the contribution of demand and supply shocks to inflation:

$$\begin{aligned}\pi_{CPI,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,t}^{IO} \\ \pi_{CPI,Demand,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,Demand,t}^{IO} \\ \pi_{CPI,Supply,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,Supply,t}^{IO} \\ \pi_{CPI,Interactions,t} &= \sum_i \omega_{PCE,i,t-1} \pi_{i,Interactions,t}^{IO}\end{aligned}$$

7. Confirm that these three contributions all add up to CPI

$$\pi_{CPI,t} = \pi_{CPI,Demand,t} + \pi_{CPI,Supply,t} + \pi_{CPI,Interactions,t}$$

8. plot them and let's see what we get! Hopefully interactions are not super large....

## 4 Earlier Incorrect Version

1. Verify that  $\sum_i \omega_{VA,i,t} P_{VA,i,t} = P_{CPI,t}$  (or that  $\sum_i \omega_{i,PCE} P_{i,t} = \sum_i \omega_{VA,i,t} P_{VA,i,t}$ )
2. Calculate value added *inflation*:

$$\pi_{VA,i,t} = \frac{P_{VA,i,t}}{P_{VA,i,t-1}} - 1$$

3. Calculate some inflation-based expenditure weights:

$$\omega_{\pi_{VA},i,t} = \frac{\omega_{VA,i,t} P_{VA,i,t-1}}{\sum_j \omega_{VA,j,t-1} P_{VA,j,t-1}}$$

based on the following logic:

$$\pi_{CPI,t} = \frac{P_{CPI,t}}{P_{CPI,t-1}} = \frac{\sum_i \omega_{VA,i,t} \frac{P_{VA,i,t-1}}{P_{VA,i,t-1}}}{\sum_j \omega_{VA,j,t-1} P_{VA,j,t-1}} = \sum_i \left( \frac{\omega_{VA,i,t} \frac{P_{VA,i,t-1}}{P_{VA,i,t-1}}}{\sum_j \omega_{VA,j,t-1} P_{VA,j,t-1}} \times \pi_{VA,i,t} \right)$$

4. Verify these weights sum to 1:  $\sum_i \omega_{\pi_{VA},i,t} = 1$ .
5. Verify that with these weights and value added inflation we recover CPI inflation  $\pi_{CPI} = \sum_i \omega_{\pi_{VA},i,t} \pi_{VA,i,t}$  (another check is whether  $\sum_i \omega_{\pi_{VA},i,t} \pi_{VA,i,t} = \sum_i \omega_{PCE,i,t} \pi_{i,t}$ .)
6. Then use these weights with the original  $D$  to calculate demand and supply:

$$\begin{aligned}\pi_{Demand} &= \sum_i D_i \omega_{\pi_{VA},i,t} \pi_{VA,i,t} \\ \pi_{Supply} &= \sum_i (1 - D_i) \omega_{\pi_{VA},i,t} \pi_{VA,i,t}\end{aligned}$$