GRAPHIC COMPARISONS BY BARS, SQUARES, CIRCLES, AND CUBES

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In an earlier issue of this Journal an attempt was made to evaluate bar charts and pie diagrams as graphic devices for showing component parts.[2] The present article deals with the relative merits of bars, squares, circles, and cubes for showing simple comparisons. Most users of statistical charts have been accustomed to believe that linear comparisons, represented by bar charts, can be judged with much more accuracy than can area comparisons, such as squares and circles, and that either is superior to volume comparisons, such as cubes. It is the purpose of this investigation to test these types of diagrams to ascertain which, if any, may be preferable from the point of view of the accuracy of the estimates made from them.

The relationships shown by the squares and circles used in this study were represented by the respective areas of the diagrams, while the relationships shown by the cubes were represented by their volumes. The fact that the actual volume of a solid can be only suggested by a two-dimension drawing may be one of the reasons why the accuracy of judgments based upon the drawings of cubes proved to be relatively low. In popular publications comparisons by circles, squares, cubes, and various pictorial means are frequently drawn on a linear basis rather than upon the proper area or volume basis. Furthermore, the reader is frequently at a loss to know upon what basis they are drawn. Careful computations have revealed that some diagrams, whether considered as linear, area, or volume comparisons, give only a very rough approximation of the figures which they undertake to present.

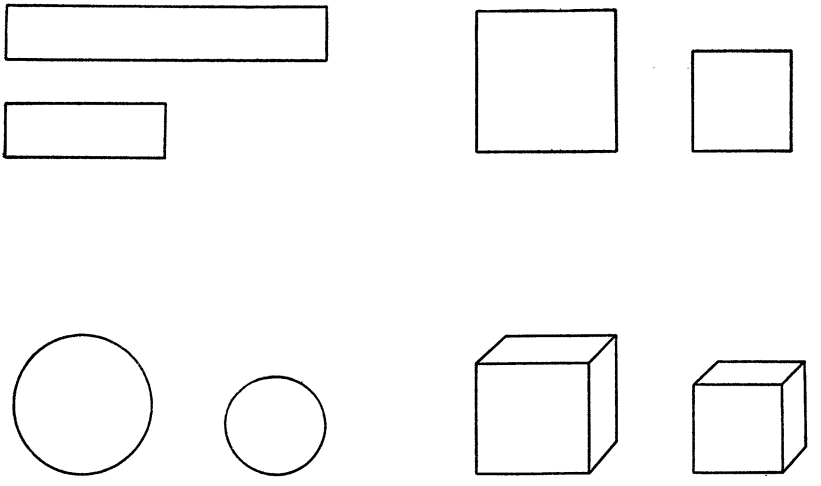
The data which follow are the results of estimates made by 550 observers and the diagrams used were 40 in number. The 40 diagrams, each on a separate card, formed nine sets. Each set consisted of a comparison by means of bars, squares, circles, and cubes showing the same ratio, as shown in Chart I; in addition an alternate arrangement of squares and circles with the diagrams centered as in Chart II, was included in two of the sets. The diagrams were presented one at a

[1] The writers wish to express their gratitude to those instructors and students who made it possible to obtain the data used in this analysis and to Mr. William Madow and Mr. Charles H. Wittmann who assisted in the computations.

[2] "Bar Charts Versus Circle Diagrams," by Frederick E. Croxton and Roy E. Stryker, this Journal, December, 1927.

CHART I

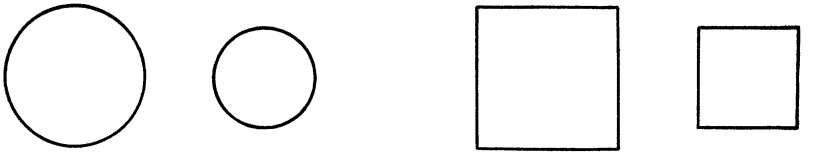
BARS, SQUARES, CIRCLES, AND CUBES SHOWING 50 TO 100 RELATIONSHIP



time to the observers in such an order that it was not obvious that the charts were in sets. For each diagram the observers undertook to estimate the size of the smaller figure in relation to the larger one. As in the earlier study referred to above, all diagrams were drawn without scales, as in Charts I and II, in order that the estimates of the observers might be based upon the diagrams alone and not influenced by the scales.

CHART II

CIRCLES CENTERED AND SQUARES CENTERED SHOWING 50 TO 100 RELATIONSHIP



Bars versus Squares. The first part of Table I presents the results obtained from comparing the estimates made of nine pairs of bars and nine pairs of squares. The mean error resulting from judging the bars was in each instance smaller than that resulting from observing the squares. In eight of the nine instances shown in the table the differences between the mean errors was significant, while in one case it was not.

Bars versus Circles. As shown in the second part of Table I, estimates based upon bars were more accurate than those based upon

circles. For each of the nine comparisons of bars and circles the advantage lay with the bars. The differences between the mean errors were significant in eight cases out of the nine.

Bars versus Cubes. Bars showed an even greater superiority over cubes than over either squares or circles. The third part of Table I indicates that in each case the mean error resulting from estimating the proportion shown by bars was smaller than the mean error resulting from estimating the cubes. In each case, also, the difference between the mean errors was significant.

TABLE I

ERRORS RESULTING FROM ESTIMATES OF BARS AND SQUARES, BARS AND CIRCLES, AND BARS AND CUBES

Percentage shown	Mean error for bars	Mean error for other diagram	Advantage in favor of:	Observed difference	Difference ÷ σ difference	Significant?
			Bars and Squares			
2.00	2.42	2.50	bars	.08	.4	no
12.50	2.83	5.07	bars	2.24	4.6	yes
16.67	2.98	5.17	bars	2.19	6.1	yes
25.00	3.04	4.36	bars	1.32	3.1	yes
33.33	2.55	8.77	bars	6.22	13.8	yes
50.00	1.54	15.58	bars	14.04	33.4	yes
66.67	3.67	12.49	bars	8.82	26.7	yes
70.00	4.44	11.92	bars	7.48	27.7	yes
90.00	3.00	4.73	bars	1.73	8.2	yes
			Bars and Circles			
2.00	2.42	2.78	bars	.36	2.1	?
12.50	2.83	6.21	bars	3.38	6.8	yes
16.67	2.98	6.37	bars	3.39	10.0	yes
25.00	3.04	6.47	bars	3.43	9.5	yes
33.33	2.55	8.72	bars	6.17	15.0	yes
50.00	1.54	9.73	bars	8.19	21.6	yes
66.67	3.67	8.98	bars	5.31	15.2	yes
70.00	4.44	9.37	bars	4.93	16.4	yes
90.00	3.00	6.52	bars	3.52	15.3	yes
			Bars and Cubes			
2.00	2.42	7.16	bars	4.74	16.9	yes
12.50	2.83	10.02	bars	7.19	13.8	yes
16.67	2.98	14.41	bars	11.43	21.2	yes
25.00	3.04	19.23	bars	16.19	24.5	yes
33.33	2.55	19.18	bars	16.63	29.7	yes
50.00	1.54	19.83	bars	18.29	42.5	yes
66.67	3.67	14.64	bars	10.97	32.3	yes
70.00	4.44	14.31	bars	9.87	31.8	yes
90.00	3.00	5.96	bars	2.96	12.9	yes

Squares versus Circles. From the data shown in the first part of Table II it would appear that there is little difference in the accuracy of estimates based on squares and circles. In five instances the mean error in estimating the squares was smaller than the mean error in estimating the circles; in two of these the difference between the

means was significant and in a third it was probably significant. In four instances the mean error in estimating the circles was smaller than the mean error in estimating the squares; in three of these the difference between the means was significant.

Squares versus Cubes. Estimates based upon squares are clearly more accurate than those based upon cubes. The second part of Table II shows that in each instance the mean error in estimating the squares was smaller than that in estimating the cubes and that in each instance the difference between the mean errors was significant.

TABLE II
ERRORS RESULTING FROM ESTIMATES OF SQUARES AND CIRCLES, AND SQUARES AND CUBES

Percentage shown	Mean error for squares	Mean error for other diagram	Advantage in favor of:	Observed difference	Difference ÷ σdifference	Significant?
			Squares and Circles			
2.00	2.50	2.78	squares	.28	1.3	no
12.50	5.07	6.21	squares	1.14	2.2	?
16.67	5.17	6.37	squares	1.20	2.9	probably
25.00	4.36	6.47	squares	2.11	4.8	yes
33.33	8.77	8.72	circles	.05	.1	no
50.00	15.58	9.73	circles	5.85	10.8	yes
66.67	12.49	8.98	circles	3.51	9.2	yes
70.00	11.92	9.37	circles	2.55	7.3	yes
90.00	4.73	6.52	squares	1.97	7.8	yes
			Squares and Cubes			
2.00	2.50	7.16	squares	4.66	14.6	yes
12.50	5.07	10.02	squares	4.95	9.5	yes
16.67	5.17	14.41	squares	9.24	15.9	yes
25.00	4.36	19.23	squares	14.87	21.2	yes
33.33	8.77	19.18	squares	10.41	15.3	yes
50.00	15.58	19.83	squares	4.25	7.5	yes
66.67	12.49	14.64	squares	2.15	5.8	yes
70.00	11.92	14.31	squares	2.39	6.8	yes
90.00	4.73	5.96	squares	1.23	5.3	yes

Circles versus Cubes. Estimates based upon circles seem to be more accurate than those based upon cubes. In Table III it may be seen

TABLE III
ERRORS RESULTING FROM ESTIMATES OF CIRCLES AND CUBES

Percentage shown	Mean error for circles	Mean error for cubes	Advantage in favor of:	Observed difference	Difference ÷ σdifference	Significant?
2.00	2.78	7.16	circles	4.38	14.6	yes
12.50	6.21	10.02	circles	3.81	7.1	yes
16.67	6.37	14.41	circles	8.04	14.1	yes
25.00	6.47	19.23	circles	12.76	19.3	yes
33.33	8.72	19.18	circles	10.46	16.1	yes
50.00	9.73	19.83	circles	10.10	18.4	yes
66.67	8.98	14.64	circles	5.66	14.5	yes
70.00	9.37	14.31	circles	4.94	13.4	yes
90.00	6.52	5.96	cubes	.56	2.3	?

that in eight instances the mean error in estimating circles was smaller than the mean error in estimating cubes and that in each of these instances the difference between the means was significant.

Squares centered versus other forms. As stated above, two diagrams of squares were included in which the squares were centered as in Chart II. As indicated in Table IV estimates made from bars appear to be more accurate than those made from squares centered, and estimates based upon squares centered are more accurate than those based upon cubes.

One reason for including squares centered was to ascertain if estimates made from them were more or less accurate than estimates made from squares drawn to the same base line. There seems to be no clear evidence in Table IV that either is superior. No conclusive statement can be made as to the relative merits of squares centered on the one hand and of circles, and circles centered on the other.

TABLE IV
ERRORS RESULTING FROM ESTIMATES OF SQUARES CENTERED AND OTHER DIAGRAMS

Percentage shown	Mean error for squares centered	Mean error for other diagram	Advantage in favor of:	Observed difference	Difference ÷ σdifference	Significant?
Squares Centered and Bars						
25.0	3.99	3.04	bars	.95	2.4	?
50.0	12.10	1.54	bars	10.56	27.1	yes
Squares Centered and Squares						
25.0	3.99	4.36	squares centered	.37	.8	no
50.0	12.10	15.58	squares centered	3.48	6.4	yes
Squares Centered and Circles						
25.0	3.99	6.47	squares centered	2.48	6.2	yes
50.0	12.10	9.73	circles	2.37	4.6	yes
Squares Centered and Circles Centered						
25.0	3.99	6.79	squares centered	2.80	6.5	yes
50.0	12.10	10.95	circles centered	1.15	2.2	?
Squares Centered and Cubes						
25.0	3.99	19.23	squares centered	15.24	22.4	yes
50.0	12.10	19.83	squares centered	7.73	14.1	yes

Circles centered versus other forms. Two sets of circles centered (as in Chart II) were also included. Table V indicates that estimates based upon bars were more accurate than those based upon circles centered and that estimates based upon circles centered were more accurate than those based upon cubes. It was thought that estimates based upon circles centered might prove to be more or less accurate than estimates based upon circles drawn to the same base line. The data of Table V show no definite proof that either is superior. No conclusive statement can be made as to the relative merits of circles centered on the one hand and of squares, squares centered, and circles on the other hand.

TABLE V

ERRORS RESULTING FROM ESTIMATES OF CIRCLES CENTERED AND OTHER DIAGRAMS

Percentage shown	Mean error for circles centered	Mean error for other diagram	Advantage in favor of:	Observed difference	Difference ÷ σdifference	Significant?
Circles Centered and Bars						
25.0	6.79	3.04	bars	3.75	9.4	yes
50.0	10.95	1.54	bars	9.41	23.5	yes
Circles Centered and Squares						
25.0	6.79	4.36	squares	2.43	5.2	yes
50.0	10.95	15.58	circles centered	4.63	8.4	yes
Circles Centered and Squares Centered						
25.0	6.79	3.99	squares centered	2.80	6.5	yes
50.0	10.95	12.10	circles centered	1.15	2.2	?
Circles Centered and Circles						
25.0	6.79	6.47	circles	.32	.8	no
50.0	10.95	9.73	circles	1.22	2.3	?
Circles Centered and Cubes						
25.0	6.79	19.23	circles centered	12.44	18.3	yes
50.0	10.95	19.83	circles centered	8.88	15.9	yes

The results of this quantitative evaluation of the relative merits of bars, squares, circles, and cubes may be summed up as follows:

(1) Estimates based upon bar charts were more accurate than estimates based upon squares, circles, or cubes.

(2) Estimates based upon squares and estimates based upon circles showed no conclusive evidence that one form of presentation is superior to the other.

(3) Estimates based upon squares were more accurate than estimates based upon cubes.

(4) Estimates based upon circles were more accurate than estimates based upon cubes.

(5) Estimates based upon squares centered and upon circles centered were more accurate than estimates based upon cubes, but less accurate than estimates based upon bars.

(6) In making comparisons by means of squares and circles it appears (upon limited evidence) that it makes no difference in the accuracy of estimates whether the figures be centered or drawn upon the same base line.

It will be noted (Chart I) that the bars used in this study were arranged one above the other and that the other figures were arranged side by side. While this is the method of arrangement most frequently encountered, it raises several points which must be left for later determination: Are estimates more accurate when based upon horizontal or vertical bars? If squares, circles, or cubes are to be used, is it preferable to arrange them side by side or one above the other? If vertical bars yield less accurate estimates than horizontal bars are they still clearly preferable to squares, circles, and cubes?