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THE RELATIVE MERITS OF CIRCLES AND BARS FOR REPRESENTING COMPONENT PARTS

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INTRODUCTION

The graphical representation of statistics has had a remarkable development in the past few years; the great value of this method, properly safeguarded, is generally recognized today. But of the possible graphic methods, many have not yet become standardized. The preliminary report of the joint committee of the national engineering and statistical organizations has done much to standardize graphic procedure, and its recommendations have deservedly been widely circulated and accepted.[1]

But there are many phases of graphic methods which this report does not mention. For example, the very important matter of the best method of representing component parts is not touched by it. The two commonly used methods for exhibiting this class of facts graphically are by means of *sectors of circles* and by *subdivisions of bars*. Various opinions as to the relative merits of these two methods have been expressed by recent writers on statistics.

OPINIONS OF AUTHORITIES

Brinton, in his excellent pioneer work on graphic methods, says:

> The circle with sectors is not a desirable form of presentation. . . . If the horizontal bar method were used as frequently as the sector method, it would be found in every way more desirable, . . . and would be read much more *quickly* [2] and *accurately* than the method involving sectors.[3]

[1] Brinton, W. C. (Chairman), *Preliminary Report of Joint Committee on Standards for Graphic Presentation*, New York. Reprinted in McCall, W. A., *How To Measure in Education*, pp. 332–341; in Kelley, T. L., *Statistical Method*, pp. 42–43; in Haskell, A. C., *How To Make and Use Graphic Charts*, pp. 71–72, etc.

[2] In the quotations which follow, italics have been inserted by the author of this paper, and are not found in the originals, except where specifically noted.

[3] Brinton, W. C., *Graphic Methods for Presenting Facts*, New York, 1914, pp. 6–7.

Karsten, in his recent detailed and exhaustive work on charts and graphs, says:

> Few readers will judge quantities by either the arc at the perimeter or the subtended angles at the center—on the contrary *most of them* will judge entirely by the *areas* of the segments. . . . The disadvantages of the pie chart are many. It is worthless for study and research purposes. In the first place the human eye cannot easily compare as to length the *various arcs* about the circle, lying as they do in different directions. In the second place, the human eye is not naturally skilled in comparing *angles*—those angles at the center of the circle, formed by the various rays or radii and subtending the various arcs. In the third place, the human eye is not an expert judge of comparative sizes or *areas*, especially those as irregular as the segments of parts of the circle. There is *no way* by which the parts of this round unit can be compared so *accurately* and *quickly* as the parts of a straight line or bar. . . . The advantage of the pie chart is psychological. It instantly commands the readers' attention. A circle is, of all geometrical patterns, the easiest resting spot for the eye. The fact is well known to advertisers. . . . Before using it in place of the *simpler and sounder* 100 per cent bar, you should carefully gauge your audience or readers, and only if you believe you have begun to strain their interest should you judiciously insert the pie chart. In a sense, it might be construed as an *insult to a man's intelligence* to show him a pie chart.[1]

Turning to the field of educational statistics, we find that McCall, in his well known manual, says:

> The sectioned-bar diagram is an even better graph for presenting component parts. It is in almost every respect superior to the sector diagram. Visual comparisons of the components are easier.[2]

Or in the field of economic statistics, Jerome says:

> The circle chart . . . is more difficult than the bar chart to read with substantial accuracy, but it has a certain popularity. . . . This type is popular for the representation of the distribution of a dollar of expense, taxes, or family budget, and is quite appropriate for that purpose. However, even in such cases the same facts could probably be shown with *equal ease* and *clarity* by the ordinary bar diagram.[3]

And finally Secrist, in the revised edition of his popular text book, is particularly severe in his detailed criticism and condemnation of the circle chart to show component parts. The short paragraph of his first edition is expanded into four pages of detailed criticism. Two brief extracts follow:

> When it [the circle] is divided into components, the parts *appear*[4] to stand in the relation of their respective chords. But this is not the case, since the smaller the sector, the longer the chord relative to its corresponding arc, and vice versa. The areas of the sectors are proportional to their respective arcs,

[1] Karsten, K. G., *Charts and Graphs*, New York, 1923, pp. 90–95.
[2] McCall, W. A., *How To Measure in Education*, New York, 1922, p. 348.
[3] Jerome, H., *Statistical Method*, New York, 1924, pp. 97, 66.
[4] The italics are in the original.

> but not to their respective chords. But it is the arcs which *cannot be easily compared*—they are circular—and relative lengths are not apparent. To be compared, they must be straightened out in the mind. The ease with which this can be done *varies inversely* with their length. All radii of a circle, of course, are equal and the lengths of the arcs are proportional to the angles at the center. But it is as difficult to compare the relative sizes of the angles as it is the lengths of the arcs.
>
> A pie diagram is a *clumsy* and *defective* method of illustrating component parts; a bar of uniform width . . . is much more satisfactory.[1]

The mathematical basis of this condemnation, comparisons by means of chords, is especially interesting in view of later results reported in this paper. Secrist further illustrates it by a detailed diagram (p. 183) of the various sectors of a four-component circle, strung along a line in different ways to show the difficulty and errors of comparisons based upon chords. This circle was used (circle M), in the experiment described below. He also quotes approvingly the greater part of the quotation given above from Karsten.

Lack of space forbids the quotation of similar criticisms of circle diagrams which are given by other recent authors in the statistical field—general, educational, or economic. Among them may be mentioned Marshall, Gavett, Alexander, Williams, and Chaddock.[2]

SUMMARY OF CRITICISMS

This group of representative quotations and similar criticisms by the authors mentioned in the last paragraph, form a very severe indictment of the use of circular diagrams to show component parts. Not a single author has been found who strongly favors their use. The criticisms expressed by the authors mentioned may be summarized as follows:

1. *Circle diagrams to show component parts cannot be read as rapidly and easily as bar diagrams.*
2. *Circle diagrams to show component parts cannot be read as accurately as bar diagrams.*

   This is the most important criticism expressed. If true, it states a fundamental fault.
3. *The cause of the inaccuracy of circle diagrams for showing component parts lies in the method by which their components are judged.*
   a. They are judged largely by *areas,* and therefore inaccurately.

[1] Secrist, H., *An Introduction to Statistical Methods* (Revised Edition), New York, 1925, pp. 181–185; First Edition, New York, 1921, p. 166.

[2] Marshall, W. C., *Graphical Methods*, New York, 1921, p. 37; Gavett, G. I., *A First Course in Statistical Method,* New York, 1925, p. 47; Alexander, C., *School Statistics and Publicity*, New York, 1919, pp. 235, 253; Williams, J. H., *Graphic Methods in Education*, Boston, 1924, p. 195; Chaddock, R. E., *Principles and Methods of Statistics*, Boston, 1925, p. 431.

b. They are judged largely by *chords,* and therefore inaccurately.

c. The human eye cannot easily or accurately compare *arcs* or *angles.*

4. *Circle diagrams enjoy a certain unintelligent popularity*—they catch attention, but are unworthy of serious use; while their popularity and psychological appeal are grudgingly admitted, their use is *"an insult to a man's intelligence"*!

It is noticeable that all of the quotations given seem to be purely matters of opinion—none show any evidence of an experimental basis of fact. It is the object of this paper to report the method and conclusions of an experiment designed to test, by objective means, the validity of the criticisms expressed above.

A page containing fifteen subdivided circles was given to each member of the class in general psychology in Whitman College. The students were instructed to think of each circle as representing 100 per cent and to write in each sector their best estimate of the percentage of the whole represented by that sector. They were told not to measure in any way, but to estimate as closely as possible, avoiding any fractional percentages; not to hurry, but to work steadily, in the order given, without skipping. At the end of five minutes each student marked the circle on which he was working. Thirteen minutes were allowed for this work. At its conclusion, four possible methods of forming judgments were outlined and explained by figures on the blackboard; the students were then asked to analyze their mental processes and to indicate, if possible, the particular method which was used by each one. The four methods were as follows:

a. By Areas of Sectors
b. By Central Angles
c. By Arcs on the Circumference
d. By Subtending Chords

Figure I is a reproduction of the sheet as used, except for the fact that the correct percentage figures have been added for reference. Each circle was 4.6 centimeters in diameter.

After an interval of three days, which included the week-end with its non-scholastic distractions, a page containing identically the same data, but represented by subdivided bar diagrams, was given to the class, with similar instructions. This page is reproduced as Figure II. The shorter bars were 10 centimeters in length, the longer ones 20 centimeters.

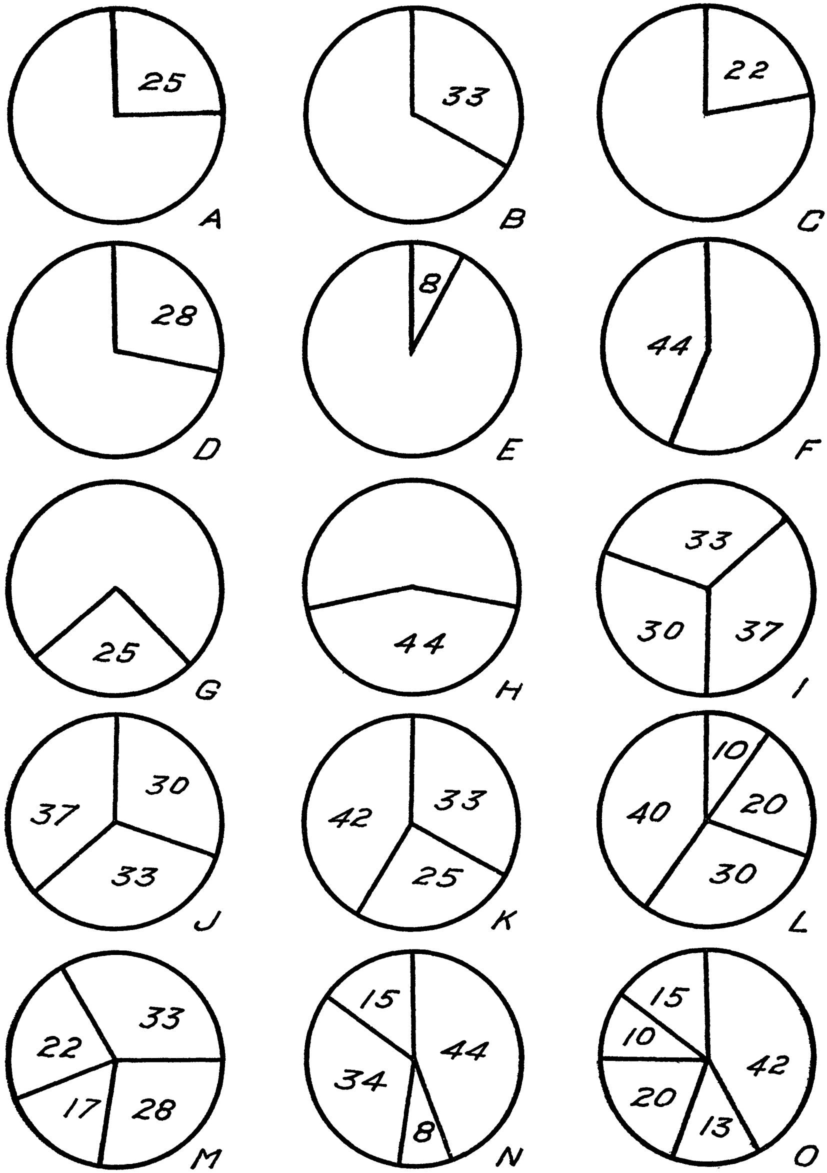

FIGURE I

The experiment was so arranged as to give to the bars all the advantage of practice and of general familiarity with the nature of the problem. A further advantage for the bars was due to their arrangement on the page, with the common point of beginning, thus facili-

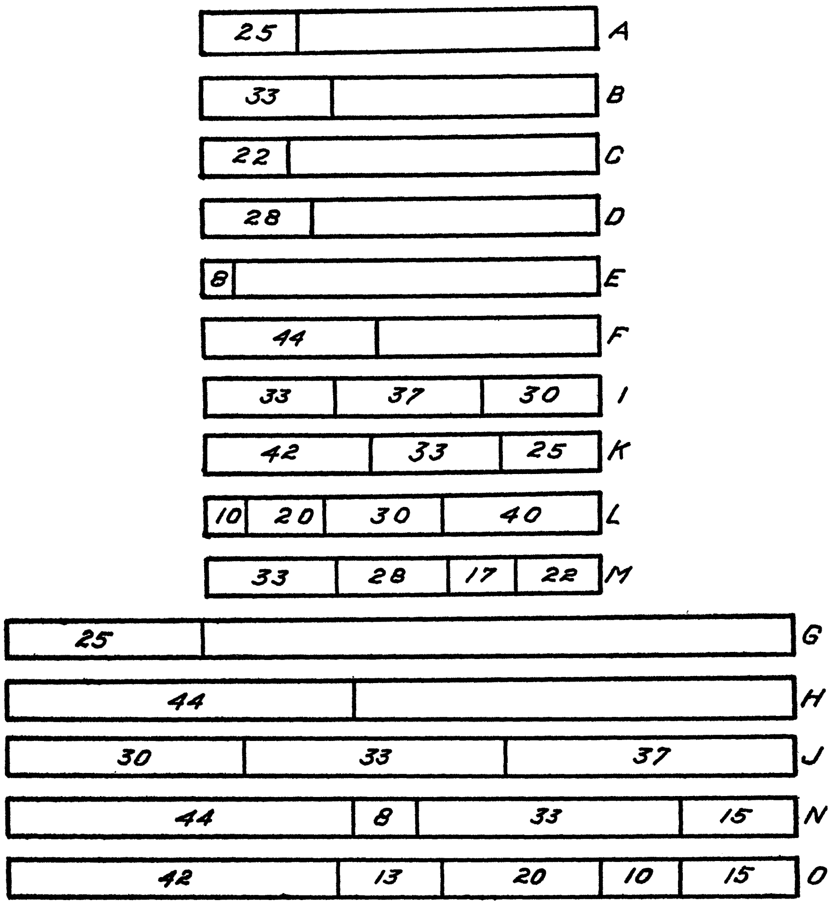

FIGURE II

tating comparisons between bars. At a subsequent meeting of the class, about three-fourths of its members said that this arrangement of the bars was a distinct aid to them in making comparisons. It was definitely planned to make the conditions such that any possible advantage would accrue to the bars so that if any superiority were shown for the circles, there could be no question from this standpoint of its reliability and significance.

On the first day the class consisted of 97 members, of whom 35 were men and 62 women. There were 51 sophomores, 38 juniors, and 8 seniors. Absences made these numbers slightly different on the second day, as indicated in the tables which follow.

RESULTS OF THE EXPERIMENT

The results of the experiment will be presented under the following heads:

1. Rapidity of Judgment
2. Accuracy of Judgment
3. Method of Judgment
4. Popularity and Appeal
5. Conclusions

1. *Rapidity of Judgment*

The rapidity score, based upon the number of circles or bars completed at the end of five minutes, is shown in Table I.

TABLE I

NUMBER OF STUDENTS COMPLETING THE GIVEN NUMBER OF CIRCLES AND BARS IN FIVE MINUTES

Number of Figures Completed in 5 Minutes	Number of Students Completing Circles	Number of Students Completing Bars
5	5	1
6	9	3
7	6	16
8	8	15
9	32	24
10	15	25
11	7	3
12	4	2
13	10	2
14	1	1
15	0	1
Total	97	93
Median	9.6	9.5
Mean	9.2	9.0
S. D. (σ)	2.1	1.5
P. E. of Mean	0.14	0.10

The advantage seems to be slightly in favor of the circles whether judged by the median or the mean. This difference would doubtless be greater, and possibly significant, if it were not for the practice effect of the first day's work. At any rate the data seem to indicate that the criticism that circle diagrams cannot be read as easily or rapidly as bar diagrams is not substantiated.

2. *Accuracy of Judgment*

For studying accuracy of judgment, the most important comparison to consider, the amount of error in "points" was marked in each figure. Thus a judgment of 22 per cent in circle A (25 per cent) was scored as an error of 3 points. Several significant methods of studying accuracy of judgment were then available. They are presented as follows:

(1) Average Error by Number of Subdivisions
(2) Average Error by Sex
(3) Average Error by Subdivisions
(4) Large Errors
(5) Comparison of Individuals
(6) Comparison of Special Groups

(1) *Average Error by Number of Subdivisions*

The mean error in judgment for each sector and subdivision of a bar was computed, the results being summarized as follows:

TABLE II

MEAN ERROR IN JUDGING CIRCLES AND BARS ACCORDING TO NUMBER OF SUBDIVISIONS INVOLVED

Circles or Bars Having	Reference Letter	Mean Error in Judging Circles	Mean Error in Judging Bars
2 Subdivisions....................	A—H	1.21	1.48
3 Subdivisions....................	I—K	1.77	1.54
4 Subdivisions....................	L—N	1.57	1.72
5 Subdivisions....................	O	1.29	2.14
Unweighted Mean..................................		1.46	1.72
Mean, weighted proportional to number of sectors.........		1.44	1.64

The advantage is clearly in favor of the circles for all groups of figures except those having three subdivisions. The general superiority of the circles for accuracy is clear, as shown by a comparison of either the weighted or unweighted mean.

It is especially interesting to notice the increasing accuracy of judgment in the case of the circles as the number of subdivisions is increased; the opposite is true in the case of the bars, where the mean error increases steadily with the number of subdivisions, until the mean error for five subdivisions is almost twice as great for the bars as for the circles.

(2) *Average Error by Sex*

The following table is similar to Table II, but divided on the basis of sex.

TABLE III

MEAN ERROR IN JUDGING CIRCLES AND BARS, BY SEX

Circles or Bars Having	Men		Women	
	Mean Error in Judging Circles	Mean Error in Judging Bars	Mean Error in Judging Circles	Mean Error in Judging Bars
2 Subdivisions..................	1.06	1.38	1.30	1.54
3 Subdivisions..................	1.64	1.43	1.84	1.62
4 Subdivisions..................	1.39	1.69	1.71	1.77
5 Subdivisions..................	1.15	2.34	1.44	1.94

For each sex, this table shows the same characteristics as Table II, as regards comparative accuracy of circles and bars. In addition it shows the greater accuracy in judgment on the part of the men. (Probable errors of the mean for men and women are given in Table VI.)

(3) *Average Error by Subdivisions*

Table IV exhibits in detail the mean error for each sector of a circle and the corresponding subdivision of a bar. The means are the same as given in Table I.

TABLE IV

MEAN ERROR, BY SUBDIVISIONS, BY CIRCLES AND BY BARS

	Figure	Per Cent	Mean Error by Circles	Mean Error by Bars	Error by Circles Less
Two Divisions............	A	25	0.09	0.72	*
	B	33	1.85	1.07	
	C	22	1.13	1.79	*
	D	28	1.30	1.73	*
	E	8	1.38	1.37	
	F	44	1.59	2.16	*
	G	25	0.42	0.60	*
	H	44	1.94	2.38	*
	Mean		1.21	1.48	*
Three Divisions...........	I	33	1.24	0.89	
		37	2.43	2.23	
		30	1.78	1.55	
	J	30	1.50	1.58	*
		33	1.31	0.63	
		37	2.30	2.01	
	K	42	2.07	2.04	
		33	2.16	1.49	
		25	1.10	1.45	*
	Mean		1.77	1.54	
Four Divisions............	L	10	1.07	0.98	
		20	1.08	0.93	
		30	1.17	1.13	
		40	1.29	1.23	
	M	33	2.24	1.85	
		28	1.68	1.77	*
		17	1.84	1.97	*
		22	1.43	1.91	*
	N	44	1.78	2.03	*
		8	1.51	1.74	*
		33	2.15	2.81	*
		15	1.64	2.23	*
	Mean		1.57	1.72	*
Five Divisions............	O	42	1.88	2.74	*
		13	1.68	2.01	*
		20	1.17	2.76	*
		10	0.92	1.17	*
		15	0.81	2.03	*
	Mean		1.29	2.14	*

This table shows that more accurate judgments were expressed by means of circles (marked *) in 20 cases; by bars in 14 cases. It shows that more accurate judgments were expressed by means of circles when there were two, four, or five subdivisions; by bars when there were three subdivisions.

Particular interest attaches to circle M and to the corresponding bar. This is the example used by Secrist in such detail to show the inaccuracy of circle representations.[1] Yet the average error for this circle was 1.80; for the corresponding bar, 1.88.

(4) *Large Errors*

Another method of comparison is to consider the number of "large" errors made by each method. "Large" is a purely relative term, but let it be assumed that errors of only one, two, or three points in judging a diagram are of little consequence; but that errors greater than three points are serious and likely to lead to bad judgments. In other words, let "large errors" be defined as those greater than three points. This is entirely arbitrary but it does not seem unreasonable. The total number of "large errors," thus defined, made by the entire class is shown in the following table.

TABLE V

TOTAL NUMBER OF LARGE ERRORS IN JUDGING CIRCLES AND BARS

Figure	Large Errors in Circles	Large Errors in Bars	Errors in Circles Fewer
A	0	7	*
B	6	5	*
C	2	9	*
D	4	7	*
E	4	3	
F	12	29	*
G	4	6	*
H	22	31	*
I	16	10	
J	12	8	*
K	29	27	
L	24	38	*
M	24	34	*
N	33	68	*
O	9	67	*
Total	201	349	*

In all except five cases (B, E, I, J, K) the superiority of the circles is clearly evident, and in those five the balance in favor of the bars is slight. It is noteworthy that no large errors were made in judging circle A, while seven were made in the corresponding bar; that more than twice as many large errors were made by bars in figures F and N; that four and a half times as many large errors were made by bars in figure C; and that seven and a half times as many large errors were made by bars in figure O, containing five subdivisions. The total number of large errors was seventy per cent greater in the case of bars than of circles.

[1] Secrist, H., *Ibid.*, (Revised Edition), p. 182.

(5) *Comparison of Individuals*

An individual accuracy score was secured by taking the average error for all of the sectors on each paper, both circles and bars. Comparisons were then made between the two papers belonging to the same individual. On account of absences on one day or the other there were only 85 sets of papers that were available for this comparison, which is shown below:

Entire Class	
Better in Circles	57
Better in Bars	28
Men	
Better in Circles	22
Better in Bars	9
Women	
Better in Circles	35
Better in Bars	19

Each sex and the class as a whole did better with circles twice as often as with bars.

(6) *Comparison of Special Groups*

A further summary of individual scores for special groups of students is given in table VI.

TABLE VI
SUMMARY OF INDIVIDUAL SCORES FOR SPECIAL GROUPS

	Circles	Bars
All Students		
Number	97	94
Mean Score	1.41	1.60
Standard Deviation (σ)	0.66	0.61
Probable Error of Mean	0.05	0.04
Men		
Number	35	36
Mean Score	1.27	1.55
Standard Deviation (σ)	0.40	0.51
Probable Error of Mean	0.05	0.06
Women		
Number	62	58
Mean Score	1.50	1.63
Standard Deviation (σ)	0.74	0.63
Probable Error of Mean	0.06	0.06
Mathematics Majors		
Number	8	8
Mean Score	1.50	1.63
Standard Deviation (σ)	0.30	0.38
Probable Error of Mean	0.07	0.09
Students Who Worked Rapidly		
Number	37	34
Mean Score	1.48	1.83
Standard Deviation (σ)	0.44	0.73
Probable Error of Mean	0.05	0.08
Statistics Class		
Number	11	12
Mean Score	1.37	1.48
Standard Deviation (σ)	0.59	0.51
Probable Error of Mean	0.12	0.10

The superiority of the circles appears in each group compared. The superiority of the men over the women is also evident.

Can students with considerable mathematical training judge either circles or bars more accurately than an unselected group? Eight of the class were mathematics majors, six men and two women. As far as conclusions can be drawn from this limited number, it seems that students skilled in mathematics judge slightly better than the general group, but not markedly so. The six men ranked 4, 6, 8, 9, 27, and 29 among the 40 men arranged according to score in judgment of circles. The two women similarly ranked 17 and 25 among the 66 women. These facts seem to indicate that circles are excellent for the general reader—that their utility and accuracy is not limited to those with special mathematical training. Further evidence along this line is suggested by the fact that a seventh grade boy with the circle diagram made a score which would have placed him in the third quartile of the college class tested; while a college professor (not of mathematics!) only succeeded in making a score which would place him in the lowest quartile!

Did the students who worked rapidly work any more accurately than those who studied the diagrams more carefully? To investigate this question the papers of the students who completed ten circles or bars in five minutes (See Table I) were summarized separately. The results indicate slightly less accuracy for the rapid working group than for the entire class, in the case of circles, but the probable errors indicate that this difference can hardly be considered significant. On the other hand, in the case of the bars, the results show significantly less accuracy for the rapid working group.

The entire experiment was also tried with the members of a small class in statistical methods, composed of juniors and seniors. The results were similar to those for the larger class, but the small number makes the results in themselves less reliable.

From every standpoint the conclusion seems to be clear that greater accuracy is indicated by circle representations than by bar representations. And this in spite of the fact that the experiment was arranged so as to give the advantage to the bars.

3. *Method of Judgment*

The third subject of special interest is the method of judgment used. Thirteen men and nineteen women thought they used two of the suggested methods, but indicated one as predominant. Two men claimed to have used all of the first three methods. Table VII gives the number claiming to use each of the four suggested methods either exclusively or predominantly.

TABLE VII

METHOD OF JUDGMENT OF SECTORS OF CIRCLES

Method	Entire Class		Men	Women
	Per cent	Number		
By Areas	25	23	6	17
By Central Angles	23	22	9	13
By Arcs	51	48	18	30
By Chords	1	1	0	1

No single means of judgment was used, although that by arcs on the circumference was most frequent. The distribution for the two sexes is essentially the same. The method by chords, the chief basis of Secrist's extended criticism, was used by only one student, a woman! One other used it as a secondary method in connection with the method of central angles. Conversation with the student who used it exclusively revealed the fact that she had had considerable special training in a similar method of judgment in her work as an artist.

What was the relative accuracy of these four methods of judgment? The answer is found in Table VIII.

TABLE VIII

ACCURACY OF JUDGMENT OF SECTORS OF CIRCLES BY DIFFERENT METHODS

Method	Number of Cases	Mean Error	S. D. (σ)	Probable Error of Mean
By Areas	23	1.40	0.43	0.06
By Central Angles	22	1.35	0.44	0.06
By Arcs	48	1.35	0.47	0.05
By Chords	1	1.03	....	....

Judgments expressed by any one of the three methods commonly used are of practically equal accuracy. There is no significant difference between them.

4. *Popularity and Appeal*

After the conclusion of the experiment, and before it was discussed with them, the students were asked the following question:

> If you were required to draw a diagram showing the relative number of students—seniors, juniors, sophomores, and freshmen—composing the entire student body of the college, would you choose a subdivided circle or a subdivided bar?

Twenty-five students expressed a choice of the bar, seventy-one of the circle.

It is not within the scope of this paper to give examples of the common use of circles to show component parts in the published literature. Reference may be made, however, to the very effective use of 127 circles in the Statistical Atlas of the United States, to their frequent use in Dr. Ayres' *War With Germany*, and in numerous recent school surveys.

Such references, however, are unnecessary since the popularity, common use, and psychological appeal of the circle diagrams is admitted. The principal basis of criticism has been their supposed inaccuracy.

5. *Conclusions*

Referring to the four criticisms summarized on pages 121–122, it may be said that the experiment described in this paper seems to justify the following conclusions:

1. Circle diagrams to show component parts *can be read fully as rapidly and easily* as bar diagrams.
2. Circle diagrams to show component parts *can be read more accurately* than bar diagrams. In addition, accuracy of judgment increases with the number of subdivisions in circles, but decreases in the case of bars.
3. Circle diagrams to show component parts are judged by different methods, approximately 50 per cent of the group considered having used arcs, 25 per cent areas, and 25 per cent central angles. But whatever the *method* of judgment, there is *no significant difference in accuracy of judgment by the different methods.*
4. In view of the three conclusions stated above, the use of circle diagrams to show component parts is worthy of encouragement; they should be recommended, not only on account of their popularity and psychological appeal, but also on the basis of scientific accuracy; their use should be considered a "*compliment to a man's intelligence.*"