The Origin and Early History of Proportion Signs
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The Origin and Early History of Proportion Signs*

By ANNA MARIA BUSSE BERGER

Proportion signs, or, to be more specific, fractions in which a certain number of notes in the numerator are made equal to a different number of notes of the same type in the denominator, are probably found for the first time in Anthonello de Caserta's *Amour m'a le cuer mis* in the MS. Modena, Biblioteca Estense, M.5.24, compiled in Bologna in 1410 or 1411.[1] The earliest theoretical explanation of proportion signs is given by Prosdocimus de Beldemandis in his *Tractatus practice de cantus mensurabilis* of 1408 (1869, 218–19). It has long been established that they came into existence to override the minim equivalence of the French system (see in particular Bent 1980, 370), yet beyond that little is known. Where did this wish to override minim equivalence originate? Why did composers suddenly make use of proportions? Precisely how were proportions used in the late fourteenth and early fifteenth centuries? For example, it is generally assumed that two consecutive proportions are cumulative, since the great late fifteenth-century theorists Johannes Tinctoris[2] and Franchinus Gaffurius (1496, Bk. IV, Ch. 5) say they are. But was this also true in the early fifteenth century? Similarly, according to Tinctoris (1978, 53) and Gaffurius (1496, Bk. IV, Ch. 5) the mensuration within the proportion derives from the mensuration sign preceding the proportion and not from the proportion itself. Was this also the case in earlier times? For example, duple mensuration followed by *sesquialtera* proportion applied on the breve level can result in imperfect or perfect breves in the *sesquialtera* section, depending on whether the original duple mensuration of the breve is

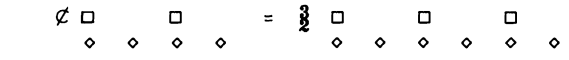
* A shorter version of the paper was read at the fifty-third annual meeting of the American Musicological Society in New Orleans in October 1987.

[1] The French pieces in the source derive from "the largely French repertory of the courts of the schismatic popes Alexander V (elected in Pisa in 1409) and his successor John XXIII." Many of the pieces were, however, composed considerably earlier. See Günther 1980a and 1970.

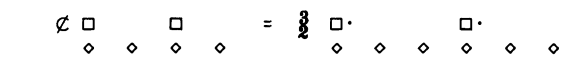
[2] See Tinctoris 1978, where all horizontal proportional relationships are cumulative.

preserved (see Example 1a), or the triple mensuration of the *sesquialtera* takes over and is applied also on the breve level (see Example 1b). In the latter case the *sesquialtera* proportion changes the original binary mensuration of the breve resulting in two perfect breves rather than three imperfect breves. In other words, imperfect time becomes perfect. While this question is of little relevance for the performance of music, it was not only an important issue for fifteenth and sixteenth century theorists, but also played a major role in the development of proportion signs.

Example 1
(a) *Sesquialtera* proportion with imperfect breves



(b) *Sesquialtera* proportion with perfect breves



The present study investigates the use of proportion signs in the theoretical treatises and the most important musical sources of the late fourteenth and early fifteenth centuries. With every theorist or composition, it attempts to find answers to the following questions:

1. Which proportions are used?
2. How are they indicated?
3. Why are only certain proportions used and what is the reason for the great variety of signs used to indicate proportions?
4. Are proportions cumulative?
5. Is the mensuration derived from the proportion sign or rather from the preceding mensuration sign?

With regard to the first question, practically all late fourteenth- and early fifteenth-century theorists describe essentially the same set of proportions (see Table 1). Prosdocimus de Beldemandis (1869, 218–19) and Ugolino of Orvieto (1960, 210–11)[3] list the following five proportions: 2:1, 3:1, 3:2, 4:3, and 9:4. The fifteenth-century anonymous treatise *Iste sunt proportiones*[4] gives the same fractions, but adds 9:8 and 8:3. Other treatises that list all or some of the same

[3] Ugolino uses the same proportions in his compositions (Seay 1955, 152–66).

[4] Scb, f. 142r-v. The treatise shows both French and Italian characteristics. I would like to thank Professor Jan Herlinger for information on the treatise. See also Gallo 1984, 344–46, who provides an excellent summary of some of these treatises. Sca, f. 48v-56r in the same manuscript describes the same proportions.

proportions are: an English treatise from the early fifteenth century (Rvat), the Spanish treatise *Venerabiles domini* (SDI), the mid-fifteenth-century treatise by Georg Erber from Aibling (see Federhofer-Königs 1969), the anonymous *Regule proportionum in quantum ad musicam pertinet* (Vnm), the British manuscript London, British Library, Lansdowne 763 (Lbl), a little known fifteenth-century Provençal treatise written in Hebrew,[5] and the German Anonymous XI, copied before 1471.[6] The musical sources of the late fourteenth and early fifteenth centuries, in particular CH[7] and ModA (edited in Apel 1970), also only make use of precisely the same set of proportions: 2:1, 3:1, 4:1, 3:2, 4:3, 9:4, and 9:8.[8]

In the second half of the fifteenth century, Guilielmus Monachus still describes the same set of proportions (1965, 19–44), although he mentions in addition the inverted versions of all of these fractions. John Hothby's *Quid est proportio*[9] also lists the usual proportions, but in his motet *Ora pro nobis* he enlarges the rhythmic possibilities by including the fractions 5:2, 5:4, and 7:4 (HOb, ff. 26v-27 and Hothby 1964, 4–7). Johannes Tinctoris (1978) alone can be credited with

[5] Anonymous 1975. The author describes 5:4 (but the subsequent explanations make clear that he means 6:4) and the very unusual 17:8, *dupla sesquioctava*, indicated by ⊙, which is normally a sign of either *sesquialtera* on the minim level when set against ○ or *dupla sesquiquarta* when set against ○; it is possible that the author meant either of those proportions. I would like to thank Talya Berger for having translated the treatise for me.

[6] Anonymous XI 1869, 474–75. He describes proportions 16:6 (which is the same as 8:3), and 9:6 (which is the same as 3:2). On p. 475 Anonymous XI gives another list which mentions only 2:1, 3:1, 4:1, 3:2, 4:3, and 9:8, saying that only these are used in performing music.

[7] For a detailed discussion of the Chantilly MS. see Ursula Günther, 1984, 87–118. Günther argues (1984, 107) that "the source of CH was written in Paris. It must then have been brought to Florence by a member of the Alberti family and copied there. If we leave aside all hypotheses, the probability remains that CH was not a southern French original but a testimony to the diffusion of the *Ars subtilior* in Italy and to the humanistic efforts of a Florentine patrician whose family found exile in France." As far as dating is concerned, Günther suggests that the manuscript is an early fifteenth-century Italian copy of late fourteenth-century music and gives the years 1393–95 as the earliest possible date. See also Günther 1980c, 663. A modern edition of the Chantilly manuscript is in Greene 1981.

[8] The following proportion signs are used in ModA: in Anthonello de Caserta's *Amour m'a le cuer mis* (ff. 32v-33r, Apel 1970:1, no.3) 9:6 and 4:2; in Comradus de Pistoria's *Se doulz espour* (f. 31v, Apel 1970:1, no. 15) 3 (that is, 3:2) and 2 (that is 2:3); the numbers 3 and 2, indicating *sesquialtera* and *dupla*, respectively, are also used in Johannes Simon Hasprois's *Ma douce amour* (f. 28r, Apel 1970:1, no. 41) and Bartholomeus de Bononia's *Que pena major* (ff. 37r-36v, Apel 1970:3, no. 301). In CH, Baude Cordier's *Belle, bonne sage* (f. 11v, Greene 1981:18, no.1) uses 8:9, Cordier's *Tout par compas* (f. 12, Greene 1981:18, no. 2) uses 3:1, 3:2, and 4:3.

[9] HOa, f. 26. The proportions listed earlier in the treatise do not refer to temporal measurements.

TABLE 1

Proportions in Theoretical Treatises, ca. 1400–1450

Theorist or Manuscript Siglum	2:1	3:1	4:1	3:2	4:3	9:8	9:4	8:3	unusual other proportions
Prosdocimus	x	x		x	x		x		
Ugolino	x	x		x	x		x		
"En alius tractatus" (Scb)	x	x		x	x	x	x	(x)	16:6 = 8:3
"Compilatio de proportionibus" (Sca)	x	x		x	x	x	x	x	
Rvat	x	x		x	x	x	x		
SDI	x			x	x	x			8:1; 9:1
Georg Erber	x	x		x	x	x	x		
Vnm	x	x		x					
Lbl	x	x	x	x	x	x			
Anonymous 1975	x	x	x	x	x	x			17:8
Anonymous XI	x	x	x	x	x	x	(x)		16:6 = 8:3, and 5:4

bringing the rhythmic proportions on a par with the harmonic ones. He describes twenty-five different proportions, all of which can, in addition, appear in inversion. Tinctoris's follower, Franchinus Gaffurius, perfected Tinctoris's system and included even more proportions.[10]

In addition to fractions, composers and theorists of the period used mensuration signs, coloration, and Italian note shapes to indicate proportions, often combining these devices to avoid ambiguity. Coloration and Italian note shapes have received considerable attention in the secondary literature,[11] and were used by composers and theorists alike to indicate the common proportions already enumerated: 2:1, 3:1, 3:2, 4:3, 9:4, and 9:8.[12] The practice of indicating proportions through mensuration signs, on the other hand, still awaits scholarly attention. In most cases a combination of different mensuration signs presupposes minim equivalence, as one might expect. Nonetheless mensuration signs were also used to indicate true proportions—that is, to override minim equivalence—often enough for us to conclude that such use represented an important practice in the late

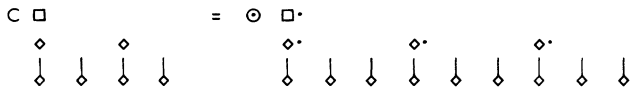
[10] Gaffurius 1496, Bk. IV. See also Clement A. Miller's detailed discussion of Gaffurius's proportions (Miller 1968).

[11] E.g., Wolf 1904, 1:289–327; Apel 1953, 403–35; Gallo 1984, 336–39.

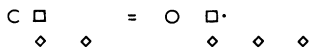
[12] See, for example, Philipoctus de Caserta's *En remirant* in ModA (ff. 34v-35r, Apel 1970:1, no. 79), which has a 9:4, 2:3, and 3:4 proportion. See also the *Tractatus figurarum*, formerly ascribed to Philippus de Caserta and recently edited in Schreur 1987.

Example 2

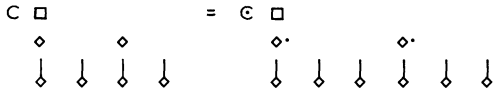
(a) *Dupla sesquiquarta* (9:4)



(b) *Sesquialtera* (3:2)



(c) *Sesquialtera* (3:2)



fourteenth and fifteenth centuries.[13] In many cases the mensuration signs are used without any additional explanations, in others a canon or coloration is added. Sometimes, too, mensuration signs are combined with Italian note shapes to remove ambiguity. It is important to stress that the proportional use of mensuration signs generally presupposes breve equality, that is, the lower note values are to be adjusted within an unchanging breve.[14] Marchettus (1961, 86, 66; Gallo 1984, 335) had already related the term *proportio* to one *tempus*. Similarly, the Hebrew treatise in Anonymous 1975 (58–76) enclosed all proportions within the central breve. As a result, the note values compared could vary: for instance, a 9:4 proportion will necessarily compare minims (see Example 2a), while a 3:2 proportion can either compare semibreves (see Example 2b) or minims (see Example 2c).

Table 2 lists all pieces which set perfect against imperfect time with the *sesquialtera* on the semibreve level, that is, presupposing breve equality as in Example 2b. All composers with the exception of Galiot use ○ to indicate *sesquialtera*. Galiot uses ⊙ instead (CH f. 37; Greene 1981:19, no. 53). Three of the pieces make use of a canon, obviously in order to avoid minim equivalence between the different mensurations.[15] Two use a red ○ to indicate the *sesquialtera* proportion.[16] Yet

[13] Berger 1986 describes the survival of this tradition in theoretical treatises in the fifteenth and early sixteenth centuries.

[14] The only sign which is not always associated with breve equality is ↄ. See below.

[15] Johannes de Janua in *Une dame requis*, (ModA, f. 12r, Apel 1970:1, no. 45, mm. 20, 32, and 65); M. de Sancto Johanne, *Je chante ung chant*, uses a red ○ set against ☾ to indicate a *sesquialtera* proportion (CH, f. 16, Greene 1981:18, no. 9); Johannes

another piece uses both the sign of perfect time, major prolation against imperfect time, major prolation and red coloration to ensure that nine semibreves are set against six (see Example 3a).[17] The same piece uses coloration in the normal way for ☾ : two perfect semibreves are replaced with three imperfect ones (Example 3b). In order to achieve the effect shown in Example 3a, the mensuration sign is absolutely necessary. On the other hand, the composer could not have used the sign without coloration, since he presupposes minim equivalence throughout the piece when perfect and imperfect time are set against each other. Finally, we have three pieces which indicate a *sesquialtera* proportion by ○ alone.[18]

Table 3 lists all pieces with imperfect time, minor prolation set against imperfect time, major prolation resulting in a *sesquialtera* proportion on the minim level when the breves or semibreves (there is no difference between breve and semibreve equality here) are equally long (as in Example 2c).[19] In three of the four cases the *sesquialtera* proportion is indicated by the mensuration sign alone; only Johannes de Janua's piece adds a canon.

The 4:3 proportion is shown by the reversed C (☽), which can follow perfect time, minor prolation (see Example 4a) or imperfect time, major prolation (see Example 4b). While all other mensuration

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Galiot, *Le sault perilleux*, (CH, f. 37, Greene 1981:19, no. 53, m. 19 in Contratenor and m. 22 in Cantus). Franchois Lebertoul indicates a *sesquialtera* proportion through ○ and a canon in *Depuis un peu un joyeux parlement* (Ob, f. 122v, Reaney 1959:2, 43, m.8).

[16] Cooke uses a red ○ in a Credo to signal a *sesquialtera* proportion (OH, ff. 78v-79, Hughes and Bent 1969:2: no. 92, m. 56); see also Damett, Credo, (OH, ff. 79v-80, Hughes and Bent 1969:2, no. 101); see also M. de Sancto Johanne, *Je chante ung chant*, (CH, f. 16, Greene 1981:18, no. 9, mm. 12, 25, and 40).

[17] Lyonel's Credo, (OH, ff. 70–71, Hughes and Bent 1969:1:2, no. 83, m. 47) also combines ○ with the red number 3 and coloration to indicate a *sesquialtera* proportion.

[18] Matheus de Perusio, *Dame que j'ayme*, (ModA, ff. 10v-11r, Apel 1970:1, no. 55, mm. 28, 51, and 71); *Ung lion say*, an anonymous piece in CH, f. 28v (Greene 1981:18, no. 35, m. 34) uses ⊙ in Contratenor and Cantus (but meaning ○) against ☵ in the Tenor with breve equality only when no semibreves occur. Of particular interest is the piece *Salve mater salvatoris*, ascribed to both Dunstable (ModB, f. 116v-117, Dunstable 1970, no. 62, m. 51) and Leonel (Tr92, f. 193v-195). In ModB the scribe uses ○ alone to indicate the *sesquialtera* proportion, in Tr 92, ff. 193–195 ○ is combined with coloration, while Tr 92, f. 215 uses only coloration.

[19] The following pieces use ☵ : ☾ as a *sesquialtera* proportion: Johannes de Janua in *Une dame requis*, (ModA, f. 12r, Apel 1970:1, no. 45, mm. 12, 39; in m. 66 ○: ⊙ is used to show a *sesquialtera* proportion); Matheus de Perusio in *Le greygnour bien*, (ModA, ff. 32r-31v, Apel 1970:1, no. 51), Cantus and Tenor are written in ☵, contratenor in ☾ with equally long semibreves; the anonymous *Ung lion say*, (CH, f. 28v, Greene 1981:18, no. 35, m. 28ff.); Damett's Gloria, (OH, ff. 33v-34, Hughes and Bent 1969:2, no. 39, m. 72).

TABLE 2

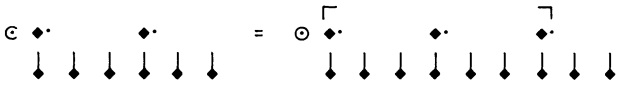
Sesquialtera on the Semibreve Level

ModA		
Johannes de Janua	*Une dame requis*	○ = C, with canon
Matheus de Perusio	*Dame que j'aym*	○ = C, without canon
Anthonello de Caserta	*Bauté parfaite*	⊙ = Ȼ, with coloration
CH		
M. de Sancto Johanne	*Je chant ung chant*	○ = C, with canon
Johannes Galiot	*Le sault perilleux*	Ȼ = C, with canon
Anonymous	*Ung lion say*	○ = C, without canon, but equal breve only when no semibreves are involved
OH		
Cooke	*Credo, ff.* 78v–79	⟦○⟧ = C, without canon
Lyonel	*Credo, ff.* 70–71	⟦○⟧ and red notes = C
Damett	*Credo, ff.* 79v–80	⟦○⟧ = C
ModB		
Dunstable	*Salve mater salvatoris*	○ = C, without canon

Square brackets indicate coloration; i.e., the symbols are written in red in the manuscript.

Example 3

(a) Anthonello de Caserta, "Bauté parfaite," (ModA, 13r, Apel 1970:I, no. 4, m. 17)



(b) Anthonello de Caserta, *Beauté parfaite*, (ModA, f. 13r, Apel 1970:1, no. 4, m. 6)

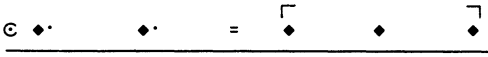


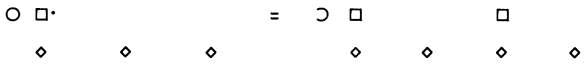
TABLE 3

Sesquialtera on the Minim Level

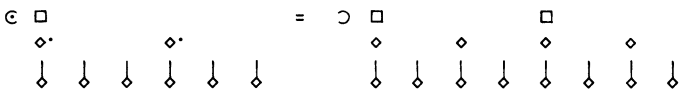
ModA		
Johannes de Janua	*Une dame requis*	C = Ȼ, with canon
Matheus de Perusio	*Le greygnour bien*	C = Ȼ, without canon
CH		
Anonymous	*Ung lion say*	C = Ȼ, without canon
OH		
Damett	*Gloria, ff.* 33v–34	C = Ȼ, without canon, but uses Ȼ instead of C and Ȼ instead of Ȼ

EXAMPLE 4

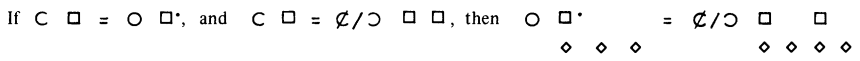
(a) *Sesquitertia* (4:3)



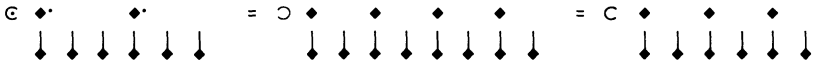
(b) *Sesquitertia* (4:3)



(c) *Sesquitertia* (4:3) and *Dupla* (2:1) with Breve Equivalence



(d) Johannes Ciconia, *Sus une Fontayne*, (ModA, ff. 28–27v, Bent and Hallmark 1985, no. 45)



(e) Credo (OH, ff. 78v–79, Hughes and Bent 1969: 2, no. 92, m. 18)

signs used as proportions presuppose breve equality, ☽ does not necessarily do so in the late fourteenth and early fifteenth century.[20] Breve equality between ☽ and ○ can be assumed only if ☽ is also used in the same piece as a sign of *proportio dupla* (see Example 4c). I have been able to find only two pieces where ☽ indicates a *proportio dupla*.[21] On the other hand, Johannes Ciconia's *Sus une Fontayne*, for example, combines ☽ with ☾ and ☉ (see Example 4d) in such a way that even though three minims of ☉ are set against four of ☽, a minim of ☾ is equal to a minim of ☉. We can thus conclude that ☽ was not always associated with breve equality in the *ars subtilior*.[22]

[20] I would like to thank Dr. Wolf Frobenius who first brought this point to my attention.

[21] Matheus de Perusio's *Le greygnour bien*, (ModA, ff. 32r-31v, Apel 1970:1, no. 51, m. 21); and the anonymous *Par le grant sens* (Wolf 1904, 2:46–48).

[22] It is likely that from the early fifteenth century on ☽ and ¢ were used interchangeably to indicate *proportio sesquitertia* and *dupla*. Alejandro E. Planchart has found in AO a version of a Gloria by Johannes Brasart that uses ☽ to indicate a 4:3 proportion, while BL and Tr 90 and Tr 93 all use ¢ (Planchart 1981, 41–42).

TABLE 4

Sesquitertia on the Semibreve and Minim Level

ModA		
Anthonello de Caserta	*Tres nouble dame*	○ = ☽, and ☾ = ☽, with canon
Johannes Ciconia	*Sus une Fontayne*	☾ = ☽, without canon
Philippot de Caserta (Johannes Galiot)	*En atendant souffrir*	☾ = ☽, without canon
Matheus de Perusio	*Dame souvrayn*	☾ = ☽, without canon
Matheus de Perusio	*A qui fortune*	○ = ☽, without canon
Matheus de Perusio	*Se puor loyaulement servir*	☾ = ☽, without canon
Anonymous	*En un vergier*	☾ = ☽, without canon
CH		
Trebor	*Quant joyne cuer*	☾ = ☽, without canon
Suzoy	*Prophilias*	☾ = ☽, without canon
Philippot de Caserta	*Par le grant senz d'Adriane*	☾ = ☽, without canon
Anonymous	*Je ne puis avoir plaisir*	○ = ☽, with Italian note shapes
Anonymous	*De tous les moys*	○ = ☽, with coloration
OH		
Anonymous	*Credo, ff. 62v–63*	○ = ☽, without canon
Bittering	*Credo, ff. 66v–67*	⊙ = ☽, without canon
Power	*Credo, ff. 68v–69*	○ = ☽, without canon
Cooke	*Credo, ff. 78v–79*	○ = ☽, without canon

☽ was used more often during this period than any other sign (see Table 4).[23] Because the sign was almost exclusively associated with the *sesquitertia* proportion, few composers found it necessary to clarify its meaning by additional signs: Anthonello de Caserta (*Tres nouble dame*) adds a canon; two pieces combine it with Italian note shapes[24] and one with coloration.[25] There are several pieces where the

[23] Anthonello de Caserta, *Tres nouble dame*, (ModA, f. 28v, Apel 1970:1, no. 8, m. 20); Johannes Ciconia, *Sus une fontayne*, (ModA, ff. 27r-26v, Apel 1970:1, no. 14, m. 1); Philippot de Caserta (Johannes Galiot), *En atendant souffrir*, (ModA, f. 20r, Apel 1970:1, no. 28, m. 48); Matheus de Perusio, *Dame souvrayne*, (ModA, f. 38r, Apel 1970:1, no. 56, m. 8); idem, *A qui fortune*, (ModA, f. 43r, Apel 1970:1, no. 61, m. 16); idem, *Se pour loyaulement servir*, (ModA, f. 43v, Apel 1970:1, no. 69, m. 26); Anonymous, *En un vergier*, (ModA, f. 18v, Apel 1970:2, no. 145, m. 1); Trebor, *Quant joyne cuer*, (CH, f. 31, Greene 1981:18, no. 40, m. 55); Suzoy, *Prophilias*, (CH, f. 35v, Greene 1981:18, no. 49, m. 11); Philippot de Caserta, *Par le grant senz d'Adriane*, (CH, f. 37v, Greene 1981:19, no. 54, m. 1); Anon., *Je ne puis avoir plaisir*, (CH, f. 24, Greene 1981:18, no. 25, m. 1); Anon., Credo, (OH, ff. 62v-63, Hughes and Bent 1969:1, no. 75, m. 72); Bittering, Credo, (OH, ff. 66v-67, Hughes and Bent 1969:1, no. 79, m. 164).

[24] Philipoctus de Caserta, *En remirant*, (ModA, ff. 43v-35r, Apel 1970:1, no. 79, m. 12); the anon. *Je ne puis avoir plaisir*, (CH, f. 24, Greene 1981:18, no. 25, m. 2).

[25] Anon. *De tous les moys*, (CH, f. 48, Greene 1981:19, no. 76, m. 32).

TABLE 5

Dupla Sesquiquarta (9:4) on the Minim Level

ModA		
Johannes de Janua	*Une dame requis*	C = ⊙, with canon
CH		
Cunelier	*Se Geneive, Tristan*	O = ⊙, with canon

TABLE 6

Sesquioctava (9:8) on the Minim Level

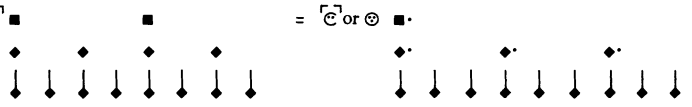
CH		
Johannes de Altacuria	*Se doit il plus*	☽ = ☾, with canon
Johannes Galiot	*Se sault perilleux*	⊙ = C, with canon; since the piece is diminished, semibreves are compared

combination of ☽ with coloration results in a 2:1 proportion when related to the initial perfect time, e.g., in an anonymous Credo (see Example 4e above).[26]

Table 5 lists two pieces with *dupla sesquiquarta*, that is 9:4— ⊙ set against C gives a 9:4 proportion on the minim level (see Example 2a).[27] Both pieces use a canon.

Two pieces in CH use the sesquioctava proportion (see Table 6). In the first piece, Johannes de Altacuria's *Se doit il plus* nine minims of ☾ are set against eight of ☽ (see Example 5). Again, the mensuration sign is elucidated by a canon. It is logical that the composer has compared ☾ to ☽, since eight minims within one breve obviously

EXAMPLE 5
Sesquioctava (9:8) in Johannes de Altacuria's "Se doit il plus" (CH, 15v, Greene 1981:18, no. 8, m. 18)



require diminution. What is surprising is that he did not use ⊙ or ⊙, even though three perfect semibreves are compared to four imperfect

[26] A similar situation obtains in *En albion de fluns*, (CH, f. 47v, Greene 1981:19, no. 75, m. 26).

[27] Johannes de Janua, *Une dame requis*, (ModA, f. 12r, Apel 1970:1, no. 45, m. 66) and Cunelier, *Se Geneive, Tristan*, (CH, f. 41v, Greene 1981:19, no. 63, m. 38).

ones. The other piece with *sesquioctava*, Galiot's *Le sault perilleux*[28] uses precisely this sign of perfect time with three dots (⊙). This piece also includes a canon.

Goscalch's *En nul estat* (CH, f. 39v, Greene 1981:19, no. 58) and Anthonello de Caserta's *Dame d'onour enqui* (ModA, f. 40v, Apel 1970:1, no.5) use two binary or ternary numbers on top of each other, the upper indicating prolation, the lower time. Johannes Wolf pointed out that such usage was described by Johannes Ciconia;[29] I have found it mentioned even earlier in the Berkeley Manuscript,[30] and then by Bartolomeus Ramis de Pareja (1901, 82) and Nicolaus Burtius (1975, 135). None of these theorists elaborate on this usage. However, from a study of the two compositions in which it appears makes clear that it was reserved for vertical and simultaneous combinations of different mensurations under breve equality. In Goscalch's *En nul estat*,[31] a breve of perfect time and major prolation ($\frac{3}{3}$) is made equal to a breve of perfect time and minor prolation ($\frac{2}{3}$) (see Example 6a), while imperfect time, minor prolation is always diminished by half—in other words, it substitutes ☽ (see Example 6b). Precisely the same temporal relationships are found in Anthonello de Caserta's *Dame d'onour enqui* with the difference that Anthonello never uses perfect time, major prolation.

It will come as no surprise that theoretical treatises also make extensive use of mensuration signs as proportions. In a revision of the *Ars (musicae)* by Johannes Boen (1972, 44) we find a table (see Figure 1)

[28] CH, f. 37, Greene 1981:19, no. 53, m. 3. Since the piece is sung in diminution, semibreves and not minims are compared.

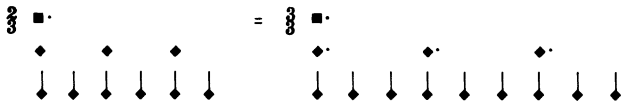
[29] Wolf 1904, 95–96. The text is printed in Anonymous de la Fage 1862, 387f. See also Wolf 1900, 202.

[30] "Ciphers of ternary and binary numbers are usually to be placed, one directly over the other. The lower signifies the tempus, the upper the prolation." ("Item solent poni cifre numeri ternarii et binarii, una sopra aliam directe. Inferior designat tempus, superior vero prolacionem." (In Goscalcus 1984, 170–71, "solent" is incorrectly translated as past tense.) The treatise is dated 1375 and therefore represents the earliest known discussion of these signs. It is striking that both the theorist, who is identified in the MS. Catania, Biblioteche Riunite Civiche e A. Ursino Recupero D 39 as "Goscalcus francigenus," and the composer identified in CH as Goscalch use these extremely rare mensuration signs. This lends further support to Klaus-Jürgen Sachs's hypothesis that the author of the Berkeley MS. was indeed the composer Goscalch (see Sachs 1974, 184). Ellsworth (Goscalcus 1984, 13–15) was able to add further evidence pointing to Goscalch's authorship, but was reluctant to decide firmly that Goscalch was the author. He seems to have been unaware that the composer and theorist both make use of these rare signs. According to Ursula Günther (1980b, 543–54) Goscalch, along with other composers represented in Chantilly, might have been a papal singer.

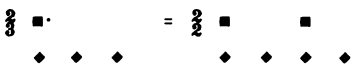
[31] See also Günther's perceptive discussion of this piece (Günther 1960, 283–85). A canon instructs the singer to sing some sections in augmentation or diminution.

Example 6

(a) Goscalch, *En nul estat*, (CH, 39v, Greene 1981:19, no. 58)



(b) Goscalch, *En nul estat*



to exemplify harmonic proportions. Yet the signs used by the scribe are exclusively associated with temporal mensurations. He uses ⊙ as a sign of *proportio sesquioctava*, just as Galiot did in *Le sault perilleux* (CH, f. 3, Greene 1981:19, no. 53). The ☾ when related to the circle with three dots stands for *sesquialtera*, which is justified in view of the fact that time has been made perfect and three dots are used to show major prolation (see Example 7). ☽ stands, as usual, for *sesquitertia*. Gallo suggests that "the working over of the treatise was connected with the theory and practice of *ars subtilior* between the end of the 14th and the beginning of the 15th century" (Boen 1972, 14).

Other theorists who use mensuration signs as proportions are: the anonymous author of an early fifteenth-century English treatise;[32] Ugolino d'Orvieto (in his compositions)[33]; the Hebrew author of Anonymous 1975[34], probably a student of Jean Vaillant who wrote

[32] Rvat, f. 51. The author uses ⊙ for *sesquioctava* and ☽ for *sesquitertia* proportion. I would like to thank Professor John Daverio for having made the microfilm available to me.

[33] Ugolino uses ⊙ (when compared to ☾) as a sign of *sesquialtera* on the minim level (see Example 2c) and ⊙ (when compared to ☾) as a sign of *dupla sesquiquarta* (see Example 2a) in his compositions. (Seay 1955, 152–66.) On the other hand, he also uses mensuration signs as proportions based on minim equivalence. For a detailed discussion, see Berger 1986, 17. It can well be imagined that a scribe placed the mensuration signs in Ugolino's compositions without Ugolino's knowledge, since we know that the composer prefered the use of fractions. See Ugolino 1960, 211, and p. 420, n. 45 below. For a discussion of Tinctoris's fractions, which suffered a similar fate, see p. 426ff. below.

[34] The author (pp. 55ff.) describes ☽ as a sign of *sesquitertia* and ⊙ as a sign of *dupla sesquioctava* (that is, 17:8; I think he meant 9:4, which is commonly associated with ⊙), and ⊙ for *sesquioctava* (that is 9:8). That the author was slightly confused can also be gathered from the fact that, on the one hand, he mentions a proportion *sesquiquarta* (that is 5:4), but on the other hand, gives an example where six and not five minims are set against four (ibid., 64–6).

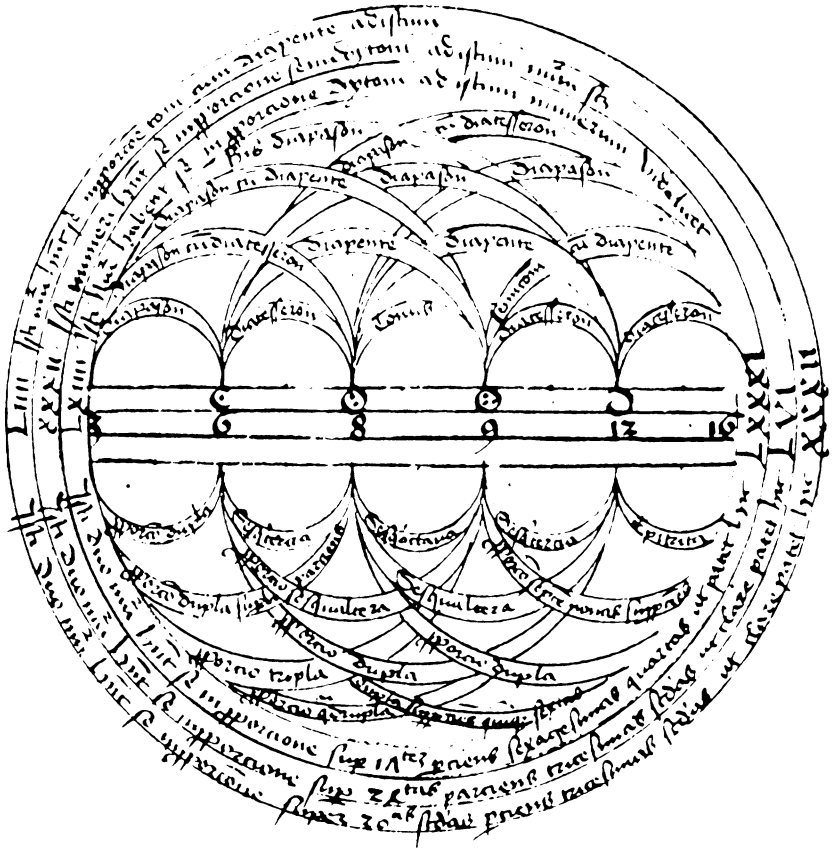


Figure 1 Johannes Boen, *Ars (musicae)*

down his treatise in the mid-fifteenth century; Coussemaker's Anonymous XI, whose treatise probably dates from the same time;[35] John

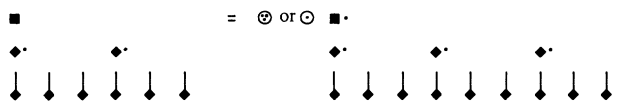
[35] Anonymous XI lists ☽ for *sesquitertia*, ☉ for *proportio sesquioctava*, that is, 9:8, and ☩ for an 8:3 proportion, which is, again based on breve equality when compared to ○ (see Example 8).

Example 8

Dupla superbipartiens tertias, 8:3

He also gives ☽ as a sign of *proportio sesquiquarta*. Neither the sign nor the proportion are mentioned anywhere else (Anonymous XI 1869, 471–75). For dating see Gushee 1980, 444.

Example 7
Sesquialtera



Hothby;[36] and the anonymous fifteenth-century author in Ccc.[37] I have traced a number of late fifteenth-century theorists who still use precisely the same set of mensuration signs to indicate proportions based on breve equality (Berger 1986). Thus, the anonymous author of PEc makes consistent use of mensuration signs as proportions, always presupposing breve equality. For example, ⊙ is used after C to produce a 9:4 proportion between minims (f. 8or, see Example 2a), and ⊙ is used after ○ to produce a 9:8 proportion between minims[38] (see Example 5).

Tinctoris, on the other hand, is extremely critical of composers who use mensuration signs to indicate proportions. But he is clearly fighting a popular tradition of indicating, for example, a *sesquialtera* proportion through the sign of major prolation or a *sesquitertia* by ☽. "But I pray them to spare me, since these signs are so frivolous, so wrong, and so far from all appearance of reason that I have not believed them worthy of example."[39] Ironically, the scribe of one source of *Proportionales musices* signals *sesquialtera* through a mensuration sign, namely imperfect time, major prolation with coloration (Fl, f. 8v). However, this does not necessarily mean that the scribe did not understand what he was writing. Tinctoris's critical remarks on the use of mensuration signs as proportions appear only towards the end of the treatise (Bk. III, Ch. 3), while the *sesquialtera* example occurs in Book I, Chapter 2. One can well imagine that the scribe realized his mistake, but was reluctant to do any corrections in order to keep a clean copy.

[36] Hothby describes a ◡ for *sesquialtera*, ◡ for *subsesquialtera*, ☽ for *sesquitertia* and ⊙ for *subsesquitertia*, and the very unusual ☉ for *sesquioctava* and ☽ for *subsesquioctava* (HOa, f. 26r).

[37] The manuscript gives ☽ for *sesquitertia*, ☽ for *dupla sesquitertia* (7:3), ☉ for *dupla sesquiquarta*, and ⊙ for *subsesquioctava* (8:9), ff. 12–13.

[38] PEc, f. 79r. In the manuscript C is not cut, either because of an error, or in deference to English usage.

[39] "Sed mihi deprecor parcant quoniam haec signa adeo frivola, adeo erronea adeoque ab omni rationis apparentia sunt remota, ut nec exemplo digna crediderim." (Tinctoris 1978:2a, 48; trans. Tinctoris 1979, 36.) Similarly, Gaffurius 1496 (Book IV, Ch. 2 and Ch. 5) disapproves of mensuration signs to indicate proportions.

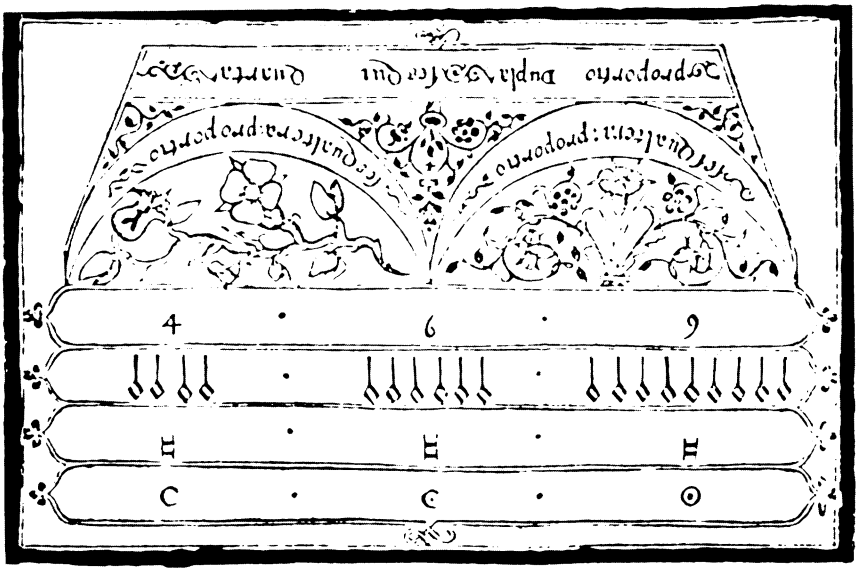


Figure 2 Diagram of mensural proportions from Chapter 16 of Spataro's *Tractato di Musica* (1531)

Giovanni Spataro inserted a diagram in his *Tractato di musica* of 1531 (Ch. 16) in which mensuration signs still have the same proportional function (see Figure 2): on the assumption of breve equality, the *sesquialtera* relationship on the prolation level is found between minims of C and C, and on the *tempus* level between C and ⊙, while the *dupla sesquiquarta* (9:4) relationship is found between minims of C and ⊙. Similarly, Pietro Aaron states in his *Lucidario in musica* of 1545 (f. 12v) that the circle after the semicircle indicates a *sesquialtera* proportion.

In sum, we have seen that there is a tradition of indicating proportions through mensuration signs in both practical and theoretical sources, which proportions result from a combination of basic mensuration signs, without or with the stroke of diminution, and presuppose that the duration of the breve remains constant.

We have further seen that early theorists and composers, with very few exceptions, used only those proportions listed in Table 1. Why did early theorists and composers use only these proportions and not the other ones later described by Tinctoris? What do the proportions in Table 1 have in common? Fourteenth- and fifteenth-century theorists certainly never restricted the number of ratios when describing the intervallic system, but instead presented the whole set as found in Boethius. One might suppose that the temporal proportions chosen corresponded to the most important harmonic intervals, but

this does not explain the prominent occurrence of 9:4. Nor could one claim that these proportions were selected because they were the easiest to perform. All of the fractions described by the early theorists do have one feature in common: they, and only they, are the proportions that can be shown as a combination of mensuration signs with breve equality assumed. The fact that the earliest theorists mentioned only those proportions that can be indicated by means of mensuration signs, and that they listed all proportions that can be thus indicated, cannot be mere coincidence. I propose that proportion signs were introduced not only to override the minim equivalence of the French notational system, but also to provide an arithmetically precise notation to circumvent the ambiguity that arises from the combination of different mensuration signs.

When and where did this happen? Gallo (1984, 334–36) has described how Franco, Marchettus, and Johannes de Muris used the term *proportio* when comparing different note values with each other. In the narrower sense described above the idea can be traced back to the writings of Johannes de Muris, who, in the second book of his *Notitia artis musicae*, written in 1321, advocated division of the breve into anywhere from two to nine semibreves of equal duration :[40] "A song will be made of 2, 3, 4, 5, 6, 7, 8, 9 equal semibreves of the same shape."[41] Another source of the concept was, of course, the Italian notational system which permitted *sesquitertia* and *sesquialtera* proportions by combining different mensurations (see Gallo 1984, 304–27 and Long 1981, 54–97). Yet the Italian notational system could not allow the occurrence, say, of a 9:4 proportion, since the imperfect time had to be one-third shorter than the perfect one, there being no provision for equality in the breve between perfect and imperfect time.[42]

In short, it can well be imagined that composers in the second half of the fourteenth century were inspired to use proportions by the writings of Johannes de Muris and, in part, by the Italian notational

[40] See Michels 1970, 106–8 and Gallo 1984, 272–78.

[41] "Fiet igitur cantus ex 2, 3, 4, 5, 6, 7, 8, 9 semibrevibus equalibus eiusdem figurae." Johannes de Muris 1972, 104. Michael P. Long (1981, 35–47) has suggested a different reading of this passage: he believes that de Muris is describing semibreves which have the same shape rather than length. However, I believe that de Muris states clearly two different things in this sentence: first, that the semibreves are equal (and with this he can only refer to their length), and second, that they do have the same shape. See also Ristory (1987, 98) who also argues that the semibreves are equally long.

[42] Gallo 1984, 312–14. I would also like to thank Professor Leeman L. Perkins for a stimulating letter on this subject.

system. But why did they select only certain divisions of the breve from de Muris's system, ignoring the division into five and seven equal semibreves entirely? The answer can be found in the notational system of the period: before the introduction of the fractions in the early fifteenth century, there was simply no method available to indicate, let us say, a 5:3 or 7:4 proportion. Composers were forced to use only those proportions which they could indicate, either through mensuration signs, different note shapes, or coloration. The author of the late fourteenth-century treatise *Tractatus figurarum*, attributed in some sources to Philip de Caserta, describes precisely this struggle to find an adequate notational tool for the technique of combining various mensurations under breve equality:

> And granted the masters instructed us in these note shapes [duplex long, long, breve, semibreve, and minim] and also in the four principal mensurations; namely, in perfect time of major prolation and in imperfect time of the same, in perfect time minor prolation and in imperfect time of the same. Yet they did not teach us how we should discant perfect time of minor prolation against imperfect time of minor prolation, and visa versa, and so on for the individual *tempora* which will clearly be shown one-by-one below. Because it would be very incongruous that it which can be performed could not be written and to show this clearly, I took care to organize this little treatise.[43]

The author of this treatise sought to overcome this problem by developing various note shapes which allow to indicate precisely the same set of proportions discussed above. But these note shapes were never used with much consistency (see von Fischer 1956, 118–21; Gallo 1984, 336–9). Walter Odington describes the situation perhaps best when he says: "There are as many inventors of new note shapes as there are writers of music." ("Quot [sunt nota]tores tot sunt novarum inventores figurarum." Odington 1970, 42.) The author of the revised Boen manuscript and the composers in CH and ModA took another path, associating mensurations signs with proportions. But the use of mensuration signs to indicate proportions was problematic, since mensuration signs were usually combined with minim

[43] "Et licet magistri instruxerunt nos in his figuris ac etiam in quatuor mensuris principalibus, videlicet in tempore perfecto majoris prolationis et in tempore imperfecto ipsius, in tempore perfecto minoris prolationis et in tempore imperfecto ipsius. Tamen non docuerunt quomodo super tempus imperfectum minoris discantare deberemus perfectum minoris, et e converso, et sic de singulis temporibus quod clare singulariter inferius patebit. Quia esset multum inconveniens quod illud quod potest pronuntiari non posset scribi et clare ostendere tractatum hunc parvulum ordinare curavi." Schreur 1987, Ch. 2.4–2.6.

equality. Some of the problems inherent in the use of coloration (Apel 1953, 126–44) were perceptively discussed by the author of the Berkeley Manuscript:

> From this, it can be deduced that sometimes the tempus alone is diminished by varying the colors, sometimes only the prolation, and sometimes both together. If one wishes to indicate diminutions by changing colors (for example), let him take care to accomplish it without becoming involved in a contradiction.[44]

Precisely because mensuration signs, note shapes, and coloration could all be interpreted differently, they were often combined with each other and with canons in the hope of eliminating any ambiguity.

It is clear that theorists were troubled by the lack of a notational device to indicate proportions unambiguously. The binary and ternary numbers written on top of each other, the upper denoting prolation and the lower time, as described by the author of the Berkeley Manuscript, Ciconia, Ramis, and Burtius, can be seen as another attempt for combining mensuration signs under breve equality, since they are exclusively reserved for showing proportional relationships. Considering all these confusing and ambiguous methods to indicate proportions, it is not astonishing that fractions were greeted with enthusiasm by Prosdocimus de Beldemandis (1966, 141–42) and Ugolino, the latter of whom saying: "And there is no shortcoming in them [fractions], but in the others [mensuration signs] there can occur error and ambiguity . . . We like the numbers better, because through them proportions can be shown better."[45] As we know, this did not prevent Ugolino or the scribes who notated his music from using mensuration signs to indicate proportions in his compositions.

The Turin Manuscript (Tn), completed in the second decade of the fifteenth century, provides further proof for the hypothesis that the introduction of the fraction resulted in the use of proportions not naturally inherent in the mensural system, that is, numbers not

[44] "Item ex hiis potest elici quod quandoque tempus solum diminuitur variando colores, quandoque prolacio tantum, et quandoque ambo simul. Caveat igitur diminuere sic voluens sub variacione colorum, ut exemplum, sibi non implicet contradiccionem." Goscalcus 1984, 30–31.

[45] "In eis namque nulla deceptio, in his autem ambiguitas cadere potest et error . . . nobis plus placet cifrarum positio qua proportionum clarior ostenditur demonstratio." Ugolino 1960, 211.

divisible by two or by three.[46] The collection includes several pieces with fractions which include the numbers five and seven, unheard of in the earlier repertory.[47]

How were all of these fractions related to one another? Were the proportions cumulative or rather related to the original mensuration sign? If the hypothesis is valid that fractions were introduced as an improved substitute for mensuration signs, it would seem logical that fractions would not be cumulative, just as mensuration signs could not be cumulative. A study of all numerical proportions encountered in ModA and CH supports this view: none of the proportions appear to be cumulative. We must distinguish between two kinds of numerical signs: first, a single number standing for a multiplex or superparticular proportion, for example 2 indicating a 2:1 proportion, or 3 indicating a 3:2 proportion; and second, a fraction. Both types occur in ModA and CH—the former often with a canon,[48] and the latter always without a canon,[49]—and both types are always related to the preceding mensuration sign, never to another proportion. Similarly, fractions in Ugolino's and Hothby's compositions are never cumulative.[50] (Neither of the two composers discusses the problem in their treatises.) The top voice of Anthonello de Caserta's *Amour m'a le*

[46] A similar expansion in the number of used proportions has been traced in the development of architecture of the fifteenth and sixteenth centuries by Naredi-Rainer (1985, 149–78, see in particular 162–67). Leon Battista Alberti, for example, uses 6:5 proportions in the façade of the Palazzo Rucellai in Florence, built around 1455, even though proportions based on the number five are only legitimized as canonic in Andrea Palladio's *Quattro libri dell'architettura* of 1570. According to Naredi-Rainer the prominence of the 5:4 (equivalent to the major third) and 6:5 (equivalent to the minor third) proportions derives from the description of the triad in music theoretical treatises of the time.

[47] See for example the ballade *Puisque ame* (Hoppin 1963:3, no. 26) which makes use of a 5:2 (m. 58) and 7:3 (m. 67) proportion. See also Hoppin 1957.

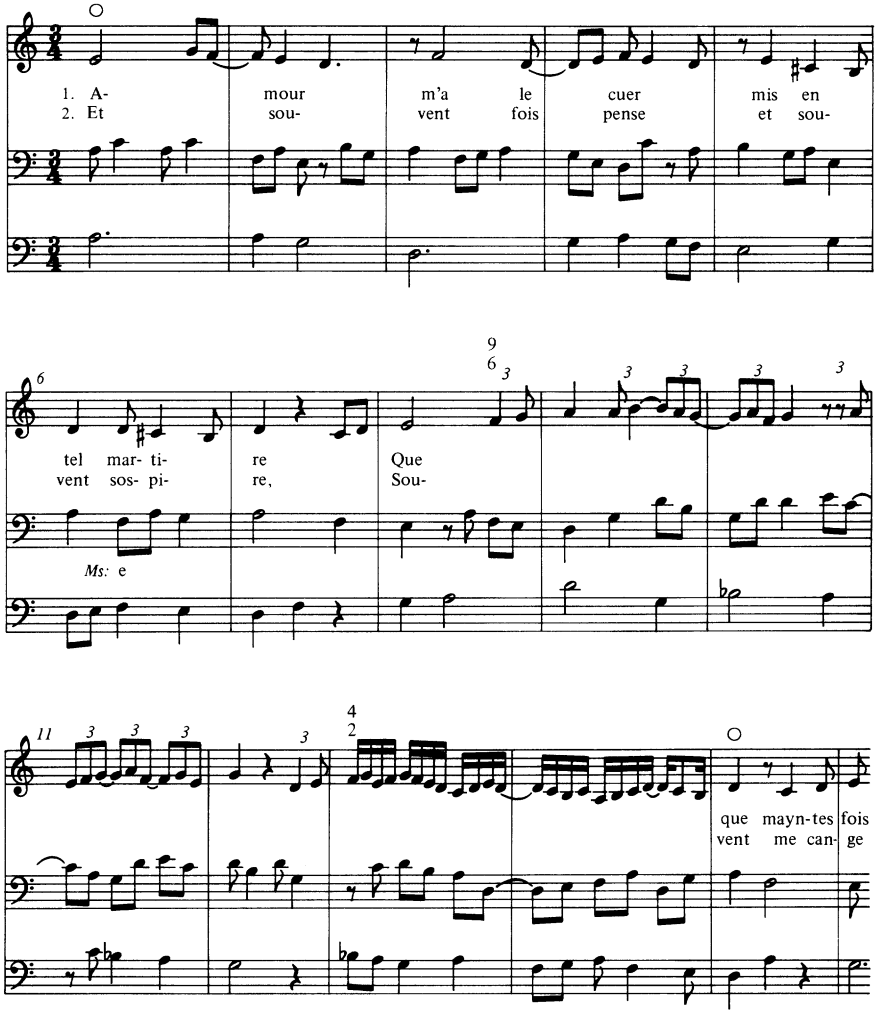
[48] Conradus de Pistoria in *Se doulz espour*, (ModA, f. 31v, Apel 1970:1, no. 15, m. 12) uses the number 3 to show *sesquialtera* and the number 2 (m. 17) to show *dupla*; he does not add a canon; the same numbers occur with a canon in Johannes Simon Hasprois' *Ma douce amour*, (ModA, f. 28r, Apel 1970:1, no. 41, m. 8 and m. 12) and Bartholomeus de Bononia's *Que pena major* (ModA, ff. 37r-36v, Apel 1970:3, no. 301, mm. 10, 24, 42, 54).

[49] Anthonello de Caserta's *Amour m'a le cuer mis*, (ModA, ff. 32v-33r, Apel 1970:1, no. 3, mm. 8 and 13); Baude Cordier's *Belle, bonne sage*, (CH, f. 11v, Greene 1981:18, no. 1, m. 44) uses an 8:9 fraction after the number 3 which is not cumulative; Cordier's *Tout par compas* (CH, f. 12, Greene 1981:18, no. 2) makes use of fractions which are always alternated with mensuration signs.

[50] For Ugolino's compositions see Seay 1955. For Hothby see HOb, ff. 26v-27 and Hothby 1964, 4–7. Similarly, the proportions indicated by a single number in the early fifteenth-century anonymous treatise in Rvat (f.51) are not cumulative. This treatise is also particularly interesting because fractions and mensuration signs are used interchangeably in the musical examples.

Example 9

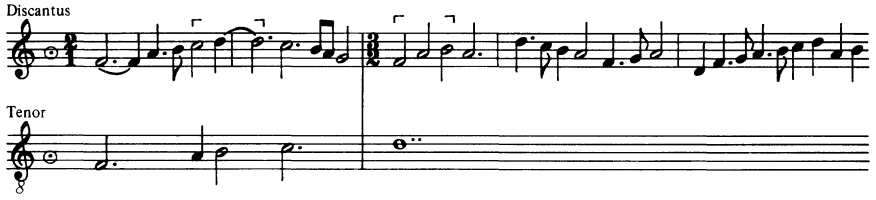
Anthonello de Caserta, *Amour m'a le cuer mis*, (ModA, ff. 32v-33r, Apel 1970:1, no. 3)



cuer mis (see Example 9), perhaps illustrates the problem best. The piece starts out in perfect time, minor prolation. After the 9:6 proportion in m. 8, nine minims are equivalent to six. When in m. 13 a 4:2 proportion is written, it is not related to the immediately preceding section—that is, not multiplied with the 9:6 fraction, as would be arithmetically correct— but related to the initial mensuration sign of perfect time, minor prolation.

Example 10

PEc, f.90v (cf.Tinctoris 1978, 54, m. 1-5)



In contrast, all examples in Tinctoris's *Proportionale musices* are cumulative. He cautions the reader that proportions can be related "by comparison to the preceding number in one and the same voice, or . . . by comparison to the notes of the other part".[51] In one of his examples (see Example 10) the discant has a 2:1 proportion followed by a 3:2 proportion.[52] The 3:2 proportion is not related to the initial perfect time, but to the 2:1 proportion, resulting in a 3:1 proportion.

Gaffurius gives perhaps the clearest description of how successive proportions are to be performed:

> Various proportions following each other in succession are reckoned according to the proportion directly preceding. If, for example, *proportio tripla* follows *proportio dupla*, *proportio sextupla* will result from reckoning the *tripla* with the first number preceding the *dupla*, as is seen in these two numbers, 1:2:6.[53]

Thus, we can conclude that even though fractions were introduced in order to provide musicians with accurate and less ambiguous

[51] "Per relationem ad numerum praecedentem in una et eadem parte . . . per relationem ad notas alterius partis . . . " Tinctoris 1978:2a, 13; trans. Tinctoris 1979, 3–4.

[52] I am indebted to Dr. Bonnie Blackburn for calling my attention to the fact that Albert Seay's transcription (Tinctoris 1978, 54) is incorrect. I should like in turn to propose an emendation to Dr. Blackburn's solution. In order to avoid "the parallel ninths which are not sanctioned in Tinctoris's rules of counterpoint" (Blackburn 1981, 42, n. 31) her first sentence should be corrected to "Seay's transcription of the first measure of the discantus should be amended to begin with a breve imperfected *ad partes* (four minims)" rather than "imperfected *ad totum* (six minims)", which would result in a 20:9 proportion rather than 18:9, that is 2:1 proportion. My transcription is based on the PEc, f. 90v).

[53] "Diverse proportiones sese invicem consequentes varias subsequentium notularum ad precedentes sana consyderatione ducunt habitudines. Namque si (exempli causa) tripla proportio in notulis immediate *dupla*m fuerit subsequuta: sexcupla illico proportio ex numerositate notularum ipsius triplae descriptae ad priorem notularum numerum qui scilicet ante *dupla*m ipsam dispositus fuerat: resultabit: quod his numeris sane percipitur 1.2.6." Gaffurius 1496, Bk. IV, Ch. 13; trans. Gaffurius 1968, 234.

notational symbols to indicate proportions, it took almost one hundred years for musicians to learn to use the fractions in an arithmetically correct way, and to forget their origin as a substitute for mensuration signs. As in many other aspects of the mensural system, the great theorist-reformers Tinctoris and Gaffurius were the ones who initiated this change and emancipated the fraction from the mensuration sign.

This point is closely related to the last question: is the mensuration in a proportion derived from the proportion itself or from the preceding mensuration sign?[54] It would seem a logical assumption that those theorists who use proportions interchangeably with mensuration signs also determine the mensuration in the proportional section from the proportion sign itself. Since mensuration signs from which the proportions presumably derived can alter the original mensurations, proportions can do so as well. Statements by both Prosdocimus and Ugolino indicate that our assumption is indeed correct. Prosdocimus says:

> Likewise you must know that these notes so diminished must be brought back to the mensurations mentioned above in the first chapter . . . for the diminished notes which are sung in *sesquialtera* proportion, that is, three for two, must be brought back to perfections of mensurations, and can be perfected, imperfected, emptied out [a black note, made white], altered, and, in short, all occurrences which can be permitted for the notes having a true perfection of mensuration can be allowed.[55]

Ugolino (1960, 210) makes a similar statement. Thus, it is not surprising that in Hothby's motet *Ora pro nobis*[56] the 6:4 section which follows a passage in C makes the semibreves perfect.

Guilielmus Monachus also makes clear that the mensuration is derived from the preceding proportion sign. In his comments on the performance of *sesquialtera* he states:

[54] Willi Apel (1953, 150) is the only scholar to recognize the problem. He assumes that the proportional mensuration always takes precedence over the original one.

[55] "Item scire debes, quod iste figure sic diminute habent reduci ad mensuras superius in primo capitulo nominatas, . . . nam figure diminute que in proportione sexquialtera cantantur, sicut tres pro duabus, habent reduci ad perfectiones mensurarum, et possunt perfici, imperfici, evacuari, alterari, et breviter omnes passiones pati quas pati possunt figure recte perfectiones mensurarum habentes." Beldemandis 1869, 219.

[56] Hothby 1964, 4–7, m. 55. Hothby often avoided confusion concerning mensurations after proportion signs by using mode signs (O2 , C 2, and OO2 etc.) which not only diminish but also show the precise mensuration on every level. See Chapter 4 of Berger forthcoming.

> *Sesquialtera* in perfect minor [perfect time, minor prolation] requires that the minims be counted in threes as the minims of imperfect [time] major [prolation] are counted, and in the same way the minims are altered before semibreves, and, just as there, the semibreves are imperfect before minims, and three minims are placed for a single *mensura* of the common semibreve . . .[57]

In other words, the mensuration is changed from minor to major prolation and the semibreve becomes perfect.

The earliest author to state explicitly that in proportions the original mensuration must be preserved is the same theorist who emancipated fractions from mensuration signs—Johannes Tinctoris, in his *Proportionale musices* of ca. 1473–4:

> It must be noted that in any proportion we ought to consider in what modus, in what tempus, and in what prolation it is made, for certain proportions are binary, as duple, quadruple, etc., certain ternary, as triple, sesquialtera, etc., certain both, as sextuple, sesquiquinta, etc., certain neither, as sesquiquarta, superbipartiens tertias, etc. They, however, cannot change the nature of the quantities in which they are made. Regardless whether any proportion be either binary, ternary, or both, or neither, always the notes must be computed according to their perfection or imperfection in respect to the modus, tempus, and prolation signature under which they fall.[58]

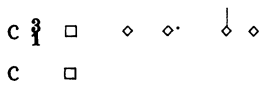
In other words, he advises the retention of the original mensurations under proportional signs, and stands therefore in opposition to all

[57] "Sexquialtera super perfecto minore exigit ut minimae numerentur ternae uti numerantur minimae temporis imperfecti maioris, et eodem modo minimae alterantur ante semibreves, et ut ibidem semibreves sunt imperfectae ante minimas ponunturque tres minimae pro singulo ictu pausae communis semibrevis, . . . " Guilielmus 1965, 21. According to Wolf Frobenius (1971, 4), Guilielmus's term for *mensura* is *ictus pausae* or *ictus pausationis*. *Pausare mensuram* must be translated as "making the rhythm of a *mensura*." He adds further that it is unclear why the word *pausa* was used in this meaning.

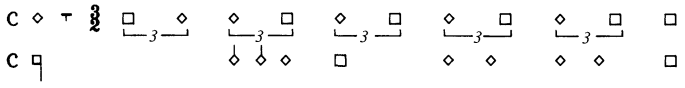
[58] "Deinde notandum est quod circa quamlibet proportionem debemus considerare in quo modo, in quo tempre et in qua prolatione fiat. Nam quaedam proportiones binariae sunt, ut dupla, quadrupla, etc., quaedam ternariae, ut tripla, sesquialtera, etc., quaedam utraque, ut sextupla, sesquiquinta, etc., quaedam neutrae, ut sesquiquarta, superbipartiens tertias, etc. Non tamen naturam quantitatum in quibus fiunt immutare possunt. Immo qualiscumque proportion sit sive binaria, sive ternaria, sive utraque, sive neutra, semper notae iuxta perfectionem aut imperfectione earum per respectum signi modalis, temporalis et prolationalis, sub quo consistunt computandae sunt . . . " Tinctoris 1978, 53; trans. Tinctoris 1979, 41.

Example 11

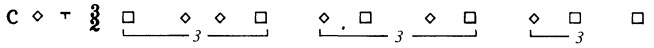
(a) cf. Tinctoris 1978, 13, m. 1-3



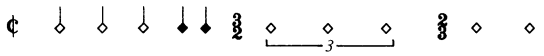
(b) cf. Tinctoris 1978, 45



(c) cf. Tinctoris 1978, 45



(d) cf. Tinctoris 1978, 43, mm. 8–10



earlier theorists.[59] However, several of Tinctoris's own examples do not follow the rules he has outlined. In *proportio tripla* (see Example 11a) he can easily retain the original mensuration, since the triple mensuration places three imperfect breves against one imperfect breve. However, in *sesquialtera* following C it is considerably more difficult to preserve the original mensuration. Before Tinctoris, theorists and musicians usually replaced two semibreves of C with three semibreves of *sesquialtera*, but this resulted in perfect time. The only way to preserve the original mensuration is to replace two imperfect breves with three. This is Gaffurius's solution, as will be seen shortly. However, both of Tinctoris's examples of C followed by $\frac{3}{2}$ make the breve perfect, as can be seen clearly from the way the *sesquialtera* section is aligned with the C in the other voice (see Example 11b). If the examples were regrouped into imperfect sesquialtered breves (see Example 11c), the last triplet would not be complete. Similarly, the preservation of the original mensuration is impossible in Tinctoris's example of ¢ followed by the *sesquialtera* (see Example 11d). Since the theorist uses only three semibreves (or one

[59] It is striking that this wish to retain the original mensuration in proportions is formulated at about the same time as is the rule of preserving the original mensuration under diminution. See Berger 1987.

perfect breve), we must interpret the *sesquialtera* as perfect time. In sum, we can conclude that Tinctoris did not preserve the original mensuration consistently in his examples, even though he advocated the practice in his text. It is possible that he felt unable always to follow his rules, because the pull of the traditional and current practice was too strong.

Tinctoris's most important follower, Franchinus Gaffurius, also demands that the original mensuration be preserved in a proportion. He is more consistent than Tinctoris in applying his rules to imperfect time:

> I also do not approve of the improper practices of many composers who, in showing *sesquialtera* by a number three, think that alteration and perfection are an augmentation of notes, and who write *tempus imperfectum* instead of *tempus perfectum* (which is absurd), and *prolatio minor* instead of *prolatio maior*, while *sesquialtera* is easily understood as a proportional diminution . . . . For a perfect breve and an altered semibreve are only found in *tempus perfectum*, whose proper sign is a circle; a perfect semibreve and an altered minim are only found in *prolatio perfecta* or *maior*, whose proper sign is a dot within the sign of *tempus*. In *tempus imperfectum*, which is shown by a semicircle, a breve always contains just two semibreves, whether normal or in *proportio diminuta* (unless a dot of augmentation is used), and a semibreve never increases in value through alteration. In the same way a semibreve of *prolatio minor* cannot become perfect nor a minim be altered.[60]

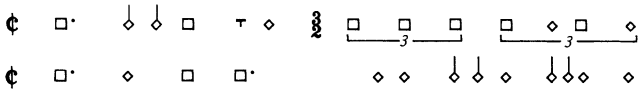
In other words, "when *proportio sesquialtera* occurs in imperfect mensurations, all notes and rests are imperfect."[61] This is shown clearly in Example 12a. Whenever Gaffurius intends a perfection

[60] "Neque assentio complurimorum coruptellae: qui quum solo ternarii numeri charactere sesqualteram describunt in notulis *tempus* imperfectum (quod absurdum est) pro perfecto: atque pro maiore prolatione minorem posuere: alterationem & perfectionem in notulis consyderantes quo facile conspicitur ex diminutione proportionis deductum esse in notulis augumentum . . . . Brevis enim perfecta & semibrevis alterata in solo tempore perfecto disponitur: cuius proprium signum est circulus. Semibrevem vero perfectam ac mininam alterabilem sola maior sive perfecta prolatio confert: huius proprium signum est punctus in signo temporis affixus. In tempore autem imperfecto quod semicirculus declarat: brevis notula duas tantum semibreves semper possidet sive recta: sive quavis Proportione diminuta: nisi punctum augumentationis susceperit: sed neque in eo semibrevis unque alterationis suscipit incrementum. Atque idcirco eodem sensu neque semibrevis in minori prolatione perfectionem acquiret: neque minima poterit alterari." Gaffurius 1496, Bk. IV, Ch. 5; trans. Gaffurius 1968, 182.

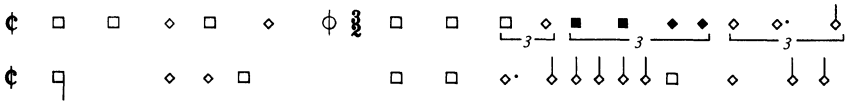
[61] "Si autem proportio sesquialtera disposita fuerit in notulis imperfectae quantitati subiectis omnes tunc notulae & pausae semper erunt imperfectae." Gaffurius 1496, Bk. IV, Ch. 5, trans. Gaffurius 1968, 182.

Example 12

(a) Gaffurius 1496, Bk. IV, Ch. 5, (cf. Miller 1968, Ex. 80)



(b) Gaffurius 1496, Bk. IC, Ch. 5 (cf. Miller 1968, Ex. 79)



within a *sesquialtera* proportion in imperfect time, he adds the circle to the proportion sign (see Example 12b).[62]

In sum, we have been able to trace the following development in fifteenth-century theoretical treatises: at first, the mensuration following the proportion sign was determined by the fraction, since the fraction substituted for a mensuration sign. The first theorists to differentiate clearly between the functions of mensuration signs and fractions were Tinctoris and Gaffurius, both of whom explained that the function of the fraction was to indicate not the mensuration but only the level of diminution. Closely correlated with this reform was, of course, their novel idea that proportions should be cumulative.

In conclusion, I want to postulate the following scenario. The idea of dividing the breves into two to nine equal parts was introduced by Johannes de Muris in 1321. This suggested to musicians who wanted to use in one piece several of the four *Ars nova prolationes* that they could relate them to one another with the breve retaining constant duration. Indeed, theorists and composers of the second half of the fourteenth century picked only those divisions of the breve which could be shown by a combination of various mensuration signs with the assumption of breve equality. The other proportions were not used because musicians lacked a notational tool to express them. The use of fractions to indicate rhythmic proportions represented a true innovation. It allowed not only a less ambiguous notation of already used proportions, but also the gradual inclusion of the proportions not naturally inherent in the mensural system of the time. However, in

[62] Gaffurius 1496, Bk. IV, Ch. 6, Example 79. Gaffurius does not preserve the original mensuration in major prolation. For a detailed discussion of Gaffurius's, Giovanni Spataro's, and other theorists' view on this subject see Ch. 5 of Berger, forthcoming.

the first half of the fifteenth century fractions were used as if they actually *were* mensuration signs—that is, like their predecessors, the mensuration signs, they were not cumulative and they determined mensuration in their own right. It took theorists of Tinctoris's and Gaffurius's stature to emancipate fractions from mensuration signs and to use these fractions in an arithmetically correct way. As a result, they were able to enlarge the number of proportions to include those which did not result from combination of various mensuration signs under breve equality, so that the system of rhythmic proportions could become as rich as that of harmonic proportions. It is puzzling that composers wrote music which made use of elaborate proportions only in the late fourteenth and early fifteenth centuries, at a time when they were still searching for a notational tool to indicate them unambiguously. Once the fraction had been associated with proportions, composers lost interest in writing music with elaborate proportions and the whole matter became a theoretical issue.

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BL Bologna, Civico Museo Bibliografico Musicale, MS. Q 15.

Ccc Cambridge, Corpus Christi College, MS. 410.11.

CH Chantilly, Musée Condé, MS. 564.

Fl Florence, Biblioteca Medicea-Laurenziana, MS. Plut. XXIX, 48.

HOa John Hothby. *Quid est proportio.* London, Lambeth Palace, MS. 446, ff. 19–30.

HOb ——. *Ora pro nobis.* Faenza, Biblioteca Comunale, MS. 117, ff. 26v-27.

Lbl London, British Library, MS. Lansdowne 763, ff. 117–23.

ModA Modena, Biblioteca Estense, MS. M.5.24.

ModB Modena, Biblioteca Estense, MS. lat. 471 (X1,11).

Ob Oxford, Bodleian Library, MS. Canonici misc. 213.

OH London, British Library, MS. Add. 57950 (olim Old Hall, St. Edmund's College, Ware, Herts.).

PadA Padua, Biblioteca Universitaria, MSS. 684 and 1475, as well as Oxford, Bodleian Library, Can. Pat. lat. 229.

PEc "Regule de proportionibus. Cum suis exemplis." Perugia, Biblioteca Comunale Augusta, MS. 1013, ff. 78–123.

Rvat Rome, Biblioteca Apostolica Vaticana, MS. Regina lat. 1146, ff. 48–51.

SDI Saint-Dié, Bibliothèque municipale, MS. 42, f. 131.

Sca Siena, Biblioteca Comunale degli Intronati, MS. L.V.30, ff. 48v-56r.
Scb Siena, Biblioteca Comunale degli Intronati, MS. L.V. 30, f. 142r-v.
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Abstract

In the late fourteenth and early fifteenth centuries rhythmic proportions were indicated through coloration, Italian note shapes, mensuration signs, and fractions. Even though Johannes de Muris had introduced in 1321 a division of the breve into from two to nine equal parts, theorists and composers used only those proportions which could be indicated by a combination of various mensuration signs with the assumption of breve equivalence: 3:2 shown by ○:𝐂 or 𝐂:𝐂; 4:3 by 𝐂:○ or 𝐂:𝐂; 9:4 by ⊙:𝐂; 9:8 by ⊙:¢; 2:1 by 𝐂:¢; 8:3 by ¢:𝐂. Other proportions were not used because musicians lacked adequate signs to indicate them. The invention of the fraction to show rhythmic proportions presented, therefore, a true innovation because it permitted the indication of proportions not naturally inherent in the mensural system. However, fractions were used until the late fifteenth century as if they were mensuration signs, that is, they were not cumulative

and they determined the mensuration of the following section. It was not until the late fifteenth century that Johannes Tinctoris and Franchinus Gaffurius emancipated proportions from mensuration signs and used fractions in an arithmetically correct way.