PS01 - Bayesian Statistics

Due date: **01/25/2025 (Note: Revised date)**

Sources: Gelman et al. (2021); Ross (2007); DeGroot and Shervish (2012)

Question 01

Three prisioners are informed by their jailer that one of them has been chosen at random to be freed while the other two will remain in prison. Prisoner A asks the jailer to tell them privately which of their fellow prisoners will remain in prison, claiming that there would be no harm in divulging the information, since she already knows that at least one will stay in prison. The jailer refuses to give the answer, pointing out that if A knew which of his fellows were to remain in prison, then her own probability of being set free would rise from $\frac{1}{3}$ to $\frac{1}{2}$. What do you think about the jailer's reasoning? (Hint: https://youtu.be/By-ESpngmsg?si=i6wJ1i1Z3Be7LF0I)

Question 02

Suppose that 5% of males and 0.25% of non-males are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? (Hint: Assume that there are an equal number of males and non-males.)

Question 03

Probability calculation for genetics (from Lindley, 1965): suppose that in each individual of a large population there is a pair of genes, each of which can be either x or X, that controls eye color. Those with xx have blue eyes, while heterozygotes (those with Xx or xX) and those with XX have brown eyes. The proportion of blue-eyed individuals is p^2 and of heterozygotes is $2p(1-p)$, where $0 < p < 1$. Each parent transmits one of its own genes to the child. If a parent is a heterozygote, the probability that it transmits the gene of type X is $\frac{1}{2}$. Assuming random mating, show that among brown-eyed children of brown-eyed parents, the expected proportion of heterozygotes is $2p/(1+2p)$. Suppose Judy, a brown-eyed child of brown-eyed parents, marries a heterozygote, and they have n children, all brown-eyed. Find the posterior probability that Judy is a heterozygote and the probability that her first grandchild has blue eyes.

Question 04

Conditional probability: approximately $\frac{1}{125}$ of all births are fraternal twins and $\frac{1}{300}$ of births are identical twins. Elvis Presley had a twin brother (who died at birth). What is the probability that Elvis was an identical twin? (You may approximate the probability of a boy or girl birth as $\frac{1}{2}$.)

Question 05

60% of families in a certain community own their car. 30% own their home. 20% own both their car and their home. If a family is randomly chosen, what is the probability that this family own a car or a house, but not both?