1 Integrals

1.1 Improper Integral Summary

 $\text{Integral } \qquad p \leq 1 \qquad \qquad p > 1 \qquad \quad \text{Value}$

 $\int_0^1 \frac{1}{x^p} \quad \text{divergent} \quad \text{convergent} \quad \frac{1}{1-p}$

 $\int_{1}^{\infty} \frac{1}{x^{p}} \quad \text{divergent} \quad \text{convergent} \quad \frac{1}{p-1}$

1.2 Comparison Theorem

If f and g are continuous and $f(x) \ge g(x) \ge 0$ for $x \ge a$ (there is some a where f is now always larger than g) then,

If $\int_a^\infty f(x)dx$ is convergent then the "smaller" integral $\int_a^\infty g(x)dx$ must be convergent too.

If $\int_a^\infty g(x)dx$ is divergent then the "larger" integral $\int_a^\infty f(x)dx$ must be divergent too.