1 Definitions

1.1 Trivial Solution

A zero vector. If $Ax = 0$ has only the trivial solution then x must be something like,

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

1.2 Symmetric Matrix

A square matrix A where $A = A^T$. Thus, $(A)_{ij} = (A)_{ji}$.

$$\begin{bmatrix} 1 & 9 \\ 9 & 2 \end{bmatrix}$$

1.3 Skew-symmetric Matrix

A square matrix A where $A^T = -A$.
All the main diagonal entries must be 0.

$$\begin{aligned} -(A_{ij}) &= (A^T)_{ij} \\ -(A_{ij}) &= A_{ji} \\ -(A_{ii}) &= A_{ii} && \text{On the diagonal, } i = j \\ A_{ii} &= 0 && \text{0 is the only value that will hold} \end{aligned}$$

2 Equivalence Theorem

If A is an $n \times n$ matrix, then the following statements are equivalent. That is, if one is true, the rest is true, as they are logically equivalent.

- A is invertible.
- $Ax = 0$ has only the trivial solution.
- The reduced row echelon form of A is I_n.
- A is expressible as a product of elementary matrices. $A = E_n E_{n-1} \dots E_1 I_n$.
- $Ax = b$ has exactly one solution for every $n \times 1$ matrix b.
- $det(A) \neq 0$.
- $\lambda = 0$ is not an eigenvalue of A.

3 Determinant Properties

3.1 Adjoint Matrices

We know the following:

$$\begin{aligned}
A \operatorname{adj}(A) &= \det(A) I \\
\operatorname{adj}(A) &= A^{-1} \det(A) I
\end{aligned}$$

We can then find the determinant of the adjoint of a matrix in terms of the determinant of the original matrix.

$$\begin{aligned}
A &= \operatorname{adj}(B) \\
A &= B^{-1} \det(B) I \\
\det(A) &= \det(B^{-1}) \det(\det(B)) \det(I) \\
\det(A) &= \det(B)^{-1} \det(B)^{n} 1 \\
\det(A) &= \det(B)^{n-1}
\end{aligned}$$