

1 Definitions

1.1 Trivial Solution

A zero vector. If $Ax = 0$ has only the trivial solution then x must be something like,

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_r \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

2 Equivalence Theorem

If A is an $n \times n$ matrix, then the following statements are equivalent. That is, if one is true, the rest is true, as they are logically equivalent.

- A is invertible.
- $Ax = 0$ has only the trivial solution.
- The reduced row echelon form of A is I_n .
- A is expressible as a product of elementary matrices. $A = E_n E_{n-1} \dots E_1 I_n$.
- $Ax = b$ has exactly one solution for every $n \times 1$ matrix b .
- $\det(A) \neq 0$.
- $\lambda = 0$ is not an eigenvalue of A .