1 Limits

1.1 e

The function e is defined as a continuous, differentiable function $f(x)$ that satisfies $f'(x) = f(x)$ for all x and $f(0) = 1$.

$$e = \lim_{n \to 0} (1+n)^{\frac{1}{n}}$$

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

2 Integrals

2.1 Improper Integral Summary

Integral	$p \leq 1$	$p > 1$	Value
$\int_0^1 \frac{1}{x^p}$	divergent	convergent	$\frac{1}{1-p}$
$\int_1^\infty \frac{1}{x^p}$	divergent	convergent	$\frac{1}{p-1}$

2.2 Comparison Theorem

If f and g are continuous and $f(x) \geq g(x) \geq 0$ for $x \geq a$ (there is some a where f is now always larger than g) then,

If $\int_a^\infty f(x)dx$ is convergent then the "smaller" integral $\int_a^\infty g(x)dx$ must be convergent too.

If $\int_a^\infty g(x)dx$ is divergent then the "larger" integral $\int_a^\infty f(x)dx$ must be divergent too.