# Homework 4

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## 1

1.

a. https://github.com/emilyjcosta5/mathematical_modeling/blob/master/assignment4/hw4_0.png

b. Drop the constant H, as it does not depend on population and multiply both sides by $dt$

$$dN = rN(1 - N/k)dt$$

Then divide both sides by $N(1 - N/k)$

$$1/(N(1 - N/k))dN = r * dt$$

Integrate both sides

$$-ln(k - N) + ln(N) = rt + c$$

Solve for N and set $m = e^{-c}$

$$N = k/(1 + m * e^{-rt})$$

3. Multiply both sides by $dt$ and move I variables to LHS

$$dI/(I(k - I)) = r * dt/k$$

Solve LHS by letting

$$1/(I(k - I)) = A/I + B/(k - I)$$

Then solve for A and B

$$A = 1/k$$

$$B = 1/k$$

Then plug into $A/I + B/(k - I)$ and then plug back in original equation

$$dI(1/I - 1/(k - I)) = r * dt$$

Now integrate both sides

$$log(I) + log(k - I) = r * t + c$$

Solve for c when $I(0) - I_0$

$$c = log(I_0(k - I_0))$$

Plug in c and rearrange equation

$$I^2 - kI + I_0(k - I_0) * e^{rt}$$

Plug into quadratic equation to solve for I(t)

$$I(t) = k \pm \sqrt{k^2 - 4I_0(k - I_0)e^{rt}}/2$$

As the I(t) approaches infinity,

$$\lim_{t \to \infty} I(t) = \infty$$

6. Plug in qEN for H(N) then distribute $rN * (1 - N/K)$ and multiply both sides by $dt/K(rN - rN^2/K - qEN)$

$$dN/(rKN - rN^2 - qENK) = dt/K$$

Integrate and simplify

$$(rK - qEK - rN)^{1/(r^2K - rqEK)}/N^{1/(rK - qEK)} = e^{(-t/K) - N_0}$$

The limit of t and is approaches infinity is 0 of the RHS, so set the LHS to 0 and solve

$$N = K(1 - qE/r)$$

So, either $K = 0$ or $qE = r$

# 2

With the given information, we can plug in $u*$ for $u^k$

$$g(u*) = g(\lim_{k \to +\infty} u^k)$$

So,

$$\lim_{k \to +\infty} g(u^k) = \lim_{k \to +\infty} u^{k+1} = \lim_{k \to +\infty} u^k = u*$$

Which shows that if a solution to this system converges, then the limit $u*$ is a fixed point.

## 3

Combine the equations to get

$$J = (b/(D + a)) * R$$

Now plug into the Romeo equation

$$(D + a) * R = b^2/(D + a) * R$$

Solve for D

$$(D^2 + 2aD + a^2 - b^2) * R = 0$$

$$D = -a \pm b$$

So, the equations will look like

$$R(t) = A_1 * e^{-(a-b)t} + A_2 * e^{-(a+b)t}$$

$$J(t) = B_1 * e^{-(a-b)t} + B_2 * e^{-(a+b)t}$$

where A and B are constant depending on the initial conditions. Ig their caution is greater than love,

$$R(\infty) = J(\infty) = \infty$$

So their love increases with time still.

## 4

For code: `https://github.com/emilyjcosta5/mathematical_modeling/blob/master/assignment4/hw4.py`
For image produced: `https://github.com/emilyjcosta5/mathematical_modeling/blob/master/assignment4/hw4.png`

## 5

Team members: Emily Costa, Max Rottenberg, Antoinette Randall
For our final project, we will be implementing the algorithm from "Fast Sampling Plane Filtering, Polygon Construction and Merging from Depth Images", which I have sent to you through Canvas. This algorithm uses linear algebra to detect the plane of best fit in a room. We will add another algorithm to determine whether a person has collapsed or fainted in the room.