# Homework 2

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## 1

See for plots and equation:
https://github.com/emilyjcosta5/mathematical_modeling/blob/master/assignment2/all_plots.png
See for code:
https://github.com/emilyjcosta5/mathematical_modeling/blob/master/assignment2/hw2_1.R

## 2

a. Given

$$Y = Y_c * N_c + E_c * dN_c/dt$$

and

$$Y = Y_0 * m^{3/4}$$

and

$$m = m_c * n_c$$

It logically follows that (i.)

$$Y_0 * m^{3/4} = Y_c * N_c + E_c * dN_c/dt$$

and

(ii.)

$$N_c = m/m_c$$

Take the derivative of Nc with respect to time (iii.)

$$dN_c/dt = 1/m_c * dm/dt$$

Then substitute (iii.) into (i.) to get

$$Y_0 * m^{3/4} = Y_c * N_c + E_c * 1/m_c * dm/dt$$

Now, solve for dm/dt and substitute using (ii.)

$$dm/dt = (m_c/E_c) * (Y_0 * m^{3/4} - Y_c * N_c)$$

Set (iv.)

$$a = Y_0 * m_c/E_c$$

and

(v.)

$$b = Y_c/E_c$$

Then reduce dm/dt and substitute using (iv.) and (v.):

$$dm/dt = a * m^{3/4} - b * m$$

b. Given (i.)

$$dm/dt = 0$$

When (i.) we know that (ii.)

$$m = M$$

and then

$$dm/dt = a * m^{3/4} - b * m = 0$$

then we can conclude that

$$a * m^{3/4} = b * m$$

Solve for m then substitute using (ii.)

$$M = (a/b)^4$$

Then plug in a and b (iii.)

$$M = (Y_0 * m_c/Y_c)^4$$

Finally, plug in b and (iii.) into original equation to get

$$dm/dt = a * m^{3/4}(1 - (m/M)^{1/4})$$

c. Let

$$r = (m/M)^{1/4}$$

and

$$R = 1 - r$$

Take the derivative of r and R

$$dr/dt = (dm/dt)(m^{-3/4}/4 * m^{1/4})$$

$$dR/dt = -dr/dt$$

Solve for $d_m/d_t$ and plug in $-d_R/d_t$ for $d_r/d_t$

$$dm/dt = -4 * M^{1/4} * m^{3/4} * dR/dt$$

Now simplify and substitute using a:

$$dR/dt = -(a/(4 * M^{1/4}) * R$$

From this, we can derive

$$ln(R/R_0) = a * t/(4 * M^{1/4}) + K$$

which is similar to

$$y = m * x + c$$

and we can conclude that

$$m = -1$$

d. The time constant is

$$4 * M^{1/4}/a$$

The letter a consists of the defined constants. So, the only variable is $M^{1/4}$. Hence, the interval between heartbeats should scale with its size (or mass) as $M^{1/4}$.

# 3

Given the fluid flow rate through the aorta (i.)

$$Q_0 = N_k * Q_k = N_k * pi * r_k^2 * u_k = N_N * pi * r_N^2 * u_N$$

and

(ii.)

$$N_N = (M/M_0)^{3/4}$$

From (i.), we know

$$N_k * pi * r_k^2 * u_k = N_N * pi * r_N^2 * u_N$$

At the base level, $N_0 = 1$. So we replace $N_0$ and drop $u_k$ and $u_N$, as they are constant. We can also drop pi from both sides.

$$r_k^2 = N_N * r_N^2$$

We can substitute using (ii.)

$$r_k^2 = (M/M_0)^{3/4} * r_N^2$$

Now solve for $r_k/r_N$

$$r_k/r_N = (M/M_0)^{3/8}$$

As $r_N$ and $M_0$ are constants, we can conclude that the radius of the aorta is proportional to the mass on the animal as $M^{3/8}$
Given

$$V = pi * p^2 * l_N * N_k = C * l_N^3 * N_N$$

We can insert

$$C * l_N^3 * N_N = C * l_k^3 * N_k$$

We set $k = 0$ and drop the constants

$$l_0^3 = N_N$$

Substitute from (ii.) and solve for $l_0$

$$l_0 = (M/M_0)^{1/4}$$

and we can conclude that the length of an aorta scales with $M^{1/4}$.
*Source: http://mukarramtahir.com/aorta.pdf*

# 4

a. When a new node is added

$$N_k * (n+1) = (k-1)_{pk-1} * n/2$$

and

$$N_k * (n) = k_{pk} * n/2$$

So,

$$N_k * (n+1) - N_k * (n) = ((k-1)_{pk-1} * n/2) - (k_{pk} * n/2)$$

b. We can rewrite (a.) as

$$(n+1)_{pk} - (n)_{pk} = ((k-1)_{pk-1}/2) - (k_{pk}/2)$$

Then rewrite again as

$$p_k(1 + k/2) = p_{k-1}((k-1)/2)$$

Solve for $p_k/p_{k-1}$

$$p_k/p_{k-1} = (k-1)/(k+2), for k >= 2$$

c. Solve for $p_k$

$$p_k = (k-1)!/((k+2)(k+1)...)$$

Cancel common factors and drop constant

$$p_k = 1/((k+2)(k+1)k)$$

which means that $p_k$ approaches $k^{-3}$