Homework 3

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1

a. We know that
r is the number of disintegration of Ra/gram of ordinary lead at time, t.

$$t_0$$

is the time the pigment was manufactured
Core: at a rate, r, the number of atoms of radioactive lead will increase because of Ra236 disintegration. λ is just a constant of proportionality. Hence,

$$dy/dt = -\lambda * y(t) + r$$

Manufacturer: Once radon is removed, the factor r is removed.

$$dy/dt = -\lambda * y(t)$$

b. Since in core there is radioactive equilibrium,

$$dy/dt = 0 - (-\lambda * y + r)$$

Which gives

$$dy(t) = r$$

Since r ranges from 0-200,

$$dy(t) = r = 0 - 200$$

c. We can integrate equation 4.10 to get

$$ln|y| = -\lambda * dt$$

Then solve for y and substitute $A = e^c$

$$y(t) = A * e^{-\lambda t_0}$$

Substitute the given conditional that $y = r/\lambda$ at initial time to solve for the constant A

$$A = r/\lambda * e^{\lambda t_0}$$

https://www.overleaf.com/project/5da0fe92305cbb0001b71cdf Now we plug in A and simplify

$$y(t) = (r/\lambda) * e^{-\lambda(t-t_0)}$$

d. Solve for λ

$$\lambda = 1.32 * 10^{-6}$$

Differentiate the equation from (c.) and plug in r and λ

$$dy(t)/dt = -8.5 = -200 * e^{-.00000132(t-t0)}$$

Solve for $t - t_0$

$$t - t_0 = 100 * 26$$

Hence, the painting is not more than 100 years old and is fake.

2

a. We can integrate equation (4.13 after substituting $V_I = V_0 e^{-ct}$) using an integrating factor e^{St} to get

$$T^* e^{St} = k * V_0 T_0 (e^{-ct} e^{St})/(S-c) + c$$

Which gives us

$$T^* = T^*(0)e^{-St} + k * V_0 T_0 (e^{-ct} e^{St})/(S-c)$$

For the final step, do as the book and assume T^* is in a quasi-steady state before the therapy

$$T^* = k * V_0 T_0 [ce^{-St} - Se^{-ct})/(S(c-S))$$

b. Use the integrating factor e^{ct} to integrate (4.14)

$$V_{NI} * e^{ct} = [NSkT_0 V_0 / S(c-S)] * [ce^{(c-S)t}/(c-S) - St]$$

Then solve for V_{NI}

$$V_{NI} = cV_0/(c-S)[c(e^{-}S^{t} - e^{-ct})/(c-S) - Ste^{-ct}]$$

3

Let T be the time for one orbit and distance covered by one orbit be 2*pi*r. We can say that velocity (v) is

$$v = 2 * \pi * r/T$$

When we plug in v, then acceleration is

$$a = (2 * \pi * r/T)^2/r = 4 * \pi^2 * r/T^2) = G * M/r^2$$

We solve for radius, r

$$r = [T^2 * G * M/(4\pi^2)]^{1/3}$$

Plug in known values to get

$$r = 3.905 * 10^8 meters$$

4

$(d/dx)(e{*}sinx/(1{+}e{*}cosx)) = (e{*}cosx{+}e^2)/(1{+}e{*}cosx)^2 = 1/(1{+}ecosx)^2{+}(e^2{-}1)/(1{+}e{*}cosx)^2$

So, (i)

$$\int_0^{2*pi} dx/(1{+}e{*}cosx)^2 = e/(e^2{-}1){*}sinx/(1{+}e{*}cosx){-}1/(e^2{-}1){*}\int_0^{2*pi} dx/(1{+}e{*}cosx)$$

To intergrate

$$\int_0^{2*pi} dx/(1 + e * cosx)$$

Substitute the following for $cosx$

$$cosx = (1 - tan^2 x/2)/(1 + tan^2 x/2)$$

Then integrate to get

$$\int_0^{2*pi} dx/(1 + e * cosx) = 2 * [tanh^{-1}((e - 1)tan(x/2))]/(\sqrt{e^2 - 1})$$

Now plug in to (i)

$$\int_0^{2*pi} dx/(1{+}e{*}cosx)^2 = e/(e^2{-}1){*}sinx/(1{+}e{*}cosx){-}1/(e^2{-}1){*}2{*}[tanh^{-1}((e{-}1)tan(x/2))]/(\sqrt{e^2 - 1})$$