Minimum Cuts and Shortest Non-Separating Cycles via Homology Covers

Jeff Erickson
Department of Computer Science
University of Illinois at Urbana-Champaign
jeffe@cs.uiuc.edu

Amir Nayyeri
Department of Computer Science
University of Illinois at Urbana-Champaign
nayyeri2@cs.uiuc.edu

Let G be a directed graph with weighted edges, embedded on a surface of genus g. We describe an algorithm to compute a shortest directed cycle in G in any given $\mathbb{Z}_2$-homology class in $2^{O(g)}n \log n$ time; this problem is NP-hard even for undirected graphs. We also present two applications of our algorithm. The first is an algorithm to compute a shortest non-separating directed cycle in G in $2^{O(g)}n \log n$ time, improving the recent algorithm of Cabello *et al.* [SOCG 2010] for all $g = o(\log n)$. The second is a combinatorial algorithm to compute minimum (s,t)-cuts in *undirected* surface graphs in $2^{O(g)}n \log n$ time, improving on previous combinatorial algorithms, and in particular the recent of Chambers *et al.* [SOCG 2009], for all $g = o(\log n)$. Unlike earlier algorithms for surface graphs that construct and search finite portions of the universal cover, our algorithms use another canonical covering space, called the $\mathbb{Z}_2$-*homology cover*.