PSTAT 122 - HW5

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13.1 An experiment was performed to investigate the capability of a measurement system. Ten parts were randomly selected, and two randomly selected operators measured each part three times. The tests were made in random order, and the data are shown below:

```
part <- factor(rep(1:10,times=6))
operator <- factor(rep(1:2,each=30))
measurement <- c(50, 52, 53, 49, 48, 52, 51, 52, 50, 47,
                 49, 52, 50, 51, 49, 50, 51 ,50, 51, 46,
                 50, 51, 50, 50, 48, 50, 51, 49, 50, 49,
                 50, 51, 54, 48, 48, 52, 51, 53, 51, 46,
                 48, 51, 52, 50, 49, 50, 50, 48, 48, 47,
                 51, 51, 51, 51, 48, 50, 50, 50, 49, 48)
head(cbind(part,operator,measurement))
```

```
##      part operator measurement
## [1,]    1        1          50
## [2,]    2        1          52
## [3,]    3        1          53
## [4,]    4        1          49
## [5,]    5        1          48
## [6,]    6        1          52
```

(a) Analyze the data from this experiment.

Sol. $H_0: \sigma^2_{\tau\beta} = 0$ vs. H_1: $\sigma^2_{\tau\beta} > 0$

```
meas.aov <- aov(measurement ~ part*operator)
summary(meas.aov)
```

```
##               Df Sum Sq Mean Sq F value   Pr(>F)
## part           9  99.02  11.002   7.335 3.22e-06 ***
## operator       1   0.42   0.417   0.278    0.601
## part:operator  9   5.42   0.602   0.401    0.927
## Residuals     40  60.00   1.500
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

According to the ANOVA table above, the p-value (0.927) is greater than $\alpha = 0.05$ which means that there is insufficient evidence to suggest that the interaction between the parts and operators significantly affects the measurement. Since the interaction was not significant, we will now test the individual factors. Our hypotheses will be $H_0: \sigma^2_\tau = 0$ vs $H_1: \sigma^2_\tau > 0$.

```
a <- 10; b <- 2; n <- 3
F_part <- 11.002/0.602
pf(F_part, (a-1), (a-1)*(b-1), lower.tail = F)
```

```
## [1] 9.39006e-05
```

Since the p-value is very small ($< \alpha = 0.05$), we would reject H_0. There is sufficient evidence to suggest that the parts significantly affect measurement.

```r
F_operator <- 0.417/0.602
pf(F_operator,(a-1), (a-1)*(b-1), lower.tail = F)
```

```
## [1] 0.7034154
```

Since the p-value is $0.7034 > \alpha = 0.05$, fail to reject H_0. There is insufficient evidence to suggest that the operators significantly affect measurement.

(b) Estimate the variance components using the ANOVA method.

$\hat{\sigma}^2 = MSE = 1.500$

$\hat{\sigma}^2_{\tau\beta} = \frac{0.602-1.500}{3} = -0.2993333$

$\hat{\sigma}^2_{\tau} = \frac{11.002-0.602}{2*3} = 1.733333$

$\hat{\sigma}^2_{\beta} = \frac{0.417-0.602}{10*3} = -0.006166667$

13.5 Reanalyze the measurement systems experiment in Problem 13.1, assuming that operators are a fixed factor. Estimate the appropriate model components using the ANOVA method.

Since operators is now a fixed factor and parts is still a random factor, this is now a mixed model. The interaction, however, is still a random effect. The ANOVA table is the same, but F-statistics may change.

$H_0 : \beta_j = 0 \forall j$ vs. $H_1 : \beta_j \neq 0$ for some j

$F_0 = \frac{MSB}{MSAB} = \frac{0.417}{0.602} = 0.692691$

```r
pf(0.692691, b-1, (a-1)*(b-1), lower.tail = F)
```

```
## [1] 0.4267823
```

Since p-value $= 0.4267823 > \alpha = 0.05$, we fail to reject H_0.

$H_0 : \sigma^2_\tau = 0$ vs. $H_1 : \sigma^2_\tau > 0$

$F_0 = \frac{MSA}{MSE} = \frac{11.002}{1.500} = 7.334667$

```r
pf(7.334667, a-1, a*b*(n-1), lower.tail = F)
```

```
## [1] 3.215389e-06
```

Since p-value $= 0.0001103954 < \alpha = 0.05$, we reject H_0.

$\hat{\sigma}^2_{\tau} = \frac{MSA-MSE}{bn} = \frac{11.002-1.500}{2*3} = 1.583667$

$\hat{\sigma}^2_{\tau\beta} = \frac{MSAB-MSE}{n} = \frac{0.602-1.500}{3} = -0.2993333$

$\hat{\sigma}^2 = MSE = 1.500$