

Emily Riehl

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RESEARCH INTERESTS	Topics in higher category theory related to homotopy theory: e.g., ∞ -categories, (∞, n) -categories, homotopy type theory	
ACADEMIC POSITIONS	Johns Hopkins University , Baltimore, MD Kelly Miller Professor of Mathematics, July 2024–present Professor, July 2022–June 2024 Associate Professor, July 2019–June 2022 Assistant Professor, July 2015–June 2019 Harvard University , Cambridge, MA Benjamin Peirce Postdoctoral Fellow and NSF Postdoctoral Fellow, mentored by Michael J. Hopkins, July 2011–June 2015	
VISITING POSITIONS	Hausdorff Research Institute for Mathematics , Bonn, Germany “Prospects of Formal Mathematics” Program Participant, May–August 2024 Institut Henri Poincaré , Paris, France Visiting Professor, Sorbonne Université, June 2023 Australian National University , Canberra, Australia Visitor, January 2023 Institut Mittag-Leffler, the Royal Swedish Academy of Sciences , Djursholm, Sweden “Higher algebraic structures in algebra, topology and geometry” Program Participant, January 2022 Mathematical Sciences Research Institute , Berkeley, CA Chern Professor, January–May 2020 Centre for Advanced Study, Norwegian Academy of Science and Letters , Oslo, Norway Fellow, May–June 2019 Centre of Australian Category Theory , Sydney, Australia Visitor, January–July 2017 Max Planck Institute for Mathematics , Bonn, Germany “Higher structures in Geometry and Physics” Program Participant, January 2016 Hausdorff Research Institute for Mathematics , Bonn, Germany “Homotopy theory, manifolds, and field theories” Program Participant, May–June 2015	

Mathematical Sciences Research Institute, Berkeley, CA

Research Member, January–May 2014

EDUCATION **University of Chicago**, Chicago, IL

Ph.D. in Mathematics, June 2011; M.A. in Mathematics, June 2009

- Thesis: “Algebraic model structures” advised by J. Peter May.

University of Cambridge, Churchill College, Cambridge, UK

Certificate of Advanced Study in Mathematics (Part III) with Distinction, June 2007

- Essay: “Higher category theory” advised by Martin Hyland.

Harvard University, Cambridge, MA

A.B. Mathematics, *magna cum laude*, June 2006

- Thesis: “Lubin-Tate formal groups and local class field theory” advised by Frank Calegari.

AWARDS
& HONORS

Invited Sectional Speaker (Topology), International Congress of Mathematics, Philadelphia 2026

Provost’s Fellows for Public Engagement, 2025

Association of American Publishers PROSE Awards Category Winner in Mathematics and Statistics, for *Elements of ∞ -Category Theory* with Dominic Verity, 2023

Simons Fellow, 2022

Fellow of the American Mathematical Society, 2022

AWM-Birman Research Prize in Topology and Geometry, 2021

Joel Dean Award for excellence in teaching, 2017, 2019

Simons Visiting Professorship at Oberwolfach, 2016

Harvard University Certificate of Teaching Excellence, 2014, 2015

National Science Foundation Mathematical Sciences Postdoctoral Research Fellowship, 2011–2014

National Science Foundation Graduate Research Fellowship, 2006–2011

Churchill Scholarship, The Winston Churchill Foundation of the United States, 2006

Barry M. Goldwater Scholarship, 2005

Harvard University Certificate of Distinction in Teaching, 2004, 2005, 2006

3rd Place in Intel Science Talent Search, 2002

GRANTS

Principal Investigator, “The formal theory of higher categories,” National Science Foundation Grant DMS-2507077, 2025–2027

Principal Investigator, “Homotopical Macrocosms for Higher Category Theory,” National Science Foundation Grant DMS-2204304, 2022–2025

Principal Investigator, “Synthetic and Constructive Mathematics of Higher Structures in Homotopy Type Theory,” AFOSR Multidisciplinary University Research Initiative grant with Mike Shulman, Steve

Awodey, Favonia, Bob Harper, and Dan Licata, 2021–2025

Principal Investigator, “Toward Mathematical Intelligence and Certifiable Automated Reasoning: From Theoretical Foundations to Experimental Realization,” ARO Multidisciplinary University Research Initiative grant with Arthur Jaffe, Mikhail Lukin, and Zhenghan Wang, 2020–2025

Johns Hopkins President’s Frontier Award, 2020

Johns Hopkins Catalyst Award, 2018

Principal Investigator, “Model-Independent Foundations for Higher ∞ -Categories,” National Science Foundation CAREER Grant DMS-1652600, 2017–2022

Principal Investigator, “Reimagining the Foundations of Infinite Dimensional Category Theory,” National Science Foundation Grant DMS-1551129, 2015–2018

AMS-Simons Travel Grant, 2014

BOOKS

E. Riehl and D. Verity, *Elements of ∞ -Category Theory*, Cambridge Studies in Advanced Mathematics 194, 2022, xvii+763pp, available from: emilyriehl.github.io/files/elements.pdf

B. Gross, J. Harris, E. Riehl, *Fat Chance: Probability from 0 to 1*, Cambridge: Cambridge University Press, 2019.

E. Riehl, *Category Theory in Context*, Aurora: Modern Math Originals, Dover Publications, 2016, xvii+240 pp. also available from emilyriehl.github.io/files/context.pdf

E. Riehl, *Categorical Homotopy Theory*, New Mathematical Monographs, 24, Cambridge University Press, 2014, xviii+352 pp. also available from emilyriehl.github.io/files/cathtpy.pdf

EDITED VOLUMES

Will machines change mathematics?, guest edited by M. Fraser, A. Granville, M. H. Harris, C. McLarty, E. Riehl and A. Venkatesh, Bull. Amer. Math. Soc. volume 61, numbers 2-3, 201–342, 373–469, 2024.

A Conversation on Professional Norms in Mathematics, edited by M. Gerbelli-Gauthier, P. E. Harris, M. A. Hill, D. Karp, E. Riehl, American Mathematical Society, 2021, 138pp.

New Directions in Homotopy Theory: Second Mid-Atlantic Topology Conference, edited by N. Kitchloo, M. Merling, J. Morava, E. Riehl, W.S. Wilson, Contemporary Mathematics 707, American Mathematical Society, 2018.

FORMALIZATION

M. Carneiro, E. Riehl, *Formalizing colimits in Cat*, to appear in ITP2025, (2025), 1–18, [2503.20704](https://arxiv.org/abs/2503.20704)

N. Kudasov, E. Riehl, J. Weinberger, *Formalizing the ∞ -categorical Yoneda lemma*, CPP 2024: Proceedings of the 13th ACM SIGPLAN International Conference on Certified Programs and Proofs, January 2024, 274–290, [2309.08340](https://arxiv.org/abs/2309.08340)

PUBLISHED RESEARCH

E. Riehl and D. Verity, *Cartesian exponentiation and monadicity*, Cah. Topol. Géom. Différ. Catég., LXV-3, (2024), 294–353, [arXiv:2101.09853](https://arxiv.org/abs/2101.09853)

S. Hazratpour, E. Riehl, *A 2-categorical proof of Frobenius for fibrations defined from a generic point*, Mathematical Structures in Computer Science, Volume 34 ,Special Issue 4: Advances in Homotopy type theory, April 2024 , pp. 258–280 [arXiv:2210.00078](https://arxiv.org/abs/2210.00078)

- P. Hackney, V. Ozornova, E. Riehl, M. Rovelli, *Pushouts of Dwyer maps are $(\infty, 1)$ -categorical*, *Algebr. Geom. Topol.* **24-4** (2024), 2171–2183, [arXiv:2205.02353](#)
- P. Hackney, V. Ozornova, E. Riehl, M. Rovelli, *An $(\infty, 2)$ -categorical pasting theorem*, *Trans. Amer. Math. Soc.* **376** (2023), 555–597 [arXiv:2106.03660](#)
- E. Riehl, M. Wattal, *On ∞ -cosmoi of bicategories*, *La Matematica* **1** (2022), 740–764, [arXiv:2108.11786](#)
- E. Riehl and D. Verity, *On the construction of limits and colimits in ∞ -categories*, *Theory Appl. Categ.* **35** (2020), no. 30, 1101–1158, [arXiv:1808.09835](#)
- E. Riehl and D. Verity, *Recognizing quasi-categorical limits and colimits in homotopy coherent nerves*, *Appl. Categ. Struct.*, **28**(4), (2020), 669–716, [arXiv:1808.09834](#)
- E. Riehl and D. Verity, *∞ -category theory from scratch*, *Higher Structures* **4**(1):115–167, 2020, [arXiv:1608.05314](#)
- R. Garner, M. Kędziołek, and E. Riehl, *Lifting accessible model structures*, *J. Topology*, **13** (2020), no. 1, 59–76, [arXiv:1802.09889](#)
- E. Riehl and D. Verity, *The comprehension construction*, *Higher Structures* **2** (2018), no. 1, 116–190, [arXiv:1706.10023](#)
- K. Bauer, B. Johnson, C. Osborne, E. Riehl, and A. Tebbe, *Directional derivatives and higher order chain rules for abelian functor calculus*, *Topology Appl. Women in Topology II: Further collaborations in homotopy theory* **253** (2018), 375–427, [arXiv:1610.01930](#)
- E. Riehl and M. Shulman, *A type theory for synthetic ∞ -categories*, *Higher Structures* **1** (2017), no. 1, 116–193, [arXiv:1705.07442](#)
- K. Hess, M. Kędziołek, E. Riehl, and B. Shipley, *A necessary and sufficient condition for induced model structures*, *J. Topology* **10** (2017), no. 2, 324–367, [arXiv:1509.08154](#)
- E. Riehl and D. Verity, *Kan extensions and the calculus of modules for ∞ -categories*, *Algebr. Geom. Topol.* **17** (2017), no. 1, 189–271, [arXiv:1507.01460](#)
- E. Riehl and D. Verity, *Fibrations and Yoneda’s lemma in an ∞ -cosmos*, *J. Pure Appl. Algebra* **221** (2017), no. 3, 499–564, [arXiv:1506.05500](#)
- M. Ching and E. Riehl, *Coalgebraic models for combinatorial model categories*, *Homol. Homotopy Appl.* **16** (2014), no. 2, 171–184, [arXiv:1403.5303](#)
- E. Riehl and D. Verity, *Completeness results for quasi-categories of algebras, homotopy limits, and related general constructions*, *Homol. Homotopy Appl.* **17** (2015), no. 1, 1–33, [arXiv:1401.6247](#)
- M. Bayeh, K. Hess, V. Karpova, M. Kędziołek, E. Riehl, and B. Shipley, *Left-induced model structures and diagram categories*, *Contemp. Math.* **641** (2015), 49–81. [arXiv:1401.3651](#)
- E. Riehl and D. Verity, *Homotopy coherent adjunctions and the formal theory of monads*, *Adv. Math.* **286** (2016), 802–888, [arXiv:1310.8279](#)
- T. Barthel, J.P. May, and E. Riehl, *Six model structures for DG-modules over DGAs: model category theory in homological action*, *New York J. Math.* **20** (2014), 1077–1159, [arXiv:1310.1159](#)
- E. Riehl and D. Verity, *The 2-category theory of quasi-categories*, *Adv. Math.* **280** (2015), 549–642, [arXiv:1306.5144](#)

- E. Riehl and D. Verity, *The theory and practice of Reedy categories*, Theory Appl. Categ. **29** (2014), no. 9, 256–301, [arXiv:1304.6871](#)
- E. Cheng, N. Gurski, and E. Riehl, *Cyclic multicategories, multivariable adjunctions and mates*, J. K-theory **13** (2014), no. 2, 337–396, [arXiv:1208.4520](#)
- A.J. Blumberg and E. Riehl, *Homotopical resolutions associated to deformable adjunctions*, Algebr. Geom. Topol. **14** (2014), no. 5, 3021–3048, [arXiv:1208.2844](#)
- T. Barthel and E. Riehl, *On the construction of functorial factorizations for model categories*, Algebr. Geom. Topol. **13** (2013), no. 2, 1089–1124, [arXiv:1204.5427](#)
- E. Riehl, *Monoidal algebraic model structures*, J. Pure Appl. Algebra **217** (2013), no. 6, 1069–1104, [arXiv:1109.2883](#)
- C. Kennett, E. Riehl, M. Roy, M. Zaks, *Levels in the toposes of simplicial sets and cubical sets*, J. Pure and Appl. Algebra **215** (2011), no. 5, 949–961, [arXiv:1003.5944](#)
- E. Riehl, *On the structure of simplicial categories associated to quasi-categories*, Math. Proc. Camb. Phil. Soc. **150** (2011), no. 3, 489–504, [arXiv:0912.4809](#)
- E. Riehl, *Algebraic model structures*, New York J. Math. **17** (2011), 173–231, [arXiv:0910.2733](#)
- J. D'Angelo, S. Kos, E. Riehl, *A Sharp Bound for the Degree of Proper Monomial Mappings Between Balls*, J. Geom. Anal. **13** (2003), no. 4, 581–593.
- E. Graham Evans, Jr. and E. Riehl, *On the intersections of polynomials and the Cayley-Bacharach theorem*, J. Pure and Appl. Algebra **183** (2003), no. 1–3, 293–298.
- PUBLISHED EXPOSITION E. Riehl, *On the ∞ -topos semantics of homotopy type theory*, Bulletin of the London Mathematical Society **56**(2):461–517, February 2024, [arXiv:2212.06937](#)
- E. Riehl, *Could ∞ -category theory be taught to undergraduates?*, Notices of the AMS **70**(5), May 2023, 727–736, [arXiv:2302.07855](#)
- E. Riehl, *Homotopy coherent structures*, Expositions in Theory and Applications of Categories, 2023, No. 1, 1–31, [arXiv:1801.07404](#), also available via AMS Open Math Notes OMN:201901.110786.
- E. Riehl, *Homotopical categories: from model categories to $(\infty, 1)$ -categories*, in: Andrew J. Blumberg, Teena Gerhardt, Michael A. Hill (eds.) Stable categories and structured ring spectra, MSRI Book Series, Cambridge University Press (2022) [arXiv:1904.00886](#)
- F. Loregian and E. Riehl, *Categorical notions of fibration*, Expositiones Mathematicae **38** (2020), no. 4, 496–514, [arXiv:1806.06129](#)
- E. Riehl, *Complicial sets, an overture*, 2016 MATRIX Annals, (2017), 49–76, [arXiv:1610.06801](#)
- E. Riehl, *The Kan Extension Seminar: An Experimental Online Graduate Reading Course*, Notices Amer. Math. Soc. **61** (2014), no. 11, 1357–1358.
- A.M. Bohmann, *A comparison of norm maps*, with an appendix by A.M. Bohmann and E. Riehl, Proc. Amer. Math. Soc. **142** (2014), no. 4, 1413–1423, [arXiv:1201.6277](#)
- POPULAR WRITING M. Fraser, A. Granville, M.H. Harris, C. McLarty, E. Riehl, A. Venkatesh, *Will machines change mathematics?*, Bulletin of the AMS, Volume 61, No. 2, April 2024, 201–202.

E. Riehl, *Should all mathematical proofs be checked by a computer?*, Lost in Space-Time Column, New Scientist, July 2023.

E. Riehl, *Infinite Math*, Scientific American, October 2021.

PREPRINTS S. Awodey, E. Cavallo, T. Coquand, E. Riehl, C. Sattler, *The equivariant model structure on cartesian cubical sets*, (2024), 1–87, [arXiv:2406.18497](https://arxiv.org/abs/2406.18497)

TEACHING **Johns Hopkins University**, Baltimore, MD

Instructor, 2015–present

- *Homotopy Type Theory* — a graduate-level topics course
- *The model-independent theory of ∞ -categories* — a graduate-level topics course
- *Category Theory in Context* — a graduate-level topics course
- *Algebraic Topology II* — a graduate-level course (twice)
- *Algebra I* — a graduate-level course (twice)
- *Algebra II* — a graduate-level course
- *Honors Algebra I* — an undergraduate-level course (twice)
- *Introduction to Abstract Algebra* — an undergraduate-level course
- *Introduction to Proofs* — a (newly developed) undergraduate-level course (thrice)
- *Calculus III* — an undergraduate-level course (twice)
- *Linear Algebra* — an undergraduate-level course
- *Computer-Verified Proof: a Hands-On Introduction to Interactive Theorem Proving* — a first year seminar course

Harvard University, Cambridge, MA

Instructor in Mathematics, 2012–2015

- *Categorical Homotopy Theory* — a graduate-level topics course
- *Topology I* — an undergraduate-level course (twice)
- *Fun and Games with Discrete Mathematics* — a one-week “Wintersession” course
- *Introduction to Mathematical Logic* — an undergraduate-level course
- *Category Theory in Context* — an undergraduate-level topics course

Teaching Fellow in Mathematics, 2011

- *Multivariable Calculus*

University of Chicago, Chicago, IL

College Fellow in Mathematics, 2008–2009

- *Elementary Number Theory*, mentor: J. Boller
- *Basic Geometry*, mentor: D. Hermann
- *Introduction to Algebraic Topology*, mentor: T. Fiore

University of Cambridge, Cambridge, UK

Supervisor in Mathematics, University of Cambridge, 2006–2007

- *Coding and Cryptography*

MENTORING **Johns Hopkins University**, Baltimore, MD

Advising, 2016–present

- *Postdoctoral Mentor*
 - Egbert Rijke, 2024–present
 - Tim Champion, 2021–present
 - Paula Verdugo, 2023–2024
 - Jonathan Weinberger, 2022–2024
 - Sina Hazratpour, 2021–2024
 - Maru Sarazola, 2021–2023
 - Martina Rovelli, 2017–2019
- *Graduate Advisor*
 - Anthony Agwu, Alvaro Belmonte, Anna Matsui, Daniel Carranza
 - Astra Kolomatskaia, “Type Theory and Higher Structures,” PhD 2024
 - Paula Verdugo, “On the homotopy theory of double categories and the equivalence invariance of formal category theory,” PhD 2024 (co-supervised with Dominic Verity)
 - David Jaz Myers “Symmetry, Geometry, Modality,” PhD 2022
 - tsilil clingman “Towards the theory of proof-relevant categories,” PhD 2022
 - Daniel Fuentes-Keuthan “Goodwillie Towers of ∞ -categories and Desuspension,” PhD 2021
- *Masters Thesis Advisor*, Lyne Moser “Derivators and basic localizers,” 2016
- *Research Advisor*, Mira Wattal “On ∞ -cosmoi of bicategories,” 2020–2021.

Harvard University, Cambridge, MA

Faculty Supervisor in Mathematics, 2012–2015

- Undergraduate senior thesis project by Marina Lehner “All Concepts are Kan Extensions,” BA 2014
- Undergraduate reading courses in category theory, simplicial homotopy theory, quasi-category theory, higher category theory, and topos theory.

University of Chicago, Chicago, IL

Directed Reading Program, mentor for undergraduates, 2008–2010

- Projects involving category theory and knot theory

Research Experience for Undergraduates, mentor, 2008–2010

- Projects involving topics in algebraic topology, category theory, and number theory

LECTURES **Mini Courses & Lecture Series**

- London Mathematical Society Hardy Lectureship, June–July 2025.
 - Aberdeen, “Elements of ∞ -category theory”
 - Manchester, “Contractibility as uniqueness”
 - Glasgow, “From the 1-categorical Yoneda lemma to the ∞ -categorical Yoneda lemma”
 - Edinburgh, “Queer in math and queering math”

- Cardiff, “Homotopy types as homotopy types”
- Cambridge, “Path induction and the indiscernibility of identicals”
- Oxford, Topos Institute, “Testing artificial mathematical intelligence”
- Birmingham, “Prospects for formalizing the theory of weak infinite-dimensional categories” and “Directed univalence for left fibrations of internal ∞ -categories”
- London, Heilbronn Institute, “A reintroduction to proofs”
- London, LMS Meeting Hardy Lecture, “Could we teach ∞ -category theory to undergraduates or to a computer?”
- Bristol, “A conversation on professional norms in mathematics”
- UCLouvain with the support of the Hoover Foundation, “Elements of ∞ -Category Theory,” October 2024.
- Higher Structures in Geometry and Mathematical Physics, Institut Henri Poincaré, “Elements of ∞ -Category Theory,” June 2023.
- UCLA Distinguished Lecture Series, “Contractibility as Uniqueness,” “Path induction and the indiscernibility of identicals,” “Arrow induction and the dependent Yoneda lemma,” May 2022.
- Logic and Higher Structures, CIRM-Luminy, “On the ∞ -topos semantics of homotopy type theory,” February 2022.
- Higher structures in homotopy theory, Isaac Newton Institute for Mathematical Sciences, “The model-independent theory of $(\infty, 1)$ -categories,” Cambridge, UK, July 2018.
- Floer homology and homotopy theory summer school, “Homotopy coherent structures,” UCLA, July 2017.
- Higher Structures in Geometry and Physics Workshop, “Weak complicial sets,” MATRIX, Melbourne, June 2016.
- Young Topologists Meeting, EPFL Lausanne, “ ∞ -category theory from scratch,” July 2015.
- University of Sheffield, “Algebraic model structures and cellularity,” April 2011.

Named Lectures

- London Mathematical Society Hardy Lectureship, June–July 2025.
- C. Dwight Lahr Lecture Series, Dartmouth, “Path induction and the indiscernibility of identicals” and “Queer in math and queering math” February 2025.
- PIMS Distinguished Lecture at the University of Regina, “A reintroduction to proofs,” February 2023.
- Hayden-Howard Lecture at the University of Kentucky, “Path induction and the indiscernibility of identicals,” September 2022.
- Hanna Neumann Lecture at the 65th Annual Meeting of the Australian Mathematical Society, “From path induction to arrow induction and the dependent Yoneda lemma,” December 2021; winner of the Blue Hat Prize for best non-student talk
- Harry E. Valentine Lecture at Kansas State, “Contractibility as Uniqueness,” September 2021.
- London Mathematical Society Lecturer, “Elements of ∞ -Category Theory,” July 2021.
- Pauline Sperry Lecture at UC Berkeley, “Contractibility as Uniqueness,” April 2021.
- Kaori Kitao Lecture at Swarthmore College, “Categorifying Cardinal Arithmetic,” October 2018.
- Friends of Harvard Mathematics Junior Faculty Lecture, “The algebra and geometry of ∞ -categories,” May 2012.
- Friends of Harvard Mathematics Lecture and Honoraria, “Lubin-Tate formal groups and local class field theory,” Harvard University, April 2006.

Public Lectures

- Natural Philosophy Symposium, “A new paradigm for mathematical proof?” May 2025.
- Data Science and AI Institute and the Krieger School of Arts and Sciences Symposium Series: The Science of AI, “Can machines think logically?” February 2025.
- Hausdorff Research Institute for Mathematics, “Queer in math and queering math: in celebration of Pride,” July 2024.
- Calgary Mathematics and Philosophy Lectures, “On the art of giving the same name to different things,” February 2023.
- Math for America Thursday Think Speaker Series, “The mathematics of social choice,” April 2022.
- Santa Fe Institute Colloquium, “On the art of giving the same name to different things,” March 2022.
- The Multiplicity Turn: Theories of Identity from Poetry to Mathematics, France-Stanford Center for Interdisciplinary Studies, “On the art of giving the same name to different things,” February 2022.
- Google Internal Seminar, “An introduction to homotopy type theory and univalent foundations,” November 2021.
- Perimeter Institute Public Lecture, “A solution to the stable marriage problem,” May 2021.
- Applied Category Theory 2020 Tutorial Day, “The Yoneda Lemma in the Category of Matrices,” July 2020.
- Lambda World, Cádiz, “A categorical view of computational effects,” October 2019.
- National Math Festival, “The mathematics of social choice,” May 2019.
- Compose Conference, New York, “A categorical view of computational effects,” May 2017.
- Women, Gender, and Sexuality Graduate Colloquium and Lecture Series, “Can mathematical proof inform queer epistemology?” Johns Hopkins University, October 2016.
- Johns Hopkins Applied Physics Laboratory Colloquium, “A solution to the stable marriage problem,” June 2016.
- Informal Dynamics and Geometry Seminar, PechaKucha: Mathematics 20x20, Harvard University, “A solution to the stable marriage problem,” March 2013.

Invited Conference Talks

- AI for Mathematics and Theoretical Computer Science, Simons Institute for the Theory of Computing & Simons Laufer Mathematical Sciences Institute, “Testing artificial mathematical intelligence,” April 2025.
- International Workshop on Algebraic Topology: Conference in Honor of Prof. Peter May, “Prospects for Computer Formalization of Infinite-Dimensional Category Theory,” March 2025.
- CPP 2025, “Prospects for Computer Formalization of Infinite-Dimensional Category Theory,” January 2025.
- Lean Together 2025, “The ∞ -cosmos project: formalizing 1 -, 2 -, \mathcal{V} -, and ∞ -category theory in Lean,” January 2025.
- Santa Fe Institute, Investigating Reality: A Philosophical, Mathematical, and Scientific Exploration, “The mathematical multiverse: the case against a unified mathematical reality,” March 2024.
- Connections Workshop: Noncommutative Algebraic Geometry, Simons Laufer Mathematical Sciences Institute, “Do we want a new foundation for higher structures?” February 2024.
- Interactions of Proof Assistants and Mathematics, Universität Regensburg, “Formalizing ∞ -category theory in the Rzk proof assistant,” September 2023.
- Higher Structures in Enumerative Geometry, Institut Henri Poincaré, “Do we want a new foundation for ‘higher structures’?” June 2023.

- A panorama of homotopy theory: a conference in honour of Mike Hopkins, “Homotopy types as homotopy types,” June 2023.
- SUMS 2023 Math and Language, “On the art of giving the same name to different things,” March 2023.
- 2022 Fields Medal Symposium: Akshay Venkatesh, “How I became seduced by univalent foundations,” October 2022.
- Workshop on Categories and Topology, UC Louvain, “Absolute lifting diagrams and formal category theory,” September 2022.
- Algebraic Topology and Topological Data Analysis: a conference in honor of Gunnar Carlsson, “Path induction and the indiscernibility of identicals,” August 2022.
- Cornell Topology Festival, “Absolute lifting diagrams and formal category theory,” May 2022.
- Categories and Companions Symposium, “The formal theory of ∞ -categories,” June 2021.
- LGBTQ+Math Day, Fields Institute “Contractibility as Uniqueness,” November 2020.
- Berkeley Undergraduate Number Theory Conference, “Categorifying cardinal arithmetic,” November 2020.
- Binghamton University Graduate Conference in Algebra and Topology, “ ∞ -category theory for undergraduates,” November 2020.
- Graduate Research Opportunities for Women 2020, “Categorifying cardinal arithmetic,” October 2020.
- University of Chicago REU, “Categorifying cardinal arithmetic,” June 2020.
- Workshop on higher structures in geometry and physics, Fields Institute, “A macrocosm principle for cartesian fibrations,” November 2019.
- International Conference on Homotopy Type Theory, Carnegie Mellon University, “The equivariant uniform Kan fibration model of cubical homotopy type theory,” August 2019.
- International Category Theory Conference, University of Edinburgh, “A formal category theory for ∞ -categories,” July 2019.
- Summer School on Higher Topos Theory and Univalent Foundations, “The synthetic approach to ∞ -category theory,” June 2019.
- The Shanks Workshop in Homotopy Theory, Vanderbilt University, “The synthetic theory of ∞ -categories,” April 2019.
- Workshop on Higher Category Approach to Certifiably Correct Quantum Information Processing Systems, “A type theory for synthetic ∞ -categories,” February 2019.
- Higher Structures, Centre International de Rencontres Mathématiques, “The complicial sets model of higher ∞ -categories,” January 2019.
- Between Topology and Quantum Field Theory, a conference in celebration of Dan Freed’s 60th birthday, UT Austin, “The complicial sets model of higher ∞ -categories,” January 2019.
- Women’s Intellectual Network Research Symposium Panel Lecture, “Categorifying Cardinal Arithmetic,” University of Virginia, September 2018.
- Vladimir Voevodsky Memorial Conference, “The synthetic theory of ∞ -categories vs the synthetic theory of ∞ -categories,” Institute for Advanced Study, September 2018.
- Invited Paper Session “Category theory for all” at the MAA Math Fest, “Categorifying cardinal arithmetic,” Denver, August, 2018.
- Higher-Dimensional Rewriting and Algebra, Federated Logic Conference 2018, “Homotopy coherent adjunctions and other structures,” July 2018.
- Joint International Meeting of the American Mathematical Society and Chinese Mathematical So-

- ciety, “A model-independent theory of ∞ -categories,” Shanghai, June, 2018.
- Infinity-Categories, Infinity-Operads, and their Applications Workshop, “A proof of the model-independence of ∞ -category theory,” Casa Matemática Oaxaca, May, 2018.
 - Keynote lecture, Women in Math and Statistics Conference, Harvard University, “Categorifying Cardinal Arithmetic,” April, 2018.
 - Keynote lecture, 43rd Annual New York Regional Graduate Mathematics Conference, Syracuse University, “What is a universal property?,” March 2018.
 - ASL Invited Address, Joint Mathematics Meetings, “A synthetic theory of ∞ -categories in homotopy type theory,” January 2018.
 - Homotopy Type Theory Special Session, Joint Mathematics Meetings, “On the directed univalence axiom,” January 2018.
 - Women in Topology Workshop, MSRI, “Foundations of $(\infty, 2)$ -category theory,” November, 2017.
 - Topology of manifolds: a conference in honour of Michael Weiss’ 60th birthday, “Homology without simplices,” University of Lisbon, June 2016.
 - Homotopy Type Theory and Univalent Foundations Workshop, “Towards a synthetic theory of $(\infty, 1)$ -categories,” Fields Institute, Toronto, May 2016.
 - Midwest Topology Seminar, Ohio State University, “Model-independent ∞ -category theory in the homotopy 2-category,” May 2016.
 - Higher structures in Geometry and Physics Opening Conference, Max Planck Institute for Mathematics, Bonn, “On model-comparison results for ∞ -categories,” January 2016.
 - Workshop on category theory and algebraic topology, Université catholique de Louvain, “Model-independent ∞ -category theory in the homotopy 2-category,” September 2015.
 - Texas Undergraduate Topology and Geometry Conference, UT Austin, “Self-similar spaces,” February 2015.
 - Topologie workshop, Mathematisches Forschungsinstitut Oberwolfach, “Toward the formal theory of (∞, n) -categories,” September 2014.
 - Institute for Basic Science, Center for Geometry and Physics, Pohang, Korea, “A universal approach to universal algebra,” August 2014.
 - Days of the Federation de Recherche en Mathématiques de Paris Centre, Institute Henri Poincaré, “The formal theory of adjunctions, monads, algebras, and descent,” June 2014.
 - Reimagining the Foundations of Algebraic Topology, MSRI, “The formal theory of adjunctions, monads, algebras, and descent,” April 2014.
 - Graduate Student Topology and Geometry Conference, Young Faculty Speaker, “Quasi-category theory you can use,” University of Texas at Austin, April 2014.
 - Committee of Academic Sponsors Postdoc Talk, MSRI, “Categorical definitions in algebraic topology,” February 2014.
 - AMS Special Session on Homotopy Theory, “Homotopy coherent adjunctions,” January 2014.
 - Conference on Type Theory, Homotopy Theory, and Univalent Foundations, Centre de Recerca Matemàtica, “Made-to-order weak factorization systems,” September 2013.
 - International Category Theory Conference, “The formal theory of homotopy coherent monads,” July 2013.
 - Special Session on Progress in Higher Categories, CMS Summer Meeting, Halifax, “The formal category theory of quasi-categories,” June 2013.
 - Workshop on Higher Dimensional Algebra, Categories and Types, University of Ljubljana, “Homotopy coherent adjunctions of ∞ -categories,” June 2012.

- Special Session on Homotopy Theory and Its Applications, Association for Women in Mathematics 40th Anniversary Conference, “Algebraic model structures,” September 2011.
- Special Session on Homotopy and Categories, CMS Summer Meeting, Edmonton, “Algebraic model structures,” June 2011.

Colloquia

- ETH Zurich, “New approaches to ∞ -categorical abstract nonsense,” June 2025.
- Applied Physics Laboratory, “Can machines think logically?,” June 2025.
- UCLouvain, “Prospects for Computer Formalization of Infinite-Dimensional Category Theory” and “Collaborative Formalizations of ∞ -Category Theory,” October 2024.
- Center for Communications Research La Jolla, “A reintroduction to proofs,” March 2024.
- Virginia Commonwealth University, “Path induction and the indiscernibility of identicals,” March 2024.
- University of Maryland, “Path induction and the indiscernibility of identicals,” October 2023.
- Rutgers, “Do we need a new foundation for higher structures?,” October 2023.
- Logic @ Texas State, “Homotopy type theory and univalent foundations,” March 2023.
- Logic @ Texas State Undergraduate Colloquium, “A reintroduction to proofs,” March 2023.
- Colorado State, “An introduction to homotopy type theory and univalent foundations,” February 2023.
- TU Darmstadt, “How I became seduced by univalent foundations,” December 2022.
- University of Pennsylvania, “ ∞ -category theory for undergraduates,” November 2022.
- Antioch College, “The mathematics of voting: or how to count the votes so that your vote counts”
- Ohio State, “Contractibility as uniqueness,” October 2022.
- UC Louvain, “Path induction and the indiscernibility of identicals,” September 2022.
- Topos Institute, “Contractibility as Uniqueness,” May 2021.
- The Institute of Mathematical Sciences, Chinese University of Hong Kong, “Elements of ∞ -Category Theory,” April 2021.
- U(M), “Categorifying Cardinal Arithmetic,” April 2021.
- MSU-UM, “Elements of ∞ -Category Theory,” April 2021.
- MATRIX Institute, “Elements of ∞ -Category Theory,” March 2021.
- Brown University, “Elements of ∞ -Category Theory,” February 2021.
- University of California at Berkeley, “Elements of ∞ -Category Theory,” November 2020.
- Florida State University, “ ∞ -category theory for undergraduates,” September 2020.
- Berkeley Logic, “ ∞ -category theory for undergraduates,” May 2020.
- Morgan State University, “Homotopy types as a foundation for mathematics,” October 2019.
- Naval Academy, “Homotopy types as a foundation for mathematics,” September 2019.
- Perimeter Institute for Theoretic Physics, “The complicit sets model of higher ∞ -categories,” January 2019.
- University of Washington, “Applications of functoriality,” April 2018.
- University of Illinois Urbana-Champaign, “Applications of functoriality,” November 2017.
- Reed College, “Homotopy types as a foundation for mathematics,” October 2017.
- University of Western Ontario, “Applications of functoriality,” September 2017.
- Women in Maths special interest group of the Australian Maths society, “On women in topology and abelian functor calculus,” July 2017.
- Macquarie University, “Functoriality in algebra and topology,” March 2017.

- Indiana University, “On the duality between ‘free’ and ‘forgetful’ constructions,” January 2015.
- University of Virginia, “On the duality between ‘free’ and ‘forgetful’ constructions,” January 2015.
- Notre Dame, “On the duality between ‘free’ and ‘forgetful’ constructions,” January 2015.
- UCLA, “On the duality between ‘free’ and ‘forgetful’ constructions,” January 2015.
- Johns Hopkins University, “On the duality between ‘free’ and ‘forgetful’ constructions,” December 2014.
- Boston College, “On the duality between ‘free’ and ‘forgetful’ constructions,” November 2014.
- Cornell University Oliver Club, “On the duality between ‘free’ and ‘forgetful’ constructions,” November 2014.
- Wesleyan University, “Homotopy coherent adjunctions,” November 2013.

Seminars

- SUSTech Club of Mathematics and Physics, “Elements of ∞ -category Theory,” March 2025.
- Formalisation of mathematics with interactive theorem provers series, “Prospects for Computer Formalization of Infinite-Dimensional Category Theory,” January 2025.
- MPIM Topology Seminar, “A constructive model of synthetic homotopy theory in classical homotopy theory,” June 2024.
- HIM Prospects of Formal Mathematics Seminar, “Formalizing ∞ -category theory in the Rzk proof assistant,” May 2024.
- Australian Categories Seminar, “Formalizing the ∞ -categorical Yoneda lemma in the Rzk proof assistant,” January 2024.
- Johns Hopkins Work In Progress seminar of the Natural Philosophy Forum, “Psychological and sociological aspects of computer formalizations vs traditional pen-and-paper mathematics,” November 2023.
- University of Maryland Geometry-Topology Seminar, “Formalizing the ∞ -categorical Yoneda lemma in an experimental proof assistant,” October 2023.
- Texas State Topology Seminar, “Arrow induction and the dependent Yoneda lemma,” March 2023.
- NYU Abu Dhabi Geometry, Topology & Physics Seminar, “ ∞ -Category Theory for Undergraduates,” December 2022.
- University of Kentucky Math Club, “Categorifying cardinal arithmetic,” September 2022.
- Masaryk University Algebra Seminar, “Elements of ∞ -Category Theory,” September 2022.
- Seminario de Categorías UNAM, “Elements of ∞ -Category Theory,” March 2022.
- Institut Mittag-Leffler, “Arrow induction and the dependent Yoneda lemma,” January 2022.
- SFB Lecture, Universität Regensburg, “Arrow induction and the dependent Yoneda lemma,” December 2021.
- George Washington University Topology Seminar, “ ∞ -category theory for undergraduates,” November 2021.
- SFSU Algebra, Geometry, and Combinatorics Seminar, “ ∞ -category theory for undergraduates,” October 2021.
- Münster Oberseminar Topologie, “Elements of ∞ -Category Theory,” April 2021.
- University Quantum Symmetries Lectures, “Elements of ∞ -Category Theory,” April 2021.
- Wales Mathematical Physics–Physical Mathematics Zoom Seminar, “Elements of ∞ -Category Theory,” April 2021.
- Trinity College Maths Society, “Categorifying Cardinal Arithmetic,” March 2021.
- Harvard Mathematical Picture Languages, “Elements of ∞ -Category Theory,” February 2021.

- Princeton Algebraic Topology, “Elements of ∞ -Category Theory,” November 2020.
- Tetrahedral Geometry/Topology, “Homotopy types as a foundation for mathematics,” November 2019.
- UC Boulder Topology, “The comprehension construction,” August 2019.
- UCLA Topology, “The synthetic theory of ∞ -categories,” April 2019.
- Penn Topology, “Homotopical categories: from model categories to $(\infty, 1)$ -categories,” April 2019.
- MIT Topology, “The synthetic theory of ∞ -categories,” November, 2018.
- University of Chicago Topology, “A model-independent theory of $(\infty, 1)$ -categories,” April 2018.
- Homotopy Type Theory Electronic Seminar Talks, “The synthetic theory of ∞ -categories vs the synthetic theory of ∞ -categories,” March 2018.
- Stanford Topology, “A synthetic theory of $(\infty, 1)$ -categories in homotopy type theory,” November 2017.
- University of Illinois Urbana-Champaign Topology, “A synthetic theory of $(\infty, 1)$ -categories in homotopy type theory,” November 2017.
- George Mason, Topology, Arithmetic, Dynamics, “Homotopy types as a foundation for mathematics,” October 2017.
- New York City Category Theory, “Towards a synthetic theory of $(\infty, 1)$ -categories,” May 2017.
- Rochester Topology, “Model-independent ∞ -category theory in the homotopy 2-category,” April 2016.
- CMU Homotopy Type Theory, “Towards a synthetic theory of $(\infty, 1)$ -categories,” April 2016.
- MIT Topology, “The formal category theory of (∞, n) -categories,” March 2015.
- Indiana University Topology, “The formal category theory of $(\infty, 1)$ -categories,” December 2014.
- University of Illinois at Urbana-Champaign Topology, “The formal category theory of $(\infty, 1)$ -categories,” December 2014.
- Georgia Tech Geometry and Topology, “On the duality between ‘free’ and ‘forgetful’ constructions,” December 2014.
- Johns Hopkins University Topology, “Toward the formal theory of higher homotopical categories,” October 2014.
- University of Western Ontario Geometry and Topology, “Homotopy coherent adjunctions, monads, and algebras,” March 2014.
- MIT Topology, “Cell complex presentations for (generalized) Reedy categories,” November 2013.
- Wesleyan University Topology, “Algebraic model categories,” November 2013.
- University of Chicago Topology, “Algebraic perspectives on (generalized) Reedy categories,” October 2013.
- Northwestern Topology, “Algebraic perspectives on (generalized) Reedy categories,” September 2013.
- Algebra/Geometry/Topology, University of Melbourne, “Homotopy coherent adjunctions and the formal category theory of quasi-categories,” July 2013.
- Stanford University Topology, “Homotopy coherent adjunctions and the formal category theory of quasi-categories,” December 2012.
- MIT Topology, “The formal category theory of quasi-categories,” November 2012.
- Wayne State University Topology, “Homotopy coherent adjunctions of quasi-categories,” November 2012.
- University of Virginia Topology, “Homotopy coherent adjunctions of quasi-categories,” November 2012.
- Midwest Topology, Northwestern University, “Lifting properties and the small object argument,”

March 2012.

- MIT Topology, “Algebraic model structures and cellularity,” December 2011.
- University of Illinois at Urbana-Champaign Topology, “Algebraic model structures,” October 2010.
- University of Illinois at Urbana-Champaign, “On the properties of Tits graphs,” August 2002.

Contributed Talks

- Hausdorff Institute Workshop: Bridging between informal and formal, “Formalizing post-rigorous mathematics,” July 2024.
- Australian Categories Seminar, “Diagram chasing with the mates correspondence,” January 2023.
- Australian Categories Seminar, “A Chevalley criterion for cartesian arrows,” January 2021.
- Australian Categories Seminar, “A formal language for model-independent $(\infty, 1)$ -category theory,” December 2020.
- CT2018, Azores, “A proof of the model-independence of $(\infty, 1)$ -category theory,” July 2018.
- Octoberfest, Carnegie Mellon University, “A synthetic theory of ∞ -categories in homotopy type theory,” October 2017.
- International Category Theory Conference, University of British Columbia, “A synthetic theory of ∞ -categories in homotopy type theory,” July 2017.
- Australian Categories Seminar, Sydney, “Homotopy coherent structures, redux,” July 2017.
- Australian Categories Seminar, Sydney, “Homotopy coherent structures,” July 2017.
- Australian Categories Seminar, Sydney, “A synthetic theory of ∞ -categories in homotopy type theory,” May 2017.
- Australian Categories Seminar, Sydney, “Algebraic and homotopical aspects of general Reedy categories,” March 2017.
- Australian Categories Seminar, Sydney, “When is a diagram cofibrant?” February 2017.
- Australian Categories Seminar, Sydney, “Cell complex presentations for Reedy categories,” February 2017.
- International Category Theory Conference, Halifax, “Higher order chain rules for abelian functor calculus,” August 2016.
- 98th Peripatetic Seminar on Sheaves and Logic, Doorn, the Netherlands, “A model-independent calculus of pointwise Kan extensions for ∞ -categories,” January 2016.
- International Category Theory Conference, “Virtual equipments for ∞ -categories,” July 2015.
- MSRI Postdoc Seminar, “Toward a formal theory of adjunctions, monads, and descent,” March 2014.
- MSRI Connections for Women: Algebraic Topology, “Limits of quasi-categories with (co)limits,” January 2014.
- Samuel Eilenberg Centenary Conference, “The formal theory of homotopy coherent monads,” July 2013.
- Basic Notions Seminar, Harvard University, “Weighted limits and colimits,” November 2012.
- Category Theory OctoberFest, Centre de Recherche en Théorie des Catégories, Montreal, “Parametrized mates,” October 2012.
- Northwestern Topology Seminar, “Homotopy coherent adjunctions,” August 2012.
- International Category Theory Conference, “Cellularity, composition, and morphisms of algebraic weak factorization systems,” July 2011.
- International Category Theory Conference, “Algebraic model structures,” June 2010.
- New York City Category Theory Seminar, City University of New York Graduate Center, “Natural weak factorization systems in model structures,” May 2010.

- The Third Morgan-Phoa Mathematics Workshop, Australian National University, Canberra, “An advertisement for natural weak factorization systems in model structures,” January 2010.

Partial list of Panels

- “Artificial Intelligence and Publishing,” Joint Mathematics Meetings, January 2025
- Connections Workshop: Noncommutative Algebraic Geometry, Simons Laufer Mathematical Sciences Institute, February 2024
- “The Future of Graduate Mathematics Textbooks,” Joint Mathematics Meetings, January 2024
- “Category theory outreach,” Topos Institute, March 2023
- “Prospects in machine assisted proof,” IPAM workshop on Machine Assisted Proofs, February 2023
- “Past, Present, Future: Constructing Queer Spaces in Mathematics,” Spectra’s LGBTQ+ in Math Conference at ICERM, August 2021
- “Developing a research program,” Lunch in the Time of Covid, August 2021
- BEAM Career Day, July 2021
- “LGBTQ in Stem,” Physics Club of the City College of New York, May 2021
- “Transition to research,” paraDIGMS 2021 Spring Conference, April 2021
- “Our stories,” OURFA²M², December 2020
- LGBTQ+Math Panel, Fields Institute, November 2020
- “Applying to Graduate School,” UC Berkeley Undergraduate Number Theory Conference, November 2020
- BEAM Career Day, July 2020
- Society for Science & the Public Alumni Brunch & Panel, March 2019
- Professional Panel, Out in STEM, November 2018
- “Out in Mathematics: Professional issues facing LGBTQ mathematicians,” Joint Mathematics Meetings, January 2018
- Homotopy Type Theory Panel, Joint Mathematics Meetings, January 2018

SELECTED
SERVICE

Special Programs

- Lead Organizer for a MSRI semester program “Higher categories and categorification,” 2020.
- Co-Organizer of the Mathematics Research Community on Homotopy Type Theory, 2017.

Conference Organization

- Co-Organizer for *Communicating Mathematics*, a workshop conceived by Katie Mann, 2022.
- Organizer of a special section in Homotopy Type Theory at the North American Annual Meeting of the Association for Symbolic Logic, 2022.
- Co-Organizer of MayDay 2019, the Midwest Topology Conference in honor of Peter May’s 80th birthday 2019.
- Co-Organizer of the Mid-Atlantic Topology Conference and Co-PI for the associated NSF grant DMS-1619569, 2016.
- Scientific Committee for various international conferences in category theory (2016, 2018, 2023, 2025) and homotopy type theory (2023).

Mentorship and Teaching

- Organizer and Lecturer for the HoTTEST Summer School, an online introduction to homotopy type theory, 2022.

- Co-Mentor for the 2018 MIT Talbot Workshop: the model-independent theory of ∞ -categories.
- Co-Organizer of the Homotopy Type Theory Electronic Seminar Talks, 2022–present.
- Organizer of the Johns Hopkins Category Theory Seminar, 2015–present.
- Organizer of the Kan Extension Seminar, an online graduate reading course in category theory, 2014, 2017.
- Co-Host of the n -Category Café, 2013–present.
- PhD Examiner for Dimitri Zaganidis, Pierre Cagne, Zeinab Bandpey, Thibaut Benjamin, Lyne Moser, Chaitanya Leena Subramaniam, Jonathan Weinberger, Antoine Allioux, Félix Loubaton, and Louis Martini, among others

Committee Service

- Member of the AMS Executive Committee, 2024–2028
- Member at Large of the AMS Council, 2023–2026
- Member of the AMS Committee on the Profession, 2023–2026
- Member of the AMS Advisory Committee on Artificial Intelligence, 2023–2025
- Member of the AMS Committee on Committees, 2023–2027
- Member of the PIMS Scientific Review Panel, 2023–2025
- Founding Board Member, Political Chair, Board of Trustees Member for Spectra: the association for LGBT mathematicians, 2015–2020, 2023, 2025–present
- Inaugural Member of BIRS Board of Equity, Diversity and Inclusion, 2021–2023
- Member of the AWM-AMS Noether Lecture Selection Committee, 2021–2023
- Member of the AMS University Lecture Series Editorial Committee, 2019–2023
- Member of the AMS Mathematics Research Communities Advisory Board, 2017–2020
- Member of the AMS Web Editorial Group, 2015–2017
- Member of the Churchill Scholarship Screening Committee, 2014
- Member of NSF review panels for the Division of Mathematical Sciences

Editorial service

- Editor for
 - Zeitschrift für Mathematische Logik und Grundlagen der Mathematik (2025–present)
 - Bulletin of the AMS (2024–present)
 - Mathematical Logic Quarterly (2023–2025)
 - the Journal of Pure and Applied Algebra (2019–2023)
 - the Journal of Homotopy and Related Structures (2016–2021)
 - Homology, Homotopy, and Applications (2015–present)
 - Cahiers de Topologie et Géométrie Différentielle Catégoriques (2014–2019)
- Referee for various journals.

Departmental service

- Director of Graduate Studies at Johns Hopkins 2025–present.
- Diversity Champion for the mathematics department at Johns Hopkins 2015–2025.
- Member and occasional chair of a rotating list of departmental committees at Johns Hopkins 2015–present.
- Member of the Qualifying Exams Committee 2011, the Colloquium Committee 2012, and the Graduate Admissions Committee 2015; Junior Advisor 2012–2013; Undergraduate Thesis Supervisor 2013–2014; Assistant Director of Graduate Studies 2014–2015 at Harvard.

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