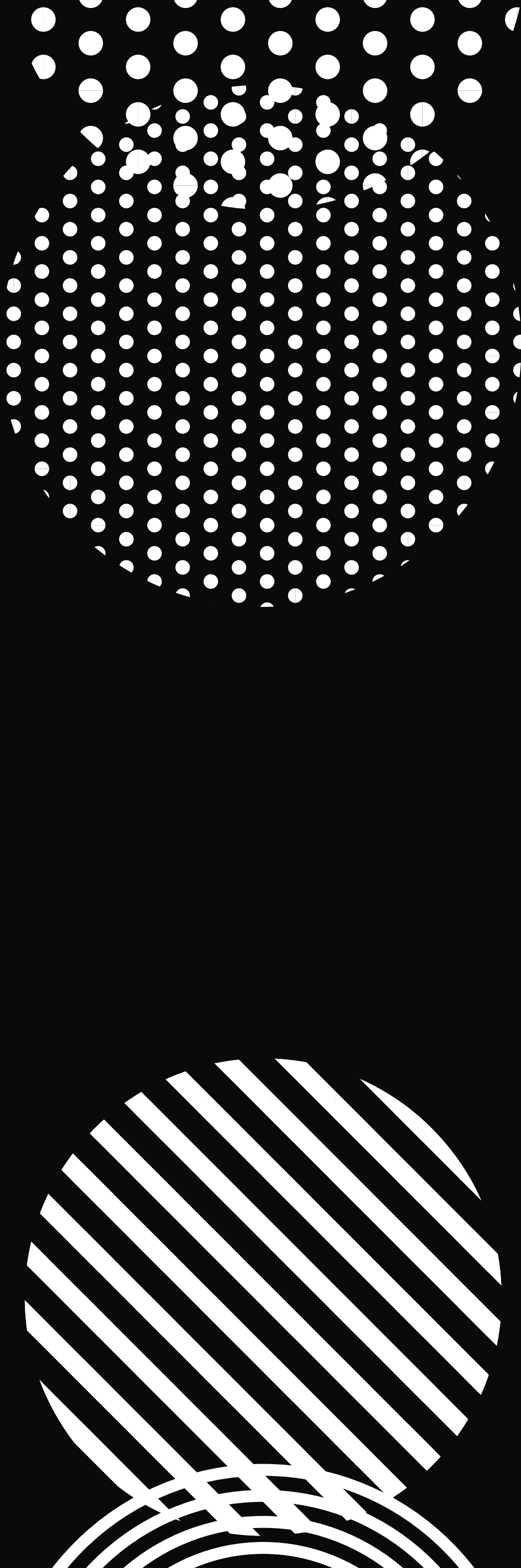


CHAPTER 1

DECISION MAKING AND PROBLEM CLASSIFICATION

- decision making process:
  1) recognize problem
  2) define goal or objective
  3) assemble relevant data
  4) identify feasible alternatives
  5) select criterion to determine best model
  6) construct model
  7) predict outcomes or consequences for each alternative
  8) choose best alternative
  9) audit result

ENGINEERING COSTS

- classification of costs:
  - ↳ fixed cost : constant & doesn't depend on level of output or activity
    - e.g. insurance costs, property taxes, salaries, insurance
  - ↳ variable cost: depends on level of output or activity
    - e.g. product supplies, fuel, raw material used in production, cost of utilities that relate to prod volume
  - ↳ total cost: sum of fixed & variable costs
  - ↳ marginal cost: variable cost to produce 1 more unit output
    - can be diff based on # units already produced
  - ↳ average cost: total cost of output or activity divided by # output units produced

- Let us assume a project with the following characteristics:
  - x units of production sold at $\boldsymbol{\rho}$ unitary price.

$FC = a_0$

$VC = b_0 x$

$TC(x) = a_0 + b_0 x$

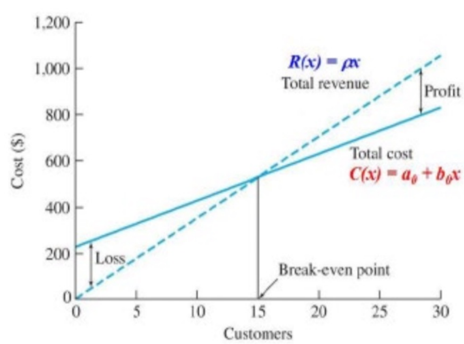
Revenue $R(x) = \rho x$

Total profit $\Omega = R(x) - TC(x)$

$\Omega = \rho x - (a_0 + b_0 x)$

Break-even $\Omega = 0$

$x = \frac{a_0}{\rho - b_0}$



- break-even point in when total profit $\Omega = 0$
  - ↳ $\Omega = 0$ or $R(x) = TC(x)$

- sunk costs: money has already been spent & can't be recovered due to past decision
  - ↳ shouldn't be considered when making decision of investing in project
  - ↳ e.g. when selling 10-yr-old car, initial cost doesn't define selling price
- opportunity costs: associated w/resource being used for alternate task
  - ↳ aka forgone opportunity costs
- incremental costs: cost diffs btwn 2 alternatives
- cash costs: require cash transaction
- book costs: don't require cash transaction & rep change of value
  - ↳ i.e. depreciation accounts
  - ↳ not considered in eng economic analysis b/c not considered transactions
- recurring costs: known, anticipated, & occurs at regular intervals

↳ e.g. maintenance costs, leasing costs
- non-recurring costs: one-of-a-kind & appears at irregular intervals
- life-cycle costs: occur over various phases of product or service life cycle

- cost estimation is process of apx total expenditure of project
  ↳ accuracy depends on project info, available info, & project estimator expertise
    - can be rough, budget, or detailed
- types of cost estimation models:
  ↳ per-unit model: estimation is made for single unit
    - total cost estimate is computed by multiplying this cost by # units
    - requires estimation of # units
  ↳ segmenting model: cost estimation task divided into individual components (i.e. segments)
    - total cost is sum of all segments
  ↳ cost indexes: historical cost data used to estimate new cost values using ratio rltnships
    - adjustments made thru use of indexes like Consumer Price Index (CPI)
    - $\frac{\text{cost at time A}}{\text{cost at time B}} = \frac{\text{index value at A}}{\text{index value at B}}$
- learning curve is percentage or rate at which output is inc due to repetition
  ↳ $T_N = T_{initial} \cdot N^b$
    - T_N is time req for N^{th} unit of prod
    - $T_{initial}$ is time req for 1^{st} unit of prod
    - N is # completed units
    - b is learning curve exponent: $b = \frac{\log(\text{learning curve percentage}\{\text{decimal}\})}{\log(2)}$
      → log(2) b/c learning curve percentage applies for doubling cumulative prod
- economic analysis often requires also considering benefits in addition to cost
  ↳ many methods used to calc cost can be used for calc benefits
- benefits are typically in future so harder to estimate
- e.g.

  - Miriam is interested in estimating the annual labour and material costs for a new production facility. She was able to obtain the following labour and material cost data:
    - **Labour costs:**
      - Labour cost index value was at 124 ten years ago and is 188 today. Annual labour costs for a similar facility were$575,500 ten years ago.
    - **Material costs:**
      - Material cost index value was at 544 three years ago and is 715 today. Annual material costs for a similar facility were $2,455,000 three years ago.

Eqn: $\frac{\text{cost at time A}}{\text{cost at time B}} = \frac{\text{index value at A}}{\text{index value at B}}$

Let AC rep annual cost.

Labour:

$$\frac{\text{AC today}}{\text{AC 10yrs ago}} = \frac{\text{index value today}}{\text{index value 10yrs ago}}$$

$$AC_{today} = AC_{10yrs}\left(\frac{\text{index}_{today}}{\text{index}_{10yrs}}\right)$$

$$= 575500\left(\frac{188}{124}\right)$$

$$= 872\,532.2581$$

$$= \$872\,532$$

Materials:

$$AC_{today} = AC_{3yrs}\left(\frac{\text{index}_{today}}{\text{index}_{3yrs}}\right)$$

$$= 2455000\left(\frac{715}{544}\right)$$

$$= 3\,226\,700.368$$

$$= \$3\,226\,700$$

- e.g.
  - Calculate the time required to produce the hundredth unit of a production run if the first unit took 32.0 minutes to produce and the learning curve rate for production is 80%.

$$b = \frac{\log(0.8)}{\log(2)} = -0.321928$$

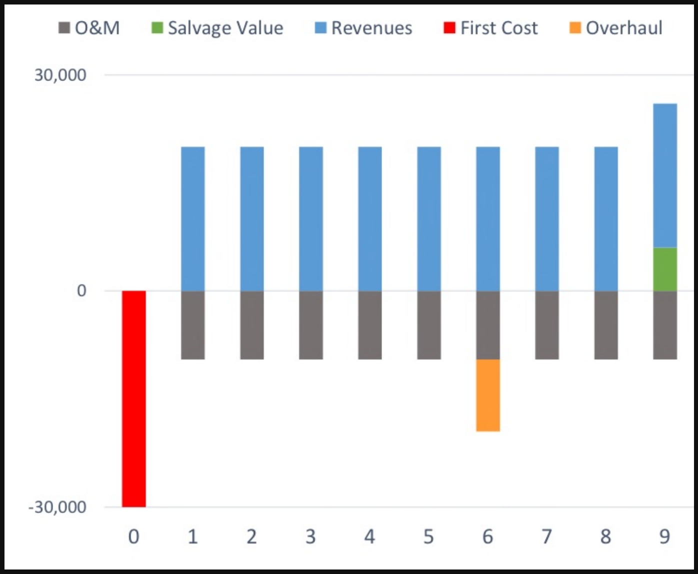
$$T_{100} = 32 \cdot 100^{-0.321928}$$

$$= 7.265973$$

$$= 7.27 \text{ min}$$

CASH FLOW

- cash flow diagrams (CFD) summarize costs (i.e. expenses) & benefits (i.e. income) occurring over some time intervals
  - ↳ rep size, sign, & timing of individual CFs
- components of CFDs:
  - ↳ x-axis rep time
  - ↳ y-axis rep size & dir of CF
    - up arrows rep +ve CFs / receipts (i.e. money received / earned)
    - down arrows rep -ve CFs / disbursements (i.e. money paid / spent)
    - #s above / belows arrows rep CF magnitudes
- when 1+ CFs occur at same period, they can be rep individually or added
  - ↳ can only directly +/- CFs when they occur at same time
- assume CFs occur at end of period or End-of-Year (EOY)
- categories of CF:
  - ↳ first cost: expenses to build or buy & install
  - ↳ operations & maintenance (O&M): annual expense (e.g. electricity, labour, & minor repairs)
  - ↳ salvage value: money received at project termination for sale or transfer of equipment
  - ↳ revenues: annual receipts due to sale of products / services
  - ↳ overhaul: major capital expenditure that occurs during asset's life
- e.g. CFD



CHAPTER 2

TIME VALUE OF MONEY

- interest indicates rent paid for use of money
  - ↳ can be from both lender & borrower's POV
- amount of money tdy is Principal Amount P
- amount later repaid is Future Amount F
- interest I can be expressed as interest rate i wrt P
- interest rate i is formally expressed per time period (i.e %/ time period)
  - ↳ e.g. annually, semi-annually, quarterly, monthly, weekly, daily, continuous
  - ↳ base unit of time is Interest Period
- interest rate gives rate of exchange btwn money at beginning of period & right to money at end
  - ↳ if right to P exchanges for right to F at end of period, where $F = P(1+i)$, then:
    - F is Future Worth (FW) of P
    - P is Present Value (PV) of F
- e.g.

> ▪ Samuel bought a one-year GIC (Guaranteed Investment Certificate) for $5,000 from a bank on May 15 last year. The bank was paying 5 percent on one-year GICs at the time. How much would Samuel cash in his certificate after one year? What would be the interest amount earned?

$F = P(1+i)$
$= 5000(1+0.05)$
$= \$5250$

$I = iP$
$= 0.05(5000)$
$= \$250$

- simple interest $I = iP$ is diff btwn amount of money lent tdy & amount of money later repaid
  - ↳ w/investment that pays simple interest, amount of interest accumulated each period depends solely on amount invested, not on prior interest earned & left in account
  - ↳ use single payment eqn: $F = P + I$

$$F = P + iP \cdot n$$
$$F = P(1 + in)$$

- P is principal amount
- F is future amount
- i is simple interest rate
- n is period of time
- $I = iP \cdot n$

- e.g.

> ▪ Find the simple interest on $4,500 at 8% per year for: a) 1 year and b) 4 years. Also, determine the future amounts in the two cases.

a) $I = iP$
$= 0.08(4500)$
$= \$360$

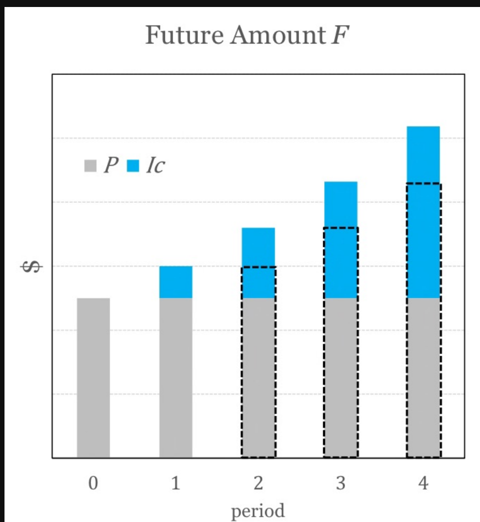
$F = P + I$
$= 4500 + 360$
$= \$4860$

b) $I = iP \cdot n$
$= 0.08(4500) \cdot 4$
$= \$1440$

$F = P + I$
$= 4500 + 1440$
$= \$5940$

when dealing w/compound interest, amount of money to be repaid for N periods is usually paid 1 period at a time & reinvested to generate additional earnings over time
- ↳ i.e loans/investments may be paid in several periods
- ↳ simple interest is calc solely on principal amount while compound interest takes into account both principal & any interest earned/paid
- ↳ final total amount is larger when large sums of money, higher interest rates, or greater #periods are involved
- for compound interest, total amount repaid at end of N periods is $F = P(1+i)^N$
  - ↳ total compound interest on loan paid at end of N periods is $I_c = P(1+i)^N - P$
  - ↳ interest period is called compounding period

Period	Amount lent	Interest	Amount owed	Simple interest
1	P	iP	$P + iP = P(1+i)$	$P(1+i)$
2	$P(1+i)$	$iP(1+i)$	$P(1+i) + iP(1+i) = P(1+i)^2$	$P(1+2i)$
3	$P(1+i)^2$	$iP(1+i)^2$	$P(1+i)^2 + iP(1+i)^2 = P(1+i)^3$	$P(1+3i)$
⋮	⋮	⋮	⋮	⋮
N	$P(1+i)^{N-1}$	$iP(1+i)^{N-1}$	$P(1+i)^{N-1} + iP(1+i)^{N-1} = P(1+i)^N$	$P(1+Ni)$



e.g.

- If you lent $2,000 for 5 years at 12%/year compound interest, how much interest would you get at the end of the 5 years?

Period	Amount lent	Interest	Amount owed	Simple interest
1	2,000	$2{,}000 * 0.12 = 240$	2,240	2,240
2	2,240	$2{,}240 * 0.12 = 268.8$	2,508.8	2480
3	2,508.8	$2{,}508 * 0.12 = 301.06$	2,809.86	2,720
4	2,809.86	$2{,}809.86 * 0.12 = 337.18$	3,147.04	2,960
5	3,147.04	$3{,}147.04 * 0.12 = 377.65$	3,524.68	3,200

$F = P(1+i)^N$
$= 2000(1+0.12)^5$
$= 3524.683366$
$= \$3524.68$

$I_c = F - P$
$= 3524.68 - 2000$
$= \$1524.68$

- as # periods inc, diff btwn accumulated interest amounts for 2 methods inc exponentially
- if interest & compounding period aren't stated, then interest rate is annual w/annual compounding
  - ↳ e.g. 12% interest means interest rate is 12% per year, compounded annually
  - ↳ e.g. 12% interest compounded monthly means interest rate is 12% per year so $\frac{12\%}{12} = 1\%$ per month
- when compounding period is not annual, problems must be solved in terms of compounding period
- nominal interest rate (i_r) states annual interest rate w/o including effect of any compounding during year
  - ↳ calc as product of interest rate per compounding period (i_s) by # m equal subperiods
    - $i_r = i_s \cdot m$ or $i_s = \frac{i_r}{m}$
- effective interest rate (i_e) is actual interest rate, found by converting given interest rate w/arbitrary compounding period (usually < 1 year) to equivalent rate w/1-year compounding period
  - ↳ $F = P(1+i_s)^m = P(1+i_e)$
  - ↳ $(1+\frac{i_r}{m})^m = (1+i_e)$

$i_e = (1 + \frac{i_r}{m})^m - 1$

- e.g

- If you lent \$2,000 for 5 years at an interest rate of 12%/year **compounded Quaterly (every 3 months)**, what future value would you get at the end of the 5 years?, and what is the effective annual interest?

$F = P(1 + \frac{i_r}{m})^{m \cdot n}$
$= 2000(1 + \frac{0.12}{4})^{4 \cdot 5}$
$= 3612.22247$
$= \$3612.22$

$i_e = (1 + \frac{i_r}{m})^m - 1$
$= (1 + \frac{0.12}{4})^4 - 1$
$= 0.12551$
$= 12.55\%$ per year

- e.g.

- If a credit card charges 1% interest every month, what are the nominal & effective interest rates per year?

$i_r = i_s \cdot m$
$= 0.01 \cdot 12$
$= 0.12$
$= 12\%$ per year

$i_e = (1 + i_s)^m - 1$
$= (1 + 0.01)^{12} - 1$
$= 0.126825$
$= 12.68\%$ per year

- if compounding periods are made infinitesimally small, then interest is compounded continuously
  - ↳ $\lim_{m \to \infty} (1 + \frac{i_r}{m})^m = e^{i_r}$
    - $F = P \lim_{m \to \infty} (1 + \frac{i_r}{m})^{mn}$
    - $F = Pe^{n i_r}$
    - $i_e = e^{i_r} - 1$
  - ↳ not used often

- e.g.

- If you lent \$2,000 for 5 years at an interest rate of 12%/year **compounded continuously**, what future value you get at the end of the 5 years?, and what is the effective annual interest?

$F = Pe^{n i_r}$
$= 2000e^{5(0.12)}$
$= 3644.237601$
$= \$3644.24$

$i_e = e^{i_r} - 1$
$= e^{0.12} - 1$
$= 0.127497$
$= 12.75\%$ per year

- summary table of eqns

Interest Type	Definition	Equation
Simple Interest	The amount of interest accumulated each period depends solely on the amount invested, not on prior interest earned and left in the account.	$F = P(1 + in)$
Compound Interest	The interest earned during each period is added to the principal sum	$P(1+i)^N$ Annual compound $P\left(1+\frac{i_r}{m}\right)^{mN}$ Sub-periods Pe^{Ni_r} Continuous
Nominal Interest rate (i_r)	The conventional method for stating the annual interest rate (i.e., it is the annual interest rate not including the effect of any compounding during the year).	$i_s = \frac{i_r}{m}$
Effective Interest rate (i_e)	The actual interest rate, found by converting a given interest rate with an arbitrary compounding period (less than a year) to an equivalent interest rate with one year compounding period	$i_e = e^{i_r} - 1$ $i_e = \left(1+\frac{i_r}{m}\right)^m - 1$

CHAPTER 3

COMPOUND INTEREST FACTORS

- compound interest factors are formulas that define mathematical equivalence for specific common cash flow patterns
- CF patterns:
  - ↳ single disbursement (money spent) or receipt (money received)
  - ↳ set of equal disbursements/receipts over sequence of periods
    - i.e. annuity
  - ↳ set of disbursements/receipts that changes by constant amount from 1 period to next in sequence of periods
    - i.e. arithmetic gradient series
  - ↳ set of disbursements/receipts that changes by constant proportion from 1 period to next in sequence of periods
    - i.e. geometric gradient series
- compound amount factor gives F that's equiv to P when interest rate is i & # compounding periods is N
  - ↳ $(\frac{F}{P}, i, N) = (1+i)^N$
    compound amount factor
  - ↳ $F = P(\frac{F}{P}, i, N)$
    $= P(1+i)^N$
  - ↳ note that time frame of each param must be same
- present worth factor gives P that's equiv to F when interest rate is i & # compounding periods is N
  - ↳ $(\frac{P}{F}, i, N) = \frac{1}{(1+i)^N}$
  - ↳ $P = F(\frac{P}{F}, i, N) = F\left(\frac{1}{(1+i)^N}\right)$
- e.g.

  ▪ If $2,000 are invested in a fixed deposit for 20 years, how much would by repaid at the end of this period if the nominal interest rate is 12%/year compounded monthly?

  - ↳ sol 1:
    $i_r = 12\%$ per year, $N = 20$, $m = 12$, $P = \$2000$, $F = ?$
    $F = P(\frac{F}{P}, i, N \cdot m)$
    $= 2000(1 + 0.01)^{20 \cdot 12}$
    $= 21785.10731$
    $= \$21\,785$
  - ↳ sol 2:
    $i_e = (1 + \frac{0.12}{12})^{12} - 1$
    $= 0.126825$
    $= 12.6825\%$
    $F = P(\frac{F}{P}, i, N)$
    $= 2000(1 + 0.126825)^{20}$
    $= 21785.09566$
    $= \$21785$
- series present worth factor gives P that's equiv to an annuity w/ disbursements/receipts in amount of A, where interest rate is i & # periods is N
  - ↳ $(\frac{P}{A}, i, N) = \frac{(1+i)^N - 1}{i(1+i)^N}$
  - ↳ $P = A(\frac{P}{A}, i, N) = A\frac{(1+i)^N - 1}{i(1+i)^N}$
- uniform series compound amount factor gives F that's equiv to series of equal size receipts/disbursements A
  - ↳ $(\frac{F}{A}, i, N) = \frac{(1+i)^N - 1}{i}$
  - ↳ $F = A(\frac{F}{A}, i, N) = A\frac{(1+i)^N - 1}{i}$
- capital recovery factor gives value of A of equal periodic payments/receipts that are equiv to P
  - ↳ $(\frac{A}{P}, i, N) = \frac{i(1+i)^N}{(1+i)^N - 1}$
  - ↳ $A = P(\frac{A}{P}, i, N) = P\frac{i(1+i)^N}{(1+i)^N - 1}$
- sinking fund factor is an account into which regular deposits are made to accumulate some amount of

money & it gives size A of repeated receipt/disbursement that's equiv to F

↳ $(\frac{A}{F}, i, N) = \frac{i}{(1+i)^N - 1}$

↳ $A = F(\frac{A}{F}, i, N) = F\frac{i}{(1+i)^N - 1}$

· e.g.

- Suppose that a recent college graduate has $3,000 available as a down payment on a new car. The graduate can afford a uniform car loan payment of no more than $500 per month for 48 months, beginning 1 month from now. Interest is 6%, compounded monthly. What is the most that the graduate can spend today on a new car?

$i_r = 6\%$ per year, $N = 4$, $m = 12$, $X = \$3000$, $A = \$500$

$i_s = \frac{6\%}{12}$

$= 0.5\%$ per month

$P = 3000 + A(\frac{P}{A}, i, N, m)$

$= 3000 + 500\left(\frac{(1+i_s)^{N \cdot m} - 1}{i_s(1+i_s)^{N \cdot m}}\right)$

$= 3000 + 500\left(\frac{(1+0.005)^{4 \cdot 12} - 1}{0.005(1+0.005)^{4 \cdot 12}}\right)$

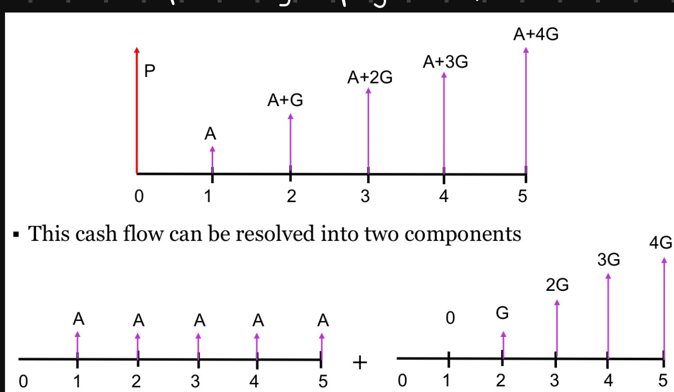
$= 24290.15889$

$= \$24290$

· arithmetic gradient series is series of receipts/disbursements that starts at 0 at end of 1st period & then inc by constant amount G from period to period

↳ e.g. increment in maintenance costs due to aging

↳ assume compounding & payment periods are same

↳ 

· suppose there's arithmetic gradient series that has G change from 1 period to next

↳ single payment present worth: $P = G(\frac{P}{G}, i, N) = G\left(\frac{(1+i)^N - iN - 1}{i^2(1+i)^N}\right)$

↳ sinking fund payment: $A = G(\frac{A}{G}, i, N) = G\left(\frac{1}{i} - \frac{N}{(1+i)^N - 1}\right)$

↳ single payment future worth: $F = G(\frac{F}{G}, i, N) = \frac{G}{i}\left(\frac{(1+i)^N - 1}{i} - N\right)$

· e.g.

- Susan wants to find the present worth of repair bills of his eight-year-old Prius over the four years that she expects to keep the car. Susan has the car in for repairs every six months. Repair costs are expected to increase by $50 every six months, over the next four years, starting with $500 six months from now, $550 six months later, and so on. What is the present worth of the repair cost over the next four years if the interest rate is 12% compounded monthly?

$i_r = 12\%$/year, $N = 4$, $m = 2\ (\frac{1 \text{ year}}{6 \text{ months}})$, $A' = 500$, $G = 50$

$i_s = \frac{12\%}{12} = 1\%$/month

$i_e = (1 + 0.01)^6 - 1$

$= 0.06152$

$= 6.152\%$ / 6 months

$A = A' + G\left(\frac{1}{i_e} - \frac{N \cdot m}{(1+i_e)^{N \cdot m} - 1}\right)$

$= 500 + 50\left(\frac{1}{0.06152} - \frac{4 \cdot 2}{(1+0.06152)^{4 \cdot 2} - 1}\right)$

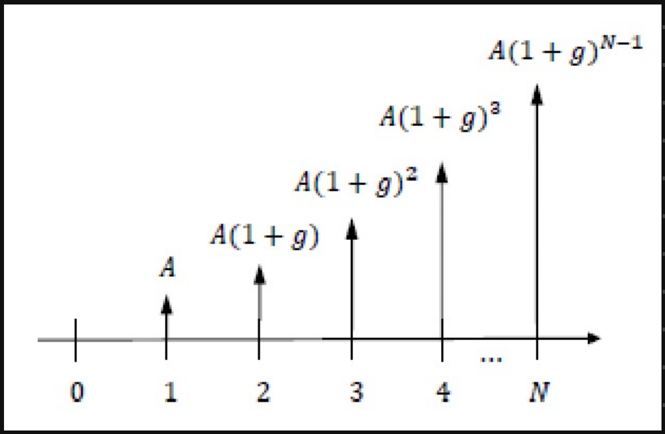
$= 659.38845$

$P = A(\frac{P}{A}, i, N)$

$= A\frac{(1+i_e)^{N \cdot m} - 1}{i_e(1+i_e)^{N \cdot m}}$

$= 659.38845\left(\frac{(1+0.06152)^{4.2}-1}{0.06152(1+0.06152)^{4.2}}\right)$
$= 4070.1469$
$= \$4070.15$

- geometric gradient series is a series of CFs that inc/dec by constant percentage each period
  - ↳ e.g. inflation, deflation, productivity improvement, increment in maintenance costs due to aging
  - ↳ base value of series is A & rate of growth is g
  - ↳ assume compounding period & payment period are same
  - ↳



- gives factor for P equiv to geometric gradient series : $(\frac{P}{A}, g, i, N) = \left(\frac{(1+i^\circ)^N - 1}{i^\circ(1+i^\circ)^N}\right)\frac{1}{1+g}$
  - ↳ growth-adjusted interest rate is $i^\circ = \frac{1+i}{1+g} - 1$
- e.g.

▪ The first-year maintenance cost for a new car is estimated to be $100, and it increases at a uniform rate of 10% per year. Using an 8% interest rate, calculate the present worth of cost of the first five years of maintenance.

$A = 100,\ g = 10\%,\ i = 8\%/\text{year},\ N = 5$

$i^\circ = \frac{1+i}{1+g} - 1$
$= \frac{1+0.08}{1+0.1} - 1$
$= -0.018182$

$(\frac{P}{A}, g, i, N) = \left(\frac{(1+i^\circ)^N - 1}{i^\circ(1+i^\circ)^N}\right)\frac{1}{1+g}$
$= \left(\frac{(1-0.018182)^5 - 1}{-0.018182(1-0.018182)^5}\right)\frac{1}{1+0.1}$
$= 4.804305$

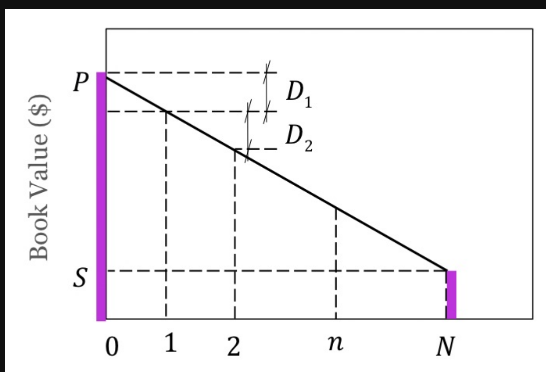
$P = A(\frac{P}{A}, g, i, N)$
$= 100(4.804305)$
$= 480.43049$
$= \$480.43$

DEPRECIATION

- depreciation is gradual dec in utility of fixed assets w/ use & time
- types of depreciation:
  - ↳ economic: use-related / time-related physical loss, functional loss
  - ↳ accounting: book & tax depreciation
- for accounting depreciation, costs of fixed assets aren't treated simply as expenses to be accounted for in year that they're acquired
  - ↳ assets are capitalized (i.e. considered a long-term investment so costs are distributed by subtracting them as expenses from gross income)
  - ↳ i.e. a way to spread out cost of assets over time it's expected to be used
- accounting/asset depreciation is systematic allocation of initial cost of asset in parts over its depreciable life
- cost basis rep total cost that's claimed as an expense over asset's life
  - ↳ includes actual cost of asset & all other incidental expenses

- useful life is estimate of duration over which asset is expected to fulfill its intended purpose
- salvage value (residual & scrap) is asset's value at end of life
  - ↳ amount recovered thru sale & trade-in can sometimes be -ve if there are disposal costs
- book value is depreciated val of asset for accounting purposes
  - ↳ calc w/ depreciation model

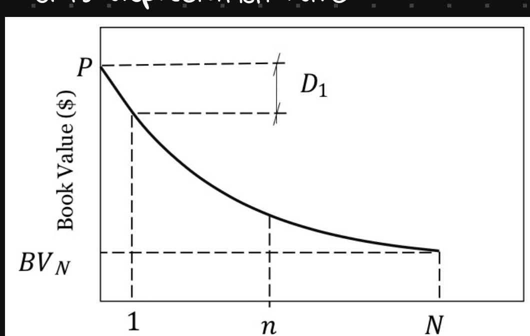
- straight-line depreciation method assumes depreciation charge D_n (rate of loss in val) of asset is constant over its useful life
  - ↳ $D_n = \frac{P-S}{N}$
    - $D_1 = D_2 = \ldots = D_n$
    - P is purchase price of asset
    - S is salvage value after N periods
  - ↳ book value BV_n after n years is $BV_n = P - n\frac{P-S}{N}$
  - ↳



- declining-balance method assumes that loss of asset's val over period of time is constant fraction of asset's book val at end of last period
  - ↳ depreciation cost in every period is constant proportion (i.e. depreciation rate) of closing BV from prev period
  - ↳ $D_n = BV_{n-1}\, d$

    $BV_n = P(1-d)^n$

    $D_n = Pd(1-d)^{n-1}$
    - D_n is depreciation charge at period n
    - BV_n is book value at end of period n
    - P is purchase price of asset
    - d is depreciation rate
  - ↳



- e.g.

Data	Value
Cost basis of the asset, P	$10,000
Useful life, N	5 years
Estimated salvage value, S	$2,000

▪ Compute the annual depreciation allowances and the book value each year of the automobile using the straight-line depreciation method

$D_n = \frac{P-S}{N}$

$= \frac{10000-2000}{5}$

$= \$1600$

$BV_1 = P - nD_n$

$= 10000 - 1600$

$= \$8400$

$BV_2 = 10000 - 2(1600)$

$= \$6800$

$BV_3 = 10000 - 3(1600)$

$= \$5200$

$BV_4 = 10000 - 4(1600)$

$= \$3600$

$BV_5 = \$2000$

e.g.

- Sarah wants to estimate the salvage value of a coffee shop 20 years after purchase.
- She feels that the depreciation is best represented using the declining-balance method, but she doesn't know what depreciation rate to use.
- She observes that the purchase price of the coffee shop was \$245,000 three years ago, and an estimate of its current salvage value (Book value) is \$180,000.
- What is a good estimate of the value of the coffee shop after 20 years?

$BV_3 = P(1-d)^3$

$180000 = 245000(1-d)^3$

$\frac{36}{49} = (1-d)^3$

$0.902337 = 1-d$

$d = 0.097663$

$S = 245000(1-0.097663)^{20}$

$= 31371.94181$

$= \$31371.94$

CHAPTER 4

- investing is giving up smth valuable now for expectation of receiving smth of greater value later
  - ↳ not all opportunities should be taken

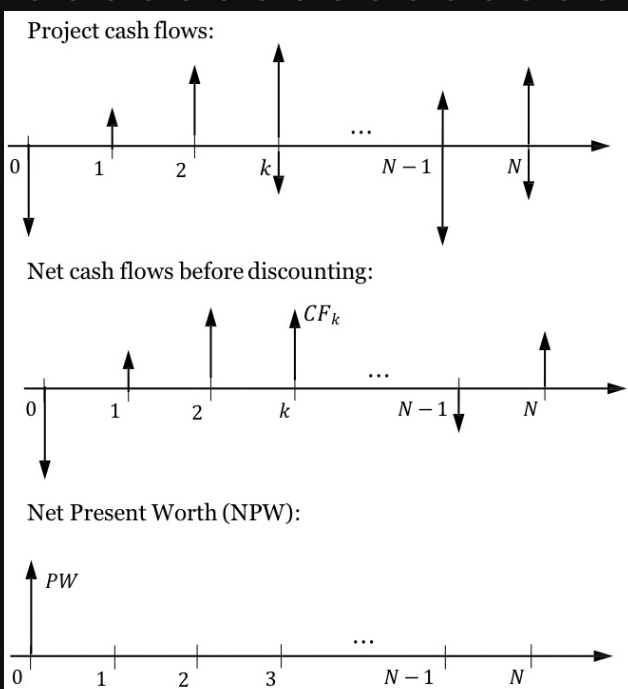
  if expected returns outweigh costs, then project (i.e. investment opportunity) may be pursued
- multiple projects in consideration:
  - ↳ Present Worth method (PW)
  - ↳ Annual Worth method (AW)
  - ↳ Future Worth method (FW)
  - ↳ payback period method
- several assumptions when using comparison methods:
  - ↳ costs & benefits are always possible to be measured in terms of money
  - ↳ future CFs are deterministic (no uncertainty)
  - ↳ sufficient funds available to implement all projects in consideration (unless otherwise stated)
  - ↳ taxes aren't applicable
  - ↳ all investments have CF at start (e.g. upfront cost, initial cost, initial investment)
- projects may be categorized in 3 groups in terms of their connection w/ other projects
  - ↳ independent: expected cost & benefits of each project don't depend on whether other one is chosen
    - e.g. buying computer vs AC, installing new production line vs updating specialized financial software
  - ↳ mutually exclusive: projects can't be implemented at same time
    - i.e. doesn't make sense to do both
    - e.g. buying computer of brand A vs brand B, 2 diff plant designs for same piece of land
  - ↳ related but not mutually exclusive: expected cost & benefits of 1 project depend on whether other is chosen
    - less common
    - e.g. hiring 1 or 2 waiters for a restaurant (A and/or B) w/ 2 diff exp lvls
- minimum acceptable rate of return (MARR) is min return, expressed as interest rate, that company is willing to obtain on investment
  - ↳ projects earning at least MARR are attractive b/c money invested is earning as much as can be earned elsewhere
  - ↳ i.e. rate of return required to get investors to invest in a business (cost of capital)
- comparison methods:
  - ↳ present worth method compares projects by looking at PW of all CFs associated w/ projects
  - ↳ future worth analysis compares projects by looking at FW of all CFs associated w/ projects
  - ↳ annual worth method is similar, but converts all CFs to a uniform series that's an annuity
  - ↳ payback period method estimates how long it takes to pay back investments
  - ↳ internal rate of return (IRR) method calcs discount rate (i.e. interest rate / required rate of return that's used to adjust FW vals to PW vals) at which net present val (NPV) of investment's CF is 0
    - higher IRRs are better b/c they indicate higher returns
  - ↳ benefit-cost method
  - ↳ sensitivity method
  - ↳ break-even method

PRESENT WORTH (PW) METHOD ANALYSIS

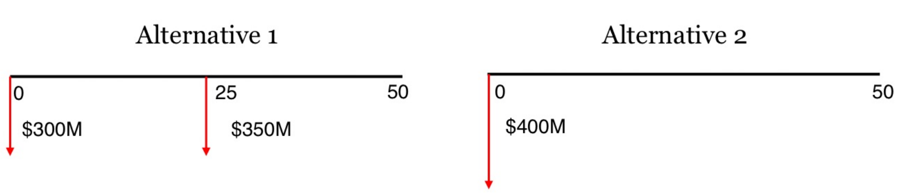
- PW analysis is used to determine present value of future money receipts & disbursements
  - ↳ aka Net Present Value (NPV) analysis
  - ↳ if future income & costs are known, then can use suitable interest rate to calc present worth of property
  - ↳ one of discounted CF techniques for project evaluation, which considers time val of money
- in PW analysis, be careful of time period covered

- in PW criterion, PW of all cash inflows is compared against worth of all cash outflows associated w/ investment project
  - ↳ diff btwn PW of these CFs determines whether project is acceptable investment
- most convenient point to calc equiv vals is usually at time $t = 0$

- procedure to calc NPV of project:
  1) determine interest rate i firm uses for investments
  2) estimate service life of project N
  3) estimate cash inflow/outflow for each period, over service life
  4) determine net CF in period k
     - $CF_k = C_{inflow,k} - C_{outflow,k}$
  5) calc discounted CFs at yr $t = 0$ using i
     - $CF_k^0 = CF_k(P/F, i, k)$
       → find $(P/F, i, k)$ on compound interest tables
  6) add all discounted CFs at yr $t = 0$ to find NPV
     - $PW(CF_k) = \sum_{k=0}^{N} CF_k(P/F, i, k)$

↳ 

- e.g.
  - Wayne County will build an aqueduct to bring water in from the upper part of the state. It can be built at a reduced size now for \$300 million and be enlarged 25 years hence for an additional \$350 million. A second alternative is to construct the full-sized aqueduct now for \$400 million.
  - Both alternatives would provide the needed capacity for the 50-year analysis period. Maintenance costs are small and may be ignored. At 6% interest, which alternative should be selected?



$PW_1 = 300M + 350M(P/F, 6\%, 25)$
$= 300M + 350M(0.233)$
$= 381.55M$

$PW_2 = 400M$

Thus, alternative 1 (2-stage construction) has smaller PW of cost & is preferred.

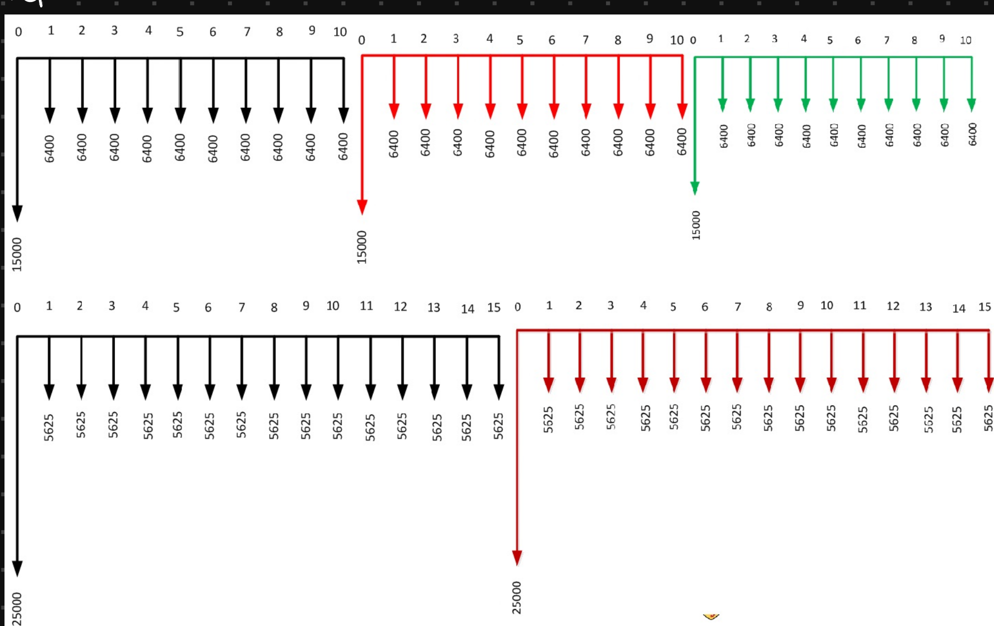
- when making PW comparisons, must always use same time period in order to take into account peak benefits & costs of each alt
- if lives of alts are unequal, can transform them into equal lives w/ either 1 of these methods:
  - ↳ repeated lives method: repeat each alt to arrive at common time period for all alts, using LCM of alts' service lives
    - assume all CFs remain same in subsequent project cycles
  - ↳ study period: define common period for all alts (usually smallest N) & consider salvage val of all alts at N
- e.g.

▪ A mechanical engineer has decided to introduce automated materials-handling equipment for a production line. She must choose between building the equipment or buying equipment off the shelf. If the MARR is 9%, which alternative is better? The data of each alternative is as follows:

	Alternative 1	Alternative 2 (off-shelf)
First cost ($)	15,000	25,000
O&M cost ($/year)	6,400	5,625
Service life	10	15

LCM of 10 & 15 is 30

↳ repeated CFs:



$$PW_1 = -15000 - 15000(P/F, 9\%, 10) - 15000(P/F, 9\%, 20) - 6400(P/A, 9\%, 30)$$
$$= -15000 - 15000(0.4224) - 15000(0.1784) - 6400(10.274)$$
$$= -89765.6$$

$$PW_2 = -25000 - 25000(P/F, 9\%, 15) - 5625(P/A, 9\%, 30)$$
$$= -25000 - 25000(0.2745) - 5625(10.274)$$
$$= -89653.75$$

Thus, alt 2 is better.

FUTURE WORTH ANALYSIS

- future worth analysis measures surplus in investment project at a time other than t = 0

↳ useful in investment situation where we need to compute equiv worth of project at end of investment period
- e.g. retirement plan where money is saved for future expenses

↳ analysis & conclusions abt FW is same as NPW w/ only diff is that it's computation of FW value

↳ $FW(CF_k) = \sum_{k=0}^{N} CF_k (F/P, i, N-k)$

· e.g.

- A mechanical engineer has decided to introduce automated materials-handling equipment for a production line. She must choose between building the equipment or buying equipment off the shelf. If the MARR is 9%, and using the FW analysis, which alternative is better? The data of each alternative is as follows:

	Alternative 1	Alternative 2 (off-shelf)
First cost ($)	15,000	25,000
O&M cost ($/year)	6,400	5,625
Service life	10	15

LCM is 30 yrs.

$FW_1 = -15000(F/P, 9\%, 30) - 15000(F/P, 9\%, 20) - 15000(F/P, 9\%, 10) - 6400(F/A, 9\%, 30)$

$= -15000(13.268) - 15000(5.604) - 15000(2.367) - 6400(136.308)$

$= -1190956.2$

$FW_2 = -25000(F/P, 9\%, 30) - 25000(F/P, 9\%, 15) - 5625(F/A, 9\%, 30)$

$= -25000(13.268) - 25000(3.642) - 5625(136.308)$

$= -1189482.5$

Thus, alt 2 is better.

CAPITALIZED COST ANALYSIS

- present sum of money that would need to be set aside now at some interest rate to yield funds required to provide service indefinitely
  ↳ e.g. when private facility needs equipment as long as it operates & life of facility is much longer than equipment's
- money set aside for future expenditures musn't decline
- interest received on money set aside can be spent, but not principal
- capitalized cost is PW w/ infinite analysis period
  ↳ capitalized cost $= A(P/A, i, N \to \infty)$

$$= A \lim_{N\to\infty} \left(\frac{(1+i)^N - 1}{i(1+i)^N} \right)$$

$$= A \cdot \frac{1}{i} \lim_{N\to\infty} \left(\frac{(1+i)^N - 1}{(1+i)^N} \right)$$

$$= A \cdot \frac{1}{i}$$

capitalized cost $= \frac{A}{i}$

· e.g.

- How much should a municipality set aside to pay $100,000 per year for maintenance on a highway if interest is assumed to be 5%? For perpetual maintenance, the principal sum must remain undiminished after making the annual disbursement.

capitalized cost $= \frac{A}{i}$

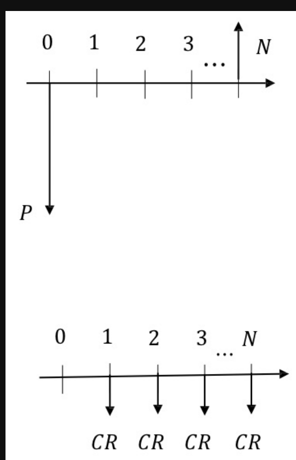
$$= \frac{100\,000}{0.05}$$

$= \$2\,000\,000$

CHAPTER 5

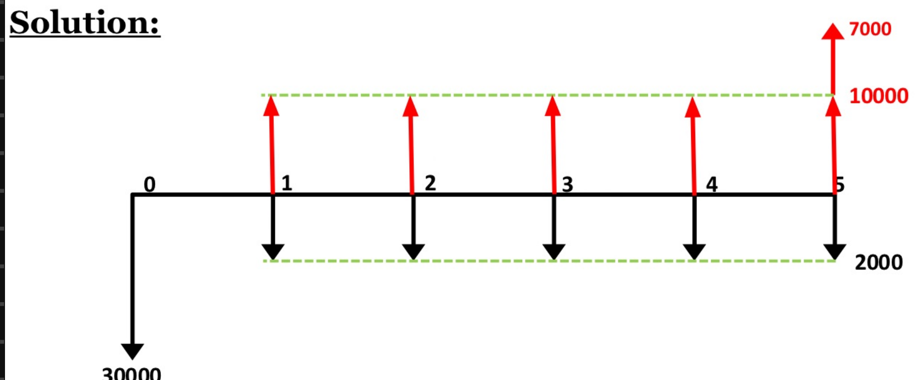
ANNUAL WORTH ANALYSIS

- AW rep measure of project profitability as equiv annual val
  - ↳ aka Equivalent Uniform Annual Worth (EUAW)
  - ↳ all receipts & disbursements are transformed to uniform series at MARR
- PW & AW yield same conclusion b/c any P can be converted into A so $AW(i) = PW(i)(A/P, i, N)$
  - ↳ use AW instead of PW is b/c concept of annual savings/payments/costs are easier, esp for annual costs
    - don't have to use repeated lives when projects have diff service lifetimes
- when mutually exclusive projects are compared in terms of annual costs, AW method is called Equivalent Uniform Annual Cost (EAUC)
  - ↳ for EAUC comparisons, project w/ least AC is preferred
  - ↳ 2 conditions for EAUC comparisons:
    - all projs have same major benefit
    - estimated val of major benefit clearly outweighs project costs
- to calc EUAW, convert single CFs / gradient series CFs to annuities using corresponding compound factors & add them up to existing annuities
  - ↳ annual equiv of init investment & salvage val over life of project is Capital Recovery Cost (CR)
    - $CR(i) = -P(A/P, i, N) + S(A/F, i, N)$
    -



- e.g. Machine costs \$30,000; annual operation and maintenance (O&M) cost is \$2000; saves \$10,000 in labor costs per year. Salvage value after 5 years is \$7000. What is the EUAW at 10% interest rate?

Solution:



$$EUAW = -30000(A/P, 10\%, 5) + (10000 - 2000) + 7000(A/F, 10\%, 5)$$
$$= -30000\left(\frac{0.1(1+0.1)^5}{(1+0.1)^5 - 1}\right) + 8000 + 7000\left(\frac{0.1}{(1+0.1)^5 - 1}\right)$$
$$= 1232.657942$$
$$= \$1232.66$$

· e.g. A mechanical engineer has decided to introduce automated materials-handling equipment for a production line. She must choose between building the equipment or buying equipment off the shelf. If the MARR is 9%, which alternative is better? Use the AW analysis:

	Alternative 1	Alternative 2 (off-shelf)
First cost ($)	15,000	25,000
O&M cost ($/year)	6,400	5,625
Service life	10	15

↳ AW assuming repeated lives:

$AW_1 = PW_1(A/P, 9\%, 30)$

$= -89765.6(0.0973)$

$= -8734.19288$

$AW_2 = PW_2(A/P, 9\%, 30)$

$= -89653.75(0.0973)$

$= \boxed{-8723.309875}$

↳ AW assuming no repeated lives (easier method):

$AW_1 = -15000(A/P, 9\%, 10) - 6400$

$= -15000(0.1558) - 6400$

$= -8737$

$AW_2 = -25000(A/P, 9\%, 15) - 5625$

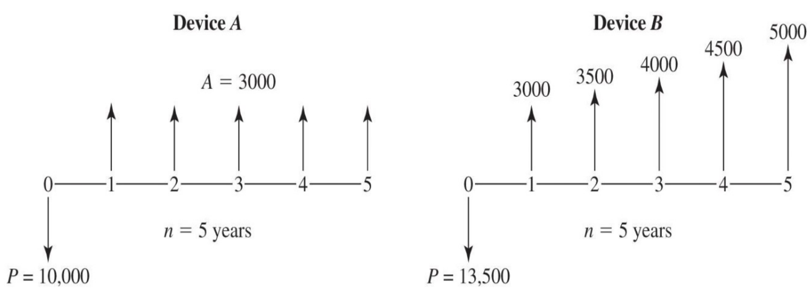
$= -25000(0.1241) - 5625$

$= \boxed{-8727.5}$

↳ annual costs of alts remain abt the same so both methods work

· e.g.

- A company is going to select only one of the following two devices. For a 7% interest rate, determine the best alternative using the AW analysis.



→ G = 500

$EUAW_A = -10000(A/P, 7\%, 5) + 3000$

$= -10000(0.2439) + 3000$

$= 561$

$EUAW_2 = -13500(A/P, 7\%, 5) + 3000 + 500(A/G, 7\%, 5)$

$= -13500(0.2439) + 3000 + 500(1.865)$

$= 639.85$

Thus, device B is better alt

PAYBACK PERIOD

- simplest method of judging economic viability of projects
- rough estimate of time it takes for investment to pay for itself
  - ↳ assume receipts are same every yr

- payback period $= \frac{\text{init investment}}{\text{annual savings}}$
  - ↳ project w/shortest payback period is preferred
- if annual savings are not constant, payback period is calc by deducting each yr of savings from 1st cost until it's recovered
- advantages:
  - ↳ easy to understand & calc
  - ↳ accounts for need of quick capital recovery
- disadvantages:
  - ↳ discriminates against long-term projects
  - ↳ ignores effect of time val of money (i.e. interest rate not considered)
  - ↳ ignores expected service life
- e.g.

Calculate the payback period for the two projects discussed in slide 8, assuming that alternative 1 generates a constant revenue equal to $10,000/year; while alternative 2, $11,000/year.

$\text{Payback period}_1 = \frac{15000}{10000 - 6400} = 4.166667 = 4.17 \text{ yrs}$

$\text{Payback period}_2 = \frac{25000}{11000 - 5625} = 4.651163 = 4.65 \text{ yrs}$

Alt 1 is preferred in this case.

- e.g.

- A firm is purchasing production equipment for a new plant. Two alternative machines are being considered for a particular operation.

	Tempo Machine	Dura Machine
Installed cost	$30,000	$35,000
Net annual benefit after all annual expenses have been deducted	$12,000 the first year, *declining* $3000 per year thereafter	$1000 the first year, increasing $3000 per year thereafter
Useful life, in years	4	8

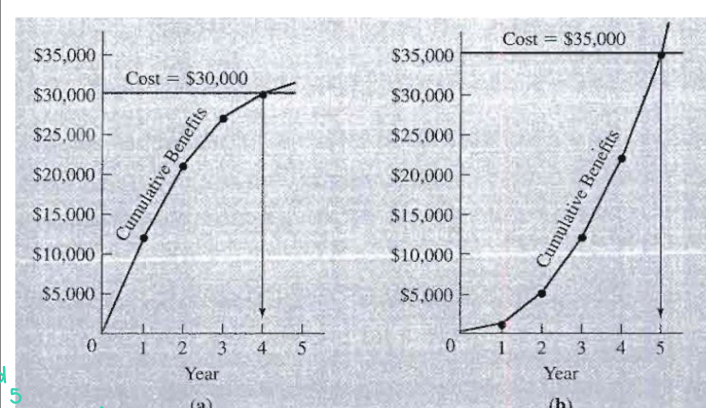
- Neither machine has any salvage value. Compute the payback period for each of the alternatives.

↳ CF for 2 alts:

Year		Tempo Machine		Dura Machine
0		−$30,000		−$35,000
1	-18000	+12,000	-34000	+1,000
2	-9000	+9,000	-30000	+4,000
3	-3000	+6,000	-23000	+7,000
4	0	+3,000	-13000	+10,000
5		0	0	+13,000
6		0		+16,000
7		0		+19,000
8		0		+22,000
		0		+57,000

if +14000, then n=5 b/c money is recovered btwn yr 4 & 5 (end yr 5 w/ +1000)

(a) Tempo machine and (b) Dura machine.



- Tempo machine has declining annual cost benefit
- Dura has inc annual benefit

To minimize payback period, Tempo is selected.

- discounted payback period is #yrs required to recover investment from discounted CFs
  - ↳ discounted payback period $= \frac{\text{first cost}}{\sum_{n} \frac{\text{Annual savings}_n}{(1+i)^n}}$

- U is period where cumulative discounted annual savings equals first cost

↳ #yrs will always be higher than std payback period

- Calculate the payback period for the two projects discussed in slide 8, assuming that alternative 1 generates a constant revenue equal to \$10,000/year; while alternative 2, \$11,000/year; and the interest rate is 9%.

Year	$15{,}000 - \sum_n^U \frac{3{,}600}{(1+0.09)^n}$	$25{,}000 - \sum_n^U \frac{5{,}375}{(1+0.09)^n}$
1	11697.2	20068.8
2	8667.2	15544.8
3	5887.34	11394.3
4	3337.01	7586.51
5	997.25	4093.12
6	-1149.31	888.19
7	-	-2052.12

↳ by interpolation:

- $PB_1 = 5.46$ yrs $\simeq$ 6 yrs
- $PB_2 = 6.3$ yrs $\simeq$ 7 yrs

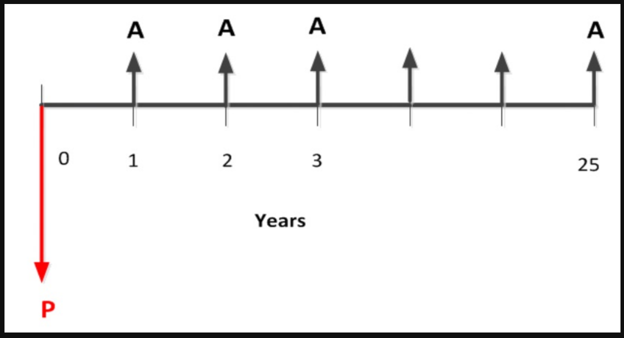
when apx PB period, always round up yr

e.g.

- Joseph wants to install a 10 kW Photovoltaic (PV) system for his house. The cost of the system is \$6/W including installation, inverter and the solar battery system. The capacity factor of the PV system in Joseph's area is 0.2.
- He is planning to join the micro feed-in-tariff (μFIT) program and sell electricity to K-W utility for \$0.4/kWh for 25 years, and he estimated that he will be using 40% of the generated power for his own use and 60% to be sold to the utility. The interest rate is 3%. K-W utility charges \$0.1/kWh (Average) to deliver electric power to the customer.

- Determine (using the appropriate method(s)) if this project is profitable for him or not.
- If Joseph invested the initial cost of the project in stocks that gives him a constant annual income instead of building the PV system for his house, calculate the annual earning from this investment, assuming the same interest rate as in the first part (3%). Using this method, show if the PV project is still profitable or not.
- Also calculate the payback period of the project.

↳ CF diagram:



$P = \frac{\$6}{W} \cdot 10000\,W$
$= \$60\,000$

Amount of kWh prod by solar PV = PV rating · capacity factor · (h/yr)
$= 10\,kW \quad 0.2 \cdot (365 \cdot 24)$
$= 17520\ kWh$

Revenue from utility = $17520\,kWh \quad 0.6 \cdot \$0.4/kWh$
$= \$4204.80/yr$

Revenue from saving electricity bill = $17520\,kWh \quad 0.4 \cdot \$0.1/kWh$
$= \$700.80/yr$

$A = 4204.8 + 700.8$
$= \$4905.60$

$P = A(P/A, i, N)$
$= 4905.6\,(P/A, 3\%, 25)$
$= 4905.6\left(\frac{(1+0.03)^{25}-1}{0.03(1+0.03)^{25}}\right)$
$= 85421.93731$
$= \$85421.94$

$NPW = 85421.94 - 60000$
$= \$25421.94$

Since NPW is +ve, project is profitable.
If money is deposited in bank for stocks:
$A = P(A/P, i, N)$
$= 60000\left(\frac{0.03(1+0.03)^{25}}{(1+0.03)^{25}-1}\right)$
$= 3445.672262$
$= \$3445.67$

Yearly revenue for PV project is \$4905.60, which is higher than \$3445.67, so PV project is more profitable

Payback period:
Annual benefits are constant so use PB formula:
$PB = \frac{60000}{4905.6}$
$= 12.23092$
$= 12.23$ yrs

↳ interpolation method:

Accumulated benefit of the project:

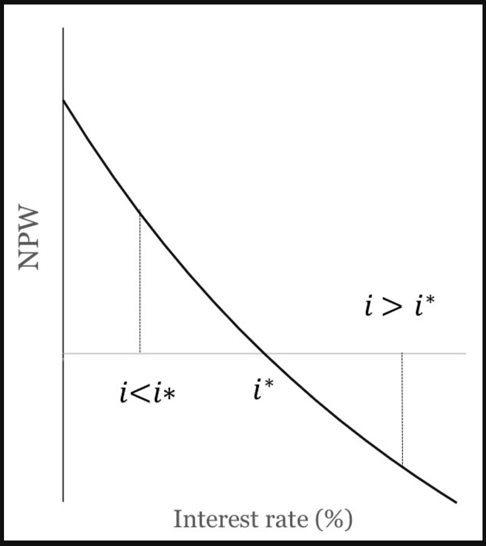
Year	Benefit	Year	Benefit
1	\$4,905	8	\$39,240
2	\$9,810	9	\$44,145
3	\$14,715	10	\$49,145
4	\$19,620	11	\$53,955
5	\$24,525	12	**\$58,860**
6	\$29,430	13	**\$63,765**
7	\$34,335	14	\$68,670

$$\frac{63765 - 58860}{13 - 12} = \frac{60000 - 58860}{x - 12}$$
$$4905 = \frac{1140}{x-12}$$
$$x - 12 = 0.232416$$
$$x = 12.232416$$
$$= 12.23\ \text{yrs}$$

CHAPTER 6

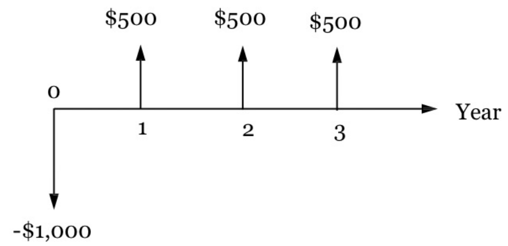
INTERNAL RATE OF RETURN ANALYSIS

- profitability of investment can be measured as rate of return per $ invested (i.e. interest rate)
- rate of return (ROR) is net gain of investment, measured as proportion of init investment over specified time period
  - ↳ e.g. investment of $1000 that pays $1100 after 1 yr has net profit of $100
    - ROR = 10%
- in ROR analysis, no interest rate in calcs
  - ↳ rate is computed from CF
- ROR is compared w/ preselected MARR
  - ↳ decision of project worthiness depends on type of project being considered (e.g. independent, mutually exclusive)
- internal ROR (IRR) on investment is interest rate i^* that makes PW of cash inflows equal to PW of cash outflows (i.e. project breaks even)
  - ↳ $\sum_{k=0}^{N} \frac{R_k}{(1+i^*)^k} = \sum_{k=0}^{N} \frac{D_k}{(1+i^*)^k}$
    - R_k is cash inflow (receipts) in period k
    - D_k is cash outflow (disbursements) in period k
    - N is #periods
    - i^* is IRR
  - ↳ internal refers to fact that IRR depends only on investment's own CFs while external factors (e.g. risk, inflation) are neglected
  - ↳ IRR is usually +ve but if it's -ve, project loses money
- to calc IRR, set disbursements equal to receipts
  - ↳ can use PW, uniform series AW, or FW
    - net PW = net FW = EUAW = 0
  - ↳ might require trial & error & interpolation to be computed
- to use IRR for independent project comparisons:
  - ↳ if $i < i^*$, project is profitable since NPW $(i < i^*) > 0$
  - ↳ if $i > i^*$, project is not profitable since NPW $(i > i^*) < 0$
  - ↳ accept all projs w/ $i^* >$ MARR & reject those w/ $i^* <$ MARR
  - ↳ assumptions:
    - projs w/ single IRR
    - projs w/ equal lives
    - no budget constraints
    - CFs generated by proj are reinvested at IRR i^*
  - ↳



· e.g.

Calculate the IRR of a project that has the following cash flow and a MARR of 10%:

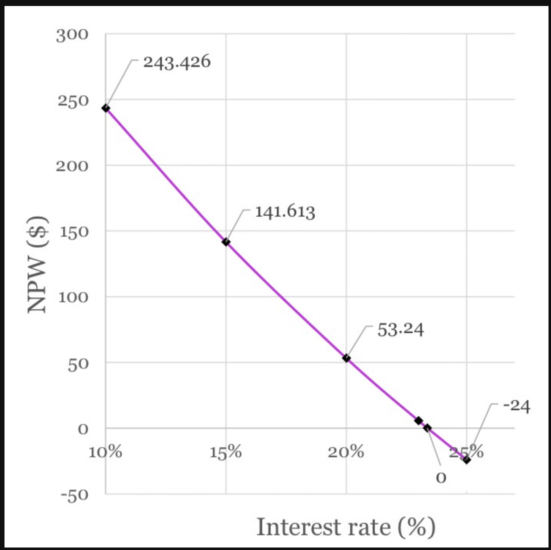


Calculate NPW = 0:

$0 = -1000 + \frac{500}{(1+i^*)^1} + \frac{500}{(1+i^*)^2} + \frac{500}{(1+i^*)^3}$

i^*	NPW i^* = 0
10%	243.42
15%	141.61
20%	53.24
25%	-24.00

> interpolate



$\frac{0.25 - i^*}{-24 - 0} = \frac{0.25 - 0.2}{-24 - 53.24}$

$\frac{0.25 - i^*}{-24} = -\frac{5}{7724}$

$0.25 - i^* = \frac{30}{1931}$

$i^* = 0.234464$

$i^* = 23.45\%$

· e.g.

An $8200 investment returned $2000 per year over a 5-year useful life. What was the rate of return on the investment? Consider a MARR of 6%

$0 = PW_{benefits} - PW_{costs}$

$0 = -8200 + 2000(P/A, i^*, 5)$

$(P/A, i^*, 5) = 4.1$

$i^* = 7\%$ (from interest tables)

↳ no interpolation is required b/c IRR is exactly 7%

INCREMENTAL INTERNAL RATE OF RETURN ANALYSIS

- PW, AW, & FW methods yield absolute $ measures of investment worth but ROR is relative measure & ignores scale of investment
- to compare 2 mutually exclusive projects A & B, write CF of B as B = A + (B - A)
  - ↳ B has 2 CF components: same CF as A & increment component (B - A)
  - ↳ B is preferred to A when ROR of increment component exceeds MARR
- w/ 2 mutually exclusive alts, ROR analysis is performed by computing incremental ROR (ΔIRR) on diff btwn alts
  - ↳ CF for diff btwn alts is computed by taking higher initial-cost alt minus lower-cost alt
  - ↳

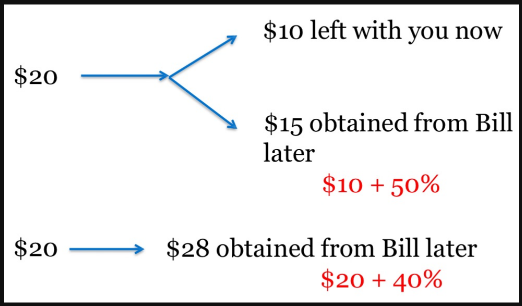
Two-Alternative Situation	Decision
ΔIRR > MARR	Choose the higher initial-cost alternative
ΔIRR < MARR	Choose the lower initial-cost alternative
ΔIRR = MARR	Choose either alternative*

- if ΔIRR = MARR, choose lowest cost alt in real life

- e.g.

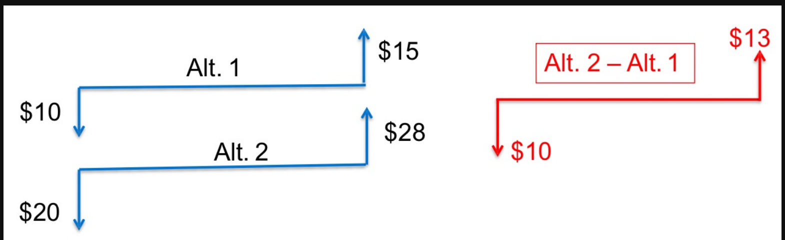
  - You have $20 in your wallet and there are two alternative ways of lending Bill some money:
    - Lend Bill $10 with his promise of a 50% return. That is, he will pay you back $15 at the agreed time.
    - Lend Bill $20 with his promise of a 40% return. He will pay you back $28 at the same agreed time.
  - You can select whether to lend Bill $10 or $20. This is a one-time situation, and any money not lent to Bill will remain in your wallet. Which alternative do you choose?
  - Assume MARR on investments is 6%

↳ 2 alts:



↳ soln 1: alt 1 gives us $25 back & alt 2 gives us $28 back; since alt 2 is more rewarding, lend Bill $20

↳ soln 2: compute ΔIRR (alt 2 - alt 1)



NPW = 0

$0 = PW_{\text{benefit of diffs}} - PW_{\text{cost of diffs}}$

$0 = -10 + 13(P/F, i, 1)$

$(P/F, i, 1) = \frac{10}{13}$

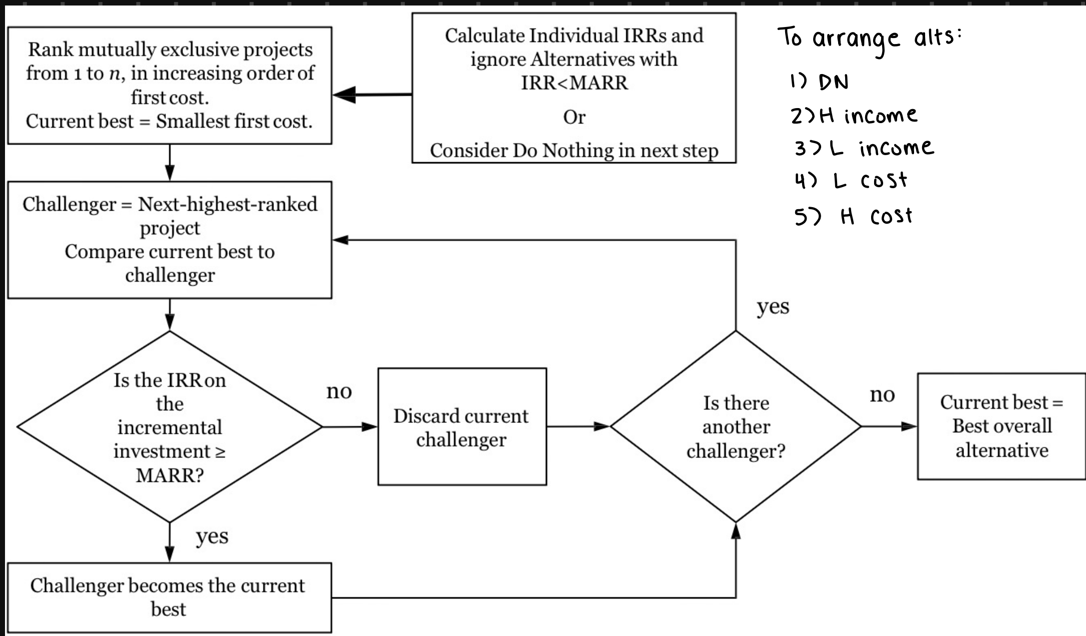
$(P/F, i, 1) = 0.76923$

$i^* = 30\%$ ← interest rate we'll gain from choosing alt 2 over alt 1

- ΔIRR = 30% is higher than MARR = 6% so investment is desirable
- since alt 2 = alt 1 + (alt 2 - alt 1) & we like (alt 2 - alt 1) increment, select alt 2 b/c it contains desirable increment

- procedure of ROR analysis for mutually exclusive alts:
  1) calc IRR for each alt & ignore any alt w/ IRR < MARR or consider Do-Nothing (DN) option as 1st alt in step 2
  2) arrange alts from lower init-cost to higher init-cost alt
     - e.g. A, B, C
  3) alt is 1st choice & alt B is challenger; apply incremental ROR for (B-A) & calc ΔIRR_{B-A}
  4) if $\Delta IRR_{B-A} > MARR$, select B to become chosen alt
     - if not, keep A as chosen alt
  5) consider alt C as new challenger & repeat steps 3-4 to decide on chosen alt
  6) repeat steps 3-5 until completing all alts

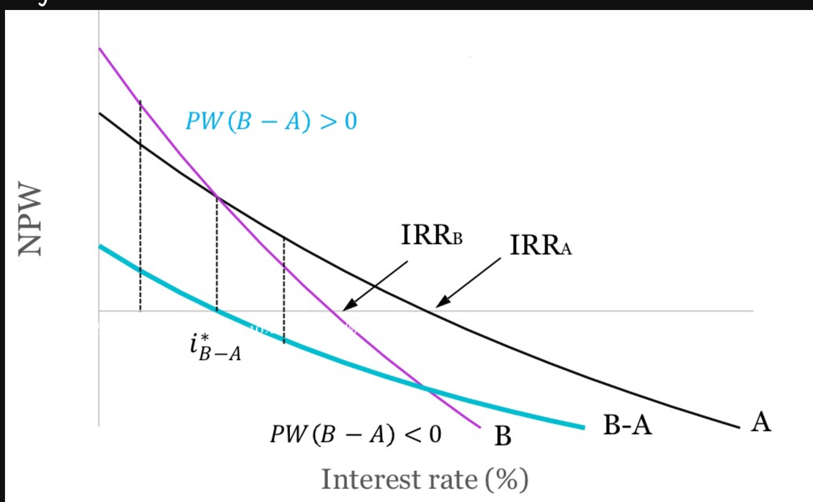
- diagram of procedure:



To arrange alts:

1) DN
2) H income
3) L income
4) L cost
5) H cost

- to use IRR for mutually exclusive projects, use incremental investment
  - ↳ $i^*_{B-A} > MARR \rightarrow PW(B-A) > 0 \rightarrow$ invest on B
  - ↳ $i^*_{A-B} > MARR \rightarrow PW(B-A) < 0 \rightarrow$ invest on A
  - ↳ e.g.



- e.g.

Consider two investments. The first costs $1,000 today and pays $2,000 in one year. The second costs $5,000 today and pays $7,000 in one year. Which is the preferred alternative? Assume a MARR of 10%.

	Investment 1	Investment 2
IRR (%)	100	40
NPW ($)	818	1,364

Choose alt 1 as one w/ least initial cost of \$1000. Choose alt 2 as one w/ largest initial cost. If ROR of incremental investment is > MARR, then alt 2 should be selected.

$0 = -(5000-1000) + (7000-2000)(P/F, i, 1)$

$0 = -4000 + 5000\left(\frac{1}{1+i^*}\right)$

$0.8 = \frac{1}{1+i^*}$

$1.25 = 1+i^*$

$i^* = 0.25$

Since $i^* = 25\% >$ MARR, alt 2 is better.

e.g.

Kitchener Meat can buy a new meat slicer system for \$ 50,000. This saves \$11,000 per year in labour and operating costs. The same system with an automatic loader is \$68,000 and will save \$14,000 per year. The life of both systems is 8 years. Which one should be chosen? MARR=12%:

		First Cost	Annual Savings
alt 1	Do nothing	0	0
alt 2	Meat slicer alone	50,000	11,000
alt 3	Meat slicer + loader	68,000	14,000

Alt 1 vs alt 2:

$0 = -50000 + 11000(P/A, i, 8)$

$0 = -50000 + 11000\left(\frac{(1+i^*)^8 - 1}{i^*(1+i^*)^8}\right)$

$i^* = 14.6\% >$ MARR $\longrightarrow$ choose alt 1

Alt 2 vs alt 3:

$0 = -(68000-50000) + (14000-11000)(P/A, i^*, 8)$

$i^* = 7\% <$ MARR $\rightarrow$ stay w/ alt 1

We choose alt 2, which is buying only the meat slicer.

e.g.

- A company needs to select a cost-saving device that has no salvage value after 5 years. Two mutually exclusive options are available:
  - Device A: Costs \$10,000 & saves \$3000 annually.
  - Device B: Costs \$13,500 & saves \$3000 the 1st year & increases \$500 annually.
- Given the MARR is 6%. Choose the best option using the ROR analysis.

Alts:

1) DN
2) A
3) B

ΔIRR_{A-DN}:

$0 = -10000 + 3000(P/A, i^*, 5)$

$i^*_{A-DN} = 15.45\% >$ MARR $\rightarrow$ choose A

ΔIRR_{B-A}:

$0 = -13500 - (-10000) + (3000-3000)(P/A, i^*, 5) + 500(P/G, i^*, 5)$

$0 = -3500 + 500(P/G, i^*, 5)$

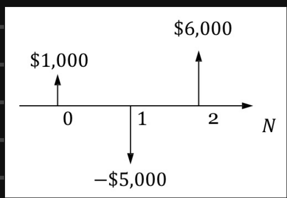
$i^*_{B-A} = 9.45\% >$ MARR $\rightarrow$ choose B

The best option is B.

CHAPTER 7

MULTIPLE INTERNAL RATE OF RETURNS PROBLEM

- when using IRR method to evaluate projs, there's a possibility of obtaining >1 IRR
  - ↳ e.g.



$0 = 1000 - 5000(P/F, i^*, 1) + 6000(P/F, i^*, 2)$

$0 = 1000 - 5000(1+i^*)^{-1} + 6000(1+i^*)^{-2}$

$0 = \frac{1000(1+i^*)^2 - 5000(1+i^*) + 6000}{(1+i^*)^2}$

$0 = 1000((1+i^*)^2 - 5(1+i^*) + 6)$

$0 = (1+i^* - 2)(1+i^* - 3)$

$0 = (i^* - 1)(i^* - 2)$

- project has 2 IRRs : 100% & 200%

- for any proj's CF of form: $CF_0 + CF_1(1+i^*)^{-1} + CF_2(1+i^*)^{-2} + \ldots + CF_N(1+i^*)^{-N}$
  - ↳ CF_N is net CF of period N
  - ↳ any i^* that solves eqn is IRR of proj
  - ↳ we solve for IRRs using N^{th} deg polynomial: $CF_0 + CF_1x + CF_2x^2 + \ldots + CF_Nx^N$
    - $x = (1+i^*)^{-1}$
    - Descartes' rule states that # +ve roots for x must be equal to m sign changes or less by even int

→

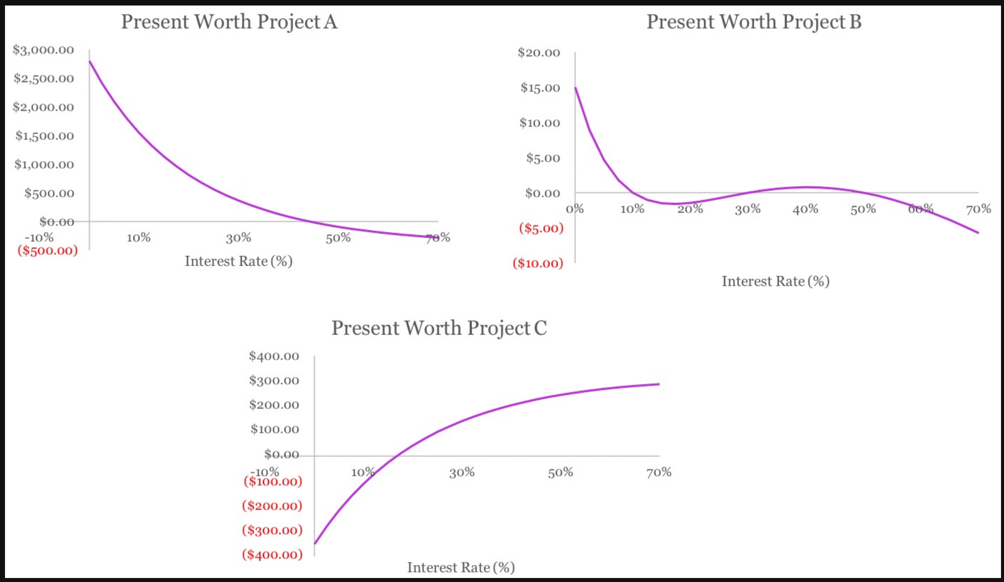
Number of sign changes, m	Number of positive values of x	Number of positive values of i
0	0	0
1	1	1 or 0
2	2 or 0	2, 1, or 0
3	3 or 1	3, 2, 1 or 0
4	4, 2, or 0	4, 3, 2, 1 or 0

SIMPLE AND NON-SIMPLE INVESTMENTS

- simple investment is where initial CFs are -ve & only 1 sign change in dir of net CFs occurring in subsequent periods
  - ↳ zero CFs aren't considered sign change
  - ↳ most projs behave like this
  - ↳ unique IRR
- non-simple investment has >1 sign change occurring in net CFs
  - ↳ e.g. projs requiring major equipment replacements within proj life, projs executed in multiple stages
  - ↳ may/may not have multiple IRRs
- e.g.

	0	1	2	3	4
Project A	-$1,000	-$500	$280	$1,500	$2,000
Project B	-$1,000	$3,900	-$5,030	$2,145	$0
Project C	$1,000	-$450	-$450	-$450	$0

↳ A is simple
↳ B is non-simple
↳ C is neither b/c it has +ve initial investment
↳



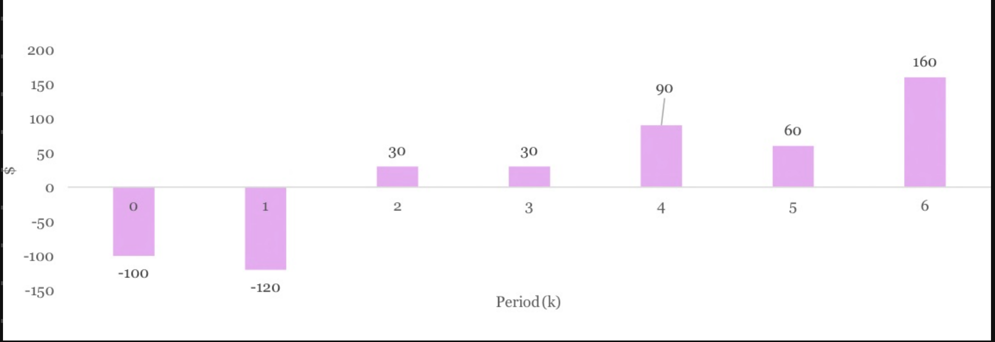
- A has $i^* = 44.24\%$ & proj should be accepted if MARR < i^*
- B's NPW crosses axis at 10%, 30%, & 50%
  - → 2 intervals of interest rates at which proj could be accepted: $i < 10\%$ & $30\% < i < 50\%$
  - → direct application of IRR method doesn't provide appropriate measure of profitability of investment w/ multiple RORs
- C is borrowing CF (i.e. money is received at period 0 & loan is paid thereafter on a regular basis
  - → single IRR

- determining if proj has multiple i^* requires computation of NPW for diff interest rates
- can predict if i^* has 1+ soln by examining CF series of investment:
  - ↳ rule 1 (Descartes' rule): # +ve real vals of i^* for proj w/ N periods is never > # sign changes in seq of net CFs CF_k
  - ↳ rule 2 (Norstrom's criterion): if series S_k (where S_k is CF accumulated till yr k) starts -vely & changes sign only once, the unique +ve i^* exists
- e.g. predict #IRR

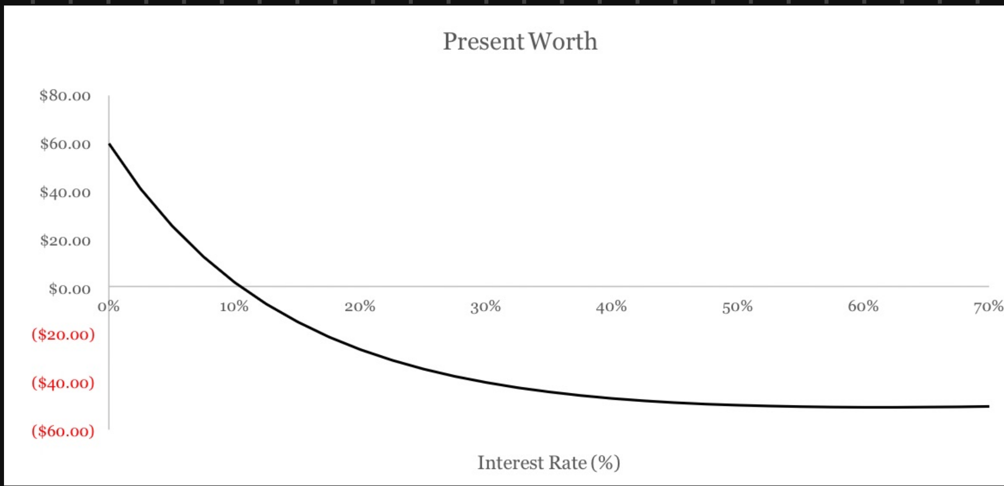
	0	1	2	3	4	5	6
Net cash flow (CF_k)	-$100	-$20	+$150	$0	+$60	-$30	+$100
Sign change			YES			YES	YES

m = 3 (# sign changes)

Calculate the cumulative cashflow:



Rule 1: m = 3 so $i^* \leq 3$
Rule 2: cumulative CF starts as -ve & only changes sign once so single i^* exists



Thus, proj has single IRR.

- e.g.

- Joseph wants to install a 10 kW Photovoltaic (PV) system for his house. The cost of the system is \$6/W including installation, inverter and the solar battery system. The capacity factor of the PV system in Joseph's area is 0.2.
- He is planning to join the micro feed-in-tariff (micro FIT) program and sell electricity to K-W utility for \$0.4/kWh for 25 years, and he estimated that he will be using 40% of the generated power for his own use and 60% to be sold to the utility. K-W utility charges \$0.1/kWh to deliver electric power to the customer.
- Use the IRR analysis to determine if this project is profitable or not. MARR for this PV project is 3%.

Amount of kWh prod by solar $= \text{PV rating} \cdot \text{capacity factor} \cdot \frac{h}{yr}$

$= 10\text{ kW} \cdot 0.2 \cdot 8760$

$= 17520 \text{ kWh}$

Revenue from selling $= 0.4\ (0.6 \cdot 17520)$

$= \$4204.80/yr$

Revenue from saving electricity bill $= 0.1(0.4 \cdot 17520)$

$= \$700.80/yr$

Annual revenue $= 4204.8 + 700.8$

$= \$4905.60/yr$

$PW = 10000 \cdot 6$

$= \$60000$

$0 = PW_{benefits} - PW_{costs}$

$0 = -60000 + 4905.6(P/A, i^*, 25)$

i	$(P/A, i, 25)$
3%	25411.5
4%	16626.3
6%	2702.36
7%	-2839.17

We use interpolation:

$$\frac{-2839.17 - 2702.36}{0.07 - 0.06} = \frac{0 - 2702.36}{i^* - 0.06}$$

$$i^* = (-2702.36)\left(\frac{0.07 - 0.06}{-2839.17 - 2702.36}\right) + 0.06$$

$= 0.064877$
$\approx 6.49\%$

IRR > MARR = 3% so proj is profitable.

EXTERNAL RATE OF RETURN

- when multiple IRRs are obtained for same proj, none are suitable as measure of profitability
  - ↳ instead consider return from money that's not invested in proj (i.e. cash at hand)
- external ROR (ERR) is ROR on proj where any CFs that aren't invested in proj are assumed to earn interest at predetermined rate (usually MARR)
  - ↳ denoted i_e^*
  - ↳ for given explicit ROR, proj can only have 1 val of its ERR
- to determine if there's cash at hand, look at proj balance, which depends on i_e^* used to discount CFs
  - ↳ if proj has seq of CFs, $CF_0, CF_1, \ldots, CF_N$, & interest rate i, there's N+1 proj balances, $B_0, B_1, \ldots, B_N$ (one at end of each period k = 1, ..., N)
  - ↳ each B_k is accumulated FV of all CFs at end of period k compounded at interest rate i
    - $B_0 = CF_0$
    - $B_1 = CF_0(1+i^*) + CF_1$
    - $B_2 = CF_0(1+i^*)^2 + CF_1(1+i^*) + CF_2$
    - $B_N = CF_0(1+i^*)^N + CF_1(1+i^*)^{N-1} + \ldots + CF_N$
  - ↳ if B_k is +ve, proj is source of funds up to next k+1 period
    - money can't be reinvested in proj, but invested somewhere else at MARR
  - ↳ if B_k is -ve, proj is like investment
    - i.e. using money until next k+1 period
- e.g. determine proj balance, using IRR as interest rate

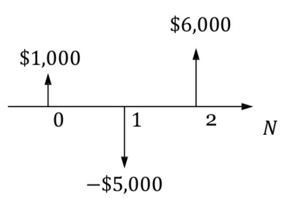
▪ When using the IRR method to evaluate projects, there is a possibility of obtaining more than one IRR.

▪ Consider the cashflow in the right:

$0 = 1{,}000 - 5{,}000(P/F, i^*, 1) + 6{,}000(P/F, i^*, 2)$

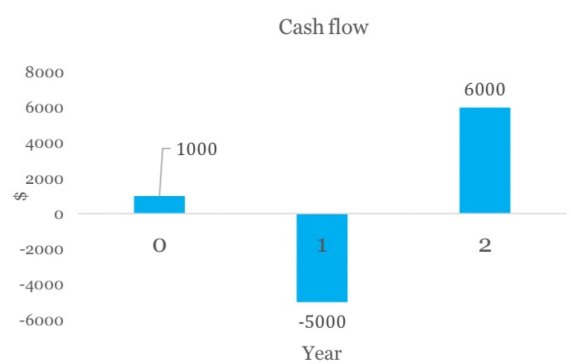
$0 = 1{,}000 - 5{,}000(1+i^*)^{-1} + 6{,}000(1+i^*)^{-2}$

$(i^* - 1)(i^* - 2) = 0$



▪ This project has two IRRs: 100% and 200%

Period k	$B_k@i^* = 100\%$	$B_k@i^* = 200\%$
0	1,000	1,000
1	1,000(1+1)-5,000=-3,000	1,000(1+2)-5,000=-2,000
2	-3,000(1+1)+6,000=0	-2,000(1+2)+6,000=0



- ↳ at end of period, $B_3 = 0$
  - \$1000 avail in yr 0 must be reinvested in proj for this to be true
  - however, proj doesn't behave as investment during 1st period
- ↳ proj balance depends on i used for discounting
- e.g. find ERR of above example, assuming MARR is 25%

1) guess ie & check proj balance

$i_e = 25\%$

$1000(F/P, 25\%, 1) - 5000$

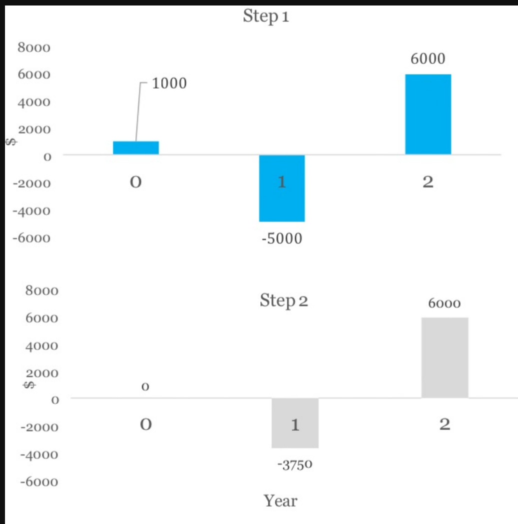
$= -3750$

2) simple investment

$-3750(F/P, i_e, 1) + 6000 = 0$

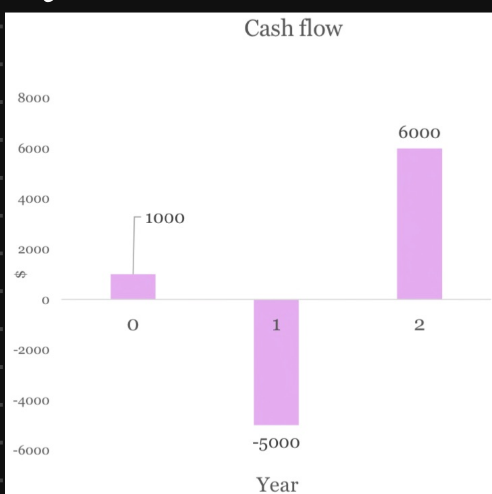
$(F/P, i_e, 1) = 1.6$

$i_e = 60\%$



- more convenient method for computing ERR is by using apx external RoR (aERR)
  - ↳ calc FV at yr N of all net receipts (CF^+) at MARR
  - ↳ calc FV at yr N of all net disbursements (CF^-) at aERR i_e^*
  - ↳ equate both vals & solve for i_e^*
    - $\sum_k CF_k^- (F/P, i_e^*, N-k) = \sum_k CF_k^+ (F/P, \text{MARR}, N-k)$

- e.g. for above example, find aERR, assuming MARR of 25%



$$1000(F/P, \text{MARR}, 2) + 6000 = 5000(F/P, i_e^*, 1)$$
$$1000(1.563) + 6000 = 5000(1 + i_e^*)$$
$$1 + i_e^* = 1.5126$$
$$i_e^* = 0.5126$$
$$= 51.26\%$$

- modified IRR (MIRR) measures attractiveness of CFs as fcn of 2 ERRs
  - ↳ external rate for investing i_{inv}
  - ↳ external rate for financing / borrowing i_{fin}
  - ↳ profit is $i_{fin} < i_{inv}$
- to obtain MIRR:
  - ↳ calc PW of all net disbursements CF_k^- using i_{fin}, then convert to FW
  - ↳ calc FW of all net receipts CF_k^+ using i_{inv}
  - ↳ find MIRR that equates PW & FW
    - $\left(\sum_k CF_k^- (P/F, i_{fin}, k)\right)(F/P, \text{MIRR}, N) = \sum_k CF_k^+ (F/P, i_{inv}, N-k)$
- e.g. for above example, find MIRR assuming $i_{inv} = 32\%$ & $i_{fin} = 25\%$

$$(5000(P/F, i_{fin}, 1))(F/P, \text{MIRR}, 2) = 1000(F/P, i_{inv}, 2) + 6000$$

$$\left(\frac{5000}{(1+0.25)^1}\right)(1+\text{MIRR})^2 = 1000(1+0.32)^2 + 6000$$
$$(1+\text{MIRR})^2 = 1.9336$$
$$\text{MIRR} = 0.391258$$
$$= 39.13\%$$

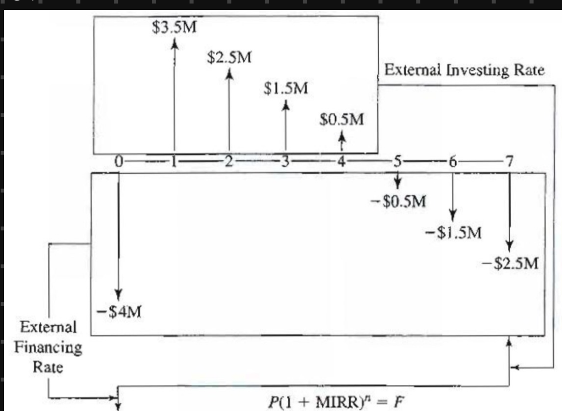
- e.g. multiple roots

- Adding an oil well to an existing field costs $4 million (4M). It will will provide of oil revenues by $3.5M on the first year, with a reduction of $1M profit each year thereafter (i.e. $2.5 M in year 2, $1.5M in year 3, etc) the project lifetime is 7 years.

- If adding the well is justified, one reason is that the oil is recovered sooner, then how many roots for the PW equation are possible?

- If the firm normally borrows money at 8% and invests at 15%,find the modified internal rate of return (MIRR).

CF:



m= 2 so # roots is 2 or 0

$PW = -4M - 0.5M(P/F, 8\%, 5) - 1.5M(P/F, 8\%, 6) - 2.5M(P/F, 8\%, 7)$

$= -4M - 0.5M(0.6806) - 1.5M(0.6302) - 2.5M(0.5835)$

$= -6.74435M$

$FW = 3.5M(F/P, 15\%, 6) + 2.5M(F/P, 15\%, 5) + 1.5M(F/P, 15\%, 4) + 0.5M(F/P, 15\%, 3)$

$= 3.5M(2.313) + 2.5M(2.011) + 1.5M(1.749) + 0.5M(1.521)$

$= 16.507M$

$PW(F/P, MIRR, 7) = FW$

$6.74435M(1 + MIRR)^7 = 16.507M$

$MIRR = 0.1364$

$= 13.64\%$

CHAPTER 8

PRIVATE VS PUBLIC SECTOR PROJECTS

- in private sector, firm pays all costs & receives all benefits
  - ↳ estimated benefits & costs are compared on PW basis using pre-determined discount rate (MARR)
- in public sector, revenue is received thru taxes & supposed to be spent in public interest
  - ↳ gov pays costs & receives few benefits
  - ↳ don't have profits
  - ↳ selection of interest rate is often issue
    - MARR is low to make them attractive
    - weighted avg cost of capital (WACC) used when there's multiple sources of funds
- comparison of characteristics:

Characteristic	Public Sector	Private Sector
Size of Investment	Larger	Some large; medium to small
Life Estimates	Typically long: 30-50 years	Shorter: 2-25 years
Annual Cash Flow Estimates	Costs and benefits no profit	Revenues, profit-cost estimates
Funding	Taxes, fees, bonds, private funds, cost sharing arrangements exist	Sale of new stocks, bonds, loans, other earnings
Interest Rate	Tends to be lower (social discount rates)	Higher: At market cost
Risk	Less perceived risks	Higher risks

- costs & benefits in public projs.
  - ↳ costs (sponsor costs):
    - construction (capital costs), O&M, & salvage vals
    - if proj generates benefits, should be subtracted from sponsor's costs
      - → e.g inc tax revenue due to higher land vals after new road construction
  - ↳ benefits (social benefits):
    - difficult to estimate
    - e.g. reduction in travel time, safety inc, jobs created
  - ↳ disbenefits (social costs).
    - unexpected -ve consequences
      - → e.g. inc noise & air pollution
- consider alt acceptable if benefit-cost ratio (BCR) $= \frac{B}{C}$

$$= \frac{PW_{benefit}}{PW_{cost}}$$

$$= \frac{AW(B)}{AW(C)} \geq 1$$

- e.g.

A firm is trying to decide which of two devices to install to reduce costs in a particular situation. Both devices cost $1000 and have useful lives of 5 years and no salvage value.

Device *W* can be expected to result in $300 savings annually. Device *Z* will provide cost savings of $400 the first year, but savings will decline by $50 annually, making the second year savings $350, the third-year savings $300, and so forth.

With interest at 7%, use the B/C method to determine which device the firm should purchase.

Device W:

$PW_C = 1000$

$PW_B = 300(P/A, 7\%, 5)$

Device Z:

$PW_C = 1000$

$PW_B = 400(P/A, 7\%, 5) - 50(P/G, 7\%, 5)$

$= 300(4.1)$
$= 1230$

$\frac{B}{C} = \frac{1230}{1000}$
$= 1.23$

$= 400(4.1) - 50(7.647)$
$= 1257.65$

$\frac{B}{C} = \frac{1257.65}{1000}$
$= 1.25765$

To maximize BCR, select device 2.

- to calc BCR:
  - ↳ PW : $BCR = \frac{PW(\text{users' benefits})}{PW(\text{sponsors' costs})}$
    - users' benefits = social benefits − social costs
  - ↳ FW : $BCR = \frac{FW(\text{users' benefits})}{FW(\text{sponsors' costs})}$
  - ↳ AW : $BCR = \frac{AW(\text{users' benefits})}{AW(\text{sponsors' costs})}$
- conventional BCR is $BCR = \frac{B-D}{C+OM}$
- modified BCR is $BCRM = \frac{B-D-OM}{C}$
  - ↳ no diff b/c although BC ratios diff, ult decision is same
  - ↳ used less
- to decide using BCR:
  - ↳ if BCR > 1, accept alt
  - ↳ if BCR < 1, reject alt
  - ↳ if BCR ≃ 1, intangible factors will sway decision
- e.g.

> - Ford Foundation expects to award $15 million in grants to public high schools to develop new ways to teach fundamentals of engineering to prepare students for university level.
> - The grant will extend over a 10-year period and create an estimated savings of $1.5 million/year in faculty salaries and student related expenses.
> - The Foundation uses a MARR of 6%/year.
> - The grants program will share Foundation funding with ongoing activities, so an estimated 200,000 $/year will be removed from other program funding.
> - To make this program successful, a $500,000/year operating cost will be incurred from the regular O&M budget.
> - Use the B/C method to determine if the grants program is economically justified.

$AW_{cost} = 15M(A/P, 6\%, 10)$
$= 15M(0.1359)$
$= 2.0385M$

$AW_{benefits} = 1.5M$

$AW_{disbenefits} = 0.2M$

$AW_{OM} = 0.5M$

$BCR = \frac{B-D}{C+OM}$
$= \frac{1.5M - 0.2M}{2.0385M + 0.2M}$
$= 0.58 < 1$

$BCRM = \frac{B-D-OM}{C}$
$= \frac{1.5M - 0.2M - 0.5M}{2.0385M}$
$= 0.392 < 1$

Using both BCR & BCRM, we reject.

- to do incremental BCR:
  1) select from 2 mutually exclusive alts X & Y
     - DN always exists
     - order based on PW/AW/FW of total cost (assume $C_x \geq C_y$)
  2) calc incremental BCR
     - $\Delta BCR = BCR(X-Y) = \frac{B_x - B_y}{C_x - C_y}$
  3) if $\Delta BCR > 1$, select X & if not, select Y
     - if $C_x = C_y$, choose proj w/ greater $PW_{benefits}$
     - if $B_x = B_y$, choose proj w/ lowest PW_{cost}

↳ if using PW/FW, must used repeated lives method / equal life model

↳ to compare multiple mutually exclusive projs:

- for each alt, find equiv PW / FW / AW for C & B - D
- order by inc total equiv costs
  - → DN is 1st option if it has non-trivial costs / benefits
- determine $\Delta BCR = \frac{\Delta(B-D)}{\Delta C}$ btwn first 2 alts
  - → if $\Delta BCR \geq 1$, elim 1 & 2 survivor
- continue until end of list

- e.g.

- A current process in a factory has two mutually exclusive alternative systems that could be implemented to improve the efficiency of this process, thus reducing the annual costs:

	System 1	System 2
Investment cost ($)	10,000	15,000
Cost Savings ($/year)	5,000	8,000

- Are these alternatives worth to implement? Consider an $i = 10\%$ and $N = 5$ years.

Alt	PW Costs	Benefits
DN	0	0
1	10 000	5000 / yr
2	15 000	8000 / yr

DN vs 1:

$\Delta PW_{costs} = 10000 - 0$
$= 10000$

$\Delta PW_{ben} = 5000(P/A, 10\%, 5) - 0$
$= 5000(3.791)$
$= 18955$

$\Delta BCR_{1-DN} = \frac{18955}{10000}$
$= 1.8955 > 1$ → choose 1

1 vs 2:

$\Delta PW_C = 15000 - 10000$
$= 5000$

$\Delta PW_B = (8000 - 5000)(P/A, 10\%, 5)$
$= 3000(3.791)$
$= 11373$

$\Delta BCR_{2-1} = \frac{11373}{5000}$
$= 2.2746 > 1$ → choose 2

Best alt is system 2.

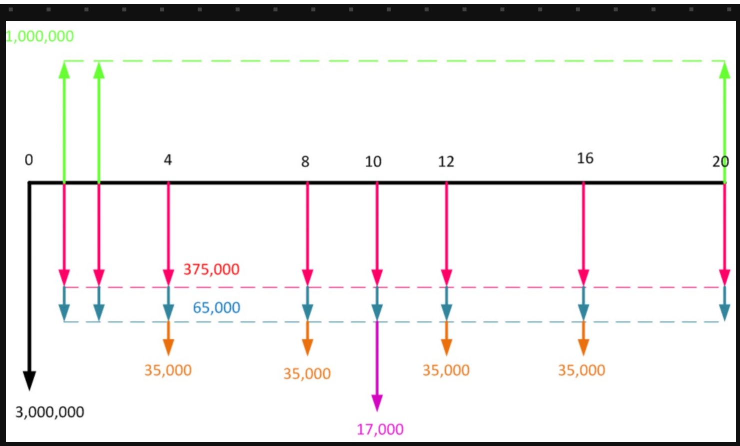
CHAPTER 9

RISK AND UNCERTAINTY

- given uncertainty in various params in economic analysis, planner must study impact of assumptions on investment
- sensitivity graphs are used to assess effect of one-at-a-time changes in key param vals of proj
  - ↳ requires base case
  - ↳ vary 1 param at a time by inc/dec it & determine its impact
  - ↳ fix all other params
- sensitivity analysis is impact of param & how much variation is necessary to affect change in outcome (i.e. alt selected)
- attitudes twd risk:
  - ↳ risk averse: fear of loss & seeking sureness
  - ↳ risk neutral: indiff to uncertainty
  - ↳ risk taker: hope to win big
- to measure risk, can use std deviation, which measures dispersion of outcomes abt expected val
  - ↳ denoted σ, which is square root of variance
- e.g.

- Cogen Corporation is replacing its current steam plant with a six-megawatt cogeneration plant that will produce both steam and electric power for operations. To move to the new system, Cogen will have to integrate a new turbogenerator and cooling tower with its current system.
- The estimated first cost of the equipment and installation is $3,000,000 though there is some uncertainty surrounding this estimate. The plant is expected to have a 20-year life and no scrap value at the end of this life.
- In addition to the first cost, the turbogenerator will require an overhaul with an estimated cost of $35,000 at the end of years 4, 8, 12, and 16; and the cooling tower will need an overhaul at the end of 10 years. This is expected to cost $17,000.
- The cogeneration system is expected to have higher annual operating and maintenance costs than the current system and will require the use of chemicals to treat the water used in the new plant. These incremental costs are estimated to be $65,000 per year.
- The incremental annual costs of wood fuel are estimated to be $375,000. The cogeneration plant will save Cogen from having to purchase 40,000,000 kilowatt-hours of electricity per year at $0.025 per kilowatt-hour, so an annual savings of $1,000,000 will be achieved.
- What is the present worth of the incremental investment in the cogeneration plant? What is the impact of a 5% and 10% increase and decrease in each of the parameters of the problem?. The MARR is given as 12%.

Cost Category	Base Case
Initial Investment	$ 3,000,000
Annual chemicals, operations, and maintenance costs	$ 65,000
Cooling tower overhaul (after 10 years)	$ 17,000
Turbogenerator overhauls (after 4, 8, 12, and 16 years)	$ 35,000
Annual wood costs	$ 375,000
Annual savings in electricity costs	$ 1,000,000
MARR	0.12



$PW = -3\,000\,000 - 65000(P/A, 12\%, 20) - 17000(P/F, 12\%, 10) - 35000((P/F, 12\%, 4) + (P/F, 12\%, 8) + (P/F, 12\%, 12) + (P/F, 12\%, 16)) - 375000(P/A, 12\%, 20) + 1000000(P/A, 12\%, 20)$

$= -3\,000\,000 + 560000(7.469) - 17000(0.322) - 35000(0.6355 + 0.4039 + 0.2567 + 0.1631)$

$= 1126\,094$

From base case, since $PW > 0$, proj is economically viable.

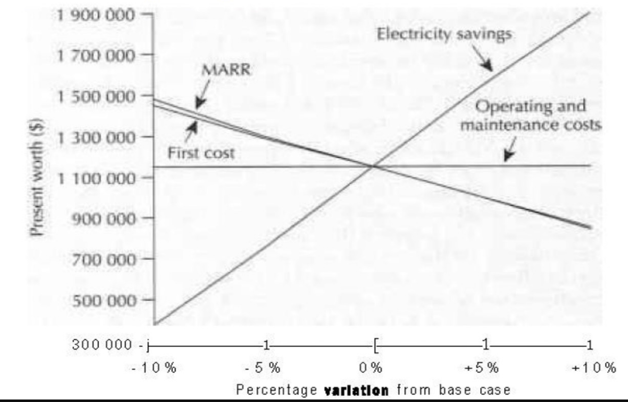
CF estimates that are $\pm 5\%$ & $\pm 10\%$ base case:

Category	-10%	-5%	Base Case	+5%	+10%
Initial investment	2,700,000	2,850,000	3,000,000	3,150,000	3,300,000
Annual O&M	58,500	61,750	65,000	68,250	71,500
Cooling tower overhaul	15,300	16,150	17,000	17,850	18,700
Turbogenerator overhaul (after 4,8,12,16 yrs)	31,500	33,250	35,000	36,750	38,500
Annual wood costs	337,500	356,250	375,000	393,750	412,500
Annual savings	900,000	950,000	1,000,000	1,050,000	1,100,000
MARR	0.108	0.114	0.12	0.126	0.132

PW considering each variation individually:

Category	-10%	-5%	Base Case	+5%	+10%
Initial investment	1,426,343	1,276,336	1,126,343	976,458	826,710
Annual O&M	1,174,894	1,150,619	1,126,343	1,102,067	1,077,792
Cooling tower overhaul	1,126,890	1,126,617	1,126,343	1,126,069	1,125,796
Turbogenerator overhaul (after 4,8,12,16 yrs)	1,131,450	1,128,897	1,126,343	1,123,789	1,121,236
Annual wood costs	1,406,447	1,266,395	1,126,343	986,291	846,239
Annual savings	379,399	752,871	1,126,343	1,499,815	1,873,287
MARR	1,456,693	1,286,224	1,126,343	976,224	835,115

Sensitivity Graph:



↳ electricity savings have greatest impact on viability of proj

↳ if individual CF estimates will fall within ±10% range, investment is economically viable

· e.g.

A 50 MW Natural Gas Combined Cycle (NGCC) plant is planned to be installed to supply the increasing peak demand in Ontario. The following estimated data is available:

Parameter	Value
Investment cost ($)	48'900,000
Capacity Factor (CF)	0.15
Hourly Ontario Energy Price (HOEP) ($/MWh)	120
Fixed annual cost ($/kW)	14.34
Fuel cost ($/MWh)	1.72
Lifetime (years)	20
MARR	10%

Revenue:

Electricity $= 0.15(50\text{ MW}) \cdot 24\text{h} \cdot 365\text{ days}$

$= 65\,700 \text{ MWh/yr}$

$A_{in} = 65700(120)$

$= 7\,884\,000\text{ /yr}$

Cost:

$A_{out} = 14.34(50\text{MW})\left(\frac{1000\text{ kW}}{1\text{ MW}}\right) + 1.72(65\,700)$

$= 830\,004\text{ /yr}$

Net annuity:

$A = A_{in} - A_{out}$

$= 7\,884\,000 - 830\,004$

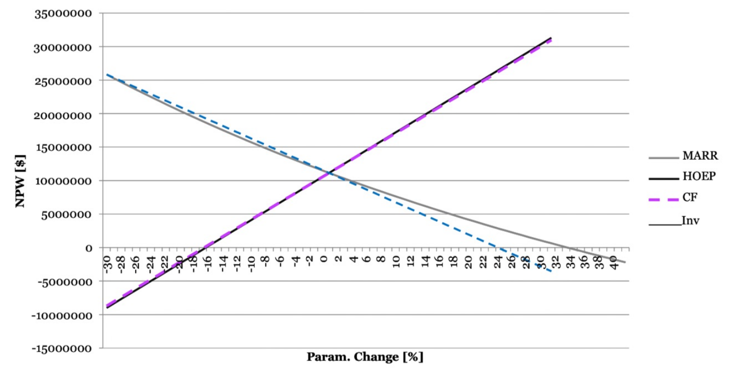
$= 7\,053\,996$

$NPW = 7053996(P/A, 10\%, 20) - 48\,900\,000$

$= 7053996(8.514) - 48\,900\,000$

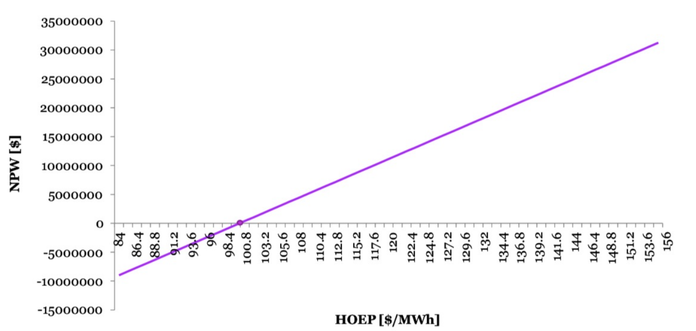
$= 11\,157\,721.94$

- Sensitivity Analysis:
  - Draw sensitivity graphs for ±30% variations in: CF, MARR, HOEP, Inv



- shortcomings of sensitivity graphs:
  - ↳ impact of param variations outside range considered may not be linear extrapolations
  - ↳ interactions btwn 2+ params aren't considered (i.e can't add up impacts)
- break-even analysis is process of varying a param & determining what val causes performance measure (NPW) to reach some threshold
  - ↳ break-even val is 0
  - ↳ e.g.

Break-even chart for the Hourly Ontario Energy Price (HOEP) for project in Example 2:



- A HOEP>100 ($/MWh) makes the project acceptable.

- e.g.

- Consider a project that may be constructed to full capacity now or may be constructed in two stages and perform a breakeven analysis.

Construction Costs:

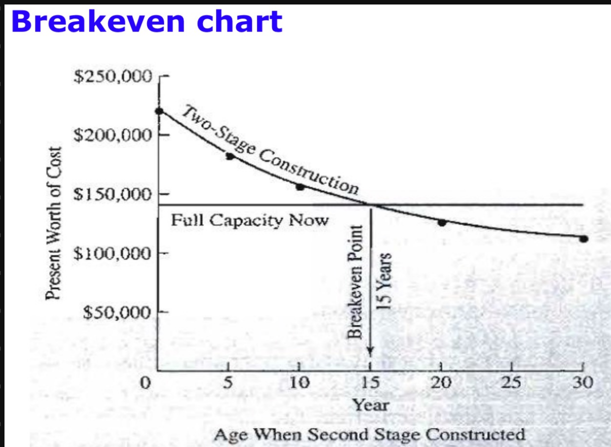
- Two-stage construction: Construct the first stage now for $100,000; then construct the second stage *n* years from now for $120,000.
- Full-capacity construction: Construct full capacity now for $140,000.

Other Factors:

- All facilities will last for 40 years, with zero salvage value
- The annual cost of operation and maintenance is the same for both two-stage construction and full-capacity construction.
- MARR is 8%

$PW_{\text{full cost}} = 140\,000$

$PW_{\text{2 stage cost}} = 100\,000 + 120\,000(P/F, 8\%, n)$



↳ yr 15 is breakeven pt
↳ decision on how to construct proj is sensitive to age at which 2nd is needed
- > 15 yrs, choose 2-stage
- < 15 yrs, choose full capacity

- for multiple indep projs, same break-even analysis carried out as NGCC example
- for mutually exclusive projs, break-even analysis gives range over which each alt is preferred
- e.g.

Westmount Waxworks is considering buying a new wax melter for its line of replicas of statues of government leaders. Westmount has two choices of suppliers: Fine detail and Simplicity. The proposals are as follows:

	Fine detail Wax Melter (A)	Simplicity Wax Melter (B)
Expected life	7 years	10 years
First cost	$200,000	$350,000
Maintenance	$10,000/year + 0.05/unit	$20,000/year + $0.01/unit
Labour	1.25/unit	$0.5/unit
Other costs	$6,500/year + 0.95/unit	$15,500/year + 0.55/unit
Salvage value	$5,000	$20,000

- Uncertainty on the number of units sold as it will depend on the stability in the world government:
  - In unsettled world scenario, sales may be as high as 200,000 units per year.
  - Average of previous year sales is 50,000 units per year.
- Uncertainty in "other costs" of project B (Simplicity Wax Melter):
  - Since Simplicity model is a new technology, costs may be as low as $0.45 per unit and as high as $0.75.
- Use annual cost (AC) to perform the Break-Even analysis. Assume repeated lives
- holds and a MARR = 15%.

Let x be # units sold. Let y be other costs per unit in proj B.

$AC_A = 200\,000(A/P, 15\%, 7) + 10000 + 6500 + (0.05 + 1.25 + 0.95)x - 5000(A/F, 15\%, 7)$

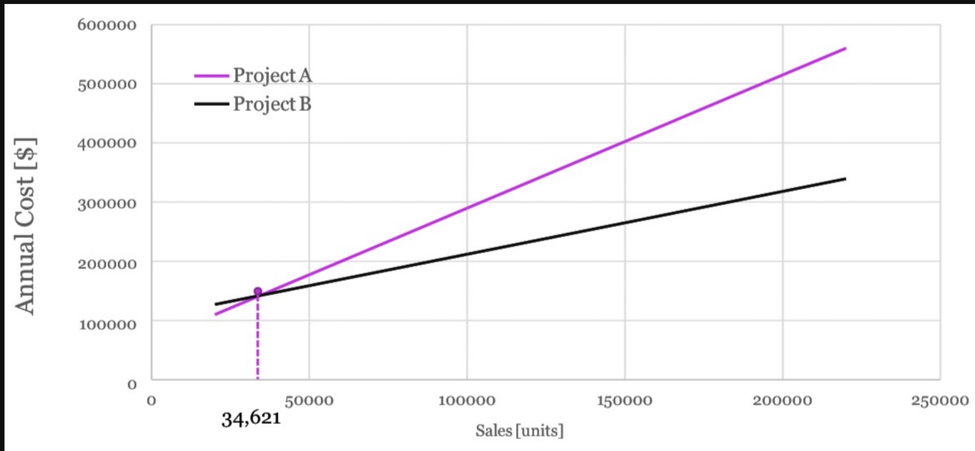
$= 200\,000(0.2404) + 16500 + 2.25x - 5000(0.0904)$

$= 64128 + 2.25x$

$AC_B = 350\,000(A/P, 15\%, 10) + 20000 + 15500 + (0.01 + 0.5 + y)x - 20\,000(A/F, 15\%, 10)$

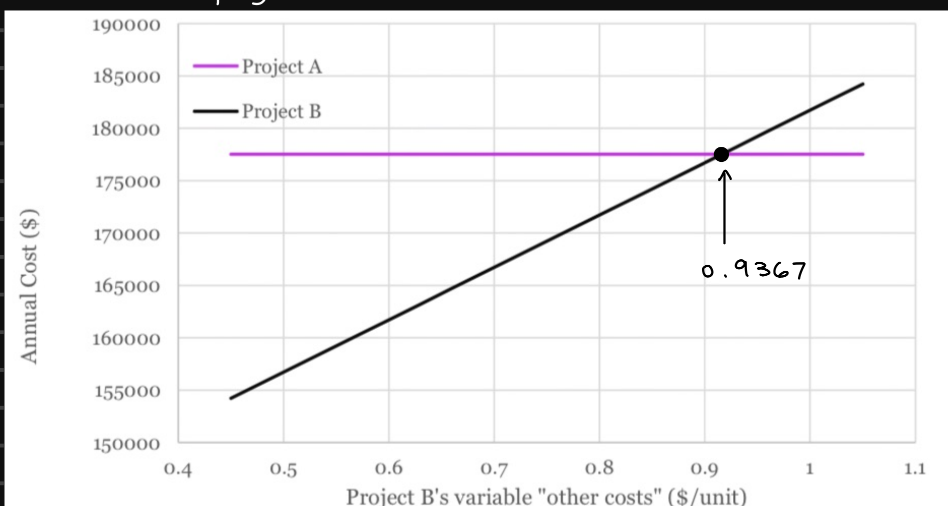
$= 350\,000(0.1993) + 35500 + (0.51 + y)x - 20\,000(0.0493)$

$= 104269 + (0.51 + y)x$



↳ if < 34621 units sold, proj A preferred
↳ if ≥ 34621 units sold, proj B preferred
↳ since avg sales were 50000 last yr & can be as high as 200 000, company will most likely face sales ≥ 34621 units

- choose proj B



↳ if other costs is < 0.9367, choose B
↳ variable price estimated to be btwn 0.45 & 0.75

- choose B to be implemented

e.g.

Three mutually exclusive alternatives are considered, with equal lives of 20 years, salvage value=0. Use the NPW analysis to determine which one should be executed if the MARR = 6%.

	A	B	C
Initial Cost	$2,000	$4,000	$5,000
Uniform Annual Benefit	$410	$639	$700

$NPW_A = -2000 + 410(P/A, 6\%, 20)$
$= -2000 + 410(11.47)$
$= 2702.7$

$NPW_B = -4000 + 639(11.47)$
$= 3329.33$

$NPW_C = -5000 + 700(11.47)$
$= 3029$

Choose B b/c NPW_B is highest.

If B was preferred at an initial cost of $4,000, then it will continue to be preferred for any smaller initial cost. How much higher can the initial cost go to still have B as the preferred alternative?

Let x be highest init cost.

$NPW_B = -x + 639(P/A, 6\%, 20)$

$3029 < -x + 639(11.47)$

$x < 4300.33$

B can go up to \$4300.33 max init cost to be preferred alt.

PROBABILITY

- random var is param/var that can take on several possible outcomes
  - ↳ e.g. flipping coin has 2 outcomes: H or T
- modeling an event that has multiple possible outcomes is accomplished by probability distribution fcn
- let X be random var that can take on m discrete outcomes $x_1, x_2, \ldots, x_m$
  - ↳ if events are mutually exclusive & collectively exhaustive (i.e. one of them must occur), prob distr fcn $p(X)$ is set of numerical measures $p(x_i)$ st $0 \le p(x_i) \le 1$ for $i = 1, \ldots, m$
    - $\sum_{i=1}^{m} p(x_i) = 1$
- product of 2 indep events is $p(A \text{ and } B) = p(A) \times p(B)$
- expected val of random var is when each outcome is weighted by its prob & results are summed
  - ↳ $\bar{x} = E(X) = \sum_{i=1}^{m} x_i \cdot p(x_i)$
  - ↳ not arithmetic mean, which is $\bar{x}_A = \frac{1}{m} \sum_{i=1}^{m} x_i$
- std deviation is:

$$\sigma = \sqrt{\sum_{i=1}^{m} (x_i - \bar{x})^2 p(x_i)}$$

$$\sigma = \sqrt{E(X^2) - (E(X))^2}$$

- e.g.

Company-XYZ has the following estimates for two grades of products:

Scenario	Annual Cost Estimate		Probability
	Grade A	Grade B	
Pessimistic	\$85,314	\$94,381	0.2
Expected	\$112,314	\$103,501	0.5
Optimistic	\$314,814	\$186,901	0.3

Determine the expected value of A and B.

$E_A = 85314 \cdot 0.2 + 112314 \; 0.5 + 314814 \; 0.3$

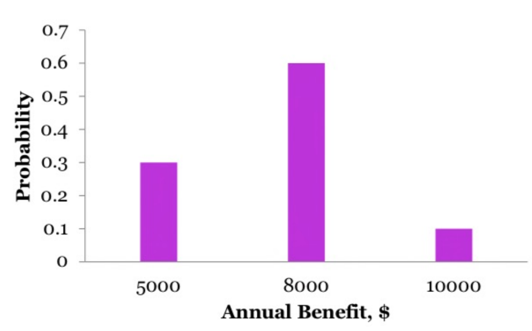
$= 167664$

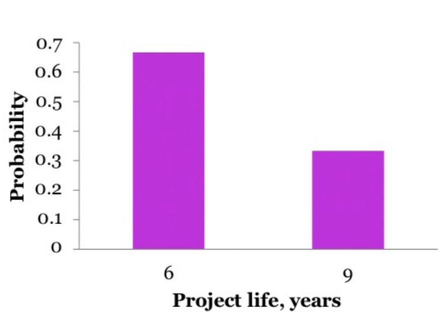
$E_B = 94381 \cdot 0.2 + 103501 \cdot 0.5 + 186901 \; 0.3$

$= 126697$

- e.g.

- The most likely value of the annual benefit from a project is \$8,000 with a probability of 0.6.
- There is a 30% probability that it will be \$5,000, and the highest benefit, that is likely, is \$10,000.
- A life of 6 years is twice as likely, as a life of 9 years.
- The project first cost is \$25,000, MARR = 10%.
  - Determine the probability distributions for annual benefit and life of the project.
  - Assume probability distribution for annual benefit and life are statistically independent, and find the probability distribution of the PW.
  - Determine the expected PW.





$p(\text{life}=6) = 2 \cdot p(\text{life}=9)$

$1 = p(\text{life}=6) + p(\text{life}=9)$

$1 = 2 \cdot p(\text{life}=9) + p(\text{life}=9)$

$1 = 3 \cdot p(\text{life}=9)$

$\frac{1}{3} = p(\text{life}=9) \longrightarrow \frac{2}{3} = p(\text{life}=6)$

State x_i (Annual Benefit, Life)	***p(x_i)***	**Net Benefit = PW(10%) – First Cost**
($5,000, 6)	0.3*0.667=0.2	$21,776 - $25,000 = -$3,224
($5,000, 9)	0.3*0.333=0.1	$28,795 - $25,000 = $3,795
($8,000, 6)	0.6*0.667=0.4	$34,842 - $25,000 = $9,842
($8,000, 9)	0.6*0.333=0.2	$46,072 - $25,000 = $21,072
($10,000, 6)	0.1*0.667=0.0667	$43,553 - $25,000=$18,553
($10,000, 9)	0.1*0.333=0.0333	$57,590 - $25,000 = $32,590
Total	1.0	

$E_{PW} = -3224 \cdot 0.2 + 3795 \cdot 0.1 + 9842 \cdot 0.4 + 21072 \; 0.2 + 18553 \cdot 0.0667 + 32590 \cdot 0.0333$

$= 10\,208.63$

$E_{\text{benefits}} = 8000 \cdot 0.6 + 5000 \cdot 0.3 + 10000 \; 0.1$

$= 7300$

$E_{\text{life}} = \frac{1}{3}(9) + \frac{2}{3}(6)$

$= 7$

$PW_{\text{expected vals}} = 7300(P/A, 10\%, 7) - 25000$

$= 7300(4.868) - 25000$

$= 10536.40$

State x_i (Annual Benefit, Life)	***p(x_i)***	**PW**	***p(x_i)*PW***	**p(x_i)*PW²**
($5,000, 6)	0.3*0.667=0.2	-$3,224	-$644.80	2,078,835
($5,000, 9)	0.3*0.333=0.1	$3,795	$379.50	1,440,202
($8,000, 6)	0.6*0.667=0.4	$9,842	$3936.80	$38,745,985
($8,000, 9)	0.6*0.333=0.2	$21,072	$4,214.40	$88,805,837
($10,000, 6)	0.1*0.667=0.0667	$18,553	$1,237.49	$22,959,061
($10,000, 9)	0.1*0.333=0.0333	$32,590	$1,085.25	$35,368,200
Total	1	EV	$10,209	189,398,120

$\sigma = \sqrt{E(X^2) - (E(X))^2}$

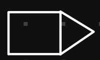
$= \sqrt{189\,398\,120 - 10209^2}$

$= 9229$

CHAPTER 10

DECISION TREES

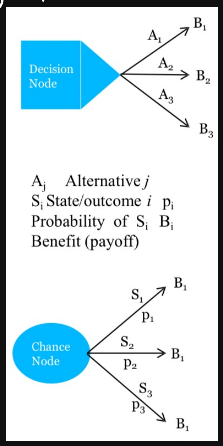
- decision trees are graphical tools for describing actions avail, events that can occur, & rltnship btwn actions & events
  - ↳ aka decision flow networks, decision diagrams
  - ↳ grow from left to right
  - ↳ usually begin w/ decision node
- symbols:
  - ↳ decision node:

 or 

  - ↳ chance node:

  - ↳ outcome node:

- decision node is where course of action is selected from set of possibilities
  - ↳ each alt is shown by branch w/ associated cost
  - ↳ each branch may end at outcome, chance, or another decision node
- chance node is when atp in process, random event is expected
  - ↳ each outcome is denoted by branch w/ associated probability
- graphical rep:



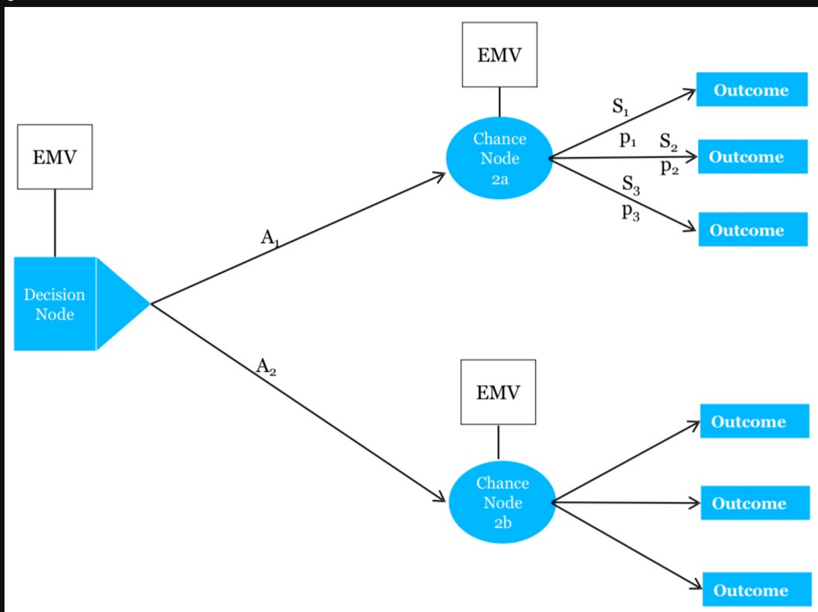
- to construct a tree:
  1) start w/ 1+ decision nodes
  2) from each decision node, all possible alts branch out:
     - to decision node w/ associated subsequent decision
     - to chance node w/ associated subsequent events
  3) each time chance node is added, appropriate status w/ their corresponding probs are denoted
  4) all branches out of are a set of mutually exclusive & collectively exhaustive consequences
  5) continue until final outcome nodes are reached
- to find soln:
  - ↳ divide tree into 2:
    - chance nodes w/ all emerging states
    - decision nodes w/ all alts
  - ↳ start w/ segments that end in outcome nodes & proceed in rev order from which it was drawn
    - i.e. rollback procedure
  - ↳ to select among alts, pick alt w/ highest expected val
    - criterion is called Expected Monetary Val (EMV)
    - at each chance node, compute EMV:

→ multiply outcomes by probs & sum everything

→ resulting EMV is outcome for branch to immediate left

- at decision node, select highest EMV from each outcome & discard rest
  - → branches of discarded alts are marked w/ " // "
- continue rolling back until leftmost node is reached
- expected val of final node is EV of overall decision

↳ graphical rep:

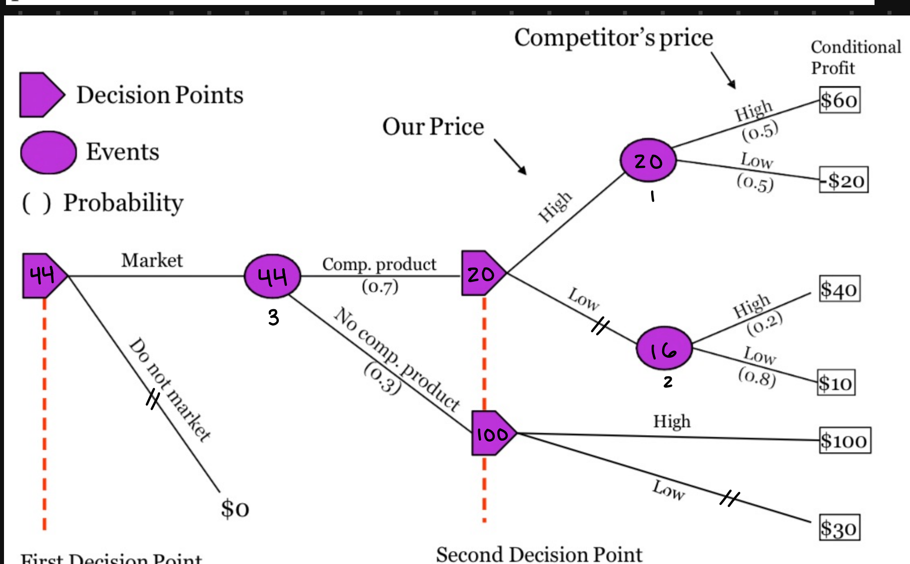


e.g.

A company is considering marketing a new product. There is a 70% chance of encountering a competitive product. The following options arise:

- With competitive product:
  - Option-1: Raise your price and see how your competitor responds:
    - If the competitor raises price, your profit will be \$60, probability=0.5.
    - If they lower the price, you will lose \$20, probability=0.5.
  - Option-2: Lower your price and see how your competitor responds:
    - If the competitor raises price, your profit will be \$40, probability=0.2.
    - If they lower the price, your profit will be \$10, probability=0.8.
- Without competitive product: You still have two options:
  - Option-1: Raise your price, your profit will be \$100.
  - Option-2: Lower your price, your profit will be \$30.

Construct a decision tree to decide whether the company should market the product or not.



$EV_1 = 0.5(60) + 0.5(-20)$
$= 20$

$EV_2 = 0.2(40) + 0.8(10)$
$= 16$

$EV_3 = 0.7(20) + 0.3(100)$
$= 44$

Market new product Whether there's a competitive product or not, raise price. EMV is $44.

e.g.

You have to invest $50,000 in financial market for one year:

- Option-1: Buy 1,000 shares of technology stock, $50 per share, hold for 1 year. Since this is a new initial public offering (IPO), there is not much research information available on the stock, hence there will be a brokerage fee of $100 for this transaction (for either buying or selling stocks).
- Assume that the stock is expected to provide a return at any of the three levels:
  - High (A): 50% return ($25,000), probability(A) = 0.25.
  - Medium (B): 9% return ($4,500), probability(B) = 0.4.
  - Low (C): -30% loss (-$15,000), probability(C) = 0.35.

- Option-2: Purchase $50,000 worth of bonds issued by Treasure Inc., a large stable financial company:
  - The $1,000 bonds cost $909.09 each, and mature in one year.
  - You can therefore buy 55 bonds.
  - These bonds do not pay interest, as they are coupon-free, so their appreciation gets taxed as capital gain.
  - There is a $150 transaction fee for either buying or selling bonds

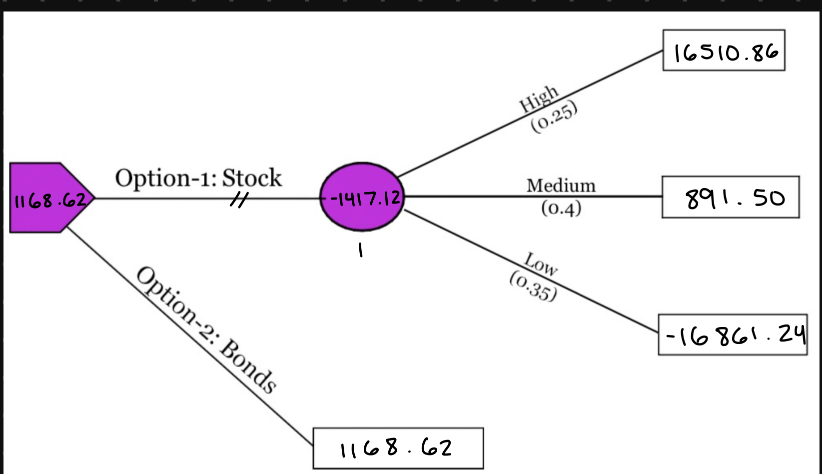
Your dilemma is which alternative to choose, to maximize financial gain.

You are not concerned about seeking professional advice on the stock before making a decision.

Assume that any capital gain is taxed at 20%, your MARR is 5% after taxes.

Determine the payoff amount at the tip of each branch.

Decision tree:



$EV_1 = 0.25(16510.86) + 0.4(891.50) + 0.35(-16861.24)$
$= -1417.12$

Option 1:

$\text{Net } CF_{A1} = 75000 - 100 - 0.2(75000 - 50000 - 100 - 100)$
$= 69940$

$NPW_{high} = -50000 - 100 + 69940(P/F, 5\%, 1)$
$= -50100 + 69940(0.9524)$
$= 16510.86$

$\text{Net } CF_{B1} = 54500 - 100 - 0.2(54500 - 50000 - 100 - 100)$
$= 53540$

$NPW_{med} = -50000 - 100 + 53540(P/F, 5\%, 1)$
$= -50100 + 53540(0.9524)$
$= 891.50$

$\text{Net } CF_{C1} = 35000 - 100 - 0.2(0)$
$= 34900$

$NPW_{low} = -50000 - 100 + 34940\,(P/F, 5\%, 1)$

$= -50100 + 34900(0.9524)$

$= -16861.24$

Option 2:

$\text{Net CF}_1 = 55(1000) - 150 - 0.2(1000 \cdot 55 - 909.09 \cdot 55 - 150 - 150)$

$= 53909.99$

$NPW = -55 \cdot 909.55 - 150 + 53909.99(P/F, 5\%, 1)$

$= -50175.25 + 53909.99(0.9524)$

$= 1168.62$

Choose option 2, which is purchasing bonds.