① sample mean..

$$m(a+bX) = a + b \times m(X)$$

$$m(x) = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$m(a+bx) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i)$$

$$= \frac{1}{N}\left(\sum_{i=1}^{N} a + \sum_{i=1}^{N} bx_i\right)$$

$$= \frac{1}{N}\left(Na + b\sum_{i=1}^{N} x_i\right)$$

$$= a + \frac{b}{N}\sum_{i=1}^{N} x_i$$

$$= a + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

$$\boxed{m(a+bx) = a + b(m(x))}$$ ✓

② covariance formula:

$$cov(x,y) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))(y_i - m(Y))$$

$$cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\left((a+by_i) - m(a+bY)\right)$$

* $m(a+bY) = a + b \cdot m(Y)$ (we just proved this in ①)

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\left(\cancel{a} + by_i - \cancel{a} - bm(Y)\right)$$

$$= \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))\, b(y_i - m(Y))$$

$$\boxed{cov(X, a+bY) = b \cdot cov(X,Y)}$$ ✓

③ $cov(a+bX, a+bX) = b^2 \cdot cov(X,X)$

$$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{n}\left((a+bx_i) - m(a+bX)\right)^2$$

$$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{n}(a+bx_i - a - bm(x))^2$$

$$cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{n}(b(x_i - m(X)))^2$$

$\boxed{cov(a+bx, a+bX) = b^2 \cdot cov(X, X)}$ ✓

★ $cov(x, X) = \frac{1}{N}\sum_{i=1}^{N}(x_i - m(x))^2 = s^2$

$\boxed{cov(a+bX, a+bX) = b^2 \cdot s^2}$ ✓

(4) if g is non-decreasing ($x \geq x'$ means $g(x) \geq g(x')$) it doesn't change the order of the data

$\Rightarrow$ median & other quantiles behave the same because they ALSO depend on order

ex) median of $g(x)$ = $g(\text{median}(x))$

$x = [1, 2, 3]$, median = 2

$g(x) = 3 + 5x$

$g(x) = [8, 13, 18]$, median = $13 = 3+5 \cdot 2$

BUT IQR and Range might not be the same.

$IQR = Q_3 - Q_1$

$\hookrightarrow IQR(g(x)) = g(Q_3) - g(Q_1)$

but only works if $g(x)$ is very linear.

same for Range = max - min.

(5) $m(g(x)) = g(m(x))$ typically speaking gives different results.

Case 1) $g(x)$ is linear

ex $x = [1, 2, 3]$ $m(x) = 2$

$g(x) = 3 + 2x$

equal

$m(g(x)) = 3 + 2 \cdot 2 = 7$

Case 2) $g(x)$ is non linear.

$x = [0, 1]$ $m(x) = 0.5$

$g(x) = x^2$

not equal.

$m(g(x)) = (0^2 + 1^2)/2 = 0.5$

$g(m(x)) = (0.5)^2 = 0.25$

$\boxed{m(g(x)) = g(m(x)) \text{ \underline{ONLY} when } g(x) \text{ is linear}}$