BMEn 2401 – Programming for Biomedical Engineers – Problem Set 2

Due Tuesday, 18 September 2018

1. (5) What is the final value of the vector v after this script executes? You may either work it out by hand or type it in.

```
v = zeros(10,1);
v(1,1) = 0;
v(2,1) = 1;
v(3,1) = v(1,1) + v(2,1);
v(4,1) = v(2,1) + v(3,1);
v(5,1) = v(3,1) + v(4,1);
v(6,1) = v(4,1) + v(5,1);
v(7,1) = v(5,1) + v(6,1);
v(8,1) = v(6,1) + v(7,1);
v(9,1) = v(7,1) + v(8,1);
v(10,1) = v(8,1) + v(9,1);
```

2. (10) Suppose that I have two vectors a and b, each of five elements. The elements of a are the numbers one through five in some permutation, and the elements of b are five arbitrary numbers. I want a vector c such any element of c is equal to an element of b as given by the permutation vector a. For example, if a = [2 4 3 1 5], then c(1) = b(2), c(2) = b(4), etc. Write a Matlab script to construct the vector c, and apply it to the vectors a = [3 2 4 5 1] and b = [8 3 2 4 7], recording the resulting c vector. NOTE: We have not yet studied iteration (“for” loops), so I made this problem small enough that they are not necessary. If you are familiar with for loops, you are welcome to use one to do the problem.

3. (20) The same effect could be achieved by constructing a permutation tensor as follows:

```
P = zeros(5,5);  % Note: This could also be zeros(5) since zeros(N) defaults to N x N
P(1,a(1)) = 1;
P(2,a(2)) = 1;
P(3,a(3)) = 1;
P(4,a(4)) = 1;
P(5,a(5)) = 1;
```

Assess the following statements as TRUE or FALSE, and if you say FALSE explain why or give a corrected version of the statement.

a. Each row of P contains four zeroes and a one.
b. Each column of P contains four zeroes and a one.
c. c = P \ b;
d. $P^T P$ = the identity matrix (zeroes everywhere except for all ones on the diagonal)

4. (20 - 5 each) Evaluate the following matrix-vector and matrix-matrix products in Matlab:

a. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$ b. $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 7 & -2 \\ -1 & 3 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 1 \\ -1 & 0 & 3 \end{bmatrix}$ c. $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 7 & -2 \\ -1 & 3 & 0 \end{bmatrix}\begin{bmatrix} 3 \\ 10 \\ -3 \end{bmatrix}$

d. $\begin{bmatrix} 6 & 7 & 7 \\ 17 & 9 & 4 \\ 5 & 1 & 0 \end{bmatrix}\begin{bmatrix} 6 \\ -3 \\ 1 \end{bmatrix}$

5. (20) The linear least squares problem is an important problem in analysis. In most cases, for a system of the form Ax = b, where A has more rows than columns, the problem Ax = b does not have a solution, but it is usually fairly straightforward to find the solution to the problem $A^TAx = A^Tb$, and it can be shown that the vector x so obtained minimizes the original residual |Ax-b|. That is, the magnitude of the vector Ax-b is minimized for the x that exactly solves $A^TAx = A^Tb$. Write a Matlab script that, given A and b, finds x by the least squares approach and also assigns to the variable R (for "residual," the term for the error) the quantity |Ax-b|. Use the apostrophe for transpose, the backslash for left division, and the *dot* command to calculate the magnitude of the vector.

6. (10) Consider the problem

$$3x + y + 3z = 7$$

$$x + 2y + 2z = 5$$

$$x + y + z = 3$$

$$2x + y + 5z = 4$$

You will notice that if we only considered the first three equations, the ordered triple (1,1,1) would solve the problem, but (1,1,1) does not solve the last equation. Similarly, the ordered triple (1,2,0) would solve the last three equations, but does not solve the first equation. It can be shown that there is no ordered triple that solves all four equations. Instead, the best you can do is to find the triple that minimizes the total squared error (i.e., $(3x+y+3z-7)^2+\ldots$) by the method of problem 2. Find the optimal solution (x,y,z) and the sum of squared error associated with that solution. Also, confirm that the sum of squared error is higher for (1,1,1) and for (1,2,0).

7. (15) Consider the circle of radius 65 centered at the origin. The points (-39,52) and (25,60) both lie on the circle. There are also two points where the circle crosses the line x = 10. I want to know the area of the triangle formed by the points (-39,52), (25,60), and the point (10,y), where y is the negative y value that puts the point on the circle. Calculate this area using a script that takes the following steps:

a. Use the fact that $x^2 + y^2 = 65^2$ to calculate the negative y value when x = 10.
b. Calculate the length of each side of the triangle.
c. Use Herod's formula from last week's problem 3c to calculate the triangle area.
d. Check your solution by rearranging the circumradius formula from last week's problem 3d to calculate the triangle area.

Turn in your Matlab script and the results.