IE 400: Principles of Engineering Management
Study Set 1 Solutions

Fall 2021-2022

Question 1.
Decision Variables:

x_{ij} =amount of land allocated by kibbutz j for crop i,
$i = 1, 2, 3$ (1:sugar beets, 2:cotton, 3:sorghum), $j = 1, 2, 3$

Model:

$$
\begin{aligned}
\max\ & 1000(x_{11} + x_{12} + x_{13}) + 750(x_{21} + x_{22} + x_{23}) + 250(x_{31} + x_{32} + x_{33}) & \\
\text{s.t. } & x_{11} + x_{12} + x_{13} \le 600 & (1)\\
& x_{21} + x_{22} + x_{23} \le 500 & (2)\\
& x_{31} + x_{32} + x_{33} \le 325 & (3)\\
& & \\
& 3x_{11} + 2x_{21} + x_{31} \le 600 & (4)\\
& 3x_{12} + 2x_{22} + x_{32} \le 800 & (5)\\
& 3x_{13} + 2x_{23} + x_{33} \le 375 & (6)\\
& & \\
& x_{11} + x_{21} + x_{31} \le 400 & (7)\\
& x_{12} + x_{22} + x_{32} \le 600 & (8)\\
& x_{13} + x_{23} + x_{33} \le 300 & (9)\\
& & \\
& (x_{12} + x_{22} + x_{32}) - 2(x_{13} + x_{23} + x_{33}) = 0 & (10)\\
& 3(x_{11} + x_{21} + x_{31}) - 2(x_{12} + x_{22} + x_{32}) = 0 & (11)\\
& 3(x_{11} + x_{21} + x_{31}) - 4(x_{13} + x_{23} + x_{33}) = 0 & (12)\\
& & \\
& x_{ij} \ge 0 \quad i = 1, 2, 3 \quad j = 1, 2, 3 &
\end{aligned}
$$

(1), (2), (3): Constraints for maximum quota limitations
(4), (5), (6): Constraints for water consumption limitations
(7), (8), (9): Constraints for the usable land

Note: Constraints (10), (11), (12) are obtained by linearizing the following equations that ensures each kibbutz will plant the same proportion of its available irrigable land:

$$\frac{x_{11} + x_{21} + x_{31}}{400} = \frac{x_{12} + x_{22} + x_{32}}{600} = \frac{x_{13} + x_{23} + x_{33}}{300}$$

Question 2.

Decision Variables:

$$\begin{aligned} x_1 &= \text{Total number of process 1} \\ x_2 &= \text{Total number of process 2} \\ a &= \text{Amount of A that is produced} \\ b_1 &= \text{Amount of B that is sold} \\ b_2 &= \text{Amount of B that is disposed} \end{aligned}$$

(Note that $b_1 + b_2$ gives the amount of B produced.)
Model:

$$\begin{aligned} \max\ & 16a + 14b_1 - 2b_2 \\ \text{s.t.}\ & a = 2x_1 + 3x_2 \\ & b_1 + b_2 = x_1 + 2x_2 \\ & 2x_1 + 3x_2 \le 60 \\ & x_1 + 2x_2 \le 40 \\ & b_1 \le 20 \\ & x_1, x_2, a, b_1, b_2 \ge 0 \end{aligned}$$

Note: One can remove the variables a and b_2 from the model by letting $a = 2x_1 + 3x_2$ and $b_2 = x_1 + 2x_2 - b_1$ (constraints 1 and 2). Then the modified model becomes:

$$
\begin{aligned}
\max\ & 16(2x_1 + 3x_2) + 14b_1 - 2(x_1 + 2x_2 - b_1) \\
\text{s.t. } & 2x_1 + 3x_2 \leq 60 \\
& x_1 + 2x_2 \leq 40 \\
& b_1 \leq 20 \\
& b_1 \leq x_1 + 2x_2 \\
& x_1, x_2, b_1 \geq 0
\end{aligned}
$$

Question 3.
Decision Variables:

c_i = Cash on hand at the end of month i, i=1,2,3,4
x_i = Amount of tons purchased at the beginning of month i, i=1,2,3,4
y_i = Amount of tons sold at the end of month i, i=1,2,3,4
I_i = Amount of tons on hand after purchase at month i, i=1,2,3,4

Parameters:

p_i = Price of buying at month i, i=1,2,3,4
s_i = Price of selling at month i, i=1,2,3,4

Model:

$$
\begin{aligned}
\max\ & c_4 \\
\text{s.t. } & I_i = I_{i-1} + x_i - y_{i-1} && i = 1, \cdots, 4 && \text{(Inventory Balance Equations)} \\
& c_i = c_{i-1} - p_i x_i + s_i y_i && i = 1, \cdots, 4 && \text{(Cash Balance Equations)} \\
& I_i \leq 100 && i = 1, \cdots, 4 && \text{(Capacity Constraint)} \\
& y_i \leq I_i && i = 1, \cdots, 4 \\
& p_i x_i \leq c_{i-1} && i = 1, \cdots, 4 \\
& I_0 = 50 \\
& c_0 = 1000 \\
& y_0 = 0 \\
& c_i, x_i, y_i, I_i \geq 0 && i = 1, \cdots, 4
\end{aligned}
$$

Question 4.

(a) **Decision Variables:**
x_i = Number of hours process i runs, i=1,2,3

$$\begin{aligned}
\max\ & 9(2x_1+2x_3)+10(x_1+3x_2)+24x_3-(5x_1+4x_2+x_3)-2(2x_1+x_2+3x_3) \\
& -3(3x_1+3x_2+2x_3) \\
\text{s.t. } & 2x_1+x_2+3x_3 \le 200 && (1) \\
& 3x_1+3x_2+2x_3 \le 300 && (2) \\
& x_1+x_2+x_3 \le 100 && (3) \\
& x_1,x_2,x_3 \ge 0 && (4)
\end{aligned}$$

(b) Remove constraints (1) and (2) and add the following constraint to the model in part (a)

$$(2x_1+x_2+3x_3)+(3x_1+3x_2+2x_3) \le 500 \implies 5x_1+4x_2+5x_3 \le 500$$

(c)

Additional Decision Variable:
y : Amount of Wonka Boxes sold

The following constraint is required:

$$y = \min\left\{2x_1+2x_3, \frac{x_1+3x_2}{2}, x_3\right\}$$

Observe that this is not a linear constraint. It **must** be linearized before it is added to the model. The equality above implies the following:

$$y \le 2x_1+2x_3, \quad y \le \frac{x_1+3x_2}{2}, \quad y \le x_3$$

Then the model becomes:

$$\begin{aligned}
\max\ & 54y-(5x_1+4x_2+x_3)-2(2x_1+x_2+3x_3)-3(3x_1+3x_2+2x_3) \\
\text{s.t. } & (1)-(4) \\
& y \le 2x_1+2x_3 \\
& 2y \le x_1+3x_2 \\
& y \le x_3 \\
& y \ge 0
\end{aligned}$$

Question 5.
Decision Variables:

x_{ij} = amount of product i produced at period j, i=1,2 j=1,...,12
I_{ij} = amount of product i stored at the end of period j, i=1,2 j=1,...,12

Parameters:

D_{ij} = demand of product i at period j, i=1,2 j=1,...,12

Model:

$$
\begin{aligned}
\min \ & \sum_{j=1}^{5}(5x_{1j} + 8x_{2j}) + \sum_{j=6}^{12}(4.5x_{1j} + 7x_{2j}) + \sum_{j=1}^{12}(0.2I_{1j} + 0.4I_{2j}) & & \\
\text{s.t. } & x_{1j} + x_{2j} \leq 120000 & & j = 1, \cdots, 9 \\
& x_{1j} + x_{2j} \leq 150000 & & j = 10, 11, 12 \\
& 2I_{1j} + 4I_{2j} \leq 150000 & & j = 1, \cdots, 12 \\
& x_{ij} + I_{i(j-1)} = D_{ij} + I_{ij} & i = 1, 2 \quad & j = 1, \cdots, 12 \\
& I_{i0} = 0 & i = 1, 2 \quad & \\
& x_{ij}, I_{ij} \geq 0 & i = 1, 2 \quad & j = 1, \cdots, 12
\end{aligned}
$$