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Q. 1

- Assume that there is a positive integer less then 1. ($k < 1$)
- Using the theorem given as $k \in \mathbb{Z}^+ \land n \in \mathbb{Z}^+ \to k^n \in \mathbb{Z}^+$.
- $k \in \mathbb{Z}^+ \land 3 \in \mathbb{Z}^+ \to k^3 \in \mathbb{Z}^+$
- Since k is the least one in the set and $k^3 < k$ this makes a contradiction.
- 1 is the least element.

Q.2

$S_{(1,1)} \Rightarrow f_{(1,1)} = 1$ is the base case.
$S_{(m,1)} \Rightarrow x_1 + x_2 \cdots x_m = 1$ assume $f_{(m,1)} = m$
$S_{(m+1,1)} \Rightarrow x_1 + x_2 \cdots x_m + x_{m+1} = 1$
for $x_{m+1} = 0$ there are m, for $x_{m+1} = 1$ there is 1 (rest is all zeros) solutions, $m+1$ in total.
So, $f_{(m,1)} = m$ is approved.

$S_{(1,n)} \Rightarrow x_1 = n$ assume $f_{(1,n)} = 1$
$S_{(1,n+1)} \Rightarrow x_1 = n + 1$
Only one solution which is $x_1 = n + 1$
So, $f_{(1,n)} = 1$ is approved.

In the last step we can say, if our assumption formula is correct for $S_{(m+1,n)}$ and $S_{(m,n+1)}$ and show that $S_{(m+1,n+1)}$ also complies, we can deduce that $S_{(m,n)}$ is correct.
In the case of $S_{(m+1,n+1)}$, x_{m+1} will be added as a term to $S_{(m,n+1)}$. For $x_{m+1} = 0$ the rest will be of number $f_{(m,n+1)}$. For $x_{m+1} = 1$ the rest will be of number $f_{(m,n)}$.
It goes on like this and :
$f_{(m+1,n+1)} = f_{(m,n+1)} + \cdots f_{(m,1)}$
Then substituting the ones after the first element, it can be written as:
$f_{(m+1,n+1)} = f_{(m,n+1)} + f_{(m+1,n)}$

$f_{(m+1,n+1)} = \frac{(n+m)!}{(n+1)! \cdot (m-1)!} + \frac{(n+m)!}{n! \cdot m!}$

$f_{(m+1,n+1)} = \frac{(n+m)!}{n! \cdot (m-1)!} \cdot (\frac{1}{m} + \frac{1}{n+1})$

$f_{(m+1,n+1)} = \frac{(n+m+1)!}{(n+1)! \cdot (m)!}$

As a conclusion, induction worked and the formula is proven.

Q.3

a.)

• –
• •
With this orientation, there are 28 possible points to choose from the table as the top point.
– •
• •
With this orientation, there are 21 possible points to choose from the table as the top point.
• •
• –
With this orientation, there are 21 possible points to choose from the table as the bottom point.
• •
– •
With this orientation, there are 21 possible points to choose from the table as the bottom point.
In total there are 91 different choice.

b.)

There are two possible distributions of 6 elements among four elements.
3 1 1 1 and 2 2 1 1.
For 3 1 1 1 there are 4 orderings and $C(6,3) \cdot C(3,1) \cdot C(2,1) \cdot C(1,1)$ combinations.
For 2 2 1 1 there are 6 orderings and $C(6,2) \cdot C(4,2) \cdot C(2,1) \cdot C(1,1)$ combinations.
Sum them $\rightarrow 4 \cdot C(6,3) \cdot C(3,1) \cdot C(2,1) \cdot C(1,1) + 6 \cdot C(6,2) \cdot C(4,2) \cdot C(2,1) \cdot C(1,1)$
1560 in total.

Q.4

a.)

Take a string of length n. Its first element can be of three types. The following string can be of two types. First, string of length $n-1$ that starts with a different digit than the first one picked and satisfies the condition (containing two consecutive same digits). It will be of amount $\frac{2}{3}a_{n-1}$.
Second, a string of length $n-1$ that starts with the same digit as the first one picked and continues with all kinds of digits. There will be 3^{n-2} such string.
$a_n = 3.(\frac{2}{3}a_{n-1} + 3^{n-2})$
$a_n = 2a_{n-1} + 3^{n-1}$

b.)

$a_1 = 0, a_2 = 3$

c.)

$r - 2 = 0$ (characteristic equation)
$r = 2 \Rightarrow a_h = c_1 2^n$
$a_p = c_2 3^n$ (insert a_p to find c_2)
$c_2 3^n = 2 \cdot 3^{n-1} + 3^{n-1}$
$c_2 = 1$
$a_p = 3^n$
Solving for the initial conditions:
$a_n = c_1 2^n + 3^n$
$a_1 = 2c_1 + 3 = 0$
$c_1 = \frac{-3}{2}$
$a_n = -3 \cdot 2^{n-1} + 3^n$