Student Information

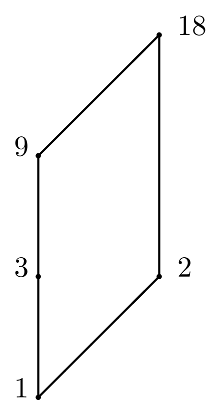
Full Name : Muhammet Emin Cihangeri
Id Number : 2448215

Answer 1

$$\begin{aligned}
G(x) &= \sum_{n=0}^{\infty} a_n \cdot x^n \\
G(x) - 1 &= \sum_{n=1}^{\infty} a_n \cdot x^n \\
&= \sum_{n=1}^{\infty} (a_{n-1} + 2^n) \cdot x^n \\
&= x \cdot \sum_{n=1}^{\infty} a_{n-1} \cdot x^{n-1} + 2x \cdot \sum_{n=1}^{\infty} 2^{n-1} \cdot x^{n-1} \\
&= x \cdot \sum_{n=0}^{\infty} a_n \cdot x^n + 2x \cdot \sum_{n=0}^{\infty} 2^n \cdot x^n \\
G(x) - 1 &= x \cdot G(x) + \frac{2x}{1-2x} \\
G(x) &= \frac{2}{1-2x} - \frac{1}{1-x} \\
&= \sum_{n=0}^{\infty} 2^{n+1} \cdot x^n - \sum_{n=0}^{\infty} x^n \\
&= \sum_{n=0}^{\infty} (2^{n+1} - 1) \cdot x^n \\
\\
a_n &= 2^{n+1} - 1
\end{aligned} \tag{1}$$

Answer 2

a)



b)

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

Yes. Because every pair in the Hasse diagram has an upperbound and a lowerbound.

d)

The symmetric closure of R is $R \cup R^{-1}$. This means $aRb \to bRa$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

e)

2 and 9 are not comparable since $2 \mid 9$ is wrong.
3 and 18 are comparable since $3 \mid 18$.

Answer 3

a)

By the matrix representation

$$\begin{bmatrix} 1 & 0 & 1 & . & 1 \\ 1 & 1 & 1 & . & 0 \\ 0 & 0 & 1 & . & 1 \\ . & . & . & . & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

All the diagonals and the upper right elements can either be 1 or 0. Then, the lower left elements will be the opposite of the mirror element of them. Meaning:
$\forall i, j(a_{ij} = 1 \to a_{ji} = 0) \land (a_{ij} = 0 \to a_{ji} = 1)$
Therefore there are $2^{\frac{n^2+n}{2}}$ anti-symmetric binary relations.

b)

Here, the only difference from 3a is that all the diagonal elements should be 1. Therefore, there are $2^{\frac{n^2-n}{2}}$ reflexive and anti-symmetric binary relations.