Student Information

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Q. 1

$$
\begin{aligned}
(A \cup B) \backslash (A \cup B) &\equiv (x \in A \vee x \in B) \wedge (x \notin (A \cap B)) \\
&\equiv (x \in A \vee x \in B) \wedge x \in (\overline{A \cap B}) \\
&\equiv (x \in A \vee x \in B) \wedge x \in (\bar{A} \cup \bar{B}) \\
&\equiv (x \in A \vee x \in B) \wedge (x \in \bar{A} \vee x \in \bar{B}) \\
&\equiv (x \in A \vee x \in B) \wedge (x \notin A \vee x \notin B) \\
&\equiv (x \in A \wedge x \notin A) \vee (x \in A \wedge x \notin B) \vee (x \in B \vee x \notin A) \vee (x \in B \wedge x \notin B) \\
&\equiv (x \in A \wedge x \notin B) \vee (x \in B \wedge x \notin A) \\
&\equiv (x \in (A \backslash B) \cup (B \backslash A) \\
&\equiv (A \backslash B) \cup (B \backslash A)
\end{aligned}
$$

Q.2

- $A = \{f \mid f \subseteq N \times \{0, 1\}\}$
- Cardinality of $N \times \{0, 1\}$ is $\mid N \mid \times \mid \{0, 1\} \mid = 2 \times \mid N \mid$, which means it is countably infinite.
- f here, is the powerset of N $\times\{0, 1\}$, then A is uncountable. (1)
- $B = \{f \mid f : \{0, 1\} \rightarrow N, fisafunction\}$
- Cardinality of this set is $\mid N \mid^{|\{0,1\}|} = \mid N \mid^2$, thus countably infinite.
- Assume A $\setminus$ B is countable. Then (A $\setminus$ B) $\cup$ B is also countable(unity of countable sets).
- This implies that A is countable, which makes a contradiction with (1). Therefore A $\setminus$ B is uncountable.

Q.3

Assume $4^n + 5n^2 logn = O(2^n)$

- for some constant c, $\mid 4^n + 5n^2 logn \mid \leq c \mid 2^n \mid$
- divide by 2^n both sides $2^n + \frac{5n^2 logn}{2^n} \leq c$
- there is no constant c that bounds 2^n , which creates a contradiction.

Q.4

$$
\begin{array}{rcll}
2x-1 & \equiv & 2(x-1)+1 & \equiv \;\Big|\; 1(mod(x-1)) \\
(2x-1)^n & \equiv & 1^n & \equiv \;\Big|\; 1(mod(x-1)) \\
\\
x & \equiv & (x-1)+1 & \equiv \;\Big|\; 1(mod(x-1)) \\
x^2 & \equiv & 1^2 & \equiv \;\Big|\; 1(mod(x-1)) \\
\\
((2x-1)^n - x^2)mod(x-1) & \equiv & (((2x-1)^n mod(x-1)) - (x^2 mod(x-1)))mod(x-1) & \\
 & \equiv & \text{1 - 1 mod (x - 1)} & \\
 & \equiv & \text{0 mod (x - 1)} & \\
\\
-x-1 & \equiv & -(x-1)-2 & \equiv \;\Big|\; -2(mod(x-1)) \\
\\
0 & \equiv & -2(mod(x-1)) & \;\Big|\; \text{merge two sides} \\
0-(-2) & = & s \times (x-1) & \\
2 & = & s \times (x-1) &
\end{array}
$$

Only holds for s = 1 (x > 2). Therefore x = 3.