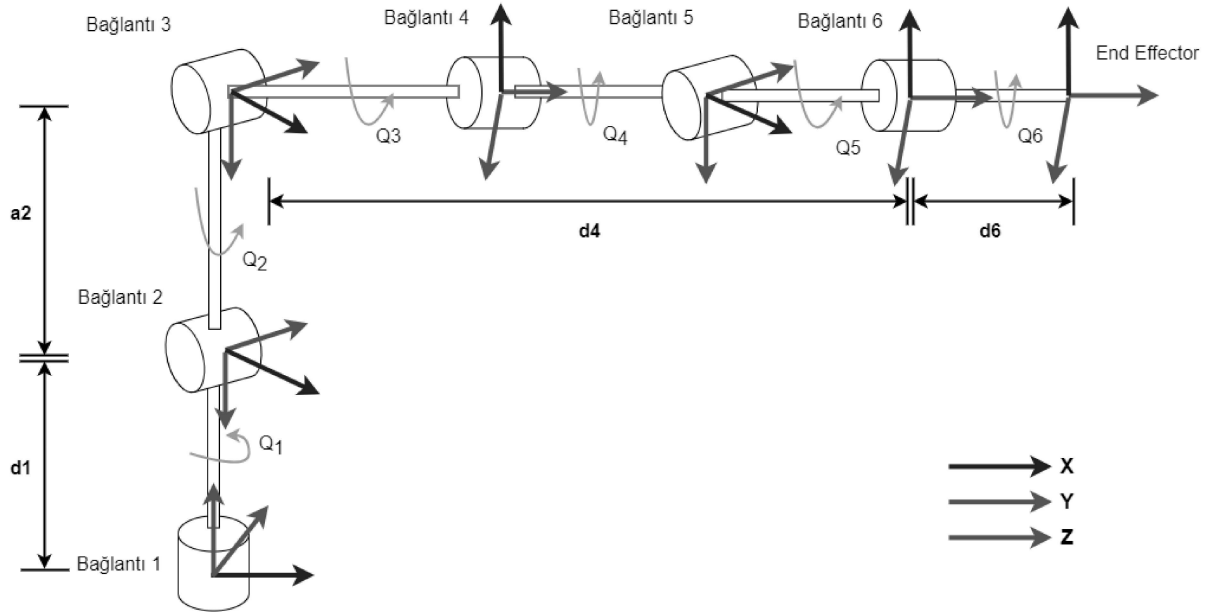


## 6-DOF Manipulator Forward and Inverse Kinematic Solutions:



### DH-Table

i	$Q_i$	$\alpha_i$	$a_i$	$d_i$
1	$Q_1$	-90	0	$d_1$
2	$Q_2$	0	$a_2$	0
3	$Q_3$	-90	0	0
4	$Q_4$	90	0	$d_4$
5	$Q_5$	-90	0	0
6	$Q_6$	0	0	$d_6$

$$T_n^{n-1} = \begin{bmatrix} \cos(Q_n) & -\sin(Q_n) \cos(\alpha_n) & \sin(Q_n) \sin(\alpha_n) & a_n \cos(Q_n) \\ \sin(Q_n) & \cos(Q_n) \cos(\alpha_n) & -\cos(Q_n) \sin(\alpha_n) & a_n \sin(Q_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow \text{General Form}$$

$$T_6^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5$$

**Position Problem:**

$$O_c^0 = O_6^0 - d_6 R_6^0(:, 3)$$

$$d_4 = a_2 = k$$

$$T_4^0(1:3,4) = \{x_c, y_c, z_c\} \rightarrow \begin{bmatrix} k \cos(q_1) (\cos(q_2 + q_3) + \sin(Q_2)) \\ k \sin(q_1) (\cos(q_2 + q_3) + \sin(Q_2)) \\ k \cos(q_2) - k \sin(q_2 + q_3) + d_1 \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

$q_1$ :

$$\frac{\cos(q_1)}{\sin(q_1)} = \frac{P_x}{P_y} \Rightarrow \frac{P_y}{P_x} = \tan(q_1) \Rightarrow \arctan(P_y, P_x) = q_1$$

$q_3$ :

$$[k(\cos(q_2 + q_3) + \sin(Q_2))]^2 + [k(\cos(q_2 + q_3) + \sin(Q_2))]^2 = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$k^2 \cos(q_2 + q_3)^2 + 2.k^2 \cos(q_2 + q_3)^2 \sin(q_2) + k^2 \sin(q_2)^2 + k.\cos(q_2)^2 - 2.k^2 \cos(q_2) \sin(q_2 + q_3) + k^2 \sin(q_2 + q_3)^2 = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$2k^2 + 2.k^2 [\cos(q_2 + q_3) \sin(q_2) - \cos(q_2) \sin(q_2 + q_3)] = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$[\cos(q_2 + q_3) \sin(q_2) - \cos(q_2) \sin(q_2 + q_3)] \Rightarrow \sin(x - y) = \sin(q_2 - q_2 - q_3) = -\sin(q_3)$$

$$2.k^2(1 + (-\sin(q_3))) = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$\sin(q_3) = -\frac{P_x^2 + P_y^2(P_z - d_1)^2}{2.k^2} + 1$$

$$q_3 = \arcsin\left(-\frac{P_x^2 + P_y^2(P_z - d_1)^2}{2.k^2} + 1\right)$$

Now  $q_1$  and  $q_3$  angles are known.

$q_2$ :

$$k^2(\cos(q_2 + q_3) + \sin(q_2))^2 = P_x^2 + P_y^2$$

$$k^2[(\cos(q_2) \cos(q_3) - \sin(q_2) \sin(q_3)) + \sin(q_2)]^2 = P_x^2 + P_y^2$$

$$k[\cos(q_2) \cos(q_3) + \sin(q_2) (1 - \sin(q_3))] = \sqrt{P_x^2 + P_y^2}$$

$$\cos(q_2) (k \cos(q_3) + \sin(q_2) (1 - \sin(q_3))) = \sqrt{P_x^2 + P_y^2}$$

$$\cos(q_2) (k \cos(q_3)) + \sin(q_2) (k(1 - \sin(q_3))) = \sqrt{P_x^2 + P_y^2}$$

$$(k \cos(q_3)) \rightarrow \{AA\}$$

$$(k(1 - \sin(q_3))) \rightarrow \{BB\}$$

$$\sqrt{P_x^2 + P_y^2} \rightarrow \{CC\}$$

$$(CC^2 - BB^2) - 2 \cdot AA \cdot CC \cdot \cos(q_2) + (AA^2 + BB^2) \cos^2(q_2) = 0$$

For  $\cos(q_2) = x$ ;

$$(AA^2 + BB^2)x^2 - 2 \cdot AA \cdot CC \cdot x + (CC^2 - BB^2)$$

$$\cos(q_2) = x$$

There exists 2 roots for x value if it is proper. Then, the reliable root value is chosen for  $\cos(q_2)$ .

$$\cos(q_2) = r$$

$$q_2 = \arccos(r)$$

**Orientation Problem:**

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(Q) & -\sin(Q) \\ 0 & \sin(Q) & \cos(Q) \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos(Q) & 0 & \sin(Q) \\ 0 & 1 & 0 \\ -\sin(Q) & 0 & \cos(Q) \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos(Q) & -\sin(Q) & 0 \\ \sin(Q) & \cos(Q) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x \cdot R_y \cdot R_z = R_6^3$$

$$R_6^3 = (R_3^0)^T \cdot R_6^0$$

$$R_6^3 = \begin{pmatrix} \cos(q_4) \cos(q_5) \cos(q_6) - \sin(q_4) \sin(q_6) & -\cos(q_6) \sin(q_4) - \cos(q_4) \cos(q_5) \sin(q_6) & -\cos(q_4) \sin(q_5) \\ \cos(q_4) \sin(q_6) + \cos(q_5) \cos(q_6) \sin(q_4) & \cos(q_4) \cos(q_6) - \cos(q_5) \sin(q_4) \sin(q_6) & -\sin(q_4) \sin(q_5) \\ \cos(q_6) \sin(q_5) & -\sin(q_5) \sin(q_6) & \cos(q_5) \end{pmatrix}$$

$$q_4 = \text{atan} \left( -\frac{\cos(q_4) \sin(q_5)}{-\sin(q_4) \sin(q_5)} \right)$$

$$q_5 = \text{atan} \left( \frac{\sqrt{(\cos(q_6) \sin(q_5))^2 - (\sin(q_5) \sin(q_6))^2}}{\cos(q_5)} \right)$$

$$q_6 = \text{atan} \left( -\frac{\sin(q_5) \sin(q_6)}{\cos(q_6) \sin(q_5)} \right)$$