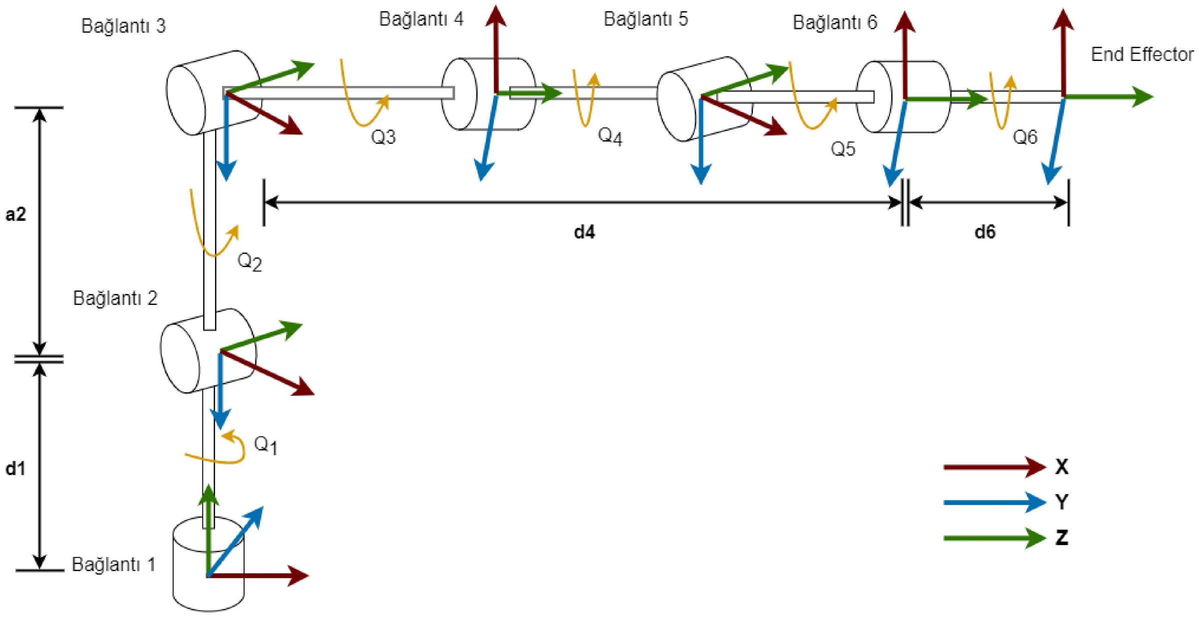
6-DOF Manipulator Forward and Inverse Kinematic Solutions:



DH-Table

i	Q_i	α_i	a_i	d_i
1	Q_1	-90	0	d_1
2	Q_2	0	a_2	0
3	Q_3	-90	0	0
4	Q_4	90	0	d_4
5	Q_5	-90	0	0
6	Q_6	0	0	d_6

$$T_n^{n-1} = \begin{bmatrix} \cos(Q_n) & -\sin(Q_n)\cos(\alpha_n) & \sin(Q_n)\sin(\alpha_n) & a_n\cos(Q_n) \\ \sin(Q_n) & \cos(Q_n)\cos(\alpha_n) & -\cos(Q_n)\sin(\alpha_n) & a_n\sin(Q_n) \\ 0 & \sin(\alpha_n) & \cos(\alpha_n) & d_n \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow General\ Form$$

$$T_6^0 = T_1^0 T_2^1 T_3^2 T_4^3 T_5^4 T_6^5$$

Position Problem:

$$O_c^0 = O_6^0 - d_6 R_6^0(:,3)$$

$$d_4 = a_2 = k$$

$$T_4^0(1{:}3{,}4) = \{x_c, y_c, z_c\} \rightarrow \begin{bmatrix} k\cos(q_1)\,(\cos(q_2+q_3)+\sin(Q_2)) \\ k\sin(q_1)\,(\cos(q_2+q_3)+\sin(Q_2)) \\ k\cos(q_2) - k\sin(q_2+q_3) + d_1 \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

q_1:

$$\frac{\cos(q_1)}{\sin(q_1)} = \frac{P_x}{P_y} => \frac{P_y}{P_x} = \tan(q_1) => \mathbf{arctan}\left(\boldsymbol{P_y}, \boldsymbol{P_x}\right) = \boldsymbol{q_1}$$

q_3:

$$[k(\cos(q_2+q_3)+\sin(Q_2))]^2 + [k(\cos(q_2+q_3)+\sin(Q_2))]^2 = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$k^2cos(q_2+q_3)^2 + 2.k^2cos(q_2+q_3)^2sin(q_2) + k^2sin(q_2)^2 + k.cos(q_2)^2 - 2.k^2cos(q_2)sin(q_2+q_3) + k^2sin(q_2+q_3)^2 = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$2k^2 + 2.k^2[\cos(q_2+q_3)\,sin(q_2) - cos(q_2)sin(q_2+q_3)] = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$[\cos(q_2+q_3)\,sin(q_2) - cos(q_2)sin(q_2+q_3)] => sin(x-y) = sin(q_2 - q_2 - q_3) = -sin(q_3)$$

$$2.k^2(1 + (-sin(q_3))) = P_x^2 + P_y^2 + (P_z - d_1)^2$$

$$sin(q_3) = -\frac{P_x^2 + P_y^2(P_z - d_1)^2}{2.k^2} + 1$$

$$\boldsymbol{q_3} = \boldsymbol{arcsin}\left(-\frac{\boldsymbol{P_x^2} + \boldsymbol{P_y^2}(\boldsymbol{P_z} - \boldsymbol{d_1})^2}{\mathbf{2}.\boldsymbol{k^2}} + \mathbf{1}\right)$$

Now q_1 *and* q_3 *angles are known.*

q_2:

$$k^2(\cos(q_2+q_3)+\sin(q_2))^2 = P_x^2 + P_y^2$$

$$k^2[(\cos(q_2)\cos(q_3)-\sin(q_2)\sin(q_3))+\sin(q_2)]^2 = P_x^2 + P_y^2$$

$$k[cos(q_2)\cos(q_3)+\sin(q_2)\,(1-\sin(q_3))] = \sqrt{P_x^2 + P_y^2}$$

$$\cos(q_2)\left(k.cos(q_3)+\sin(q_2)\,(1-\sin(q_3))\right) = \sqrt{P_x^2 + P_y^2}$$

$$\cos(q_2)\,(k.\cos(q_3))+\sin(q_2)\left(k(1-\sin(q_3))\right) = \sqrt{P_x^2 + P_y^2}$$

$$(k.\cos(q_3)) \to \{AA\}$$

$$\left(k(1-\sin(q_3))\right) \to \{BB\}$$

$$\sqrt{P_x^2 + P_y^2} \to \{CC\}$$

$$(CC^2 - BB^2) - 2.AA.CC.\cos(q_2) + (AA^2 + BB^2)\cos(q_2)^2 = 0$$

$For\cos(q_2) = x;$

$$\boldsymbol{\left(AA^2 + BB^2\right)x^2 - 2.AA.CC.x + \left(CC^2 - BB^2\right)}$$

$$\cos(q_2) = x$$

There exists 2 roots for x value if it is proper. Then, the reliable root value is chosen for $\cos(q_2)$.

$$\cos(q_2) = r$$

$$\boldsymbol{q_2 = \mathbf{arccos}\,(r)}$$

Orientation Problem:

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(Q) & -\sin(Q) \\ 0 & \sin(Q) & \cos(Q) \end{bmatrix}$$

$$R_y = \begin{bmatrix} \cos(Q) & 0 & \sin(Q) \\ 0 & 1 & 0 \\ -\sin(Q) & 0 & \cos(Q) \end{bmatrix}$$

$$R_z = \begin{bmatrix} \cos(Q) & -\sin(Q) & 0 \\ \sin(Q) & \cos(Q) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x . R_y . R_z = R_6^3$$

$$R_6^3 = (R_3^0)^T . R_6^0$$

$$R_6^3 = \begin{pmatrix} \cos(q_4)\cos(q_5)\cos(q_6) - \sin(q_4)\sin(q_6) & -\cos(q_6)\sin(q_4) - \cos(q_4)\cos(q_5)\sin(q_6) & -\cos(q_4)\sin(q_5) \\ \cos(q_4)\sin(q_6) + \cos(q_5)\cos(q_6)\sin(q_4) & \cos(q_4)\cos(q_6) - \cos(q_5)\sin(q_4)\sin(q_6) & -\sin(q_4)\sin(q_5) \\ \cos(q_6)\sin(q_5) & -\sin(q_5)\sin(q_6) & \cos(q_5) \end{pmatrix}$$

$$\boldsymbol{q_4 = \mathbf{atan}\left(-\frac{cos(q_4)\,sin(q_5)}{-\,sin(q_4)\,sin(q_5)}\right)}$$

$$\boldsymbol{q_5 = \mathbf{atan}\left(\frac{\sqrt{(cos(q_6)\,sin(q_5))^2 - (sin(q_5)\,sin(q_6))^2}}{cos(q_5)}\right)}$$

$$\boldsymbol{q_6 = \mathbf{atan}\left(-\frac{sin(q_5)\,sin(q_6)}{cos(q_6)\,sin(q_5)}\right)}$$