

smartcab_analysis

August 2, 2016

1 Project 4: Train a Smartcab to Drive

Reinforcement Learning

Machine Learning Nanodegree (Udacity) Project submission by Edward Minnett (ed@methodic.io).
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```
In [1]: import support as sp
        from IPython.display import display
        %matplotlib inline
```

1.1 Implement a Basic Driving Agent

To begin, your only task is to get the smartcab to move around in the environment. At this point, you will not be concerned with any sort of optimal driving policy. Note that the driving agent is given the following information at each intersection:

- The next waypoint location relative to its current location and heading.
- The state of the traffic light at the intersection and the presence of oncoming vehicles from other directions.
- The current time left from the allotted deadline.

To complete this task, simply have your driving agent choose a random action from the set of possible actions (None, 'forward', 'left', 'right') at each intersection, disregarding the input information above. Set the simulation deadline enforcement, `enforce_deadline` to False and observe how it performs.

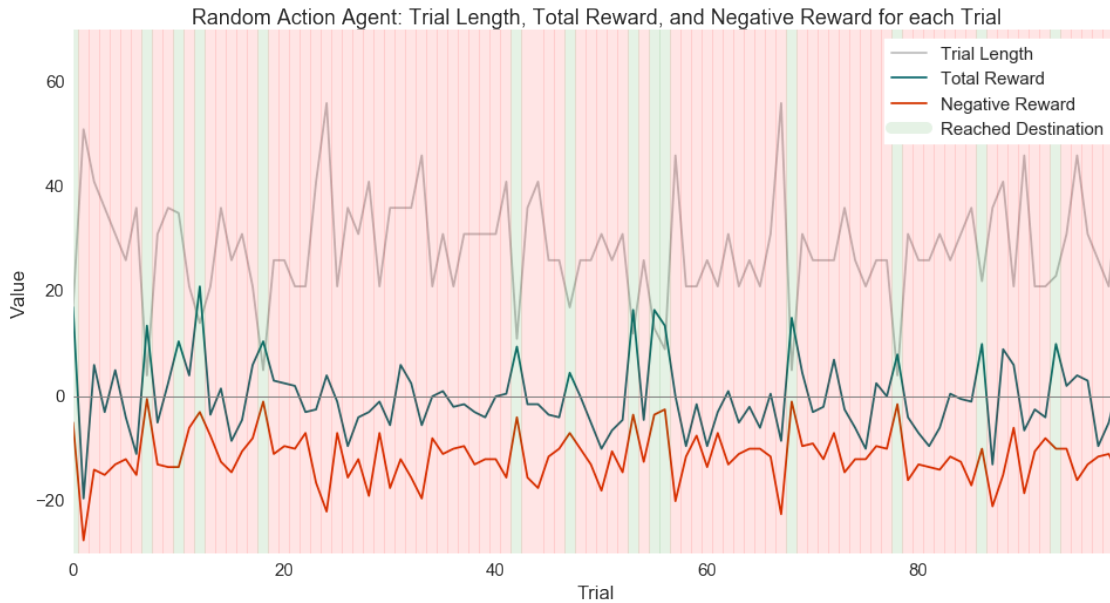
QUESTION: Observe what you see with the agent's behaviour as it takes random actions. Does the smartcab eventually make it to the destination? Are there any other interesting observations to note? During the many iterations that I watched, the number of steps taken before reaching the destination varied dramatically. A few times it reached the destination very quickly, but for most of the iterations, the agent moved around very inefficiently. This isn't surprising given the random choice of action taken. A more interesting observation is that the agent spends more time stuck at an intersection than it does moving around. At face value, it feels as though this can't be right as 3 out of 4 actions result in movement, but when those actions are combined with the state of the traffic lights and the placement of other agents within the environment, more actions than not result in no movement.

The following statistics and chart illustrate the results of simulating 100 trials using the Random Action policy.

```
In [2]: sp.display_random_agent_stats()
```

The destination was reached in 14 out of 100 trials.

	count	mean	std	min	25%	50%	75%	max
total_reward	100	-0.17	7.131244	-19.5	-4.5	-1.50	3.25	21.0
negative_reward	100	-11.59	4.859844	-27.5	-14.5	-11.75	-9.50	-0.5
trial_length	100	28.17	10.259319	4.0	21.0	26.00	36.00	56.0



As you would expect given that the policy was a random selection from the 4 possible actions without taking account for any information about the agent's environment or state, the results are quite poor. The agent incurs a significant negative penalty in almost all of the trials and only happens to reach the destination in 14 out of 100 trials.

1.2 Inform the Driving Agent

Now that your driving agent is capable of moving around in the environment, your next task is to identify a set of states that are appropriate for modelling the smartcab and environment. The main source of state variables are the current inputs at the intersection, but not all may require representation. You may choose to explicitly define states, or use some combination of inputs as an implicit state. At each time step, process the inputs and update the agent's current state using the `self.state` variable. Continue with the simulation deadline enforcement `enforce_deadline` being set to `False`, and observe how your driving agent now reports the change in state as the simulation progresses.

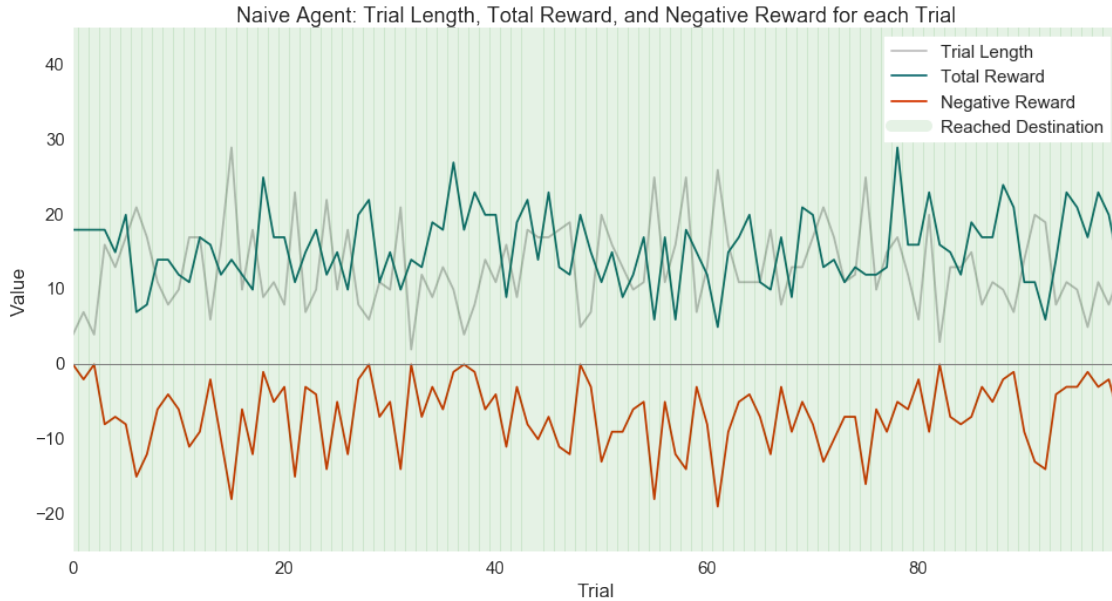
QUESTION: What states have you identified that are appropriate for modelling the smartcab and environment? Why do you believe each of these states to be appropriate for this problem?

My initial implementation of the informed driving agent was very naive. I wanted to see how the agent performed if it simply performed the action suggested by the route planner. This would be the equivalent of letting your sat-nav drive your car without any input from the environment. A bad idea certainly, but this naive agent will offer a baseline performance that less naive strategies can be compared against. 100 iterations using the Naive policy resulted in a sample of trial statistics. This results for this sample are as follows:

```
In [3]: sp.display_naive_agent_stats()
```

The destination was reached in 100 out of 100 trials.

	count	mean	std	min	25%	50%	75%	max
total_reward	100	15.46	4.822967	5	12.00	15	18.25	29
negative_reward	100	-6.84	4.602854	-19	-9.25	-6	-3.00	0
trial_length	100	12.99	5.591290	2	9.00	12	17.00	29



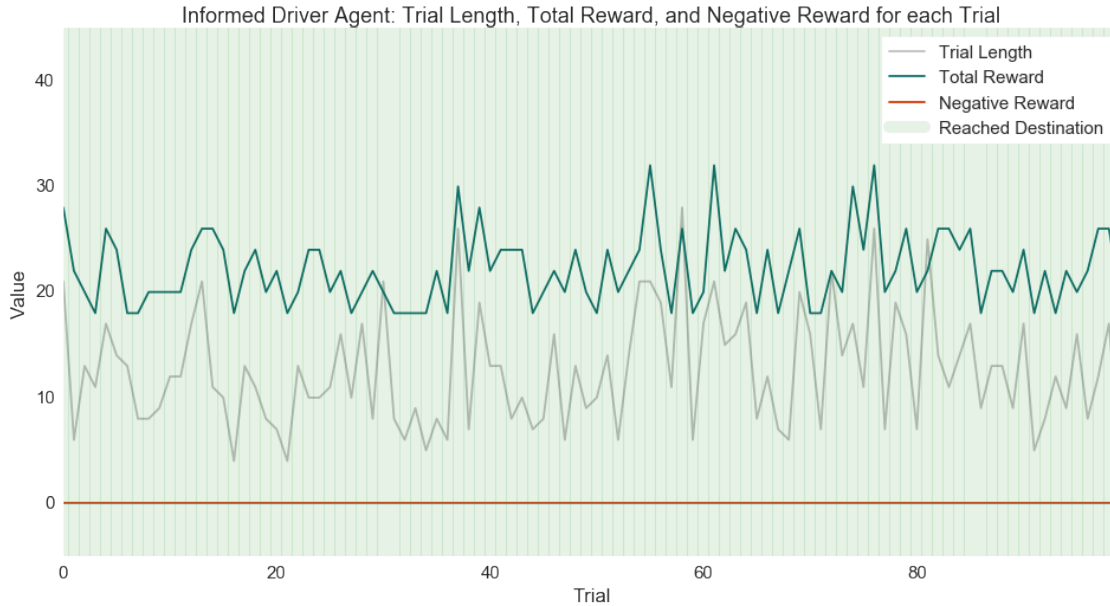
Even though the only information this agent considered was the direction suggested by the route planner, it did successfully reach the destination in all 100 trials. For the most part, the trial length (number of steps taken in the trial) is significantly lower than the Random Action Agent, but the agent did incur significant negative rewards for the large majority of the trials. Given that the number of steps taken were not a consideration, meaning that every trial received the 12 point reward for reaching the destination, the naive strategy performed quite poorly.

A natural improvement on the Naive Agent is to combine the naive policy with logic that determines whether the desired agent action is legal within the context of the environment inputs and the ‘rules of the road’. This Informed Driver policy results in a much better performing agent. A simulation with 100 trials using this policy resulted in the following statistics:

```
In [4]: sp.display_informed_driver_agent_stats()
```

The destination was reached in 100 out of 100 trials.

	count	mean	std	min	25%	50%	75%	max
total_reward	100	22.12	3.479464	18	20	22	24	32
negative_reward	100	0.00	0.000000	0	0	0	0	0
trial_length	100	12.50	5.448566	4	8	12	16	28



Given the Informed Driver agent is given all of the information it needs to navigate the environment, it is not a surprise that the total reward for each trial is so high, the trial length is so low, and that every trial resulted in the agent reaching the destination without incurring a negative penalty.

This analysis requires the knowledge of the outcomes for each trial, but this knowledge purely acts as meta information to analyse the agent performance and isn't used to influence the behaviour of the agent.

The policy for the Informed Driver Agent only requires two pieces of information about its state.

1. **What direction should I travel to receive the maximum reward?** This is necessary to know where to travel to move closer to the destination.
2. **Is the ideal direction of travel a legal move?** This is necessary to avoid incurring a negative reward.

The first state is given by the output of the route planner and the second is determined by applying the 'rules of the road' to the desired direction of travel. This logic was hard coded within the agent and encoded information such as 'you can not move when the traffic light is red unless it is to turn right when there is neither a car to the left moving forward or an oncoming car turning left'.

The simplicity of this state space results in 6 possible states (2 each representing whether the move is legal for each of the 3 possible movements).

If the ideal direction of travel would not result in a legal move, the agent takes no action (an action of 'None').

This policy ensures the maximum reward without incurring any negative rewards.

OPTIONAL: How many states in total exist for the smartcab in this environment? Does this number seem reasonable given that the goal of Q-Learning is to learn and make informed decisions about each state? Why or why not?

```
In [5]: num_traffic_light_states = len(['red', 'green'])
num_oncoming_traffic_states = len(['forward', 'left', 'right', None])
num_left_traffic_states = len(['forward', 'left', 'right', None])
num_right_traffic_states = len(['forward', 'left', 'right', None])
num_desired_direction_of_travel_states = len(['forward', 'left', 'right'])

total_size_of_the_state_space = (num_traffic_light_states *
```

```

num_oncoming_traffic_states *
num_left_traffic_states *
num_right_traffic_states *
num_desired_direction_of_travel_states)

print "Total state space size: {}".format(total_size_of_the_state_space)

```

Total state space size: 384

Given that there are 4 possible agent actions, this state space will result in Q being represented by a matrix with 4 columns and 384 rows. This is well within the realm of reasonability for Q-Learning as an algorithm especially Q-Learning only requires looking up and setting values within the matrix. I'm sure Q-Learning could support much, much, larger state / action spaces especially given the possibilities offered by improving the performance of searching the matrix. For example if the state space was significantly large, Q could be broken down into multiple matrices or tables divided by one or more aspects of the state space.

It is worth noting that in the context of this project where there are only three other agents within the environment, the Q matrix is likely to be quite sparse. The probability that there are agents occupying all three opposing directions of traffic at an intersection is very low and this would have to happen many times within a simulation for the agent to encounter all of the permutations within the state space. Each simulation would have to include far more trials than the default 100 or many more other agents to even come close to filling the entire Q matrix for this project. A sparse Q matrix should suffice to reach the optimal policy given that a high performing agent only needs to learn the optimal actions for the highest probability states and 100 trials should be enough for the sample probability of states to reflect the population probabilities.

1.3 Implement a Q-Learning Driving Agent

With your driving agent being capable of interpreting the input information and having a mapping of environmental states, your next task is to implement the Q-Learning algorithm for your driving agent to choose the best action at each time step, based on the Q-values for the current state and action. Each action taken by the smartcab will produce a reward which depends on the state of the environment. The Q-Learning driving agent will need to consider these rewards when updating the Q-values. Once implemented, set the simulation deadline enforcement `enforce_deadline` to `True`. Run the simulation and observe how the smartcab moves about the environment in each trial.

QUESTION: What changes do you notice in the agent's behaviour when compared to the basic driving agent when random actions were always taken? Why is this behaviour occurring? My initial implementation of the Q-Learning Agent used the small (6 value) state space and hard coded logic to ensure the agent would experience enough of the state space quickly enough to debug the implementation. Once I was convinced that the implementation was significantly robust, I then increased the state space first to 24 states and then to the full 384 by slowly taking information encoded within the 'rules of the road'. The agent that attempts to learn the Q values for the full state space has no prior knowledge of what constitutes a 'legal' movement. I also refactored the hard coded exploration logic to use a 'probability of exploration' function dependent upon the ϵ exploration parameter. The 'probability of exploration' function is as follows:

```

In [6]: def exploration_probability(epsilon, deadline):
        n = max(1, N(s,a)) # N(s,a) is the number of times the state-action pair has been encountered
        eagerness_to_explore = epsilon * deadline / n
        # The eagerness_to_explore value can easily be larger than 1 which will result in an exploration
        return min(1, eagerness_to_explore)

```

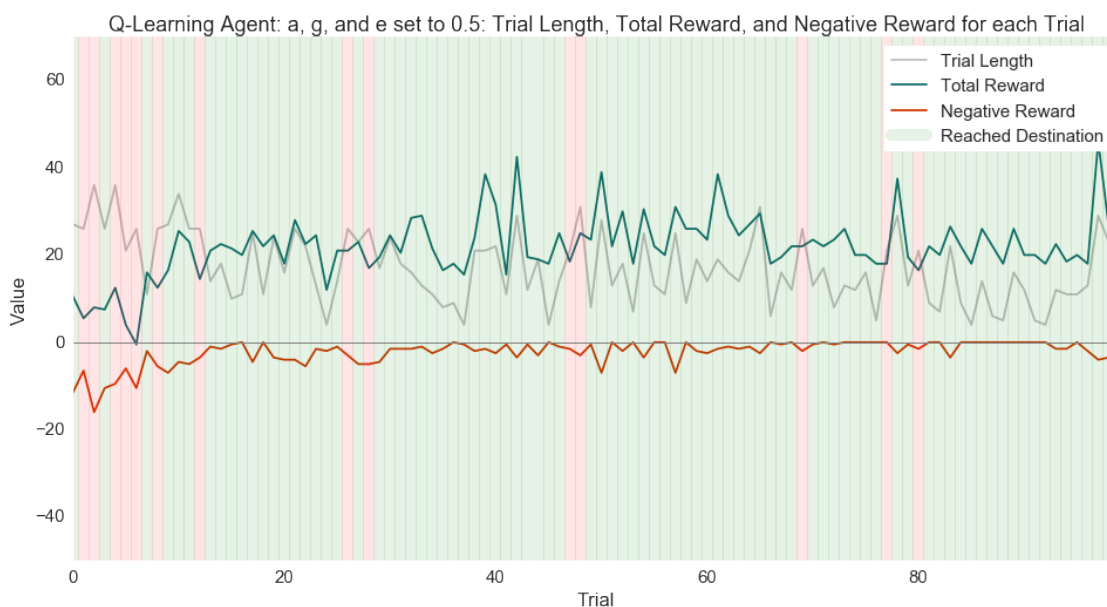
If $N(s,a)$ is small and / or the deadline is high, then the probability of exploration will be 1 or nearly 1. This ensures that the agent is more likely to explore at the beginning of a trial, but less so at the end, and also if the state-action pair has not been experienced very often, a situation where $Q(s,a)$ may be unstable. This function also ensures that the probability of exploration decays when a state-action pair has been encountered many times and is likely to have a stable Q value.

A sample of 100 trials with α , γ , and ϵ set to 0.5 (the arbitrary default values I used while developing the implementation) resulted in the following statistics:

```
In [7]: sp.display_stats_for_the_q_learning_agent_with_params(0.5)
```

The destination was reached in 86 out of 100 trials.

	count	mean	std	min	25%	50%	75%	max
total_reward	100	21.860	7.335427	-0.5	18.0	21.75	25.125	46
negative_reward	100	-2.295	2.911224	-16.0	-3.5	-1.50	0.000	0
trial_length	100	16.960	8.079054	4.0	11.0	16.00	24.000	36



The observed behaviour of the Q-Learning Agent is that in the first set of trials, the agent behaves randomly with more trials than not failing to reach the destination, but after 15 trials or so this likelihood flips. After the 15th trial, the agent behaves a lot more consistently with only 7 trials out of 85 where the agent fails to reach the destination and incurring fewer negative rewards along the way. It is by the 15th trial that the agent has presumably encountered the most common states enough times to develop reasonably stable Q values for those state-action pairs. The large ϵ value means the agent continues to explore and make mistakes pretty much all of the way through the 100 trials. I don't believe this agent is defined by the optimal parameter values, but we will come to that next.

1.4 Improve the Q-Learning Driving Agent

our final task for this project is to enhance your driving agent so that, after sufficient training, the smartcab is able to reach the destination within the allotted time safely and efficiently. Parameters in the Q-Learning algorithm, such as the learning rate (alpha), the discount factor (gamma) and the exploration rate (epsilon) all contribute to the driving agent's ability to learn the best action for each state. To improve on the success of your smartcab:

Set the number of trials, `n_trials`, in the simulation to 100. Run the simulation with the deadline enforcement `enforce_deadline` set to `True` (you will need to reduce the update delay `update_delay` and set the display to `False`). Observe the driving agent's learning and smartcab's success rate, particularly during the later trials. Adjust one or several of the above parameters and iterate this process. This task is complete once you have arrived at what you determine is the best combination of parameters required for your driving agent to learn successfully.

QUESTION: Report the different values for the parameters tuned in your basic implementation of Q-Learning. For which set of parameters does the agent perform best? How well does the final driving agent perform? In order to find optimal (or at least semi optimal) values for α , γ , and ϵ , I wrapped the execution of an agent simulation with 100 trials in logic to support a grid search for the parameters. The grid search generated simulation statistics for the 512 combinations of α , γ , and ϵ taking using the 8 values 0.01, 0.03, 0.05, 0.07, 0.1, 0.3, 0.5, 0.7. This search was far from exhaustive and a search with more possible values may end up with a different optimal result. Executing the 512 simulations took nearly two hours and increasing the search space would have increased the run time dramatically. The metric used to determine the optimal parameters was derived as follows.

The rough definition of a successful simulation is quite easy to define; the simulation that learns $Q(s,a)$ quickly enough while minimising the negative reward incurred. Quantifying this into a score that can be minimised is substantially more difficult and some what subjective. I began by defining a penalty score that was the product of the average of the the last 5 trials where a negative reward was incurred and the average of the last two trials where the agent failed to reach the destination. This metric was found to be flawed as, by chance, 32 simulations resulted in the agent reaching the destination in all 100 trials. Since the penalty score was a product, all 32 simulations received a penalty score of 0 so this wouldn't work. I then settled on the sum of the two previously described values instead of a product and decreased the number of trials averaged when examining negative rewards from 5 to 2. This effectively penalised simulations more when they incurred negative rewards toward the end of the simulation. Unfortunately, because the act of exploration in Q-Learning is stochastic and each set of parameters was simulated once, this metric could rule out potentially optimal parameters simply based on 'bad luck'. Even though this metric is still flawed, I couldn't think of a better way of objectively determining the optimal parameters from the grid search.

This process resulted in the following optimal parameters:

```
In [9]: optimal_df, optimal_parameters = sp.find_optimal_parameters()
        print "The optimal parameters are: {}".format(optimal_parameters)
```

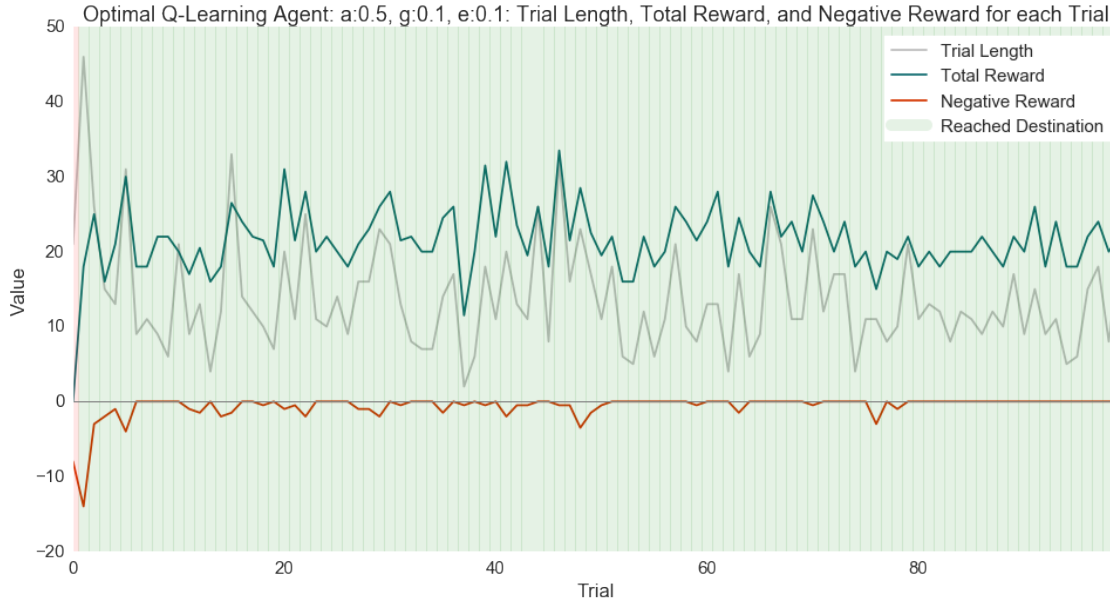
The optimal parameters are: {'alpha': 0.5, 'gamma': 0.1, 'epsilon': 0.1}

The data for the simulation using these parameters are as follows:

```
In [10]: optimal_q_learning_title = "Optimal Q-Learning Agent: a:{}, g:{}, e:{}".format(
        optimal_parameters['alpha'], optimal_parameters['gamma'], optimal_parameters['epsilon'])
        sp.display_trial_stats(optimal_df, optimal_q_learning_title, -20, 50)
```

The destination was reached in 99 out of 100 trials.

	count	mean	std	min	25%	50%	75%	max
total_reward	100	21.47	4.495126	0	18.75	21.25	24	33.5
negative_reward	100	-0.65	1.753064	-14	-0.50	0.00	0	0.0
trial_length	100	13.70	7.168252	2	9.00	12.00	17	46.0



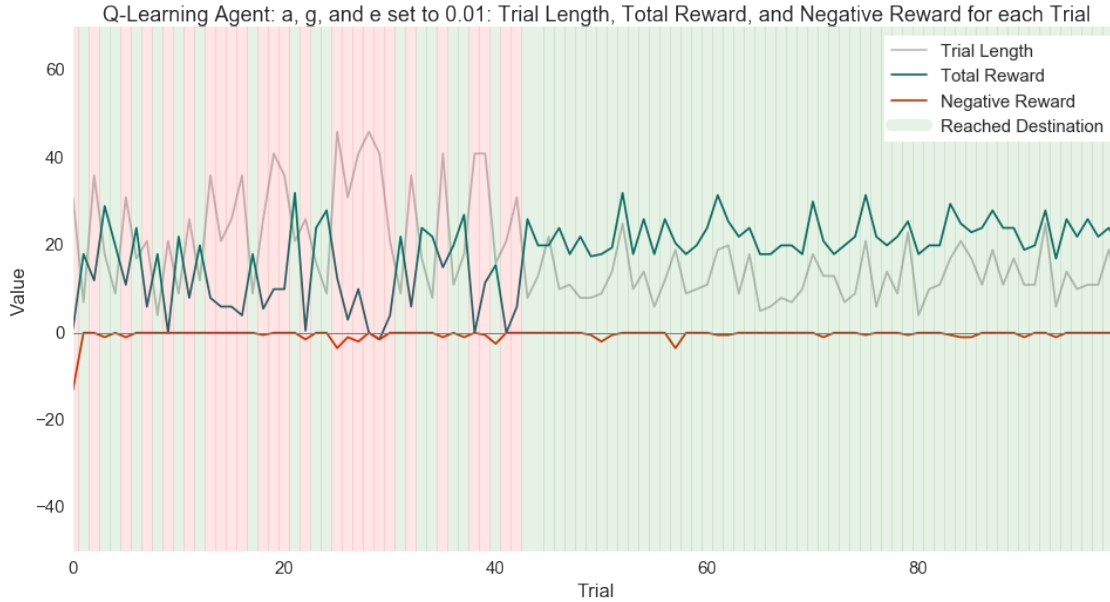
The Q-Learning agent with these parameters performed remarkably well. It reached the destination in all but the first trial and apperas to have reached reasonably optimal values of $Q(s,a)$ in only 6 trials. It didn't incur any negative rewards after the 79th trial (though, to some extent, this is down to chance). It is interesting to compare the summary statistics for this simulation and that of the Informed Driver simulation. The mean total reward per trial is only slightly less for the Q-Learning agent yet the Q-Learning agent has no prior knowledge of the 'rules of the road'. This very clearly illustrates the power of Q-learning to find the optimal policy given enough experince with the sate, action, reward loop for the most frequently encountered states.

For the sake of comparison, it will be interesting to compare this performance to the agents using the extreme parameter values (where α , γ , and ϵ are all set to 0.01 or 0.7).

```
In [11]: sp.display_stats_for_the_q_learning_agent_with_params(0.01)
```

The destination was reached in 74 out of 100 trials.

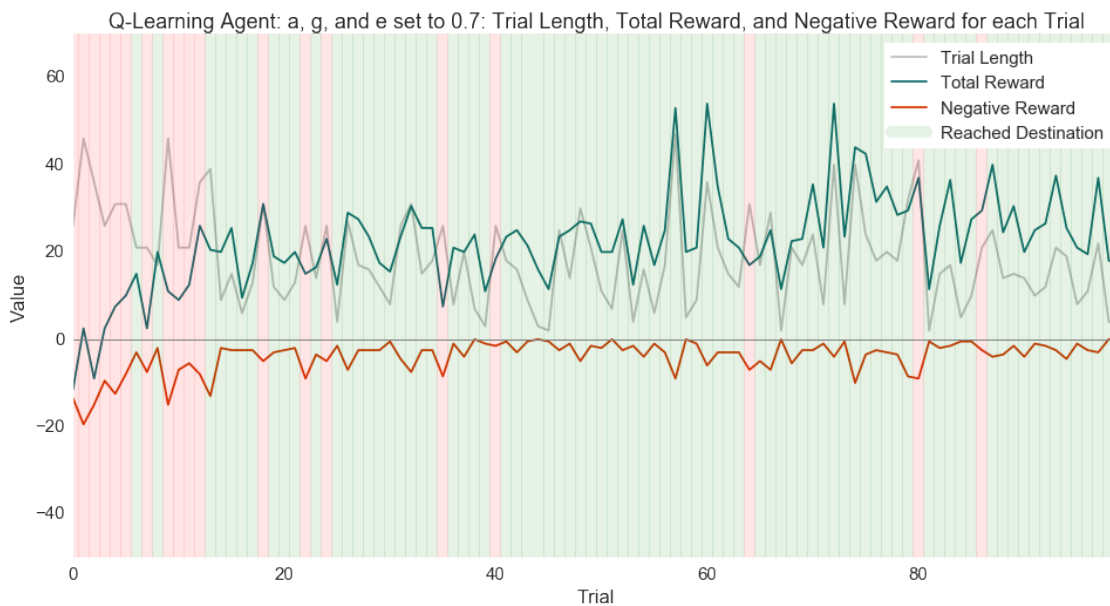
	count	mean	std	min	25%	50%	75%	max
total_reward	100	18.14	8.387424	-1.5	14.375	20	24.00	32
negative_reward	100	-0.44	1.439486	-13.0	-0.500	0	0.00	0
trial_length	100	17.81	10.769106	4.0	9.750	14	21.25	46



In [12]: `sp.display_stats_for_the_q_learning_agent_with_params(0.7)`

The destination was reached in 81 out of 100 trials.

	count	mean	std	min	25%	50%	75%	max
total_reward	100	22.315	10.960594	-11.5	17.375	22.0	26.625	54
negative_reward	100	-3.930	3.769240	-19.5	-5.000	-2.5	-1.500	0
trial_length	100	18.640	10.861404	2.0	10.000	17.0	26.000	47



When the Q-Learning agent operates with very small values for α , γ , and ε (0.01), the agent does eventually learn the optimal policy though, for the 1 simulation using these parameter values, it took 42 trials to get there. After the 42nd trial this agent always reached the destination and incurred negative rewards very infrequently (as ε was so small). Though this may still be impressive, it took the agent 7 times longer to learn the policy than it did with the optimal parameter values.

When the Q-Learning agent operates with very large values for α , γ , and ε (0.7), the agent performs far worse than either with small values or the optimal values. The large ε value means the agent incurs negative rewards throughout the 100 trials with very few exceptions. Even though the performance did improve after the 14th trial, it still failed to reach the destination in the 80th and 86th trials.

QUESTION: Does your agent get close to finding an optimal policy, i.e. reach the destination in the minimum possible time, and not incur any penalties? How would you describe an optimal policy for this problem? I think the Q-Learning agent with an α of 0.5, γ of 0.1, and ε of 0.1 does find an optimal policy. It reached the destination in all trials except the first and it avoided incurring a negative reward for the final 21 trials, but I think we can go a step further in this analysis. I believe the optimal policy is the one used by the Informed Driver Agent. The policy used by that agent minimises the trial length and negative rewards while maximising the total reward (for a detailed description of the policy, please see the appropriate section higher up in this report). More specifically, the Informed Driver policy follows the following rules:

- When the desired direction is ‘forward’:
 - And the traffic light is ‘green’: return ‘forward’ (Case 1)
 - Else: return ‘None’ (Case 2)
- When the desired direction is ‘left’:
 - And the traffic light is ‘green’ AND if there is oncoming traffic, it is travelling ‘left’: return ‘left’ (Case 3)
 - Else: return ‘None’ (Case 4)
- When the desired direction is ‘right’:
 - And the traffic light is ‘green’ OR the traffic on the left is NOT travelling ‘forward’: return ‘right’ (Case 5)
 - Else: return ‘None’ (Case 6)

By examining the Q and N matrices built up during the simulation of the optimal Q-Learning agent, we can see if the agent found the optimal policy.

In [13]: `sp.optimal_q_and_n_less_empty_rows()`

State encoding:

```
tl: Traffic light
o:  Oncoming traffic
r:  Traffic coming from the right
l:  Traffic coming from the left
dd: Desired direction
```

NB: Please note that states that were not experienced by the agent are not displayed.

Q(s,a):

	State	forward	right	left	None
47	tl:green_o:forward_r:None_l:None_dd:left	0.000	0.000	0.000	0.106
141	tl:green_o:left_r:None_l:None_dd:forward	1.611	-0.248	-0.140	0.118
142	tl:green_o:left_r:None_l:None_dd:right	0.000	0.000	-0.242	0.000
143	tl:green_o:left_r:None_l:None_dd:left	0.000	-0.141	0.000	0.000
153	tl:green_o:None_r:forward_l:None_dd:forward	0.000	0.000	0.000	0.007

154	tl:green_o:None_r:forward_l:None_dd:right	0.000	1.105	0.000	0.000
155	tl:green_o:None_r:forward_l:None_dd:left	0.000	-0.200	0.000	0.000
165	tl:green_o:None_r:right_l:None_dd:forward	1.662	-0.243	-0.139	0.000
166	tl:green_o:None_r:right_l:None_dd:right	0.000	1.105	0.000	0.000
167	tl:green_o:None_r:right_l:None_dd:left	0.000	0.000	1.007	0.000
177	tl:green_o:None_r:left_l:None_dd:forward	1.101	-0.140	-0.143	0.000
180	tl:green_o:None_r:None_l:forward_dd:forward	1.638	-0.268	-0.142	0.054
182	tl:green_o:None_r:None_l:forward_dd:left	-0.245	-0.142	0.000	0.000
183	tl:green_o:None_r:None_l:right_dd:forward	1.000	0.000	0.000	0.000
186	tl:green_o:None_r:None_l:left_dd:forward	1.104	-0.144	0.000	0.000
188	tl:green_o:None_r:None_l:left_dd:left	-0.142	0.000	1.007	0.000
189	tl:green_o:None_r:None_l:None_dd:forward	2.064	-0.369	-0.297	0.180
190	tl:green_o:None_r:None_l:None_dd:right	-0.339	2.201	-0.278	0.136
191	tl:green_o:None_r:None_l:None_dd:left	-0.336	-0.297	2.033	0.158
231	tl:red_o:forward_r:None_l:right_dd:forward	-0.500	0.000	0.000	0.000
237	tl:red_o:forward_r:None_l:None_dd:forward	-0.591	-0.222	-0.646	0.000
239	tl:red_o:forward_r:None_l:None_dd:left	-0.500	-0.140	0.000	0.000
286	tl:red_o:right_r:None_l:None_dd:right	0.000	0.000	0.000	0.110
333	tl:red_o:left_r:None_l:None_dd:forward	-0.669	-0.244	-0.750	0.000
334	tl:red_o:left_r:None_l:None_dd:right	-0.500	0.000	-0.500	0.000
345	tl:red_o:None_r:forward_l:None_dd:forward	0.000	0.000	-0.500	0.000
357	tl:red_o:None_r:right_l:None_dd:forward	0.000	-0.145	0.000	0.000
371	tl:red_o:None_r:left_l:None_dd:left	-0.500	0.000	0.000	0.000
372	tl:red_o:None_r:None_l:forward_dd:forward	-0.498	0.000	0.000	0.000
375	tl:red_o:None_r:None_l:right_dd:forward	0.000	0.000	-0.499	0.000
378	tl:red_o:None_r:None_l:left_dd:forward	0.000	0.000	0.000	0.003
381	tl:red_o:None_r:None_l:None_dd:forward	-0.872	-0.432	-0.873	0.065
382	tl:red_o:None_r:None_l:None_dd:right	-0.757	2.151	-0.858	0.161
383	tl:red_o:None_r:None_l:None_dd:left	-0.750	-0.289	-0.750	0.117

N(s,a):

	State	forward	right	left	None
47	tl:green_o:forward_r:None_l:None_dd:left	0	0	0	1
141	tl:green_o:left_r:None_l:None_dd:forward	2	1	1	3
142	tl:green_o:left_r:None_l:None_dd:right	0	0	1	0
143	tl:green_o:left_r:None_l:None_dd:left	0	1	0	0
153	tl:green_o:None_r:forward_l:None_dd:forward	0	0	0	1
154	tl:green_o:None_r:forward_l:None_dd:right	0	1	0	1
155	tl:green_o:None_r:forward_l:None_dd:left	0	1	0	0
165	tl:green_o:None_r:right_l:None_dd:forward	2	1	1	0
166	tl:green_o:None_r:right_l:None_dd:right	0	1	0	0
167	tl:green_o:None_r:right_l:None_dd:left	0	0	1	0
177	tl:green_o:None_r:left_l:None_dd:forward	1	1	1	2
180	tl:green_o:None_r:None_l:forward_dd:forward	2	2	1	1
182	tl:green_o:None_r:None_l:forward_dd:left	1	1	0	3
183	tl:green_o:None_r:None_l:right_dd:forward	1	0	0	0
186	tl:green_o:None_r:None_l:left_dd:forward	1	1	0	0
188	tl:green_o:None_r:None_l:left_dd:left	1	0	1	3
189	tl:green_o:None_r:None_l:None_dd:forward	302	2	7	3
190	tl:green_o:None_r:None_l:None_dd:right	3	55	3	3
191	tl:green_o:None_r:None_l:None_dd:left	6	3	70	9
231	tl:red_o:forward_r:None_l:right_dd:forward	1	0	0	0
237	tl:red_o:forward_r:None_l:None_dd:forward	2	2	2	0

239	tl:red_o:forward_r:None_l:None_dd:left	1	1	0	1
286	tl:red_o:right_r:None_l:None_dd:right	0	0	0	1
333	tl:red_o:left_r:None_l:None_dd:forward	2	1	2	0
334	tl:red_o:left_r:None_l:None_dd:right	1	0	1	1
345	tl:red_o:None_r:forward_l:None_dd:forward	0	0	1	0
357	tl:red_o:None_r:right_l:None_dd:forward	0	1	0	0
371	tl:red_o:None_r:left_l:None_dd:left	1	0	0	0
372	tl:red_o:None_r:None_l:forward_dd:forward	1	0	0	0
375	tl:red_o:None_r:None_l:right_dd:forward	0	0	1	0
378	tl:red_o:None_r:None_l:left_dd:forward	0	0	0	1
381	tl:red_o:None_r:None_l:None_dd:forward	6	3	3	485
382	tl:red_o:None_r:None_l:None_dd:right	5	71	5	9
383	tl:red_o:None_r:None_l:None_dd:left	2	4	2	139

Case 1: (rows 141, 153, 165, 177, 180, 183, 186, 189)

In all of these rows except row 153, the largest $Q(s,a)$ value was for the optimal action. The states represented by rows 153, 165, 183, 186 have incomplete $Q(s,a)$ values as the state was not experineced enough times. Row 189 represents the state where the traffic light is ‘green’ and the desired direction is ‘forward’ without any other traffic at the intersection. This is the most commonly experineced state (by a very large margin) for this case and the optimal action was returned for 302 out of 314 trial steps.

Case 2: (rows 231, 237, 333, 345, 372, 375, 378, 381)

All of these rows except 381 have incomplete $Q(s,a)$ values as the state was not experineced enough times. Row 381 represents the state where the traffic light is ‘red’ and the desired direction is ‘forward’ without any other traffic at the intersection. This is the most commonly experineced state (by a very large margin) for this case and the optimal action was returned for 485 out of 497 trial steps.

Case 3: (rows 143, 155, 167, 182, 188, 191)

All of these rows except 191 have incomplete $Q(s,a)$ values as the state was not experineced enough times. Row 191 represents the state where the traffic light is ‘green’ and the desired direction is ‘left’ without any other traffic at the intersection. This is the most commonly experineced state (by a very large margin) for this case and the optimal action was returned for 70 out of 88 trial steps.

Case 4: (rows 47, 239, 371, 383)

All of these rows except 383 have incomplete $Q(s,a)$ values as the state was not experineced enough times. Row 383 represents the state where the traffic light is ‘red’ and the desired direction is ‘left’ without any other traffic at the intersection. This is the most commonly experineced state (by a very large margin) for this case and the optimal action was returned for 139 out of 147 trial steps.

Case 5: (rows 142, 154, 166, 190, 286, 334, 382)

All of these rows except 190 and 382 have incomplete $Q(s,a)$ values as the state was not experineced enough times. Row 190 represents the state where the traffic light is ‘green’ and the desired direction is ‘right’ without any other traffic at the intersection where row 382 represents the same state except the traffic light is ‘red’. The optimal action was returned for 55 out of 64 trial steps when the traffic light was ‘green’ and 71 out of 90 trial steps when the traffic light was ‘red’.

Case 6: This case was never experienced by the agent.

Upon closely analysing the values within $Q(s,a)$ and $N(s,a)$ matrices, I don’t think it would be correct to claim that the Q-Learning agent with an α of 0.5, γ of 0.1, and ε of 0.1 learned the optimal policy in its entirety. This is not an indication that the parameters are not indeed optimal or that the agent performed badly, but is instead an indication of how sprasely the agent experined the state space when the environment only contains 3 other agents and a simulation is made up of only 100 trials. Too many of the states were only experined a handful of times. The one state that comprises case 6 in the optimal policy description was never experienced.

If we step away from analysing the whole state space and instead focus on the 5 states that represent the state of the agent in a very large majority of trial steps (the sum of $N(s,a)$ these 5 states out number all other states by 2 orders of magnitude), then the optimal policy was learned. In all 5 of these states, the optimal action is returned.

1.5 Conclusion

This project very clearly illustrates just how powerful Q-Learning can be for agents operating in environments with well defined state-action-reward cycles. This is especially true if the state space is explored enough to populate the $Q(s,a)$ matrix for the most often experienced states.

1.5.1 Resources

The production of this project was aided by information found on the following websites (various authors and contributors)

- [Udacity.com](https://www.udacity.com)
- [StackOverflow.com](https://stackoverflow.com)
- [Seaborn Documentation](#)

The following book was also referenced:

Russell, S., and P. Norvig. "Reinforcement Learning." In *Artificial Intelligence: A Modern Approach*. S.l.: Pearson Education Limited, 2010.