1. **4 / 4 puntos**

For a vector $\mathbf{x} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$ and the subspace U spanned by the basis vectors $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$, which of the following statements are true?

You can use the formula slide that comes with the corresponding lecture.

- [x] The projection matrix is symmetric.

Correcto
Projection matrices are always symmetric.

- [x] The projection matrix is $\frac{1}{6}\begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$

Correcto
Well done!

- [ ] The coordinates of the projected point with respect to $\mathbf{b}_1, \mathbf{b}_2$ are $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

- [ ] The projection matrix is not symmetric.

- [ ] The projection of $\mathbf{x}$ onto U is $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

- [ ] The rank of the projection matrix is 1.

- [x] The coordinates of the projected point with respect to $\mathbf{b}_1, \mathbf{b}_2$ are $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$.

Correcto
Excellent job!

☐ The projection matrix is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$

☑ The projection of $\mathbf{x}$ onto U is $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$

Correcto
Well done.

2. Project $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$ onto the subspace spanned by $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. **1 / 1 punto**

You can use the formula slide that comes with the corresponding lecture.

○ $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

○ $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

◉ $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$

○ $\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$

Correcto
Absolutely! The original vector is already in the subspace, so the projection has no effect.

3. **1 / 1 punto**

1. Project $\begin{bmatrix}12\\0\\0\end{bmatrix}$ onto the subspace U_1 spanned by $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}$.
2. Project the result from 1. onto the subspace spanned by $\begin{bmatrix}-10\sqrt{6}\\-4\sqrt{6}\\2\sqrt{6}\end{bmatrix}$.

   What is the final projection?

Hint: For step 2. you do not necessarily need to compute anything.

You can use the formula slide that comes with the corresponding lecture.

- ○ $\begin{bmatrix}5\\2\sqrt{6}\\-1\sqrt{6}\end{bmatrix}$
- ○ $\begin{bmatrix}5\\2\sqrt{6}+1\\-\sqrt{6}+2\end{bmatrix}$
- ◉ $\begin{bmatrix}10\\4\\-2\end{bmatrix}$

Correcto

Good job! The first projection already lies in the second subspace. Therefore, the second projection does not do anything.