1. **Select the characteristic polynomial for the given matrix.**

1 / 1 punto

$$\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix}$$

- (selected) $\lambda^2 - 8\lambda + 15$
- $\lambda^2 + 8\lambda + 15$
- $\lambda^2 - 8\lambda - 1$
- $\lambda^3 - 8\lambda + 15$

Correcto

Correct! $\lambda^2 - (2+6)\lambda + (2 * 6 - 1(-3)) = 0$

2. **Select the eigenvectors for the previous matrix in Q1, as given below:**

1 / 1 punto

$$\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix}$$

- $\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix}$
- $\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

- [x] $\binom{1}{3}, \binom{1}{1}$

- [ ] $\binom{1}{1}, \binom{1}{1}$

Correcto

Correct! You first find the eigenvalues for the given matrix:$\lambda = 5, \lambda = 3$. Now you solve the equations using each of the eigenvalues.

For $\lambda = 5$, you have $\begin{cases} 2x + y = 5x \\ -3x + 6y = 5y \end{cases}$, which has solutions for $x = 1, y = 3$. Your eigenvector is $\binom{1}{3}$.

For $\lambda = 3$, you have $\begin{cases} 2x + y = 3x \\ -3x + 6y = 3y \end{cases}$, which has solutions for $x = 1, y = 1$. Your eigenvector is $\binom{1}{1}$.

3. Which of the following is an eigenvalue for the given identity matrix.

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$$ID = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- [ ] $\lambda = 2$

- [x] $\lambda = 1$

- [ ] $\lambda = -1$

Correcto

Correct! The eigenvalue for the identity matrix is always 1.

4. Find the eigenvalues of matrix A·B where:

1 / 1 punto

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hint: What type of matrix is B? Does it change the output when multiplied with A? If not, focus only on one of the matrices to find the eigenvalues.

- (●) $\lambda_1 = 4, \lambda_2 = 1$
- ( ) $\lambda_1 = 3, \lambda_2 = 1$
- ( ) $\lambda_1 = 4, \lambda_2 = 2$
- ( ) Eigenvalues cannot be determined.

Correcto

Correct!

$$A \cdot B = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot 1 + 2 \cdot 0 & 1 \cdot 0 + 2 \cdot 1 \\ 0 \cdot 1 + 4 \cdot 0 & 0 \cdot 0 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$$

Since the second matrix is an identity matrix, you wouldn't need to solve the above multiplication since identity matrix does not change the result.

The eigenvalues of A are the roots of the characteristic equation $\det(A - \lambda I) = 0$.

By solving $\lambda^2 - 5\lambda + 4 = 0$, you get $\lambda_1 = 4, \lambda_2 = 1$.

5. Select the eigenvectors, using the eigenvalues you found for the above matrix A·B in Q4.

1 / 1 punto

- ○ $\vec{v_1} = (2,3); \vec{v_2} = (2,3)$
- ◉ $\vec{v_1} = (2,3); \vec{v_2} = (1,0)$
- ○ $\vec{v_1} = (2,0); \vec{v_2} = (1,0)$
- ○ $\vec{v_1} = (1,3); \vec{v_2} = (1,0)$

Correcto

Correct!

For $\lambda = 4$, you have $\begin{cases} x + 2y = 4x \\ 0x + 4y = 4y \end{cases}$, which has solutions for $x = 2, y = 3$. Your eigenvector $\vec{v_1}$ is $\binom{2}{3}$.

For $\lambda = 1$, you have $\begin{cases} x + 2y = x \\ 0x + 4y = y \end{cases}$, which has solutions for $x = 1, y = 0$. Your eigenvector $\vec{v_2}$ is $\binom{1}{0}$.

6. Which of the vectors span the matrix $W = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 5 \\ 3 & -2 & -1 \end{bmatrix}$?

1 / 1 punto

- ◉ $\vec{V_1} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \vec{V_2} = \begin{bmatrix} 3 \\ 2 \\ -2 \end{bmatrix} \vec{V_3} = \begin{bmatrix} 0 \\ 5 \\ -1 \end{bmatrix}$
- ○ $V1 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} V2 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} V3 = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$

Correcto

Correct! There are linearly independent columns that span the matrix, which individually form three vectors $\vec{V_1}, \vec{V_2}, \vec{V_3}$.These vectors span the matrix W.

7. **Given matrix P select the answer with the correct eigenbasis.**

1 / 1 punto

$$P = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Hint: First compute the eigenvalues, eigenvectors and contrust the eigenbasis matrix with the spanning eigenvectors.

- ○ $$Eigenbasis = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

- ○ $$Eigenbasis = \begin{bmatrix} 0 & -1 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

- ◉ $$Eigenbasis = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Correcto

Correct! After solving the characteristic equations to find the eigenvalues, you should get $\lambda_1 = 1$ and $\lambda_2 = 2$.

The eigenvector for $\lambda_1 = 1$ is $\vec{V_1} = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$.

The eigenvectors for \lambda_2 = 2 are $\vec{V_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \vec{V_3} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$.

The eigenvectors form the eigenbasis: $\begin{bmatrix} 0 & 0 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

8. Select the characteristic polynomial for the given matrix.

1 / 1 punto

$$\begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix}$$

- ○ $-\lambda^3 + 2\lambda^2 + 9$
- ○ $\lambda^3 + 2\lambda^2 + 4\lambda - 5$
- ◉ $-\lambda^3 + 2\lambda^2 + 4\lambda - 5$
- ○ $-\lambda^2 + 2\lambda^3 + 4\lambda - 5$

Correcto

Correct! The characteristic polynomial of a matrix A is given by $f(\lambda) = det(A–\lambda I)$.

First, you find the following:

$$\begin{bmatrix} 3 & 1 & -2 \\ 4 & 0 & 1 \\ 2 & 1 & -1 \end{bmatrix} \cdot \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Now you compute the determinant of the result:

$$det \begin{pmatrix} 3-\lambda & 1 & -2 \\ 4 & -\lambda & 1 \\ 2 & 1 & -1-\lambda \end{pmatrix} = -\lambda^3 + 2\lambda^2 + 4\lambda - 5$$

9. You are given a non-singular matrix A with real entries and eigenvalue i.

1 / 1 punto

Which of the following statements is correct?

- ○ i is an eigenvalue of $A^{-1} + A$.
- ○ i is an eigenvalue of $A^{-1} \cdot A \cdot I$.
- ◉ $1/i$ is an eigenvalue of A^{-1}.

Correcto

Correct! You know that the eigenvalues of a matrix A are the solutions of its characteristic polynomial equation det(A−λI)=0.