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1. On your next mission, while collecting rock samples, you observe a new crystal structure containing carbon, which could be key to life! You utilize the third spacecraft, Ingenuity, and meticulously collect enough rock samples to distribute within the weight limits of each spacecraft.

You place 2 basalt samples, 1 meteorite, and 5 crystal rock samples into the Perseverance rover, which all weigh 20 grams.

You then distribute 1 basalt, 2 meteorites, and 1 crystal into the Curiosity rover, with a weight of 10 grams in total.

Lastly, you place 2 basalt samples, 1 meteorite, and 3 crystals to Ingenuity, which together weigh 15 grams. Each rock sample is represented with variables b for basalt, m for meteorite, and c for crystal structures.

Which of the following systems of equations represents the correct information in the above system of sentences?



$$\begin{cases} 2b + m + 5c = 20 \\ b + 2m + c = 10 \\ 2b + m + 3c = 15 \end{cases}$$



$$\begin{cases} 2b + m + 5 = 20 \\ b + 2m + c = 10 \end{cases}$$



$$\begin{cases} m + 2b + 5 = 20 \\ 2b + m + c = 10 \\ b + 2m + 3c = 15 \end{cases}$$



$$\begin{cases} 2b + m + 5c = 100 \\ b + 2m + c = 23 \\ 2b + m + 3c = 35 \end{cases}$$

✓ **Correcto**

Correct! This system of equations represents the weights of each rock sample noted with variables b for basalt, m for meteorite, and c for the crystal structure. The first equation represents the weight of the samples on the Perseverance rover, the second on the Curiosity rover, and the third equation for Ingenuity.

2. Which of the following matrices represents the system of equations?

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$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

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$$\begin{bmatrix} 2 & 1 & 5 & 20 \\ 1 & 2 & 1 & 10 \\ 2 & 1 & 3 & 15 \end{bmatrix}$$

☒

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

✓ **Correcto**

Correct! This is the representation of the system of equations in matrix form.

3. Calculate the determinant of the matrix that represents the system of equations above. Is the matrix singular or non-singular?

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Hint: To find the determinant, apply the method described in the lecture [The determinant \(3x3\)](#).

- ☐ -6, Singular
- ☒ -6, Non-singular
- ☐ 6, Singular
- ☐ 0, Singular

**Correcto**

Well done! You have correctly calculated the determinant and identified the non singularity of the matrix.

4. Determine if the matrix found in Question 2 has linearly dependent or independent rows.

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- ☐ It cannot be determined.
- ☐ Linearly dependent.
- ☒ Linearly independent.

**Correcto**

Well done! The matrix has linearly independent rows. You cannot obtain one row by using row operations on the other rows.

5. How much does each rock sample weigh?

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Solve the system of equations for each of the spacecraft.

- ☐ basalt = 6g, meteorite = 1g, crystal = 3g
- ☒ each rock sample = 2.5g
- ☐ basalt = 1.5g, meteorite = 3.5g, crystal = 2.5g
- ☐ basalt = 2.5g, meteorite = 2.5g, crystal = 5.5g

✓ **Correcto**

Correct! The system of equations has a unique solution at the point (2.5, 2.5, 2.5) where $b = 2.5$, $m = 2.5$, and $c = 2.5$. In other words, the basalt rock sample weighs 2.5 grams, the meteorite rock sample weighs 2.5 grams, and the crystal sample weighs 2.5 grams.

6. You are given the following matrix with the values for rock samples weighs in the Perseverance and Curiosity rovers, consisting of the first and second row respectively.

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For which values in Ingenuity does the matrix have linearly dependent rows?

$$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ \mathbf{x} & \mathbf{y} & \mathbf{z} \end{bmatrix}$$

(Notation for weight samples: x - basalt, y - meteorite, z - crystal.)

- ☒ $x = 3, y = 3, z = 6$
- ☐ $x = 1, y = 2, z = 3$
- ☐ $x = 1, y = 3, z = 3$

✓ **Correcto**

Way to go! By adding the first two rows, specifically the samples of Perseverance and Curiosity, you get the values $x = 3$ by $(2+1)$, $y = 3$ by $(1+2)$, and $z = 6$ by $(5+1)$. For these values, the matrix has linearly dependent rows.

7. In an effort to optimize the weight distribution, your colleague suggests you split the rocks into the represented matrix.

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$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{bmatrix}$$

Calculate the determinant in the system of equations and determine if the matrix is singular or non-singular.

- ☒ 0, Singular
- ☐ 5, Non-singular
- ☐ 0, Non-singular

**Correcto**

Correct! The determinant for the given matrix is 0, therefore the matrix is singular.

8. **Select which of the following are true for non-singular matrices.**

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In a non-singular matrix, rows are linearly independent.

**Correcto**

Correct! Non-singular matrices have linearly independent rows.



In a non-singular matrix there is only a unique solution for the represented system of equations.

**Correcto**

Correct! Since the rows are linearly independent in the non-singular matrix, you can find a unique solution for the represented system of equations.

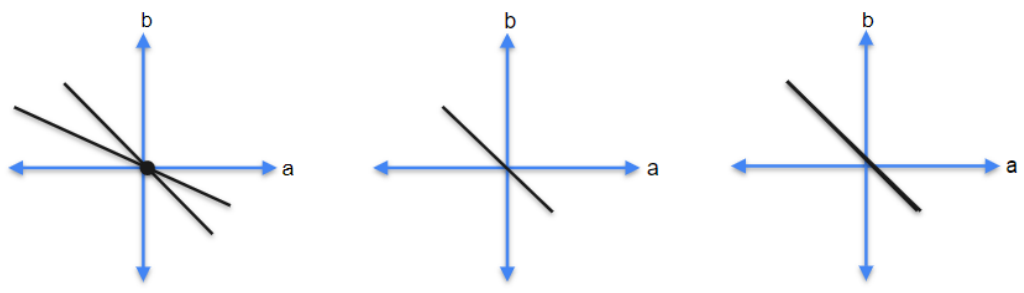
- ☐ In a non-singular matrix, rows are linearly dependent.
- ☐ In a non-singular matrix a row can be a multiple of the other one.

9. To train your AI assistant to classify systems of equations, you're now asked to select the correct sequence of graphs that represents a system of linear equations with:

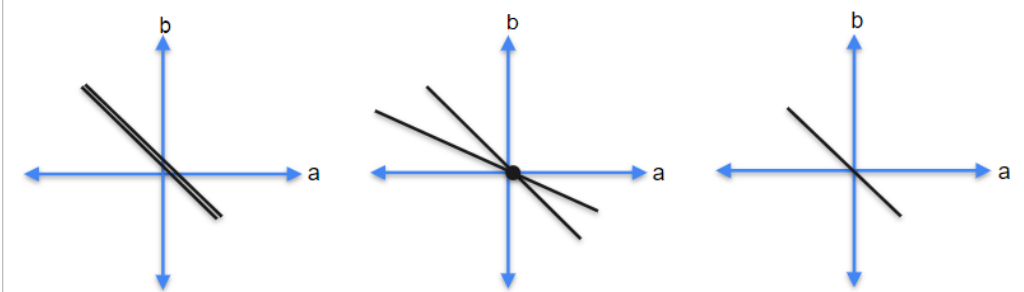
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1. zero solutions, 2. just one solution, 3. infinitely many solutions.

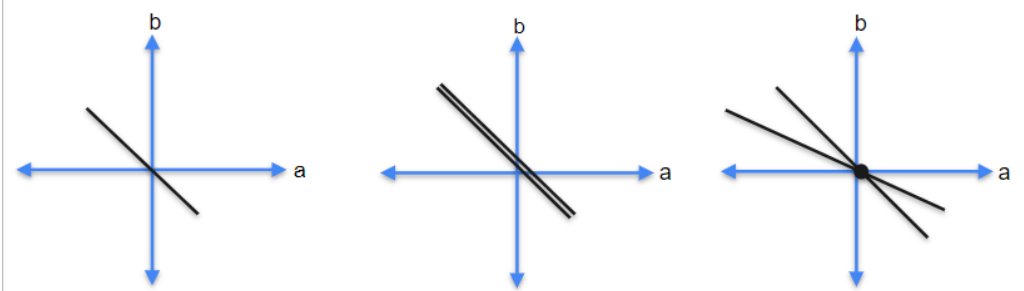
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✓ **Correcto**

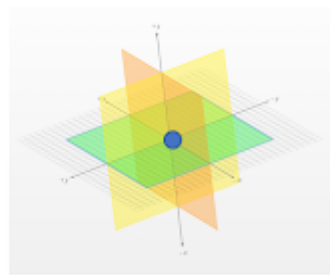
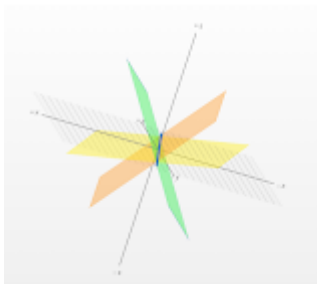
Correct! You can visually determine how many solutions a system of linear equations has following these rules: If the lines are parallel, the system has no solutions; if the lines intersect at just one point, the system has just one solution and the solution is the point where they intersect. If the lines totally overlap, the system has infinitely many solutions.

10. Select the correct sequence of graphs that shows:

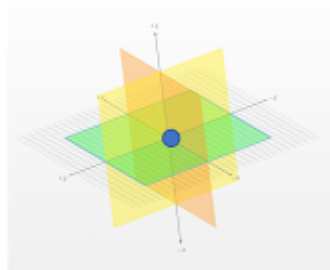
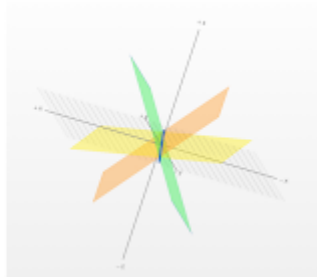
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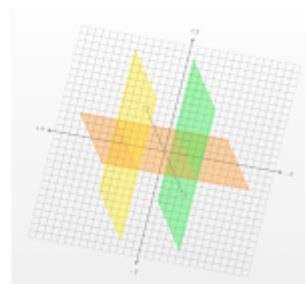
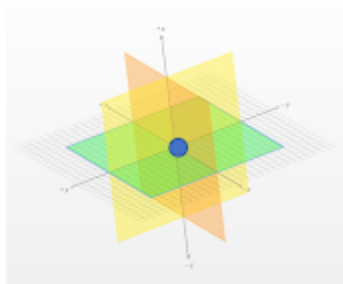
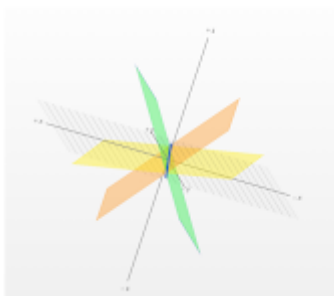
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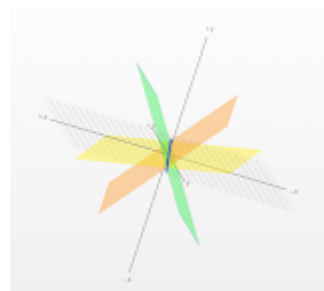
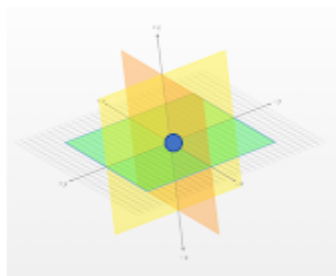


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**Correcto**

Correct! The first graph has no solution because there is no place where the three planes intersect. The second graph has one solution, the origin $(0, 0, 0)$. And finally, you can see that in the third graph, there are infinitely many solutions (points where the lines intersect).