1. In this quiz, you will calculate the Jacobian matrix for some vector valued functions.

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For the function $u(x,y) = x^2 - y^2$ and $v(x,y) = 2xy$, calculate the Jacobian matrix $J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix}$.

- ( ) $J = \begin{bmatrix} 2x & 2y \\ 2y & 2x \end{bmatrix}$
- ( ) $J = \begin{bmatrix} 2x & 2y \\ -2y & 2x \end{bmatrix}$
- (•) $J = \begin{bmatrix} 2x & -2y \\ 2y & 2x \end{bmatrix}$
- ( ) $J = \begin{bmatrix} 2x & -2y \\ -2y & 2x \end{bmatrix}$

Correcto
Well done!

2. For the function $u(x,y,z) = 2x + 3y$, $v(x,y,z) = cos(x)sin(z)$ and $w(x,y,z) = e^x e^y e^z$, calculate the Jacobian matrix $J = \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{bmatrix}$.

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- (•) $J = \begin{bmatrix} 2 & 3 & 0 \\ -sin(x)sin(z) & 0 & cos(x)cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$
- ( ) $J = \begin{bmatrix} 2 & 3 & 0 \\ sin(x)sin(z) & 0 & -cos(x)cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

○ $J = \begin{bmatrix} 2 & 3 & 0 \\ -cos(x)sin(z) & 0 & -sin(x)cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

○ $J = \begin{bmatrix} 2 & 3 & 0 \\ cos(x)sin(z) & 0 & -sin(x)cos(z) \\ e^x e^y e^z & e^x e^y e^z & e^x e^y e^z \end{bmatrix}$

Correcto
Well done!

3. Consider the pair of linear equations $u(x, y) = ax + by$ and $v(x, y) = cx + dy$, where a, b, c and d are all constants. Calculate the Jacobian, and notice something kind of interesting!

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○ $J = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$

○ $J = \begin{bmatrix} b & c \\ d & a \end{bmatrix}$

● $J = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

○ $J = \begin{bmatrix} b & c \\ a & d \end{bmatrix}$

Correcto
Well done!

A succinct way of writing this down is the following:

$$\begin{bmatrix} u \\ v \end{bmatrix} = J \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

This is a generalisation of the fact that a simple linear function $f(x) = a \cdot x$ can be re-written as $f(x) = f'(x) \cdot x$, as the Jacobian matrix can be viewed as the multi-dimensional derivative. Neat!

4. For the function $u(x, y, z) = 9x^2y^2 + ze^x$, $v(x, y, z) = xy + x^2y^3 + 2z$ and $w(x, y, z) = cos(x)sin(z)e^y$, calculate the Jacobian matrix and evaluate at the point $(0, 0, 0)$.

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- ○ $J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix}$

- ○ $J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

- ◉ $J = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

- ○ $J = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

Correcto
Well done!

5. In the lecture, we calculated the Jacobian of the transformation from Polar co-ordinates to Cartesian co-ordinates in 2D. In this question, we will do the same, but with Spherical co-ordinates to 3D.

1 / 1 punto

For the functions $x(r, \theta, \phi) = rcos(\theta)sin(\phi)$, $y(r, \theta, \phi) = rsin(\theta)sin(\phi)$ and $z(r, \theta, \phi) = rcos(\phi)$, calculate the Jacobian matrix.

- ○ $J = \begin{bmatrix} rcos(\theta)sin(\phi) & -rsin(\theta)sin(\phi) & rcos(\theta)cos(\phi) \\ rsin(\theta)sin(\phi) & r^2cos(\theta)sin(\phi) & sin(\theta)cos(\phi) \\ cos(\phi) & -1 & -rsin(\phi) \end{bmatrix}$

- ◉ $J = \begin{bmatrix} cos(\theta)sin(\phi) & -rsin(\theta)sin(\phi) & rcos(\theta)cos(\phi) \\ sin(\theta)sin(\phi) & rcos(\theta)sin(\phi) & rsin(\theta)cos(\phi) \\ cos(\phi) & 0 & -rsin(\phi) \end{bmatrix}$

○ $$J = \begin{bmatrix} rcos(\theta)sin(\phi) & -sin(\theta)sin(\phi) & cos(\theta)cos(\phi) \\ rsin(\theta)sin(\phi) & cos(\theta)sin(\phi) & sin(\theta)cos(\phi) \\ rcos(\phi) & 0 & -sin(\phi) \end{bmatrix}$$

○ $$J = \begin{bmatrix} r^2cos(\theta)sin(\phi) & -sin(\theta)sin(\phi) & cos(\theta)cos(\phi) \\ rsin(\theta)sin(\phi) & rcos(\theta)sin(\phi) & rsin(\theta)cos(\phi) \\ cos(\phi) & 1 & rsin(\phi) \end{bmatrix}$$

Correcto

Well done! The determinant of this matrix is $-r^2sin(\phi)$, which does not vary only with θ.