SW Implementation

Introduction

The goal of the fifth lab is to design and implement a band pass filter with certain specifications. The central frequency f_0 should be between 2 and 5 MHz, the band pass width $(f_{c1} - f_{c0})$ should be equal to 5% of the central frequency, A_{pass} should be less than 3 dB, and A_{stop} should be equal to or higher than 30 dB. In the chosen design, the central frequency is 3 MHz, meaning that the passband width is 150 kHz. A second order Butterworth filter is designed to achieve least error, and it is given an input AC voltage to observe the desired output. The filter involves 2 inductors and 2 capacitors connected in series and parallel orders between the load and source resistors, and the desired specifications are satisfied.

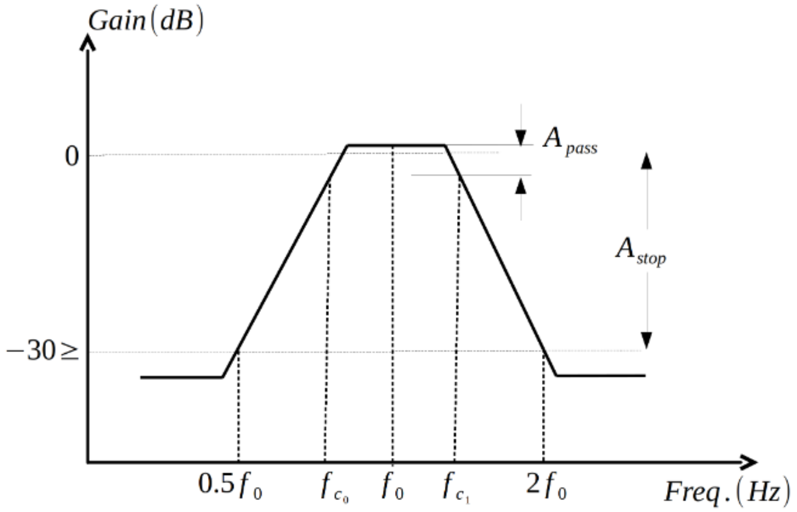

Figure 1: Specified Frequency Response

Analysis

In this lab, a second order Butterworth filter is implemented using the general design in Figure 2.

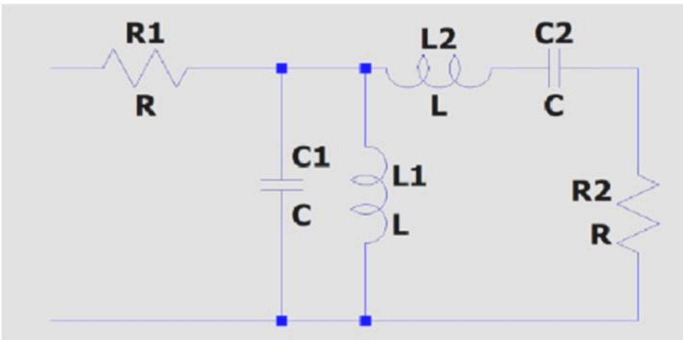

Figure 2: Butterworth Filter, Second Order

Using this design, the power transfer from one 50 Ω resistor to one another is going to be observed, and finding the capacitor and inductor values is necessary. In order to make this design completely, firstly a second order Butterworth low pass filter is necessary. Its cut off frequency, which is -3 dB should equal 150 kHz meaning the passband width. Later, the found circuit that involves a single capacitor and a single inductor should be tuned to finalize a band pass filter, where the central frequency of 3 MHz ensures resonance. The initial step of finding the low pass filter values uses Butterworth coefficients and analyzes them over the circuit in Figure 4.

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

Figure 3: Low Pass Butterworth Coefficients

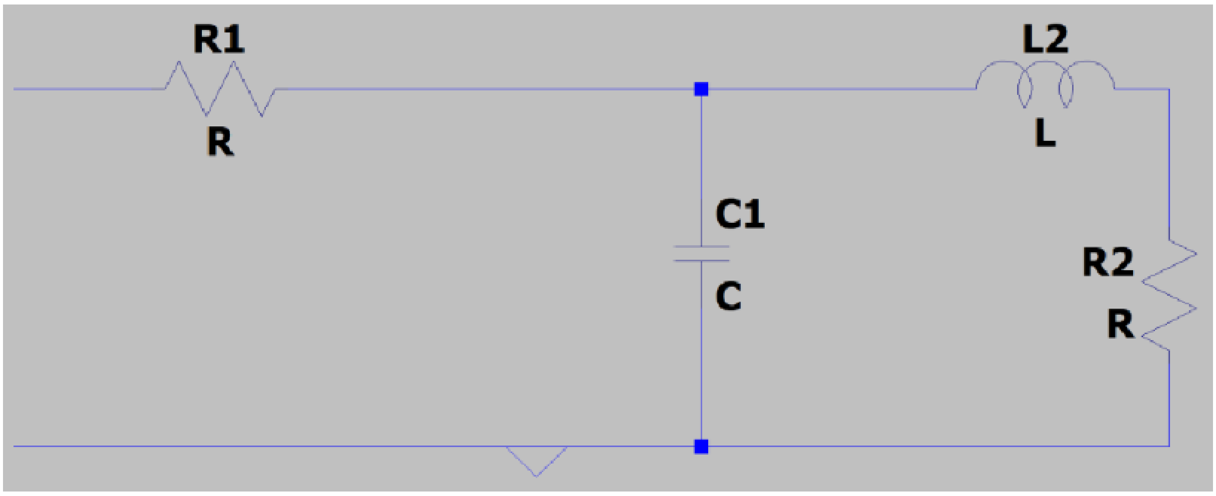

Figure 4: General Low Pass Butterworth Filter

Using the Butterworth coefficients for low pass filters, the following calculations for finding the capacitance and inductance values can be made. The coefficient for second order circuits is 1.4142 for both circuit elements and 150 kHz is put into 2πf as the cut off frequency.

$$C_1 = \frac{1.4142}{2 * \pi * 150\ kHz * R_L}$$

$$L_1 = \frac{1.4142 * R_L}{2 * \pi * 150\ kHz}$$

Knowing that the load resistance is 50 Ω;

$$C_1 = \frac{1.4142}{2 * \pi * 150\ kHz * 50\ \Omega} = 30\ pF$$

$$L_1 = \frac{1.4142 * 50\ \Omega}{2 * \pi * 150\ kHz} = 75\ \mu H$$

(1)

These values help obtain the low pass filter circuit in Figure 5, and now the circuit should be tuned to resonate at 3 MHz by placing a parallel inductor to the 30 pF capacitor and a series capacitor to the 75 μH inductor.

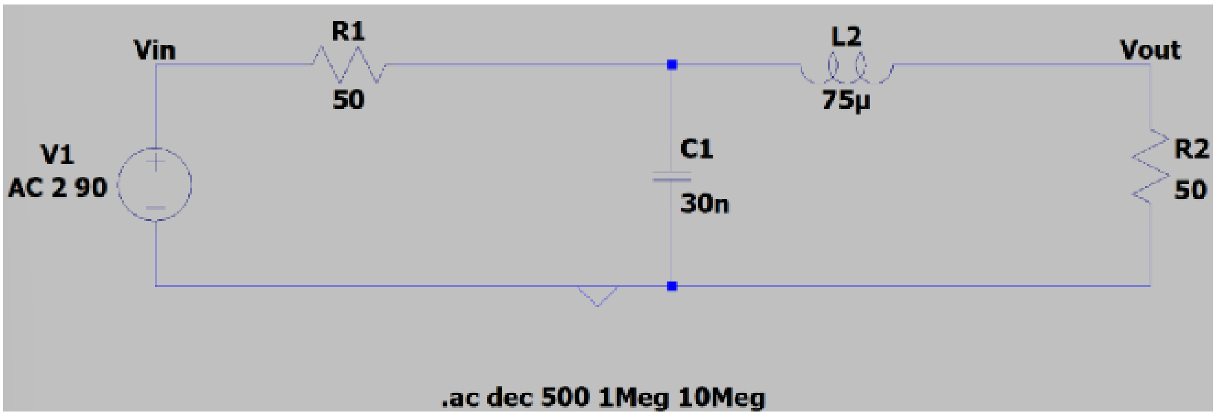

Figure 5: Circuit of the First Step, Low Pass Filter

To find the values for the new circuit elements, the following calculations are going to be made. Here, the angular frequency ω which equals 2πf will use the frequency of 3 MHz.

$$L_2 = \frac{1}{\omega^2 * C}$$

$$C_2 = \frac{1}{\omega^2 * L}$$

Plugging in the found values;

$$L_2 = \frac{1}{(2 * \pi * 3\ MHz)^2 * 30\ pF} = 94\ nH$$

$$C_2 = \frac{1}{(2 * \pi * 3\ MHz)^2 * 75\ \mu H} = 37.7\ pF$$

Making use of these values, implementations of a band pass filter on both LTSpice and hardware will be made, the simulation will reveal the gain directly but the following formula will be used for the hardware implementation, where the voltage values are peak to peak values.

$$Gain = 20\log\left(\frac{V_o}{V_{in}}\right) \tag{2}$$

Simulations

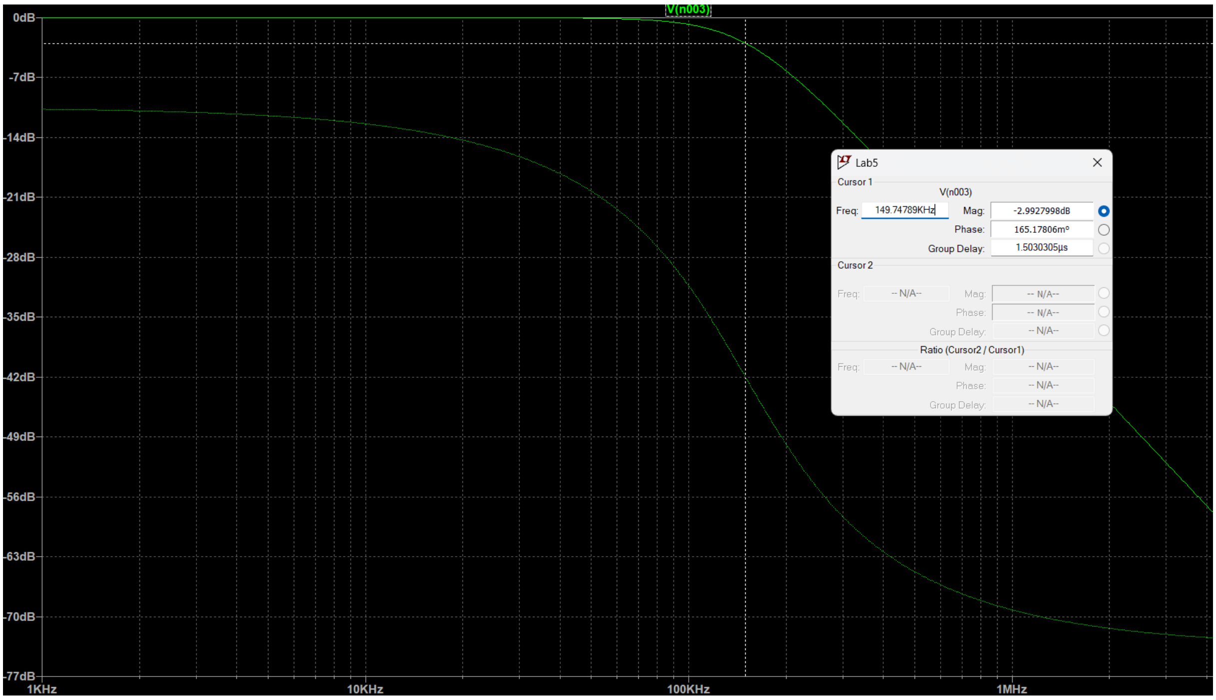

Figure 6: Low Pass Filter Results

Having found that the low pass filter's -3 dB cut off frequency is 150 kHz, the complete second order Butterworth filter is implemented on the simulation.

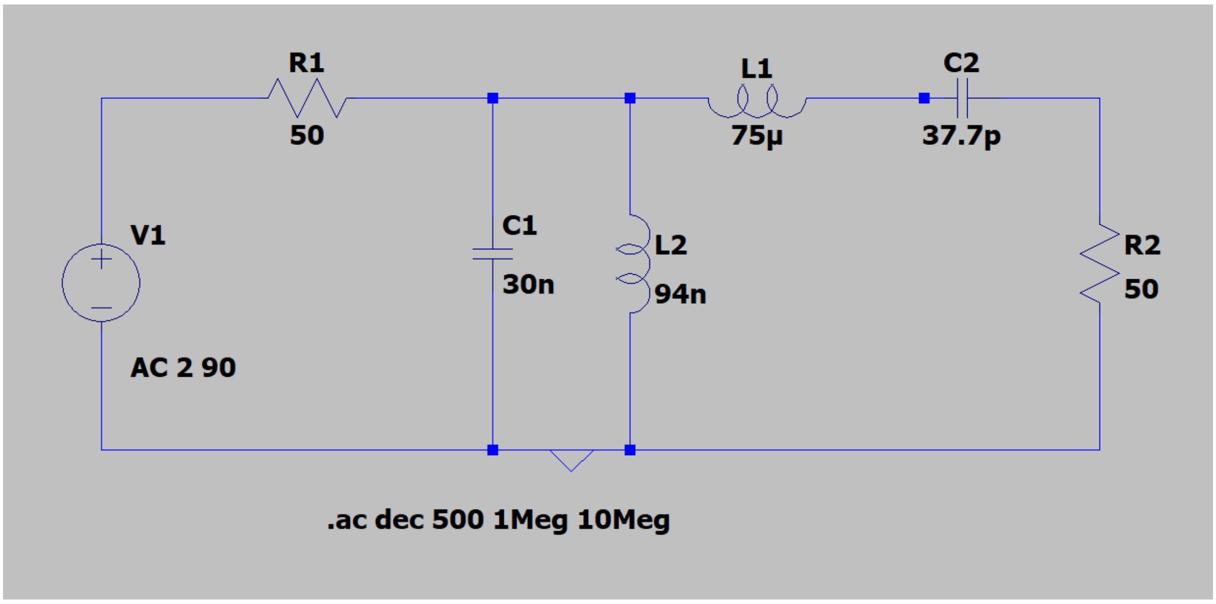

Figure 7: Complete Second Order Butterworth Filter

The gains at 3 MHz (center frequency), cut off frequencies and stopband frequencies are evaluated, and the frequency difference between the cut off points is checked in the following figures.

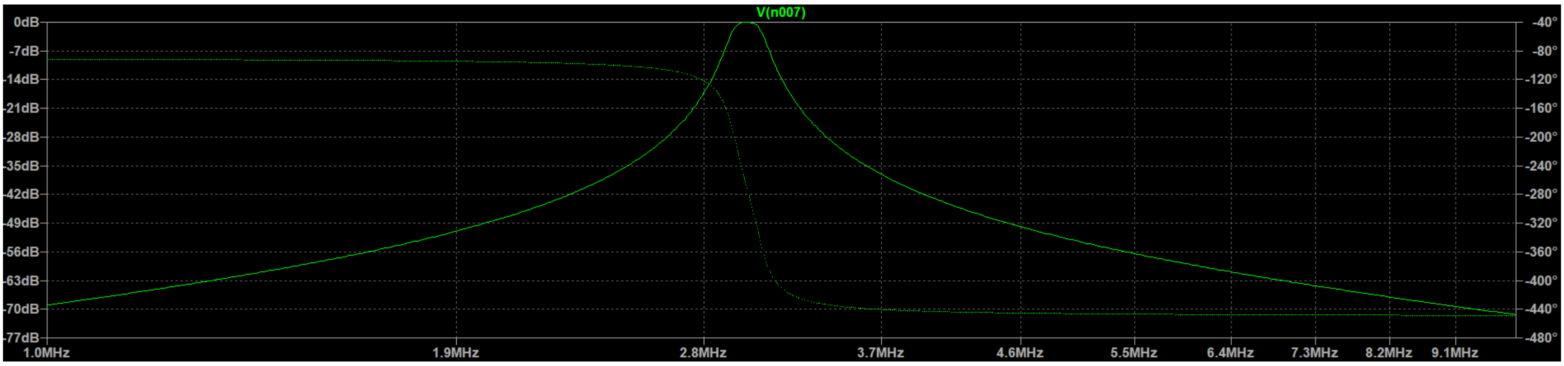

Figure 8: Gain Graph of Load

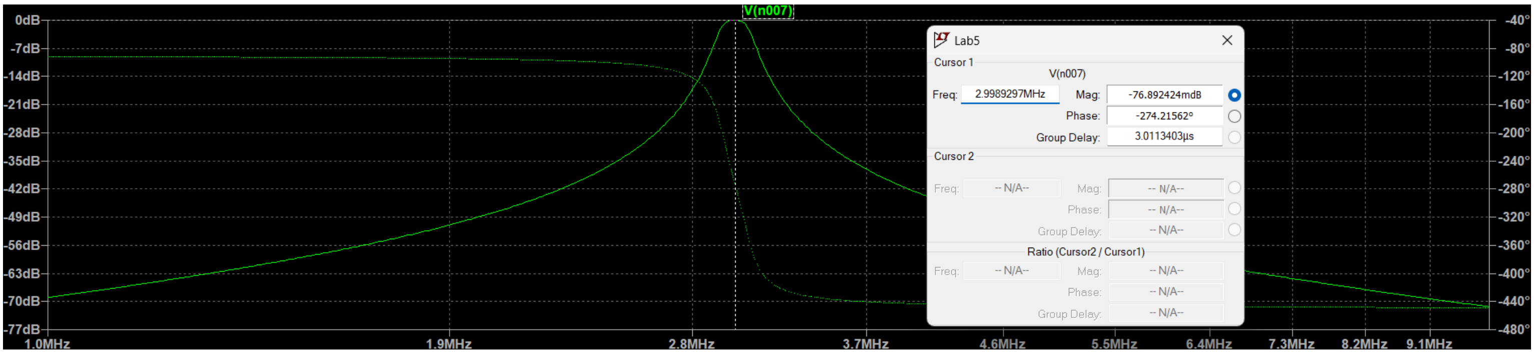

Figure 9: Gain at Center Frequency

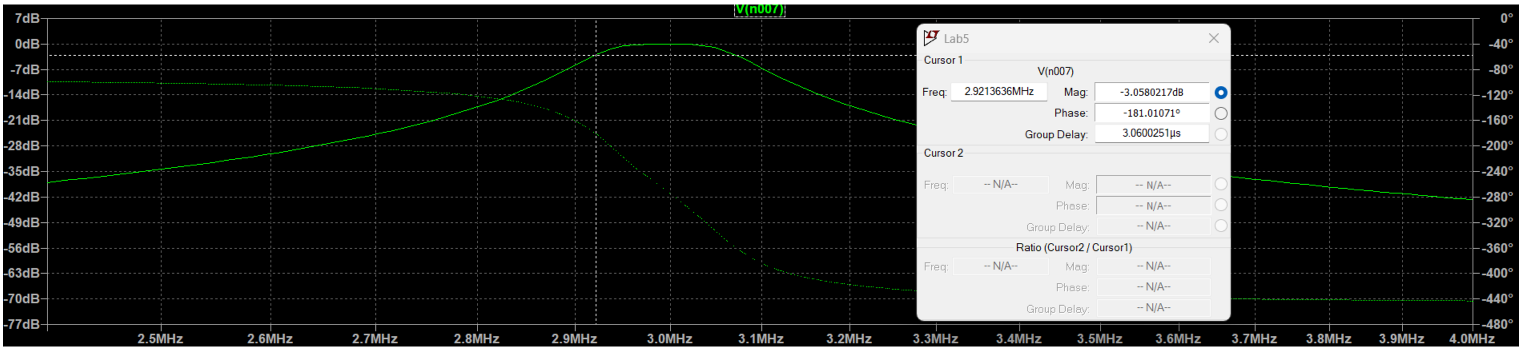

Figure 10: Gain at Lower Cut Off Frequency

Figure 11: Gain at Higher Cut Off Frequency

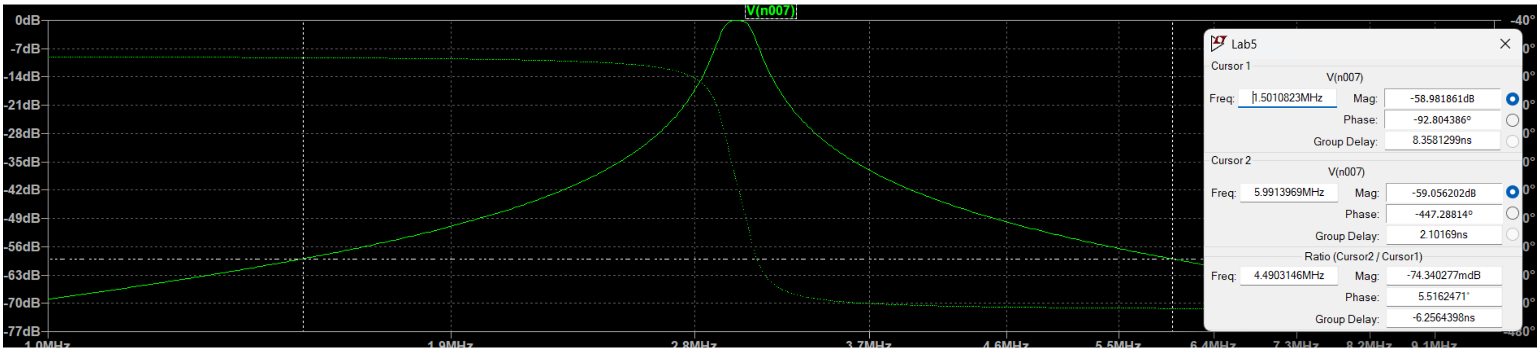

Figure 12: Gains at Stopband Frequencies

	Desired Value	Observed Value	Error
Center Frequency Gain (dB)	-	0.077	Result is as desired.
Passband Width (MHz)	0.15	0.149	0,67%
Gain at Lower Stopband Frequency (dB)	≤-30	-43.98	Result is as desired.
Gain at Higher Stopband Frequency (dB)	≤-30	-45.06	Result is as desired.
Gain Variation of Cut Off Frequencies (dB)	≤3	|-3.05 + 0.077| =2,975	Result is as desired.

Table 1: SW Implementation Results

HW Implementation

In order to obtain the found values for the capacitors and inductors, several components had to be soldered in series or parallel since the found values do not exist in the lab. The 94 nH inductor was obtained by looping a wire around a toroid, and later the whole circuit was soldered together as in Figure a1.

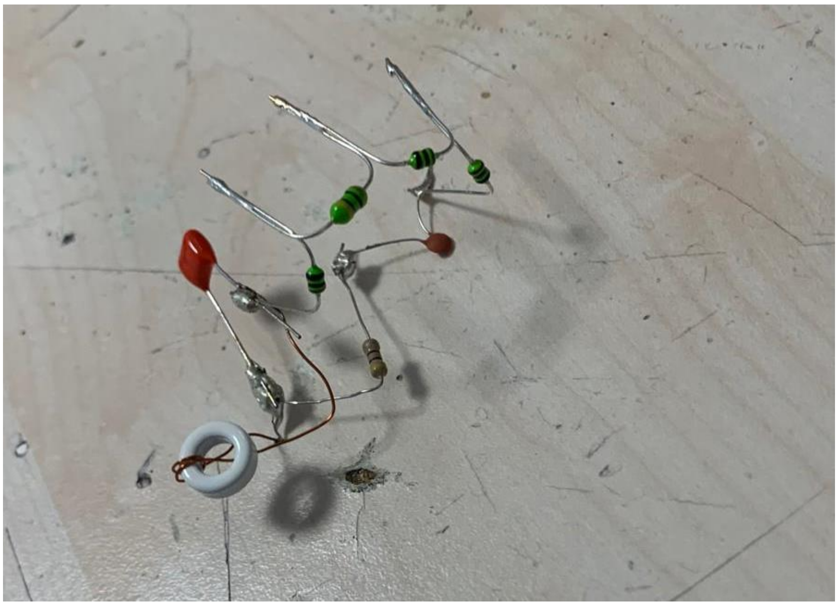

Figure a1: Soldered Circuit

Later, the circuit was put through an input AC voltage and the peak to peak voltages at the load and source resistances were observed on the oscilloscope. The found values were put under formula (2), and the frequency response of the circuit was evaluated.

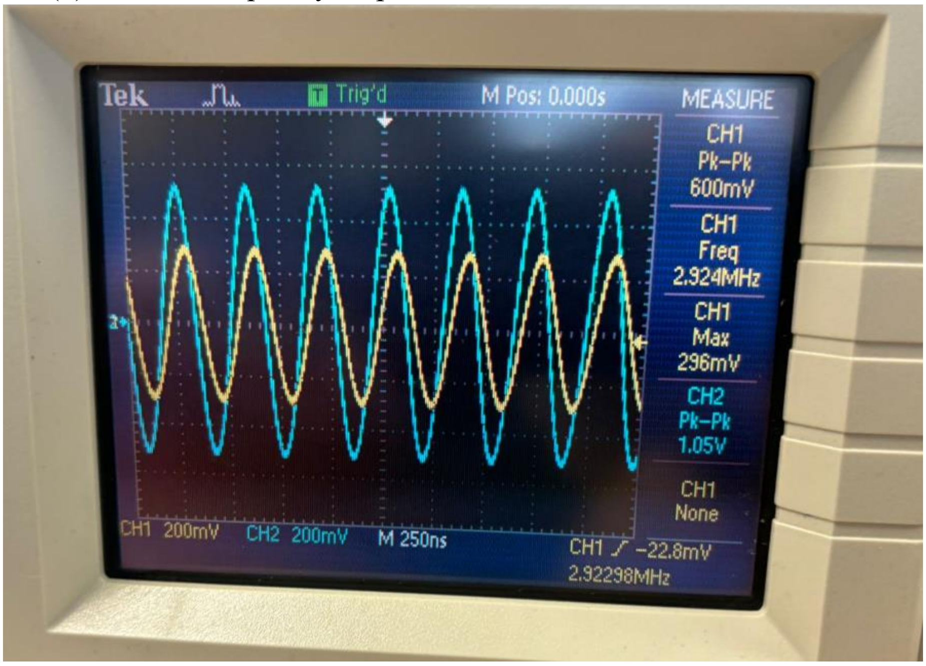

Figure a2: Output at High Cut Off Frequency

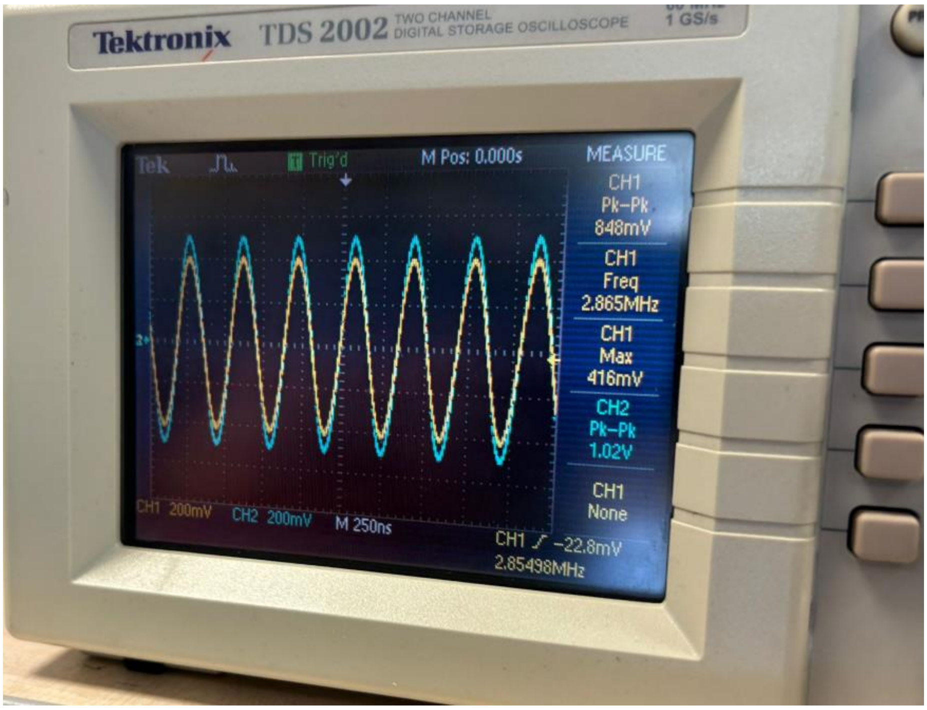

Figure a3: Output at Central Frequency

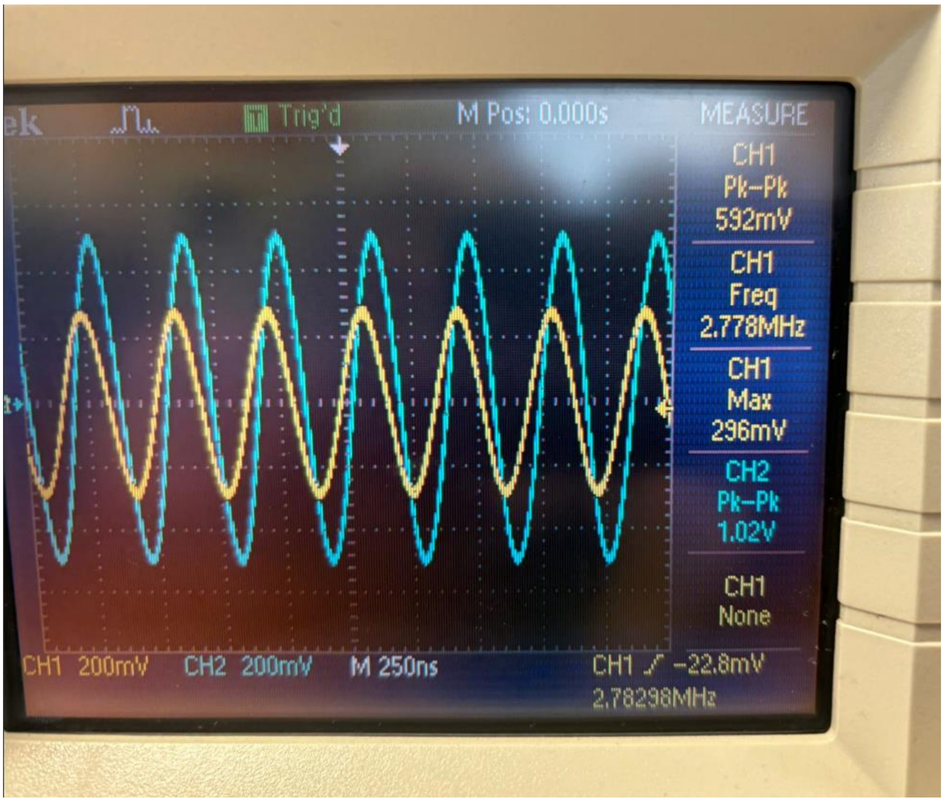

Figure a4: Output at Low Cut Off Frequency

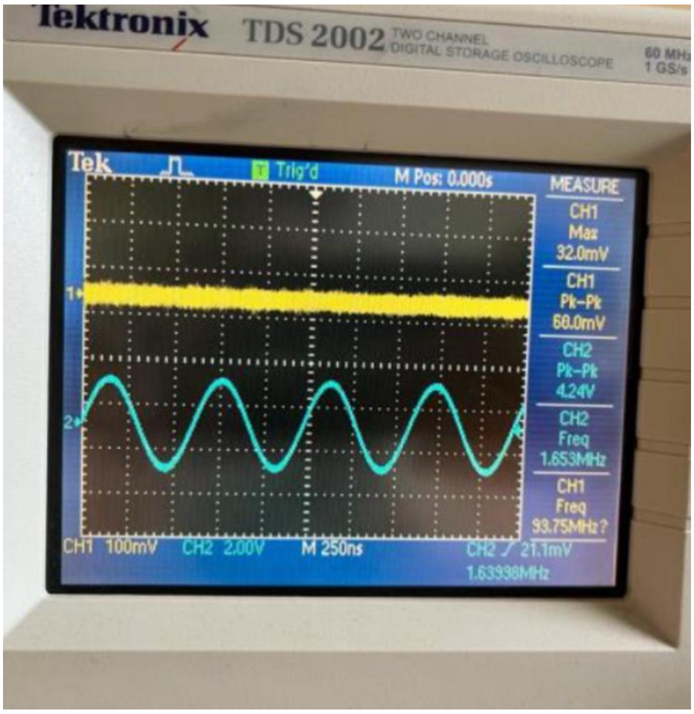

Figure a5: Output at Low Stopband Frequency

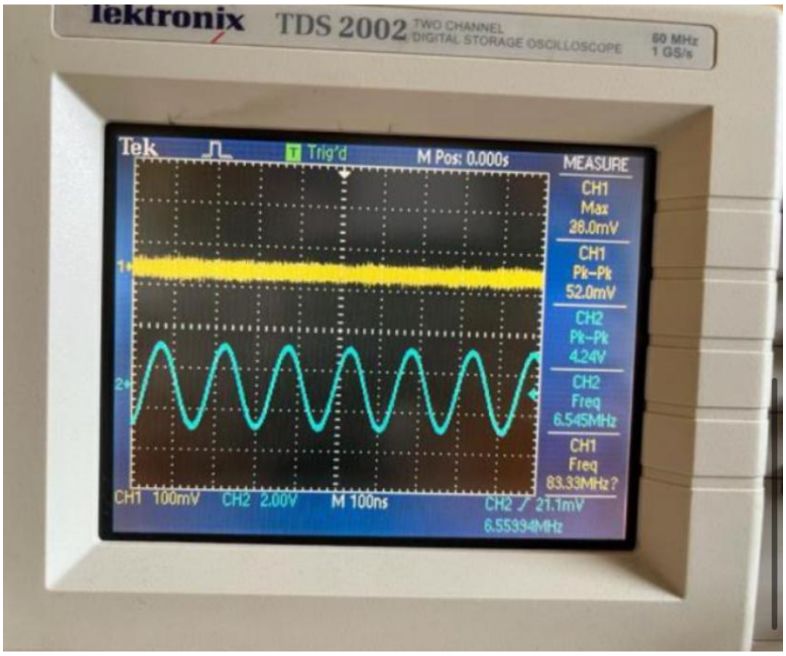

Figure a6: Output at High Stopband Frequency

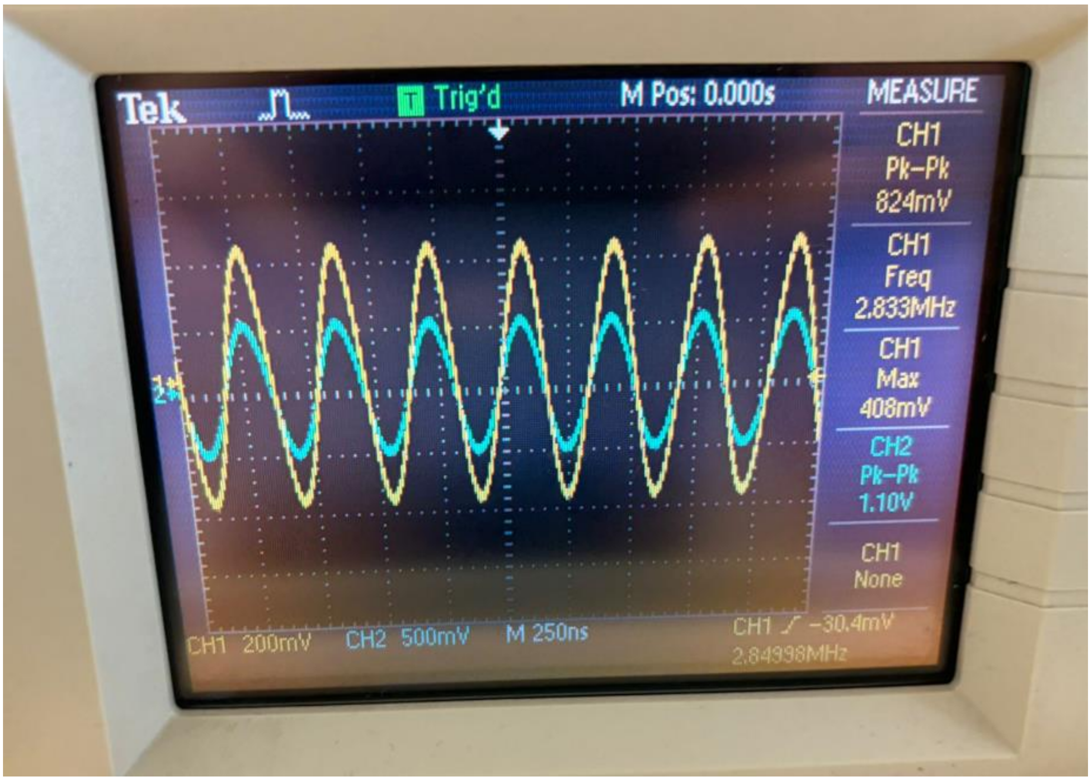

Figure a7: Output at 2.845 MHz

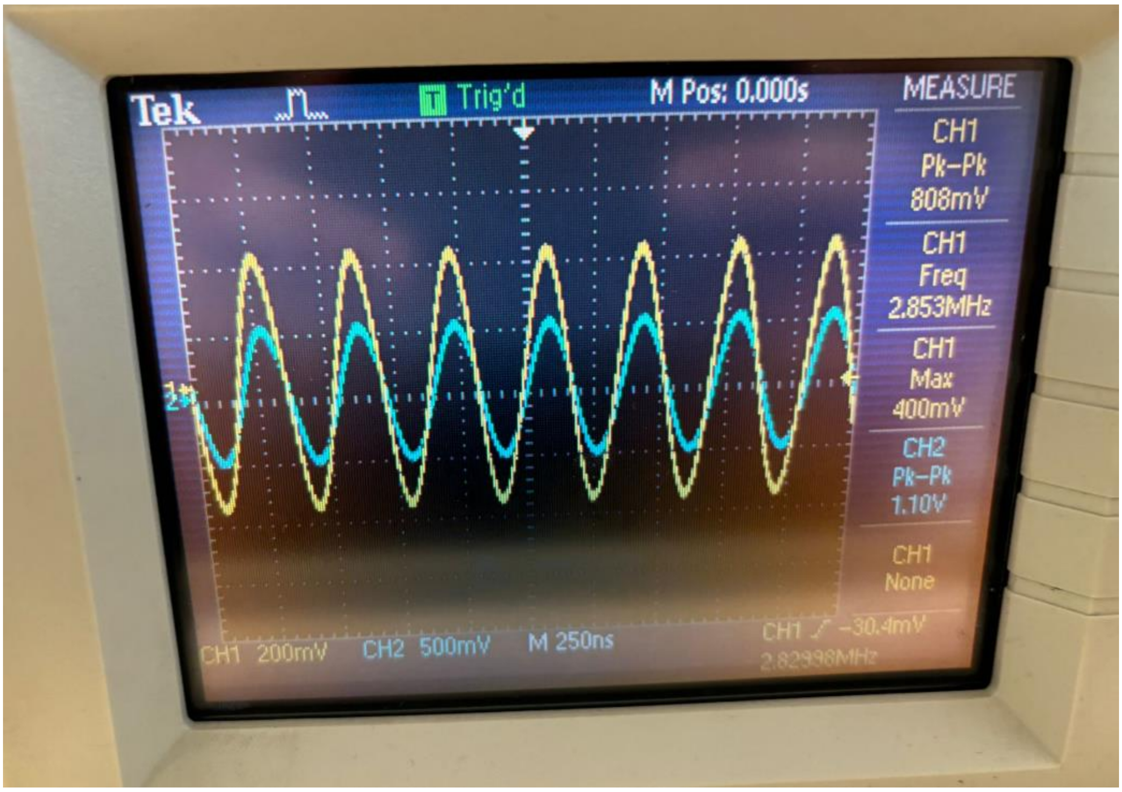

Figure a8: Output at 2.83 MHz

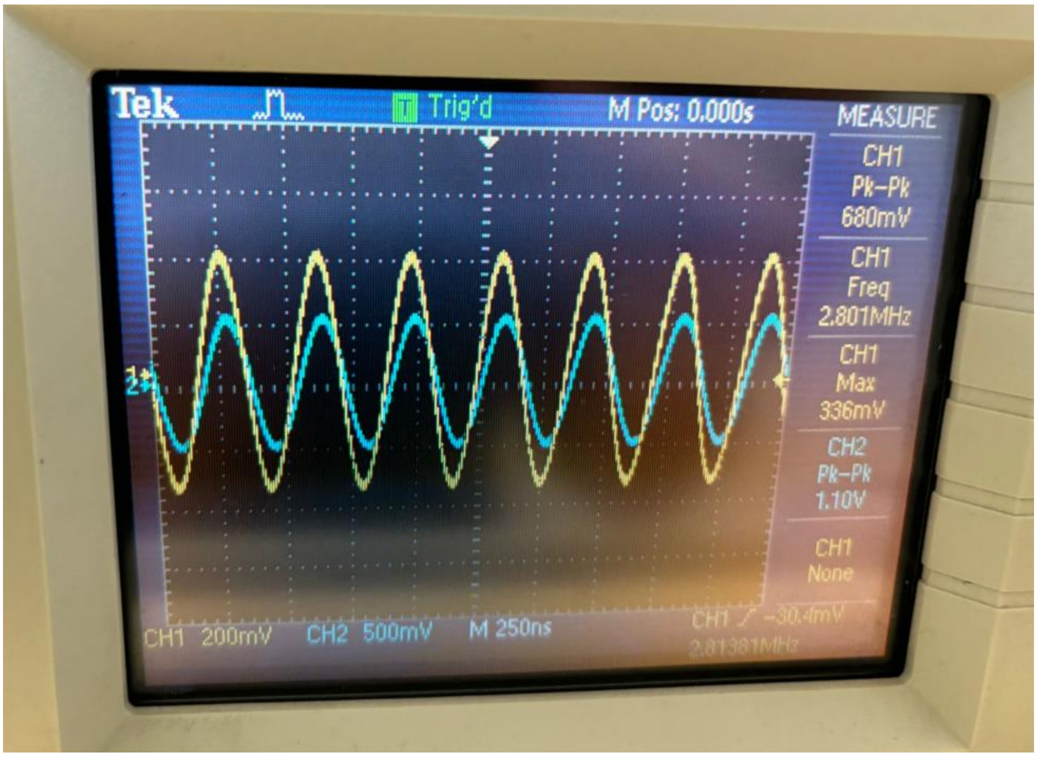

Figure a9: Output at 2.82 MHz

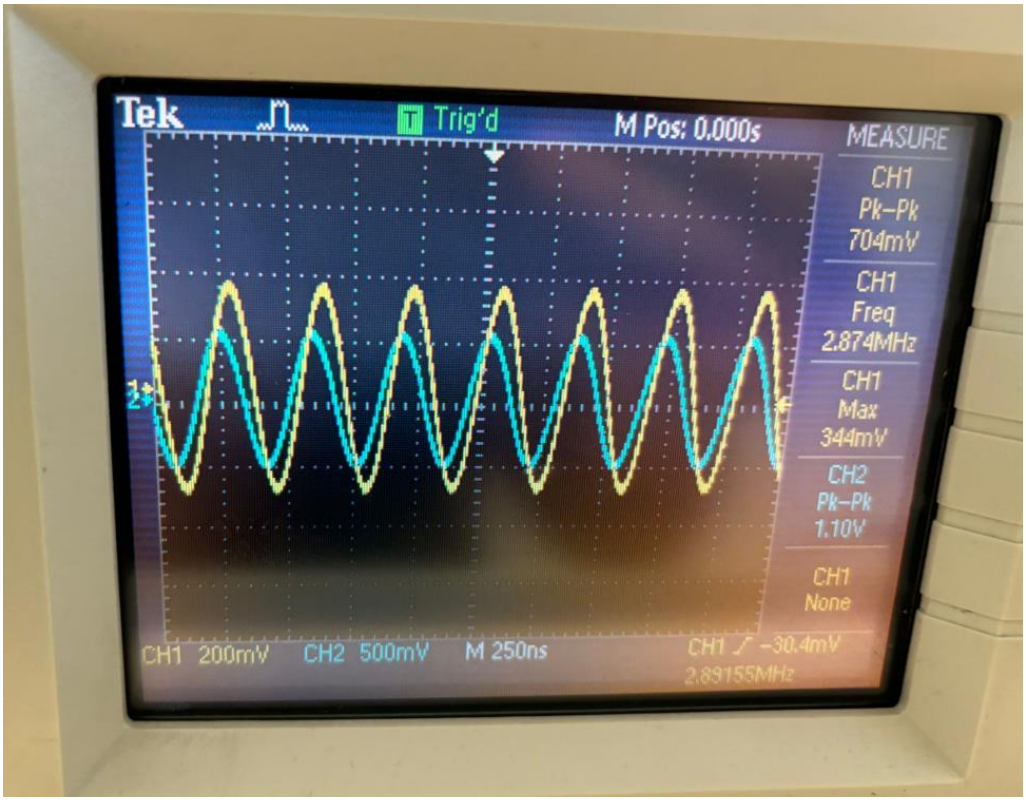

Figure a10: Output at 2.89 MHz

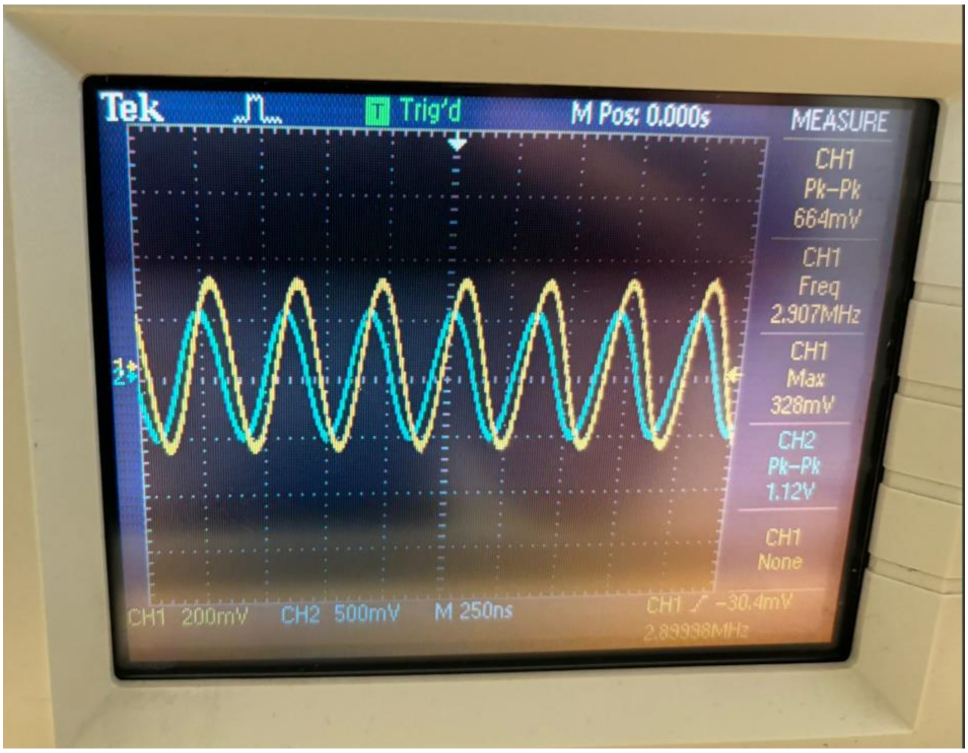

Figure a11: Output at 2.9 MHz

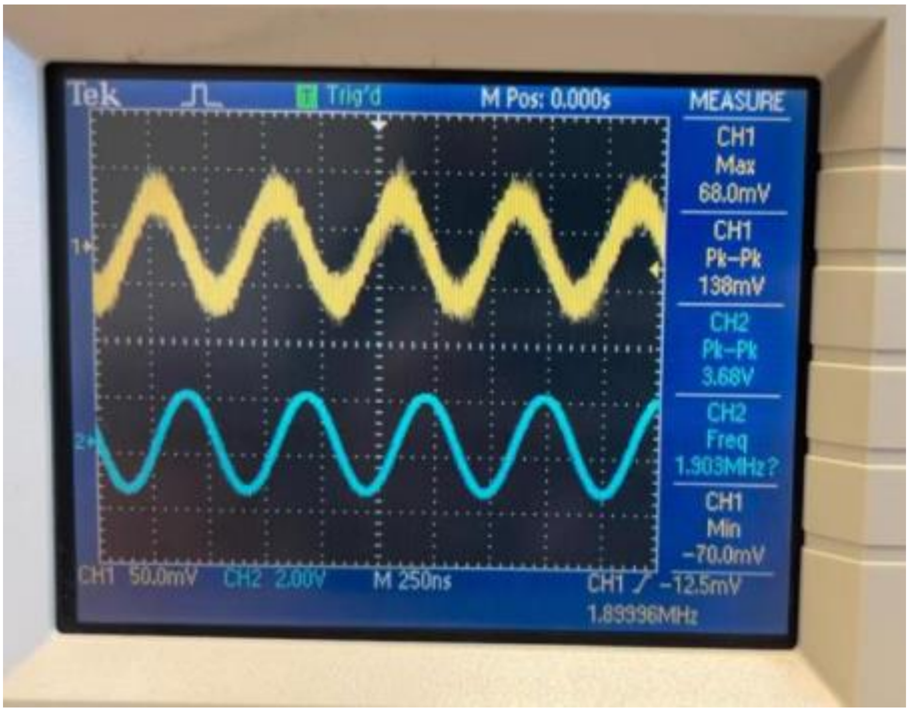

Figure a12: Output at 1.899 MHz

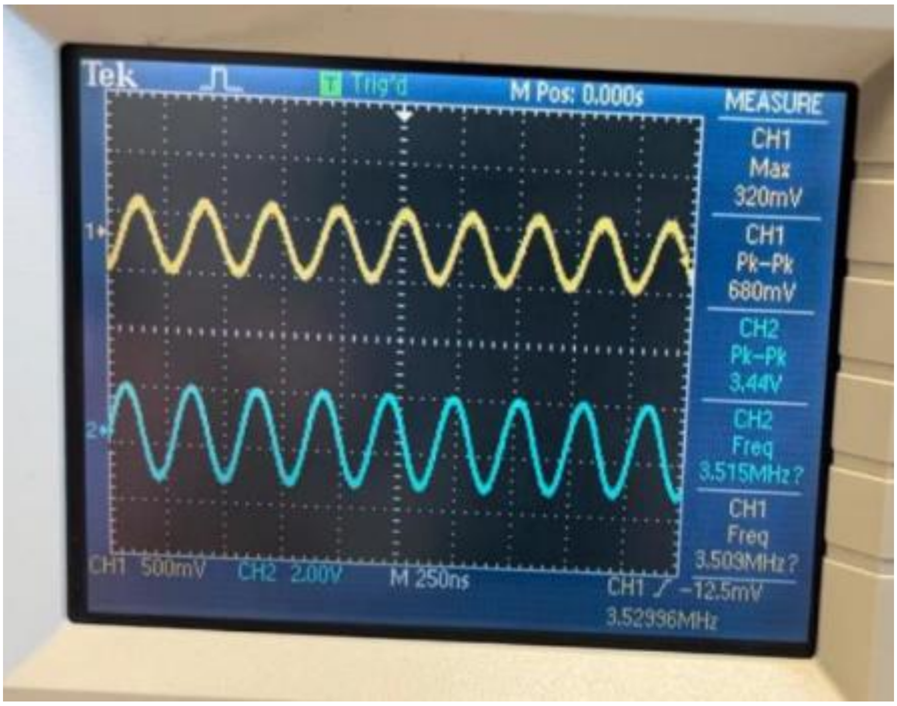

Figure a13: Output at 3.529 MHz

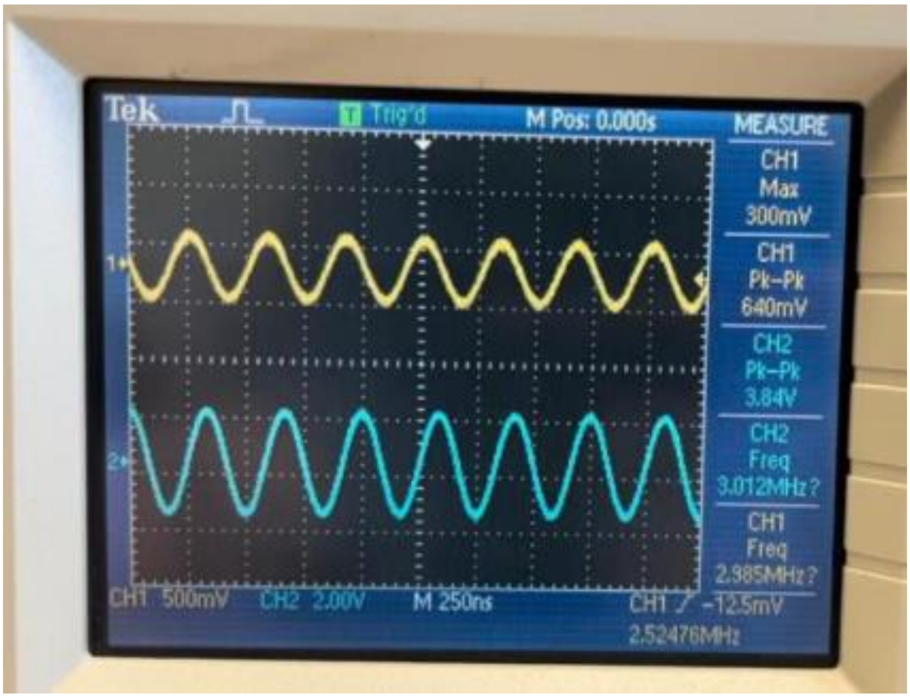

Figure a14: Output at 2.524 MHz

Making use of the observed voltage values and using formula (2), the gain for each frequency in dB is calculated.

Frequency (MHz)	Voltage at Source Resistor (V)	Voltage at Load Resistor (V)	Gain (dB)
2.783 (low cut off)	1.1	0.596	-5.323
2.814	1.1	0.689	-4.063
2.83	1.1	0.808	-2.679
2.845	1.1	0.824	-2.509
2.855 (central)	1.1	0.848	-2.26
2.892	1.1	0.704	-3.876
2.9	1.12	0.664	-4.541
2.923 (high cut off)	1.1	0.6	-5.264

Table 2: Passband Region Results

Frequency (MHz)	Voltage at Source Resistor (V)	Voltage at Load Resistor (V)	Gain (dB)
1.425	4.24	0.06	-36.984
1.899	3.68	0.138	-28.519
2.524	3.84	0.300	-22.144
3.529	3.84	0.680	-15.036
5.7	4.24	0.052	-38.227

Table 3: Stopband Region Results

	Observation Results	Desired Results	Error Percentage
Center Frequency (MHz)	2.855	3	4.83%
Passband Width (MHz)	140	150	6.67%
Gain Variation of Cut Off Frequencies (dB)	|-5.3–(-2.26)| = 3,063	≤3 dB	2.1%
Low Stopband Frequency Gain (dB)	-36.984	≤-30 dB	Result is as desired.
High Stopband Frequency Gain (dB)	-38.227	≤-30 dB	Result is as desired.

Table 4: HW Implementation Results

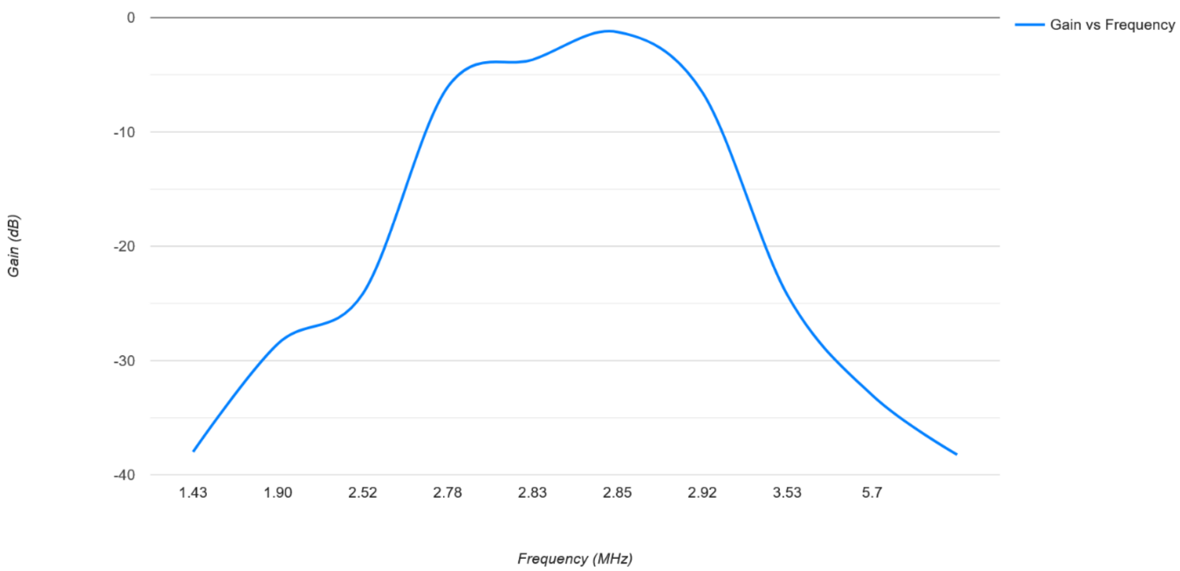

Figure a15: Gain vs Frequency Graph

Conclusion

The software results carried minimal error and the results were exactly as desired and calculated. The selected center frequency of 3 MHz was satisfied, the passband width had the only error of 0.67% and the graphs were as desired. The hardware lab though had higher errors and the center frequency was found as 2.85 MHz carrying a 4.83% error. This might be due to the tolerances and inner resistances of the lab equipment and problems that may have occurred while soldering, all the way to electrical noise in the lab and lab equipment errors.

The center frequency had a gain of -2.26 in the hardware part against 0.077. This meant that the cut off frequencies also had the same difference in between the software and hardware implementations However, the passband width stayed the same and the desired conditions were as asked. The second order Butterworth filter satisfied the desired results and revealed that a tuning process and designing a proper Butterworth filter provides efficient bandpass filters.

The graph at the final had distortions and this could be due to similar reasons in the lab equipment. In order to prevent these one option is creating higher order Butterworth filters, another ones could be further optimizing the hardware circuit by making further analysis of capacitance, inductance and resistance values. This lab was useful in terms of creating bandpass filters in simulations and physical circuitry, and it taught about how they are necessary and useful in engineering.

References

https://www.eeeguide.com/second-order-low-pass-butterworth-filter/

https://www.electronics-tutorials.ws/filter/second-order-filters.html