EEE 431 - Digital Communications

Computational Assignment 2

Due: December 13, 2025, 23:59

In this assignment, you will implement various topics you learned in the class using MATLAB. Whenever a question requires plotting, numerical computation, or simulation, use MATLAB. Certain parts will also require analytical work. Use a fixed random seed in your code so that your results are reproducible. For parts with multiple levels, repeat the experiment and report your results for each level in relevant sections.

Your report should contain all the relevant information about details of your result and your comments. The specific format is up to you, but please properly label each figure, include relevant captions, point to the right results in your explanations, etc. The report should include a title page, a brief introduction, an outline, and any references used. The references used should be cited within the report wherever they are used. The report must be typed using an advanced word processor (e.g. LaTeX, Word, etc.) and submitted as a PDF file on the course Moodle site.

Do your own work for all the parts. Your codes will be checked using software for authenticity.

Please follow the following naming convention while uploading: `{LastName}_{FirstName}.pdf` and `{LastName}_{FirstName}.m` (single `.m` file)

1 Part I

Consider a binary antipodal modulation scheme using a pulse $p(t)$ of duration T, where we use $p(t)$ when the symbol is 0, and $-p(t)$ when the symbol is 1. You are free to choose your own pulse $p(t)$ other than a rectangular pulse. Pick the sampling period as $T_s = T/20$ or a similar value, ensuring that no aliasing occurs.

(a) Obtain an orthonormal signal space representation for this modulation scheme over the interval $[0, T)$. Plot the basis function(s). Represent both $p(t)$ and $-p(t)$ within this signal space.

(b) Based on the signal space, show the optimal correlation receiver's structure. Specify the ML decision rule and derive the theoretical symbol error probability as a function of $\gamma_s = E_s/N_0$ (dB).

(c) Discuss whether the sampling period affects N_0.

(d) Implement the proposed receiver in part (b) and, using it, compute the symbol error probability as a function of γ_s in MATLAB using Monte Carlo simulation with at least 10^6 samples. Repeat the computation for at least 10 uniformly spaced values of γ_s such that the probability of error ranges from 0.5 to 10^{-4}.

(e) Plot both the theoretical and the estimated symbol error probability versus γ_s. Use the `semilogy` command in MATLAB for this. Comment on the results.

(f) Now suppose that the two symbols are not equally likely. Let the probability of generating symbol 1 be 0.25. Show the optimal correlation receiver for this case and specify the correct decision rule. Is the receiver in part (b) still optimal? If not, explain why.

(g) Implement the proposed receiver in part (f) and compute the symbol error probability as a function of γ_s in MATLAB using Monte Carlo simulation with at least 10^6 samples. Repeat the computation for at least 10 uniformly spaced values of γ_s such that the probability of error ranges from 0.5 to 10^-4.

(h) Repeat the simulation using the receiver in part (b) with the symbol distribution given in part (f). Plot both receivers' estimated symbol error probability versus γ_s. Use the `semilogy` command in MATLAB for this. Comment on the results. If there is a difference in performance, explain why.

2 Part II

Consider the following constellation with 4 symbols:

$$\boldsymbol{s}_1 = [A, A]^\top, \boldsymbol{s}_2 = [A, -A]^\top, \boldsymbol{s}_3 = [-A, -A]^\top, \boldsymbol{s}_4 = [-A, 0]^\top, \quad A \in \mathbb{R} .$$

(a) Specify the ML decision rule and derive the union bound on the symbol error probability with respect to $\gamma_s = E_s/N_0$ (dB).

(b) Compute the symbol error probability as a function of γ_s in MATLAB using Monte Carlo simulation with at least 10^6 samples. Repeat the computation for at least 10 uniformly spaced values of γ_s such that the probability of error ranges from 0.5 to 10^{-4}.

(c) Plot both the theoretical union bound and the estimated symbol error probability versus γ_s. Use the `semilogy` command in MATLAB for this. Comment on the results.

(d) Suppose that we change the last symbol in the constellation as $\boldsymbol{s}_4 = [-A, A]^\top$. Repeat the parts (a)-(c) for this constellation. Comment on the results. Is one of the constellations better than the other one in terms of performance?

(e) Using the new constellation, compute the bit error probability as a function of $\gamma_b = E_b/N_0$ (dB) in MATLAB using Monte Carlo simulation with at least 10^6 samples. Repeat the computation for at least 10 uniformly spaced values of γ_b such that the probability of error ranges from 0.5 to 10^{-4}.

(f) Repeat part (e) using Gray coding. Plot both the estimated bit error probabilities for natural and Gray coding versus γ_b. Use the `semilogy` command in MATLAB for this. Comment on the results. If there is a difference in performance, explain why.