EEE 431 - Digital Communications

Computational Assignment 1

Due: November 16, 2025, 23:59

In this assignment, you will implement various topics you learned in the class using MATLAB. Whenever a question requires plotting, numerical computation, or simulation, use MATLAB. Certain parts will also require analytical work. Use a fixed random seed in your code so that your results are reproducible. For parts with multiple levels, repeat the experiment and report your results for each level in relevant sections.

Your report should contain all the relevant information about details of your result and your comments. The specific format is up to you, but please properly label each figure, include relevant captions, point to the right results in your explanations, etc. The report should include a title page, a brief introduction, an outline, and any references used. The references used should be cited within the report wherever they are used. The report must be typed using an advanced word processor (e.g. LaTeX, Word, etc.) and submitted as a PDF file on the course Moodle site.

Do you own work for all the parts. Your codes will be checked using software for authenticity.

Please follow the following naming convention while uploading: {LastName}_{FirstName}.pdf and {LastName}_{FirstName}.m (single .m file)

1   Part I

Let X be discrete memoryless source whose whose samples are generated according to the following probability density function (PDF):
$$f_X(x) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Suppose that you have access to a uniform random variable $U \sim \text{Uni}[0, 1]$ on the unit interval. Determine a function $g(\cdot)$ such that $X = g(U)$ has $f_X(x)$ as its PDF.

(b) Using $g(\cdot)$, generate 100 realizations of X in MATLAB starting with U. Plot the histogram of realizations and compare it to $f_X(x)$. Repeat the process for 100000 realizations. Compare the two histograms and comment on how they differ.

(c) Consider uniform quantizers with $N \in \{8, 16, 64\}$ levels on the interval $[-1, 1]$. Calculate the corresponding MSE distortion values, and the resulting signal to quantization noise ratio (SQNR) analytically.

(d) Perform uniform quantization on MATLAB using the previously generated 100000 realizations of X. Compare the resulting MSE distortion values and the SQNR with results in the previous part. Comment on your findings.

(e) Determine the probability of each possible quantization output based on their frequency of occurrence and plot the resulting probability mass function (PMF). What do you observe?

(f) Implement Lloyd-Max quantizer with $N \in \{8, 16, 64\}$ levels (using the iterative approach described in class and in the textbook). Explain the implementation.

(g) Perform Lloyd-Max quantization on MATLAB using the previously generated 100000 realizations of X. Compare the resulting MSE distortion values and the SQNR with with the uniform quantization experiment.

(h) Determine the probability of each possible quantization output based on their frequency of occurrence and plot the resulting PMF. Compare with the uniform quantization experiment.

2   Part II

Choose one grayscale image (preferably close to 1:1 resolution, width $\approx$ height) for this part and load it on MATLAB. Let W and H denote the width and height of the image in terms of pixels, respectively. Repeat this part for `foliage.tif` image on Moodle as well.

(a) Plot the histograms of the pixel values and generate a third or fourth order polynomial to fit the histogram. Then, normalize the resulting function to estimate the PDF of the pixel values of the source image. If the polynomial fit results in negative values for some range of pixel values, then set these to a small but positive constant.

(b) Perform uniform quantization on MATLAB with $N \in \{2, 4, 8, 16, 32\}$ levels to quantize the pixel values. Plot the quantized images and compute the resulting squared error distortions. Estimate the resulting SQNRs. Plot SQNR versus quantization levels.

(c) Compute 2D FFT of the original image on MATLAB and plot it. Repeat this for quantized versions. How does the quantization noise appear in the spectrum?

(d) For $N = 8$, apply the following low-pass filter to the quantized image:

$$H(u, v) = \begin{cases} 1, & |u| < 0.1 \times W, |v| < 0.1 \times H, \\ 0, & \text{otherwise} \end{cases}$$

where u and v are spatial frequency components. Plot the resulting image. Apply this filter to the original image as well. Compare the two resulting images and comment on the results.