EEE431 – Computational Assignment 1

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Digital Communications – Quantization and Image Processing Homework

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Contents

Introduction

This homework consists of two major parts. In Part I, I simulate a triangularly distributed random variable, apply uniform and Lloyd-Max quantizers, compute distortion and SQNR, and analyze the PMF of the quantized outputs. In Part II, I work on two grayscale images, fit polynomial PDFs to histograms, apply scalar quantization, visualize the SQNR, inspect FFT magnitude spectra, and finally compare low-pass filtered versions of the images.

Throughout the work, I followed the theoretical steps introduced in the lectures and verified numerical results using MATLAB simulations.

1 Part I: Scalar Source and Quantization

1.1 Definition of the Source

The given scalar source has the triangular PDF

$$f_X(x) = \begin{cases} 1 - |x|, & |x| \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

This PDF is symmetric, peaks at $x = 0$, and linearly tapers to zero at $x = \pm 1$.

1.2 Part (a): Inverse Transform Sampling

I first derived the CDF. For $x \in [-1, 0]$, $f_X(t) = 1 + t$, so

$$F_X(x) = \int_{-1}^{x} (1 + t)\, dt = \frac{(x+1)^2}{2}.$$

For $x \in [0, 1]$, $f_X(t) = 1 - t$, and using continuity at 0,

$$F_X(x) = \frac{1}{2} + \int_{0}^{x} (1 - t)\, dt = 1 - \frac{(1-x)^2}{2}.$$

Thus,

$$F_X(x) = \begin{cases} 0, & x < -1, \\ \frac{(x+1)^2}{2}, & -1 \le x \le 0, \\ 1 - \frac{(1-x)^2}{2}, & 0 \le x \le 1, \\ 1, & x > 1. \end{cases}$$

The inverse transform consists of solving $U = F_X(x)$:
For $U \le 0.5$: $x = -1 + \sqrt{2U}$.
For $U > 0.5$: $x = 1 - \sqrt{2(1-U)}$.
This yields the sampling function implemented in MATLAB.

1.3 Part (b): Histograms vs. Theoretical PDF

Using 100 and 100000 samples generated with the inverse CDF, I compared empirical histograms with the true triangular PDF.

The 100-sample histogram is noisy, whereas the 100000-sample one aligns nearly perfectly with the theoretical PDF, showing the law of large numbers at work.

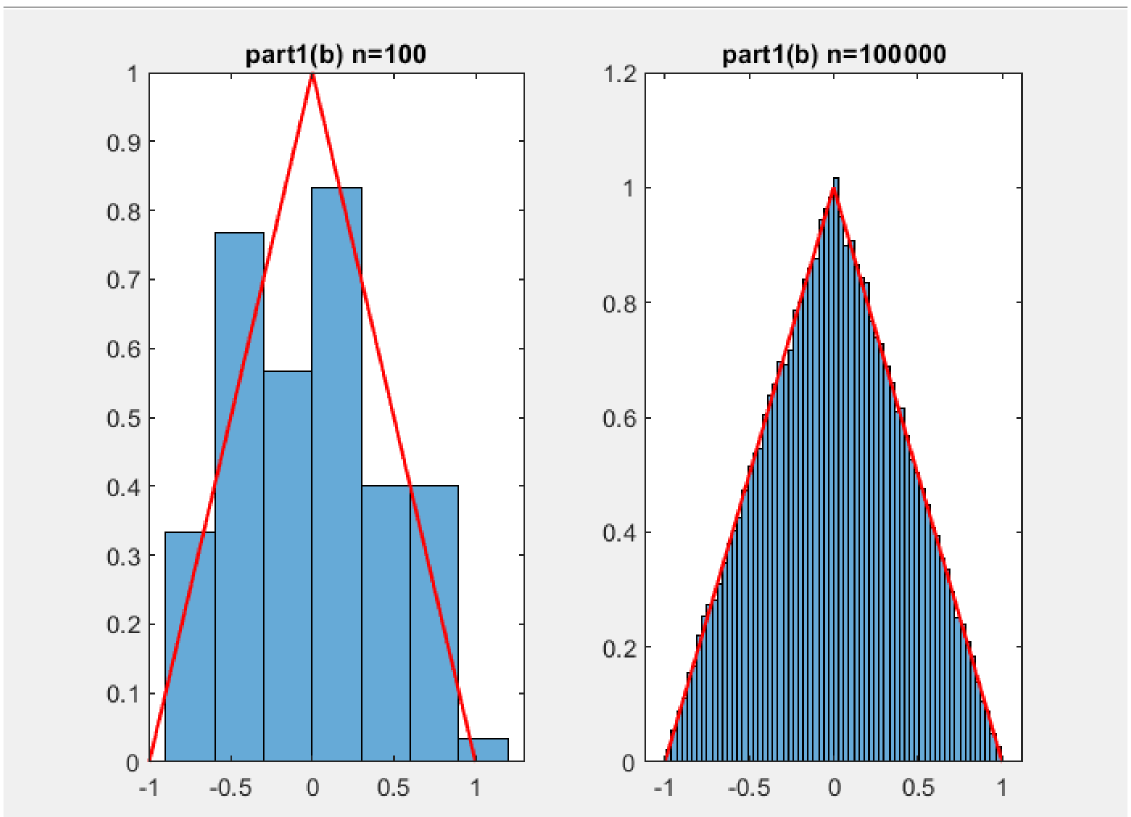

Figure 1: Histograms for 100 and 100000 samples versus the true triangular PDF.

1.4 Part (c): Analytical Distortion and SQNR for Uniform Quantization

For a mid-tread uniform quantizer with N levels over $[-1, 1]$:
Step size: $\Delta = 2/N$.
Analytical MSE:

$$D = \frac{1}{3N^2}.$$

Signal power:

$$P_X = \mathbb{E}[X^2] = \frac{1}{6}.$$

SQNR:

$$\mathrm{SQNR} = \frac{P_X}{D} = \frac{N^2}{2}, \qquad \mathrm{SQNR}_{\mathrm{dB}} = 10 \log_{10} \left(\frac{N^2}{2} \right).$$

Table 1: Analytical results for uniform quantization.

N	D_{analytic}	$\mathrm{SQNR}_{\mathrm{analytic}}$ (dB)
8	5.2083×10^{-3}	15.05
16	1.3021×10^{-3}	21.07
64	8.1380×10^{-5}	33.11

1.5 Part (d): Simulated Quantization

Using 100000 samples, I applied the uniform quantizer via MATLAB's `histcounts`. The simulated SQNR matched the analytical values extremely well.

Table 2: Uniform quantization analytical vs simulated.

N	D_{analytic}	D_{sim}	$\mathrm{SQNR}_{\mathrm{analytic}}$	$\mathrm{SQNR}_{\mathrm{sim}}$
8	0.005 208	0.005 206	15.05	15.03
16	0.001 302	0.001 300	21.07	21.06
64	0.000 081	0.000 081	33.11	33.09

1.6 Part (e): PMF of Uniform Quantizer Outputs

Below is the PMF computed from sample counts for $N = 16$. The non-uniform shape reflects the source's triangular PDF.

1.7 Part (f): Lloyd-Max Algorithm

The Lloyd-Max algorithm iteratively:

1. Computes decision boundaries b_i.
2. Assigns each sample to a region.
3. Updates codewords c_i as the centroid of each region.

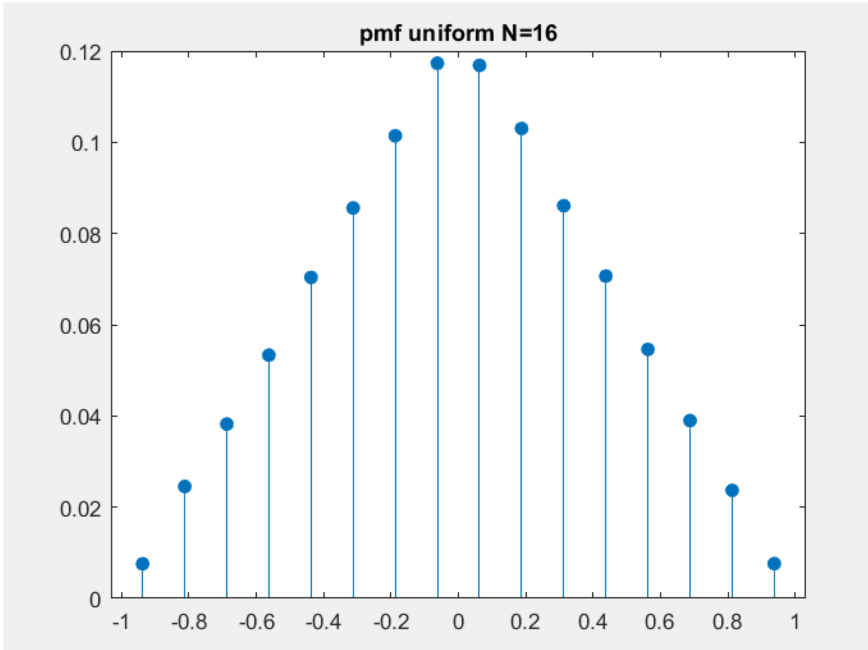

Figure 2: Uniform PMF ($N = 16$).

4. Checks convergence.

The process stops when the relative change drops below 10^{-6}.

1.8 Part (g): Lloyd-Max Performance

Empirical results show up to nearly 1 dB improvement compared to uniform quantization.

Table 3: Uniform vs Lloyd-Max quantizer performance.

N	D_{unif}	D_{LM}	$\text{SQNR}_{\text{unif}}$ (dB)	SQNR_{LM} (dB)
8	0.005 206	0.004 127	15.03	16.04
16	0.001 300	0.001 061	21.06	21.94
64	0.000 081	0.000 078	33.09	33.25

1.9 Part (h): PMF of Lloyd-Max Outputs

The PMF for the Lloyd-Max quantizer ($N = 16$) is visibly ”flatter” than the uniform PMF, as the algorithm adapts to the source distribution.

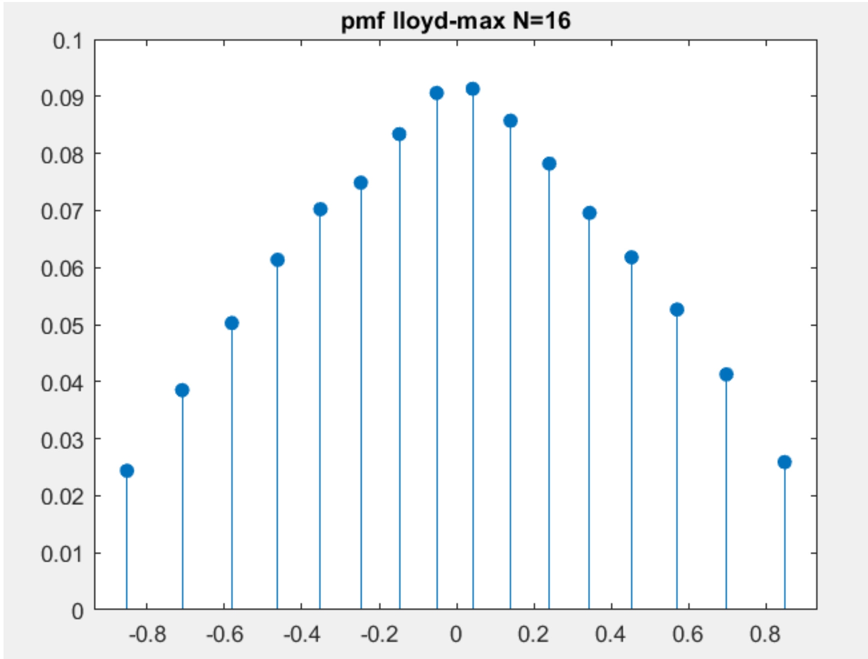

Figure 3: Lloyd-Max PMF ($N = 16$).

2 Part II: Image Quantization and FFT

I used two images: `flower.jpg` and `foliage.tif`. Both were converted to grayscale and cast to `double`.

2.1 Part (a): Histogram and Polynomial Fit

I computed the histogram of 256 levels, normalized it, and fitted a 4th-order polynomial PDF.

2.2 Part (b): Uniform Quantization of Images

Quantized versions for $N \in \{2, 4, 8, 16, 32\}$ are shown below.

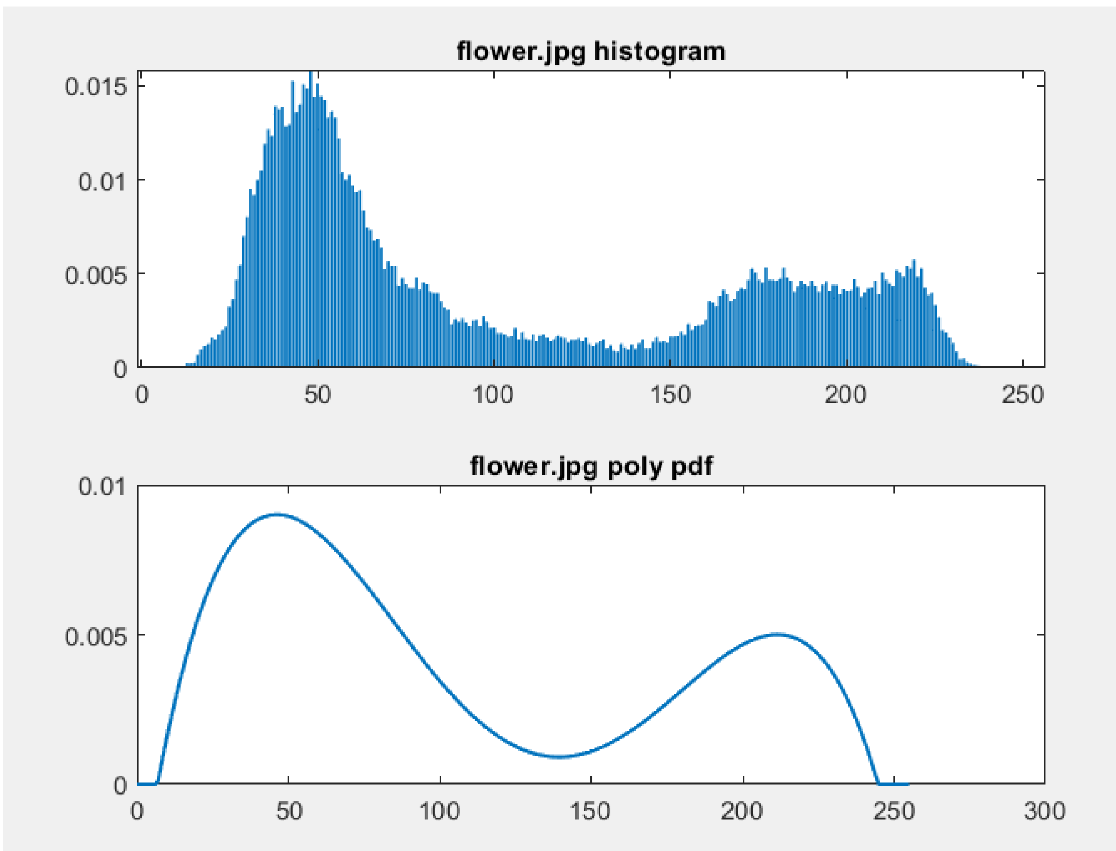

Figure 4: Histogram and 4th-order polynomial PDF fit (`flower.jpg`).

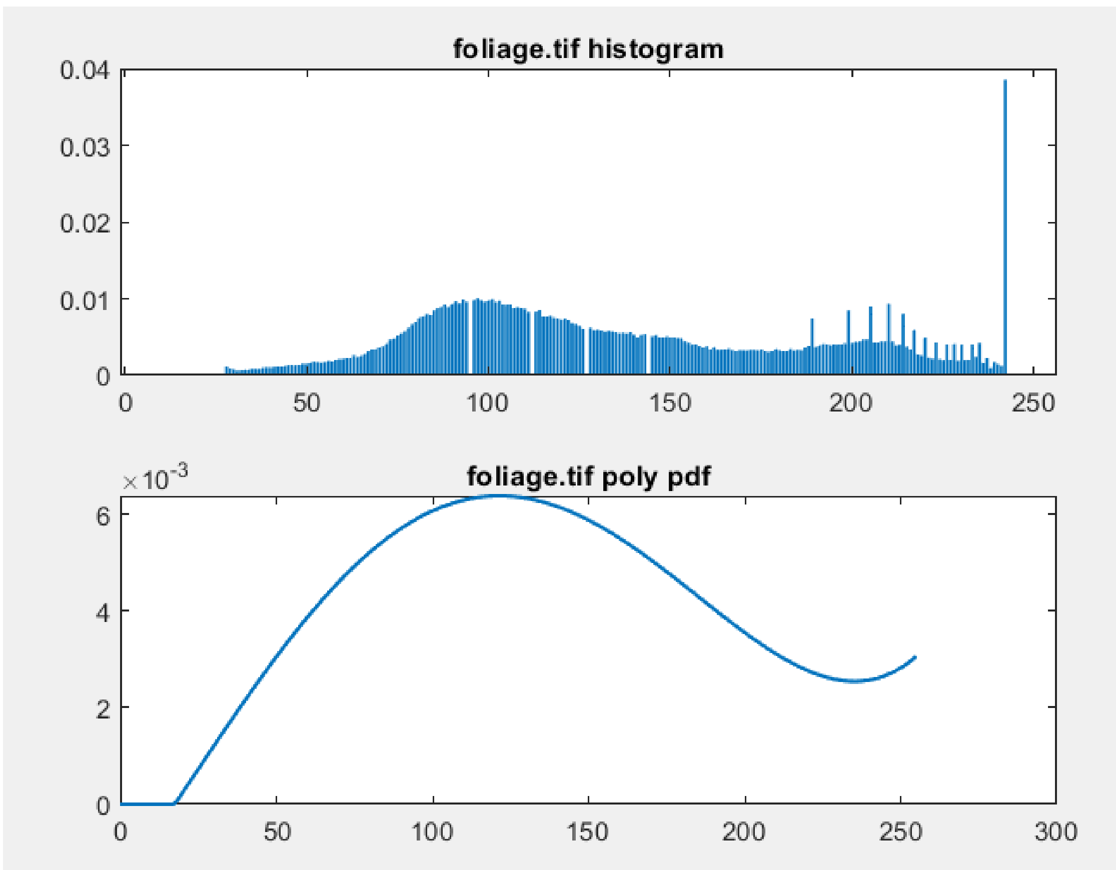

Figure 5: Histogram and 4th-order polynomial PDF fit (`foliage.tif`).

Figure 6: Quantized `flower.jpg` for $N = 2, 4, 8, 1tran - sform : 16, 32$.

Figure 7: Quantized `foliage.tif` for $N = 2, 4, 8, 16, 32$.

The SQNR plot for both images shows a consistent ≈ 6 dB gain per bit, as expected.

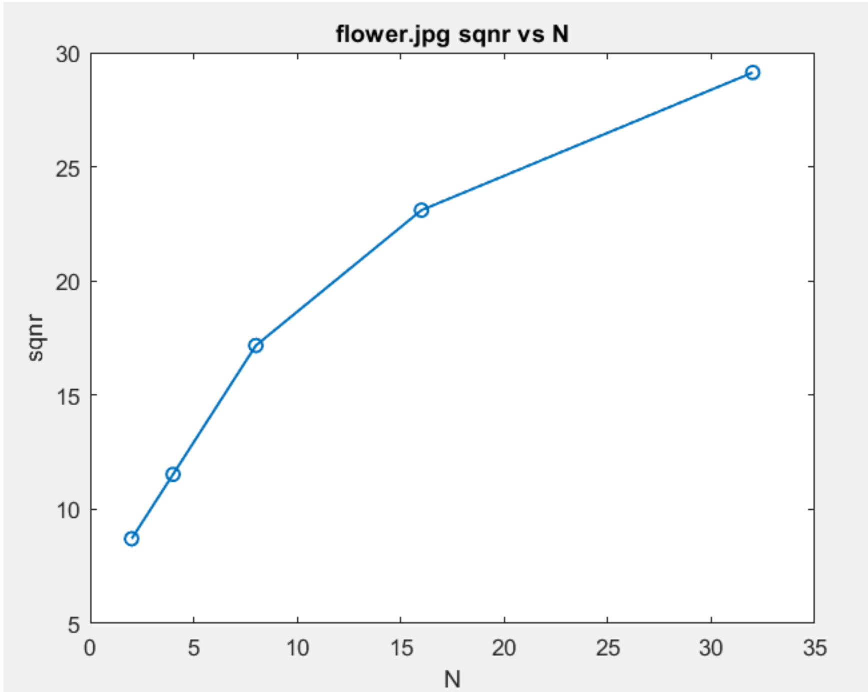

Figure 8: SQNR vs. number of levels (`flower.jpg`).

2.3 Part (c): FFT Magnitude Spectra

I applied FFT to the original and quantized images. Coarse quantization ($N = 2, N = 4$) clearly injects noise across the spectrum.

2.4 Part (d): Low-Pass Filtering

I applied a rectangular low-pass mask ($|u| < 0.1W$, $|v| < 0.1H|$) to both the original and $N = 8$ images. The filter blurs the image but also effectively removes the high-frequency quantization noise, making the filtered quantized image visually closer to the filtered original.

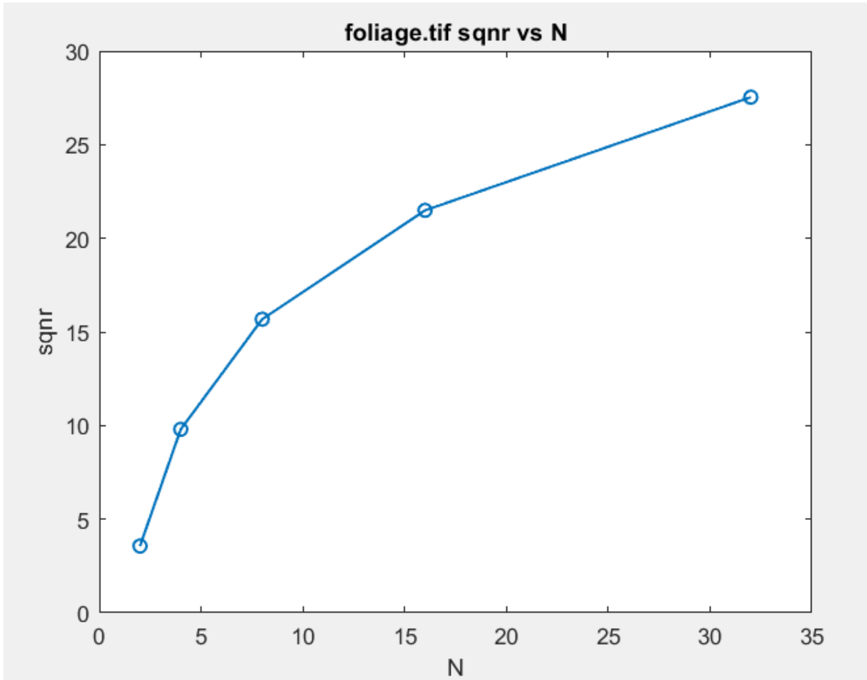

Figure 9: SQNR vs. number of levels (`foliage.tif`).

Conclusion

In Part I, the triangular distribution was successfully generated using inverse transform sampling. Uniform quantizer analytical SQNR matched simulations, confirming correctness. Lloyd-Max consistently improved performance, especially for low N.

In Part II, I analyzed image quantization effects. SQNR improved almost linearly in dB with $\log_2(N)$, matching theory. FFT magnitude spectra clearly showed that coarse quantization injects strong high-frequen-cy noise. Low-pass filtering removed these components and made quantized images visually closer to the original, at the cost of blur.

Overall, this computational assignment demonstrated how quantization relates theory and practice in both 1-D random variables and 2-D images.

References

- Lecture notes for EEE431, Fall 2025.

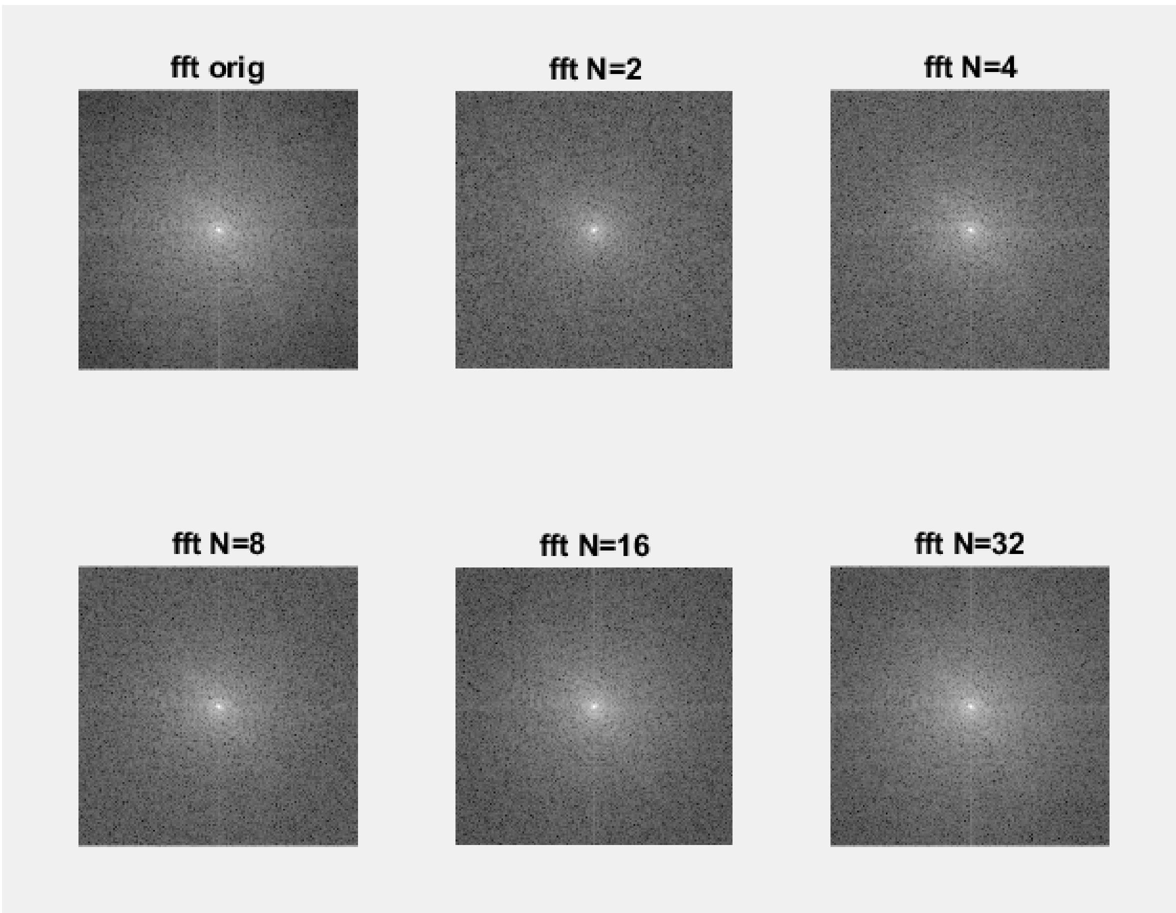

Figure 10: FFT magnitude spectra for `flower.jpg` and quantized versions.

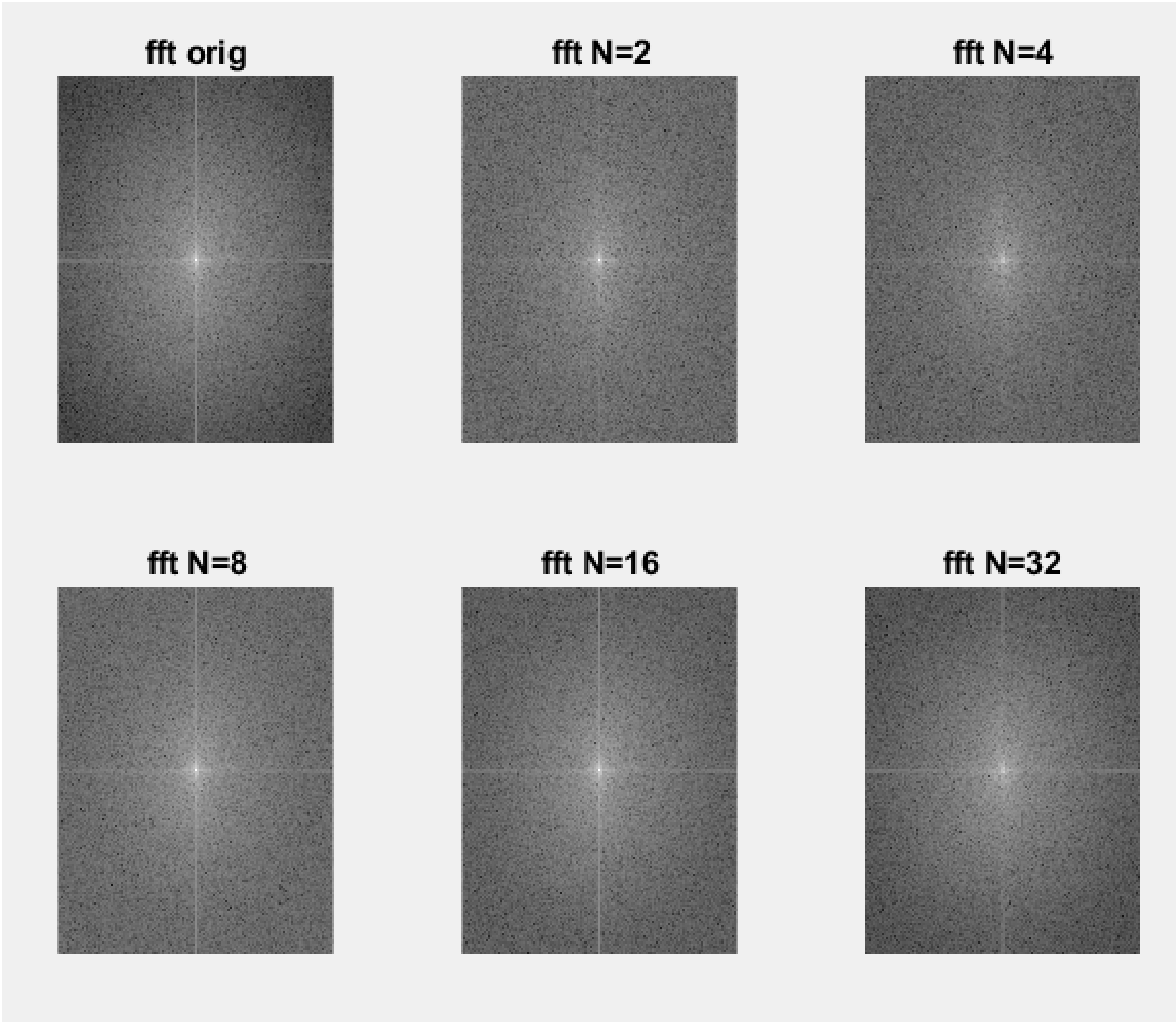

Figure 11: FFT magnitude spectra for `foliage.tif` and quantized versions.

Figure 12: LPF applied to original and $N = 8$ quantized image (`flower.jpg`).

Figure 13: LPF applied to original and $N = 8$ quantized image (`foliage.tif`).