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MATLAB CODE

Numeric Part

```
clc
clear all
close all


for q = 1:6
    if q == 1
        conv1 = convolution(@h, @s1, 16);

        figure()
        stem(0:29, conv1(41:70));
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['y', num2str(q), '[n]'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of y', num2str(q), '[n]'], 'FontSize', 14, 'FontWeight', 'bold');

        disp(['y', num2str(q), '[n]: ', num2str(conv1(41:70))]);


    elseif q == 2
        conv2 = convolution(@h, @s2, 16);

        figure()
        stem(0:29, conv2(41:70));
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['y', num2str(q), '[n]'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of y', num2str(q), '[n]'], 'FontSize', 14, 'FontWeight', 'bold');

        disp(['y', num2str(q), '[n]: ', num2str(conv2(41:70))]);


    elseif q == 3
        conv3r = convolution(@h,@s3r,16);
        conv3im = convolution(@h,@s3im,16);
        figure()
        subplot(2,1,1);
        stem(0:29, conv3r(41:70));
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['Re(y', num2str(q), '[n])'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
```

```
        axis tight
        title(['Graph of Re(y', num2str(q), '[n])'], 'FontSize', 14, 'FontWeight', 'bold');

        subplot(2,1,2);
        stem(0:29, conv3im(41:70),'r');
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['Im(y', num2str(q), '[n])'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of Im(y', num2str(q), '[n])'], 'FontSize', 14, 'FontWeight', 'bold');
        disp(['Re(y', num2str(q), '[n]): ', num2str(conv3r(41:70))]);
        disp(['Im(y', num2str(q), '[n]): ', num2str(conv3im(41:70))]);


    elseif q == 4
        conv4 = convolution(@h, @s1, 16);

        figure()
        stem(0:29, conv4(41:70));
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['y', num2str(q), '[n]'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of y', num2str(q), '[n]'], 'FontSize', 14, 'FontWeight', 'bold');

        disp(['y', num2str(q), '[n]: ', num2str(conv4(41:70))]);


    elseif q == 6
        conv6r = convolution(@h,@s6r,16);
        conv6im = convolution(@h,@s6im,16);
        figure()
        subplot(2,1,1);
        stem(0:29, conv6r(41:70));
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['Re(y', num2str(q), '[n])'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of Re(y', num2str(q), '[n])'], 'FontSize', 14, 'FontWeight', 'bold');

        subplot(2,1,2);
        stem(0:29, conv6im(41:70),'r');
        ax = gca;
```

```
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['Im(y', num2str(q), '[n])'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of Im(y', num2str(q), '[n])'], 'FontSize', 14, 'FontWeight', 'bold');
        disp(['Re(y', num2str(q), '[n]): ', num2str(conv6r(41:70))]);
        disp(['Im(y', num2str(q), '[n]): ', num2str(conv6im(41:70))]);


    else
        conv5 = convolution(@h, @s5, 16);

        figure()
        stem(0:29, conv5(41:70));
        ax = gca;
        ax.XAxis.LineWidth = 1.5; %thicken x-axis
        ax.YAxis.LineWidth = 1.5; %thicken y-axis
        axh.XAxisLocation = 'origin';
        axh.YAxisLocation = 'origin';
        xlabel('n', 'FontSize', 12, 'FontWeight', 'bold');
        ylabel(['y', num2str(q), '[n]'], 'FontSize', 12, 'FontWeight', 'bold');
        grid on
        axis tight
        title(['Graph of y', num2str(q), '[n]'], 'FontSize', 14, 'FontWeight', 'bold');

        disp(['y', num2str(q), '[n]: ', num2str(conv5(41:70))]);
    end
end



%convolution func that takes response func, input func and range as inputs
function y = convolution(h, x, range)
    y = zeros(1, 81);
    for i = -40:40
        value = zeros(1,33);
        for j = range * (-1) : range
            value(j + 17) = x(j)* h(i - j);
        end
        y(i + 41) = sum(value);
    end
end

%functions for the input signals and the response function
function x1 = s1(n)
    if (n >= 0 && n <= 8)
        x1 = 3;
    else
        x1 = 0;
    end
end

function x2 = s2(n)
```

```
    if (n >= 0 && n <= 4)
        x2 = 3;
    elseif (n >= 5 && n <= 8)
        x2 = -3;
    elseif (n >= 9 && n <= 13)
        x2 = -6;
    else
        x2 = 0;
    end
end


%using Euler's formula (e^j(1/3) = cos(1/3) + j*sin(1/3))
function x3 = s3r(n)
    if(n >= 2 && n <= 20)
        x3 = cos(n / 3);
    else
        x3 = 0;
    end
end
function x3 = s3im(n)
    if(n >= 2 && n <= 20)
        x3 = sin(n / 3);
    else
        x3 = 0;
    end
end

function x4 = s4(n)
    if(n >= 2 && n <= 20)
        x4 = (-3) * sin(n / 3);
    else
        x4 = 0;
    end
end

function x5 = s5(n)
    if(n >= 2 && n <= 20)
        x5 = 2 * cos(n / 3);
    else
        x5 = 0;
    end
end

%using Euler's formula again
function x6 = s6r(n)
    x6 = real(s1(n) + 2i.* s2(n));
end
function x6 = s6im(n)
    x6 = imag(s1(n) + 2i.* s2(n));
end

%unit step function
function uso = us(n)
    if(n >= 0)
        uso = 1;
    else
        uso = 0;
    end
end

%response function
function res = h(n)
    res = us(n-4).*(7/8).^n;
end
```

Analytical Part

```
clc
clear all
close all

arrayc = 0:100;
out1 = zeros(1, length(arrayc));
out2 = zeros(1, length(arrayc));
out3 = zeros(1, length(arrayc));

for n = 4:50
    if n > 12
        out1(n+1) = 3 * sum((7/8).^(n-8:n));
    else
        out1(n+1) = 3 * sum((7/8).^(4:n));
    end
end

for n = 4:50
    if n > 8 && n < 13
        out2(n+1) = 3 * sum((7/8).^(4:n)) - 6 * sum((7/8).^(4:n-5));
    elseif n <= 8
        out2(n+1) = 3 * sum((7/8).^(4:n));
    elseif n >= 13 && n < 18
        out2(n+1) = 3 * sum((7/8).^(n-8:n)) - 6 * sum((7/8).^(4:n-5));
    else
        out2(n+1) = 3 * sum((7/8).^(n-8:n)) - 6 * sum((7/8).^(n-13:n-5));
    end
end


for n = 6:50
    if n < 25
        out3(n+1) = ((cos(1/3) + 1i * sin(1/3    ))^n) * sum(((7/8) * cos(1/3) - 1i * (7/8) * sin(1/3)).^(4:n-2));
    else
        out3(n+1) = ((cos(1/3) + 1i * sin(1/3))^n) * sum(((7/8) * cos(1/3) - 1i * (7/8) * sin(1/3)).^(n-20:n-2));
    end
end

out4 = -3 * imag(out3);
out5 = 2 * real(out3);
out6 = out1 + 2i * out2;
outs = {out1, out2, out3, out4, out5, out6};
listt = {'y1[n]', 'y2[n]', 'y3[n]', 'y4[n]', 'y5[n]', 'y6[n]'};

%plotting loop
for m = 1:2
    if m == 1
        for i = 1:6
            figure;
            stem(arrayc, outs{i}, 'r', 'filled', 'LineWidth', 1.5);
            title(['Graph of', listt{i}], 'FontSize', 14, 'FontWeight', 'bold');
            xlabel('n', 'FontSize', 12);
            ylabel(listt{i}, 'FontSize', 12);
            grid on;
            xlim([0 50]);
        end
    else
        for i = 1:6
            figure;
            stem(arrayc, imag(outs{i}), 'r', 'filled', 'LineWidth', 1.5);
            title(['Graph of Imaginary Part for ', listt{i}], 'FontSize', 14, 'FontWeight', 'bold');
            xlabel('n', 'FontSize', 12);
            ylabel(['Imag(', listt{i}, ')'], 'FontSize', 12);
            grid on;
            xlim([0 50]);
        end
    end
end
```

ANSWER TO QUESTION ABOUT CAUSALITY AND STABILITY

The given system is causal since h[n] = 0 for n < 0 and there is no input that involves later data. The system is also stable as the following is bounded.

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=4}^{\infty} (7/8)^n = \frac{1}{1-7/8} + \sum_{n=0}^{3} (7/8)^n < \infty$$

PLOTS (left numeric, right analytic)

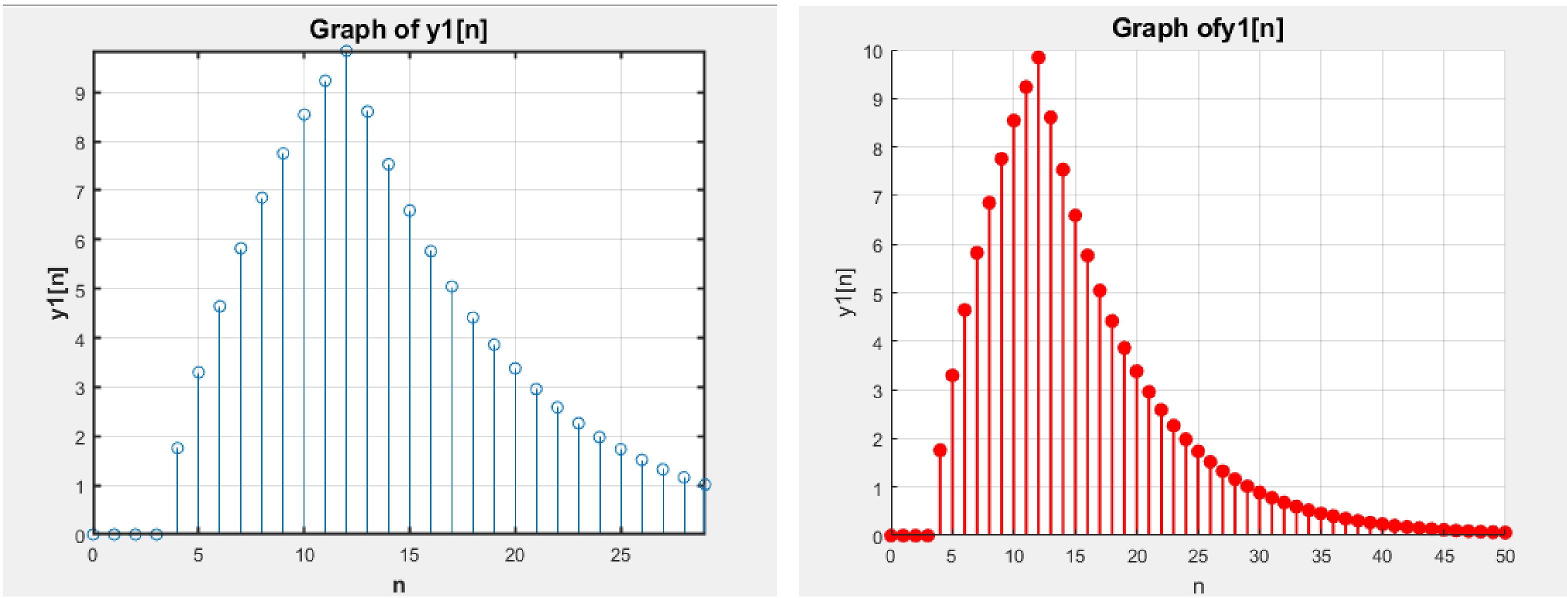

Figure 1: Graphs for x1[n]

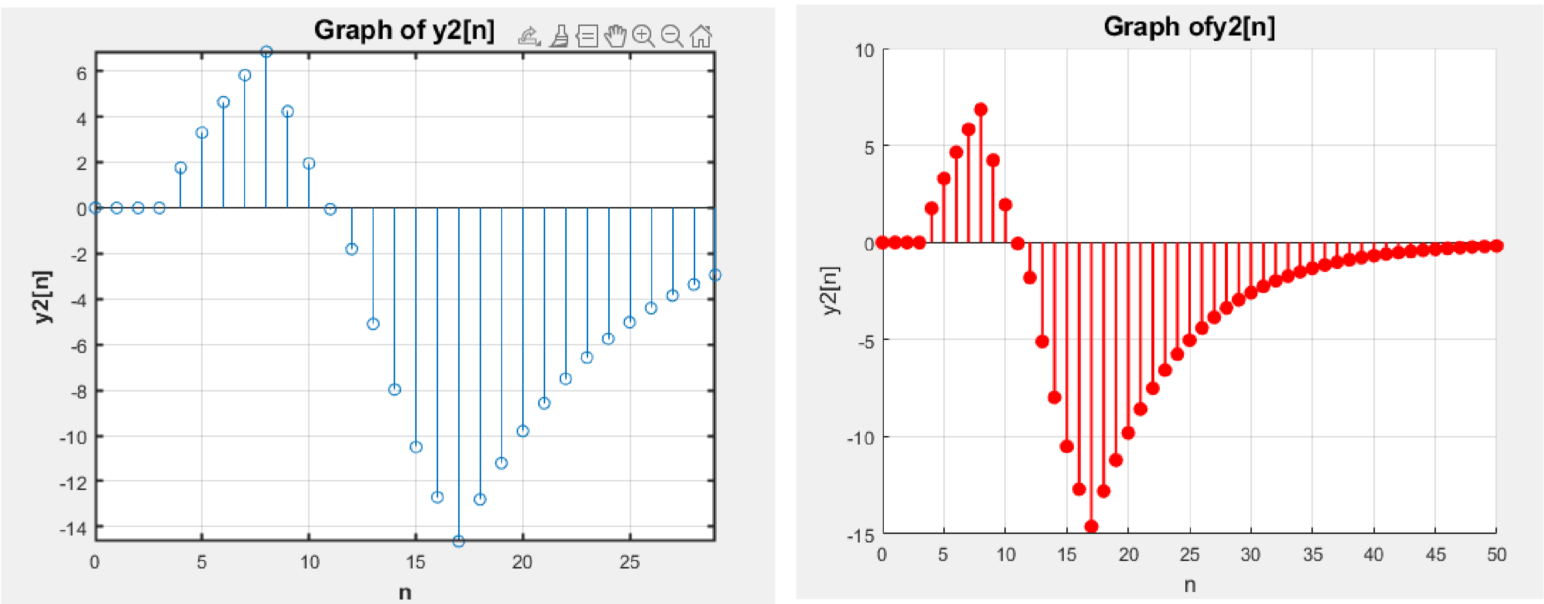

Figure 2: Graphs for x2[n]

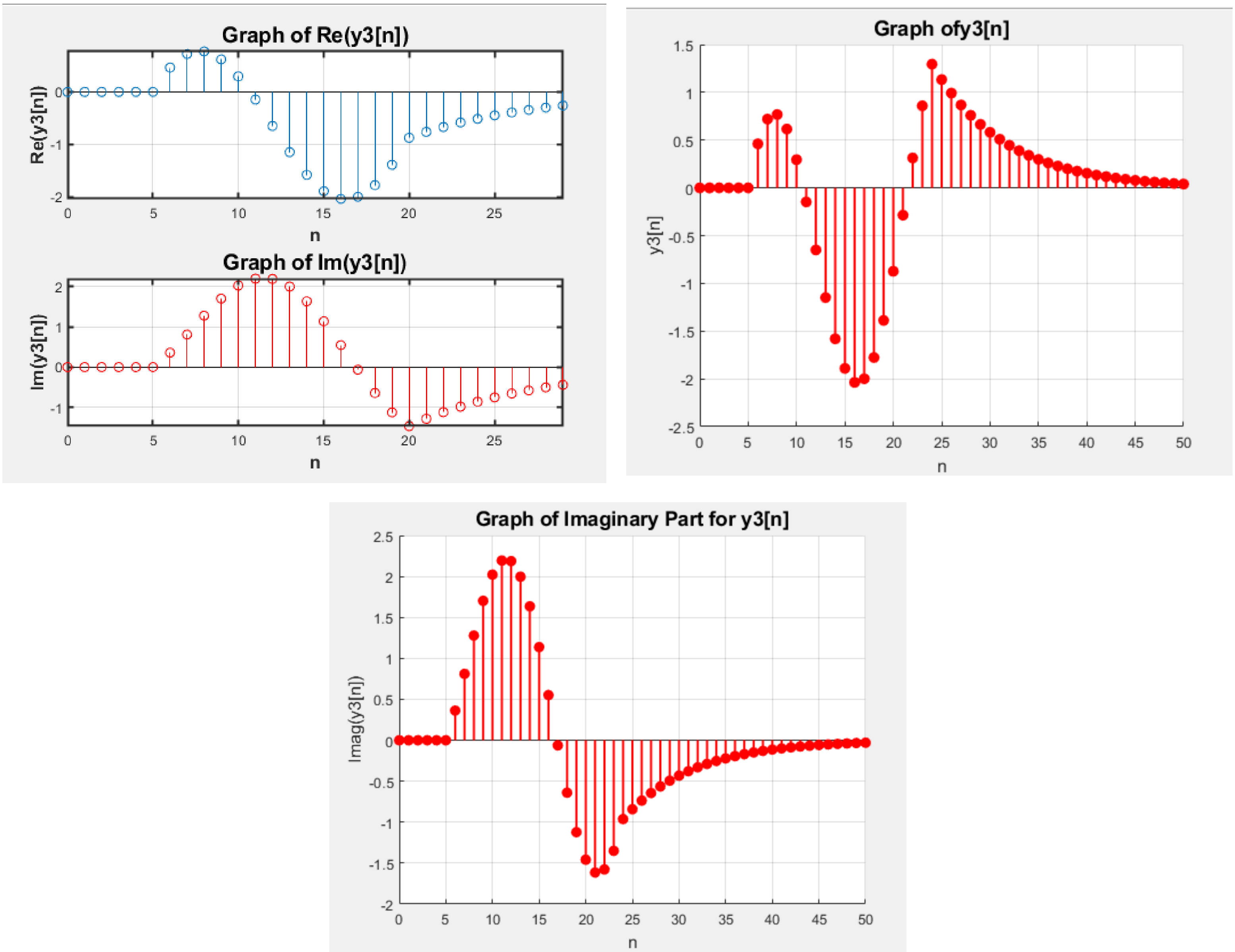

Figure 3: Graphs for x3[n]

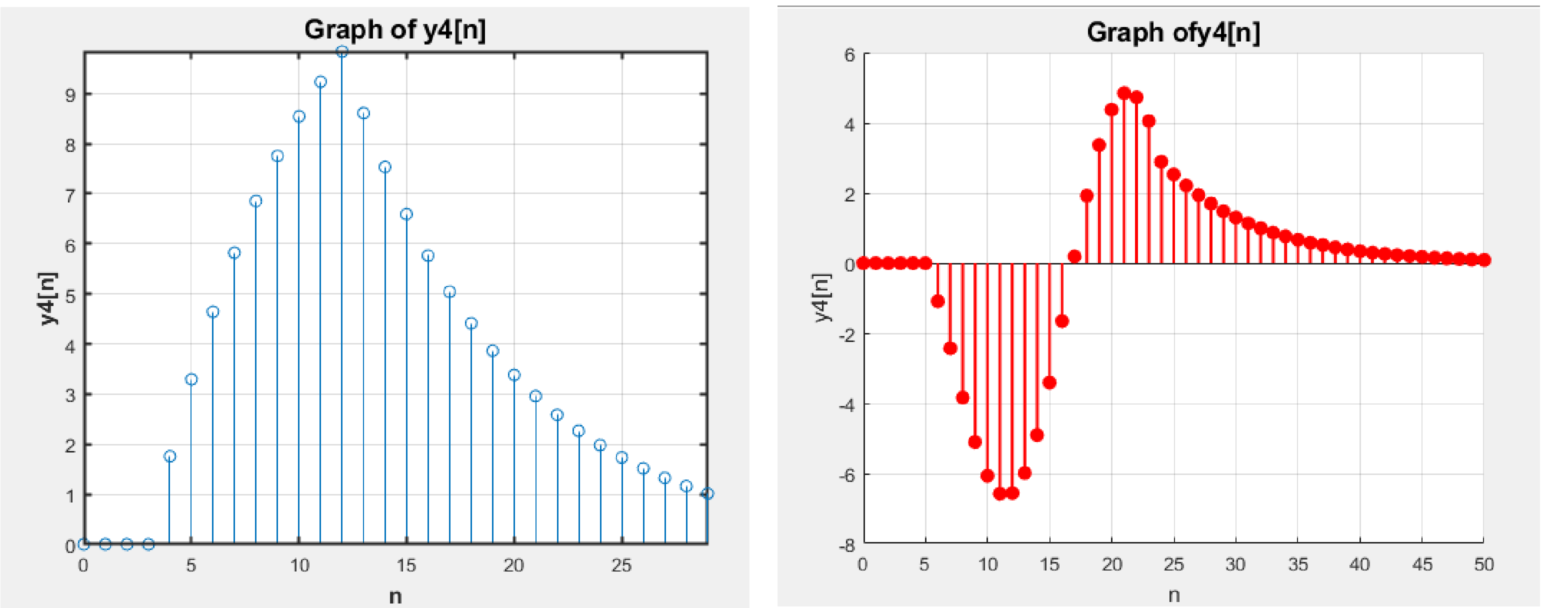

Figure 4: Graphs for x4[n]

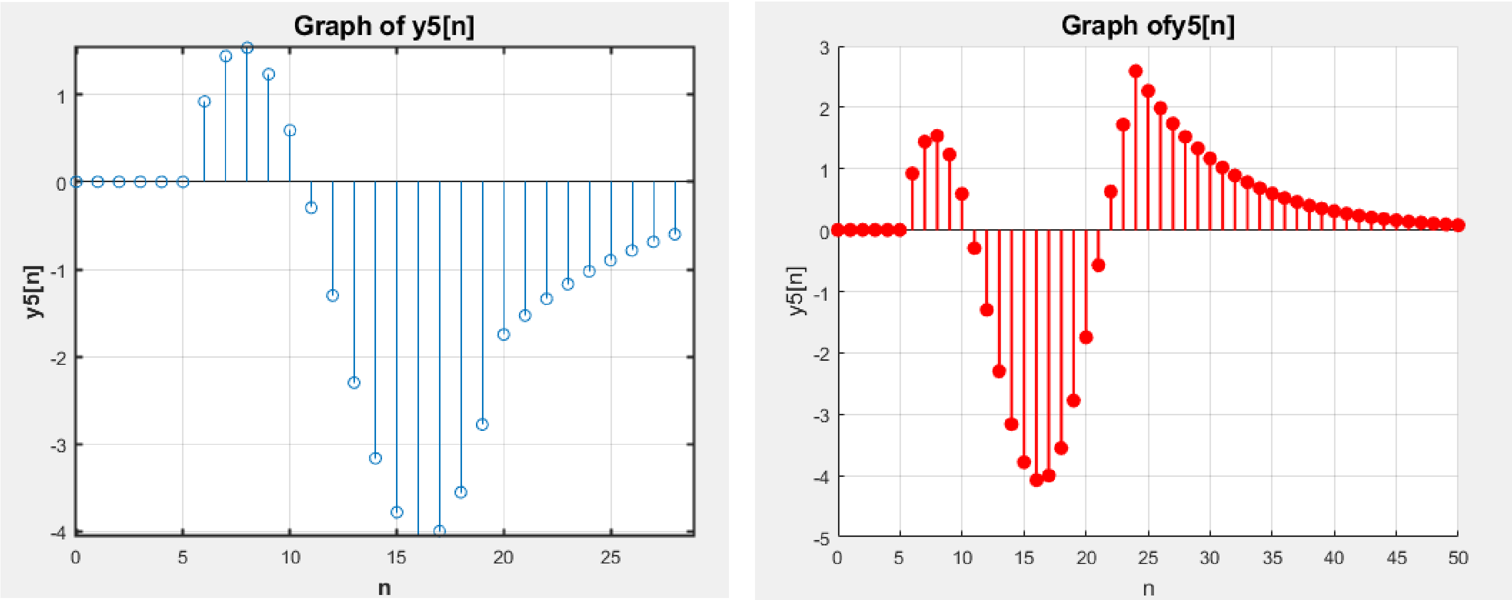

Figure 5: Graphs for x5[n]

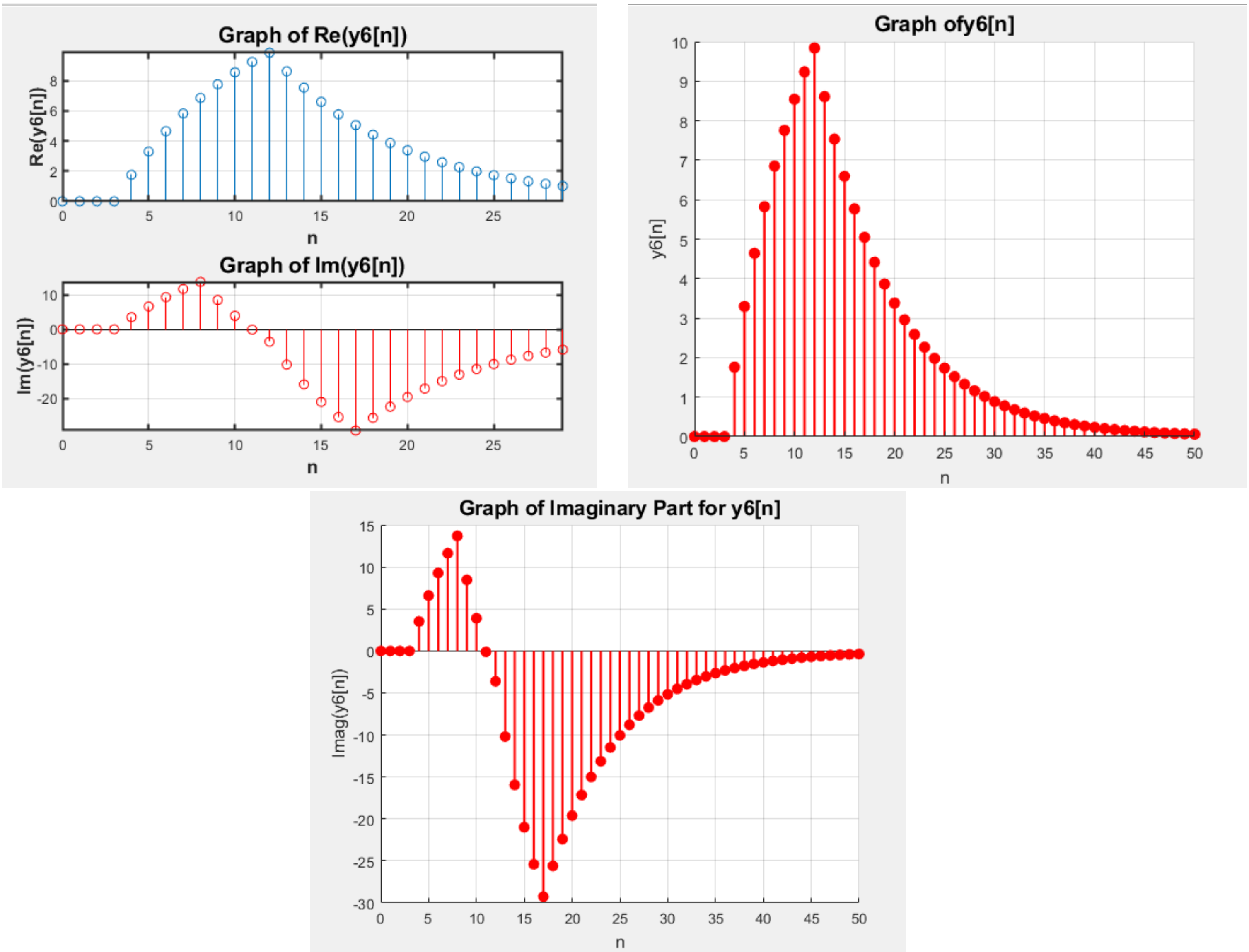

Figure 6: Graphs for x6[n]

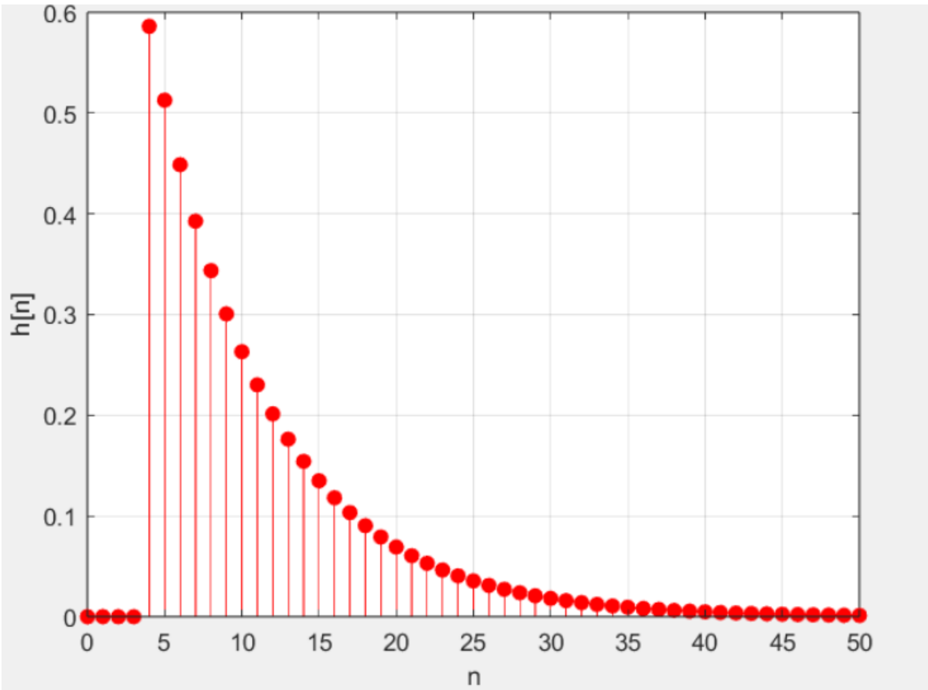

Figure 7: Graph of h[n]

The found solutions are equal for all numeric and analytic methods from the two MATLAB files. This lab was useful in terms of observing that properties of LTI systems make solving convolutions simpler, the fact that such systems are linear and time invariant lets us use one system's output to simply reach the convolution of another system, as this was the case for c), d), e) and f) of the manual.

HAND WRITTEN SOLUTIONS

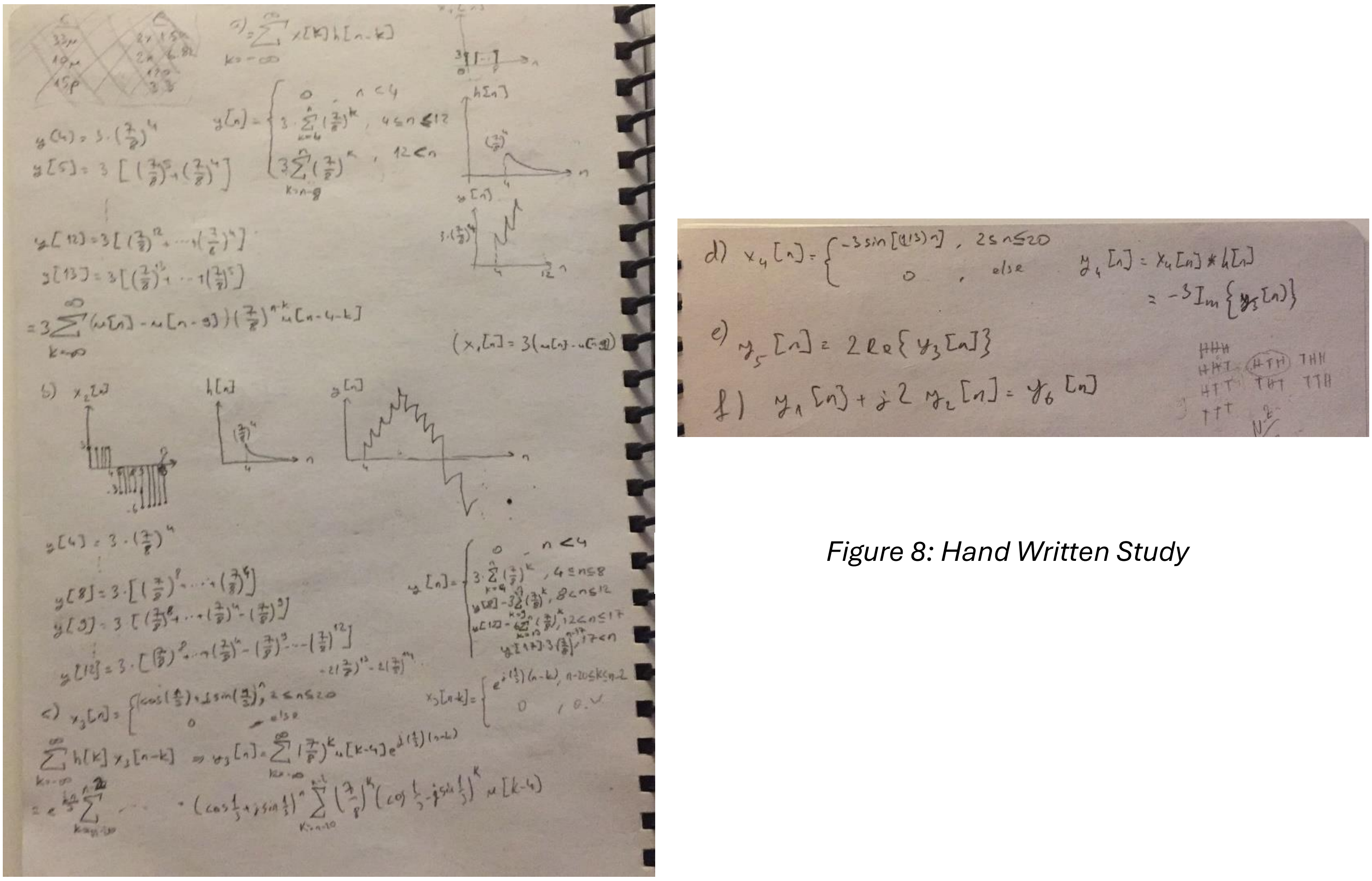

Figure 8: Hand Written Study

Printed Results (same for both)

y1[n]: 0 0 0 0 1.7585 3.2973 4.6437 5.8217 6.8526 7.7545 8.5438 9.2343 9.8386 8.6088 7.5327 6.5911 5.7672 5.0463 4.4155 3.8636 3.3806 2.9581 2.5883 2.2648 1.9817 1.734 1.5172 1.3276 1.1616 1.0164

y2[n]: 0 0 0 0 1.75854 3.29727 4.64366 5.82175 6.85257 4.23746 1.94923 -0.0529697 -1.80489 -5.09637 -7.97641 -10.4965 -12.7015 -14.6309 -12.802 -11.2018 -9.80155 -8.57636 -7.50431 -6.56628 -5.74549 -5.0273 -4.39889 -3.84903 -3.3679 -2.94691

Re(y3[n]): 0 0 0 0 0 0 0.46067 0.7198 0.76772 0.61564 0.29475 -0.147 -0.64993 -1.149 -1.5808 -1.8904 -2.0373 -1.9995 -1.7764 -1.3881 -0.87351 -0.76432 -0.66878 -0.58518 -0.51204 -0.44803 -0.39203 -0.34302 -0.30015 -0.26263

Im(y3[n]): 0 0 0 0 0 0 0.36248 0.81042 1.2789 1.7025 2.0227 2.1937 2.1875 1.9968 1.6355 1.1372 0.55146 -0.062048 -0.63986 -1.122 -1.4585 -1.2762 -1.1167 -0.97708 -0.85494 -0.74807 -0.65456 -0.57274 -0.50115 -0.43851

y4[n]: 0 0 0 0 1.7585 3.2973 4.6437 5.8217 6.8526 7.7545 8.5438 9.2343 9.8386 8.6088 7.5327 6.5911 5.7672 5.0463 4.4155 3.8636 3.3806 2.9581 2.5883 2.2648 1.9817 1.734 1.5172 1.3276 1.1616 1.0164

y5[n]: 0 0 0 0 0 0 0.92135 1.4396 1.5354 1.2313 0.5895 -0.29401 -1.2999 -2.298 -3.1616 -3.7809 -4.0746 -3.9991 -3.5528 -2.7761 -1.747 -1.5286 -1.3376 -1.1704 -1.0241 -0.89606 -0.78405 -0.68605 -0.60029 -0.52526

Re(y6[n]): 0 0 0 0 1.7585 3.2973 4.6437 5.8217 6.8526 7.7545 8.5438 9.2343 9.8386 8.6088 7.5327 6.5911 5.7672 5.0463 4.4155 3.8636 3.3806 2.9581 2.5883 2.2648 1.9817 1.734 1.5172 1.3276 1.1616 1.0164

Im(y6[n]): 0 0 0 0 3.51709 6.59454 9.28732 11.6435 13.7051 8.47491 3.89846 -0.105939 -3.60979 -10.1927 -15.9528 -20.9929 -25.403 -29.2618 -25.6041 -22.4036 -19.6031 -17.1527 -15.0086 -13.1326 -11.491 -10.0546 -8.79778 -7.69806 -6.7358 -5.89383

>>