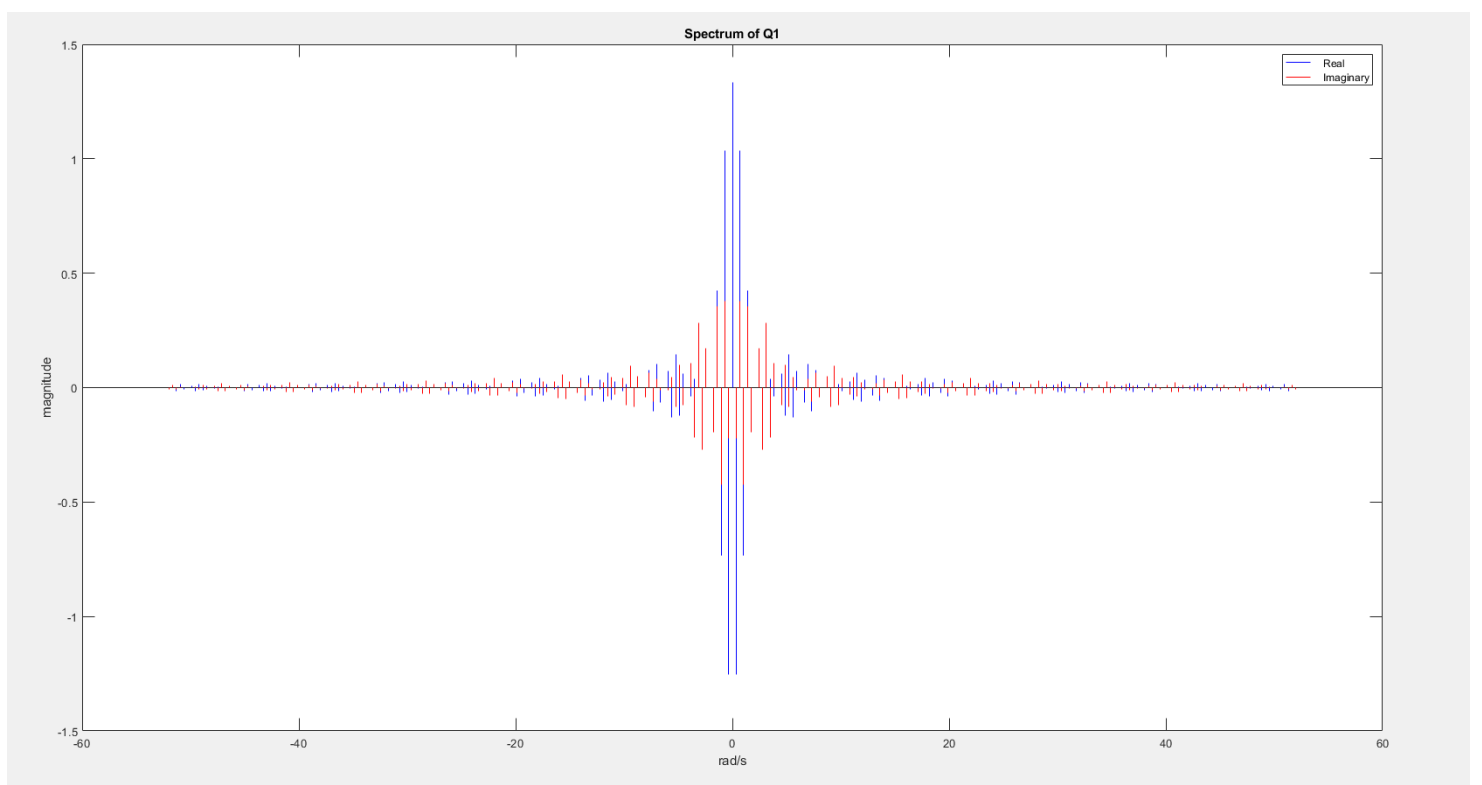
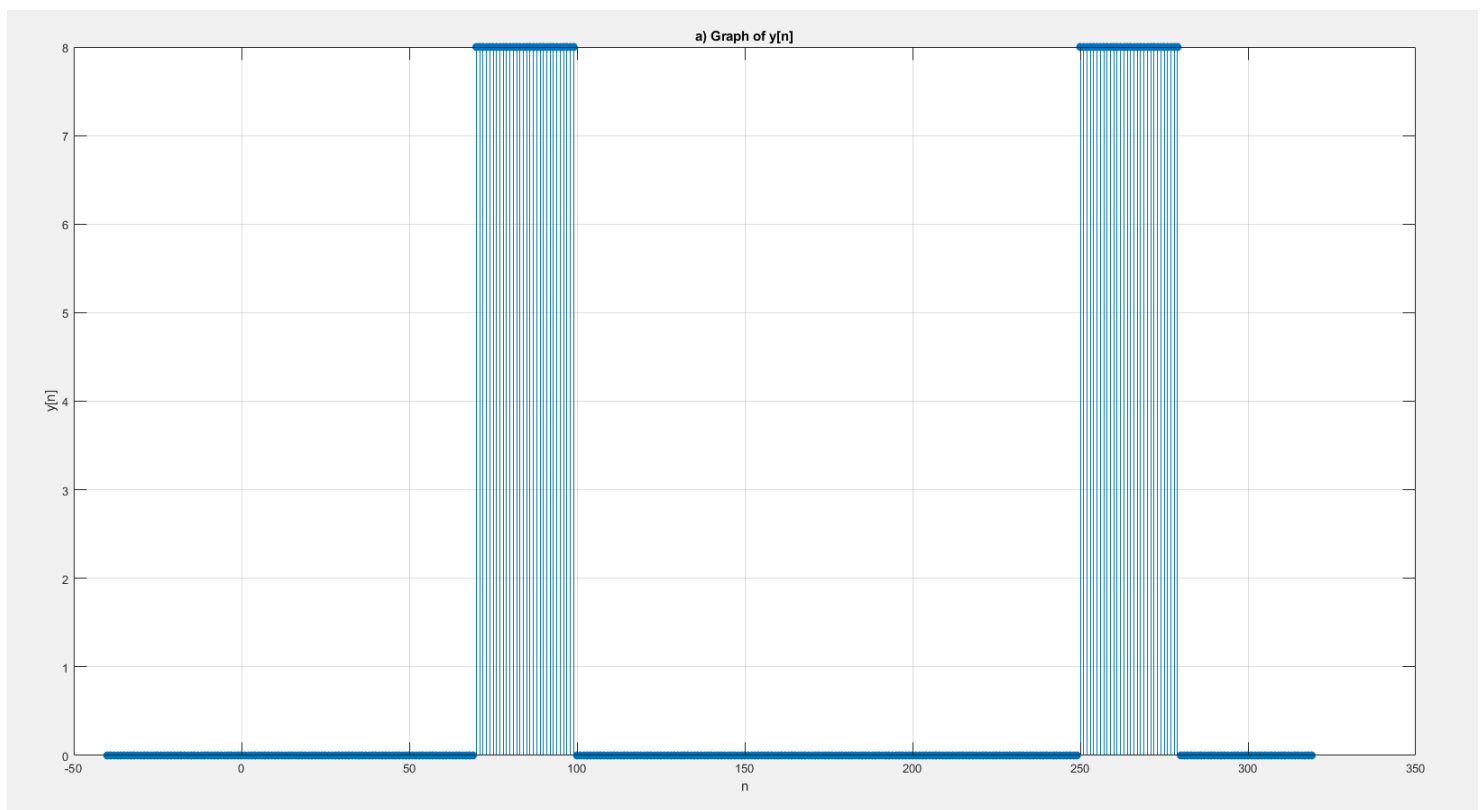
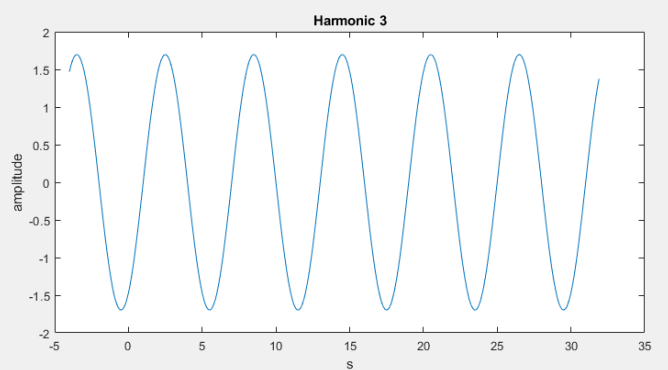
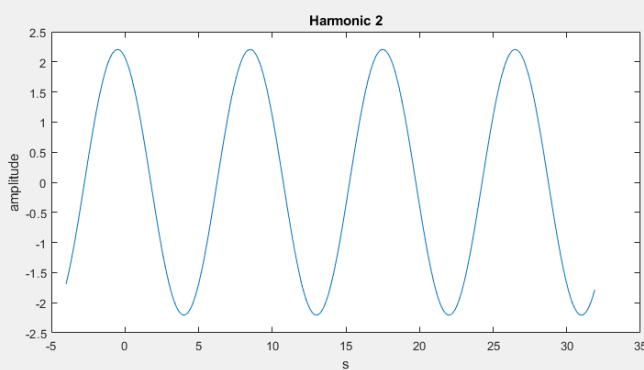
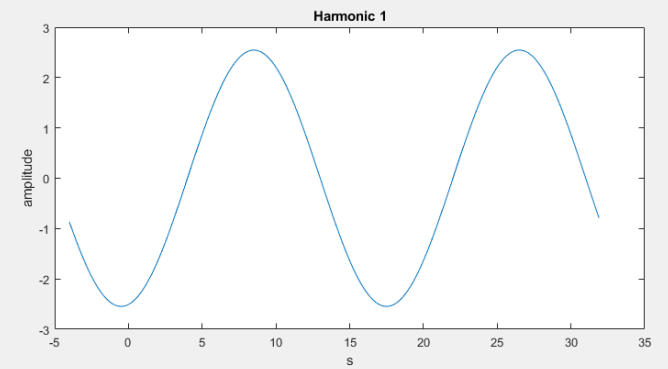
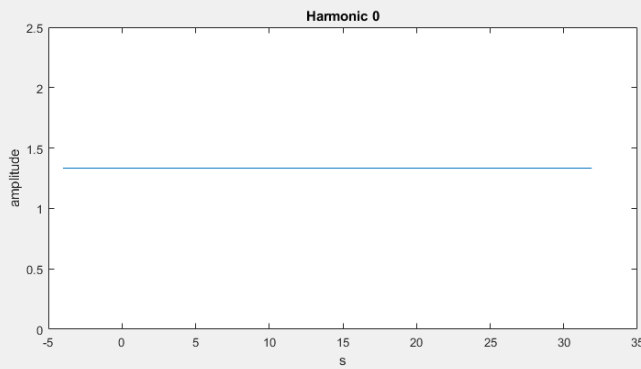
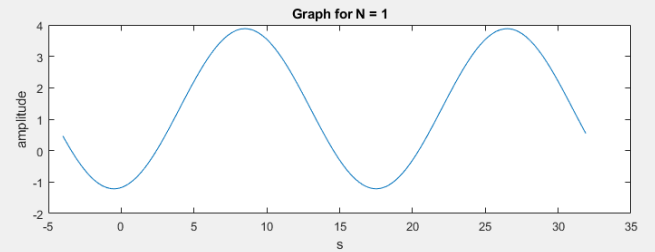
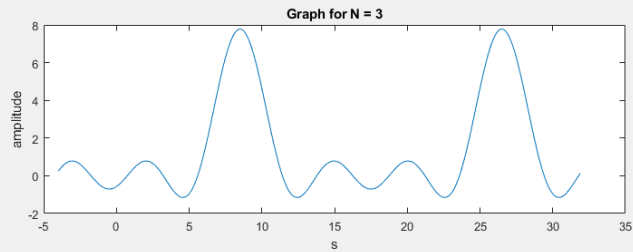
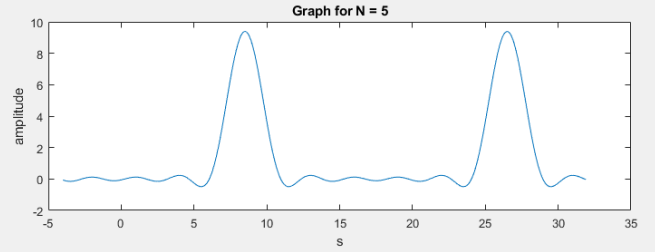
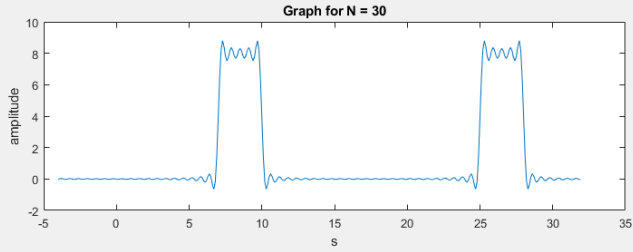
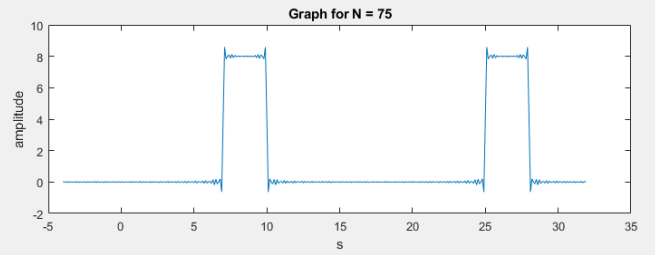
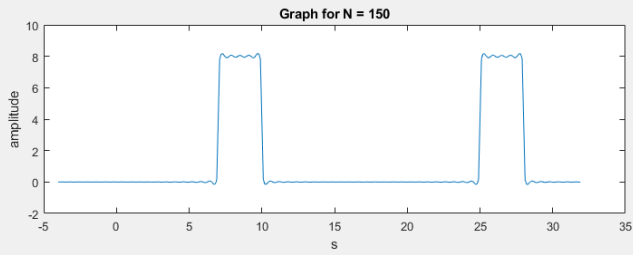
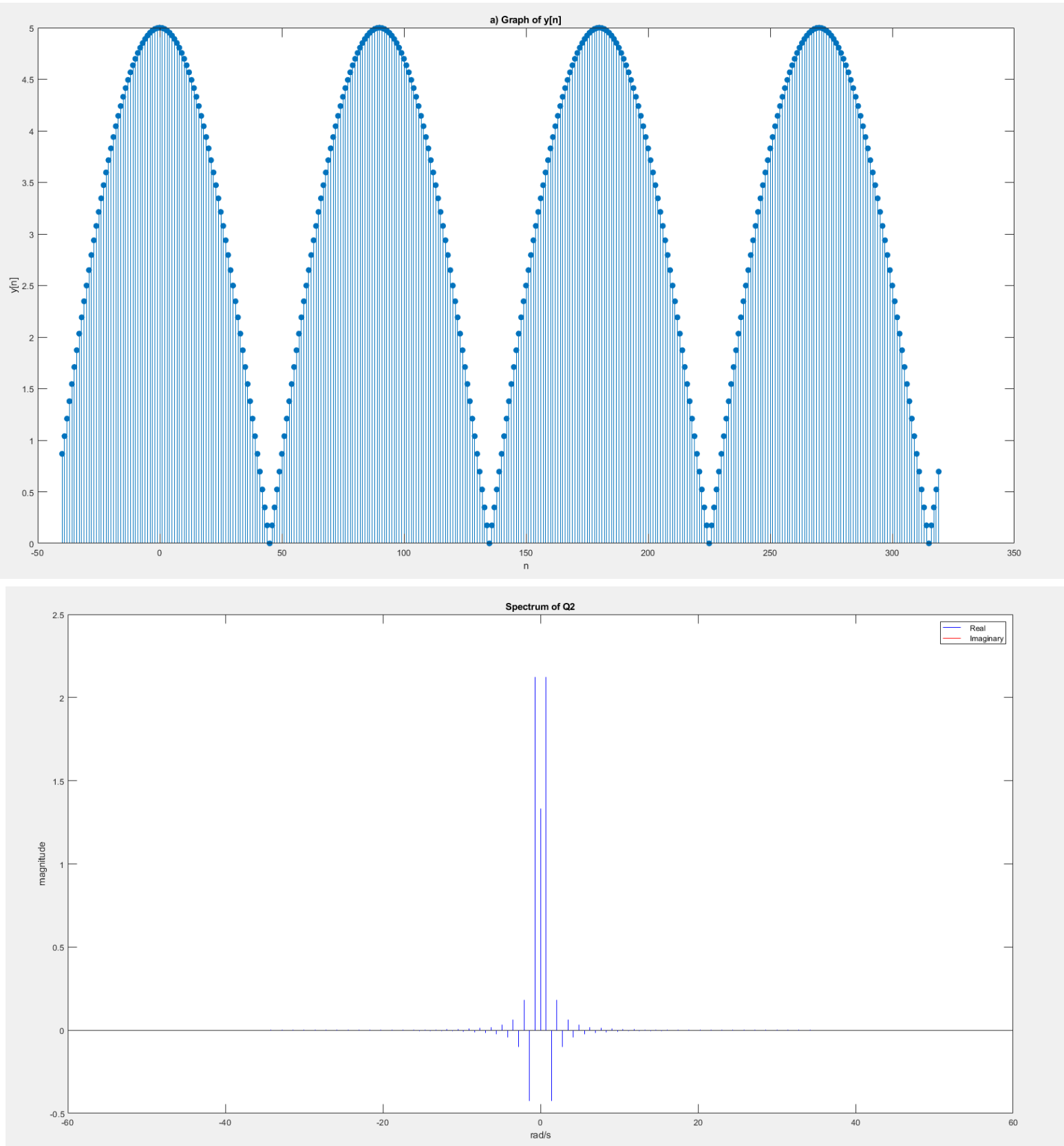


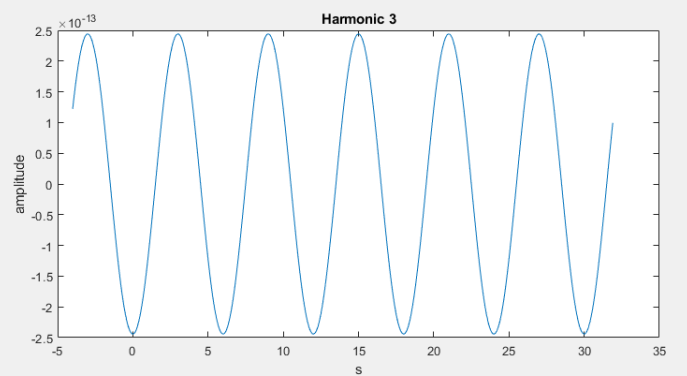
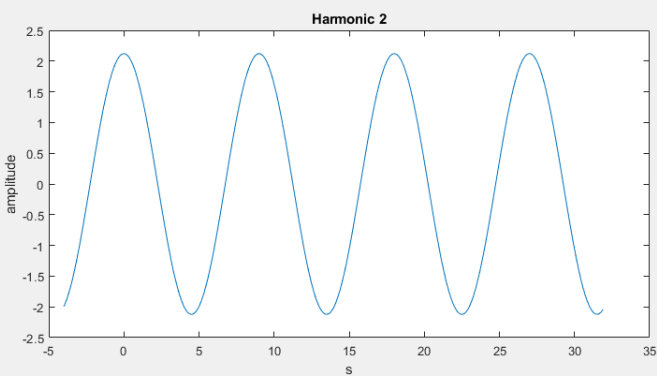
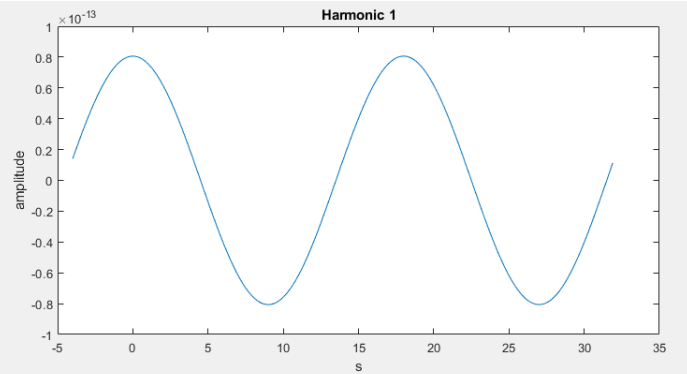
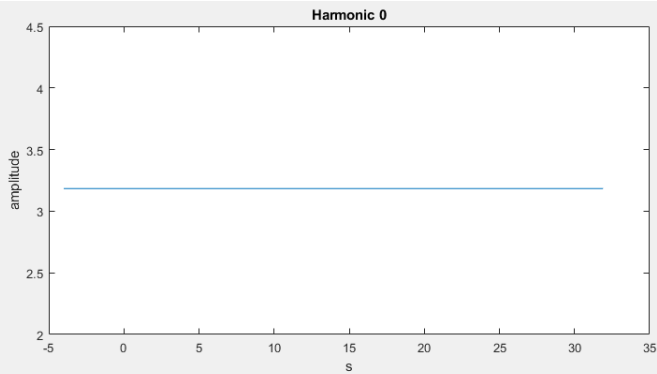
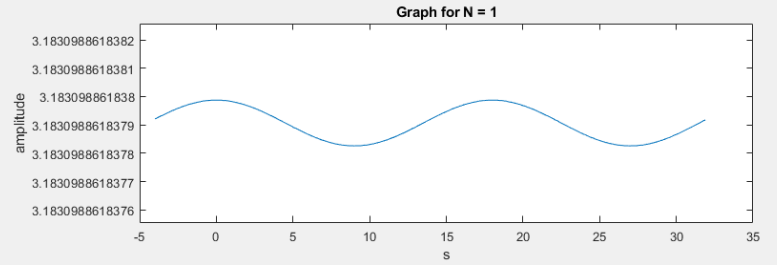
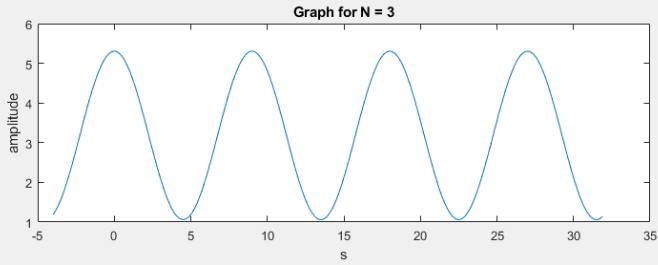
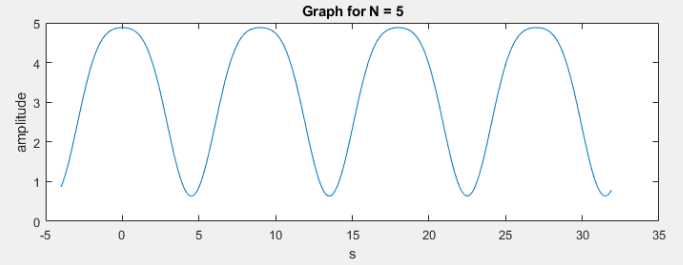
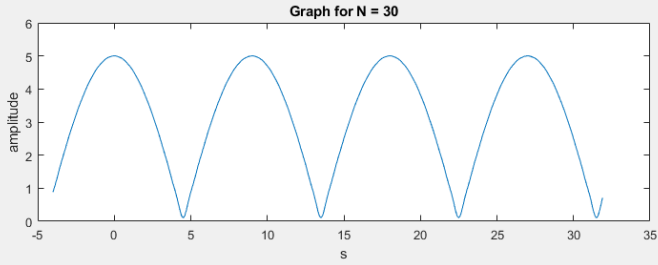
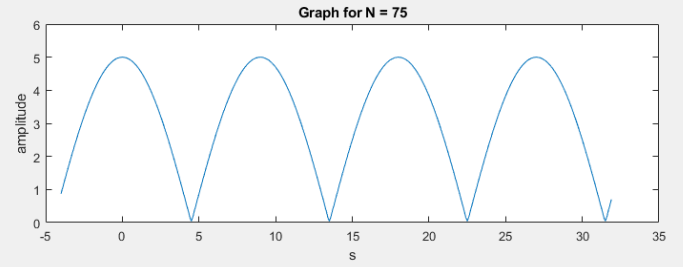
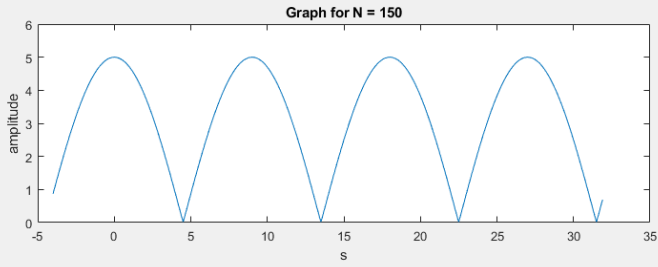
MATLAB PLOTS**Figures 1, 2: First Two Graphs for Q1**



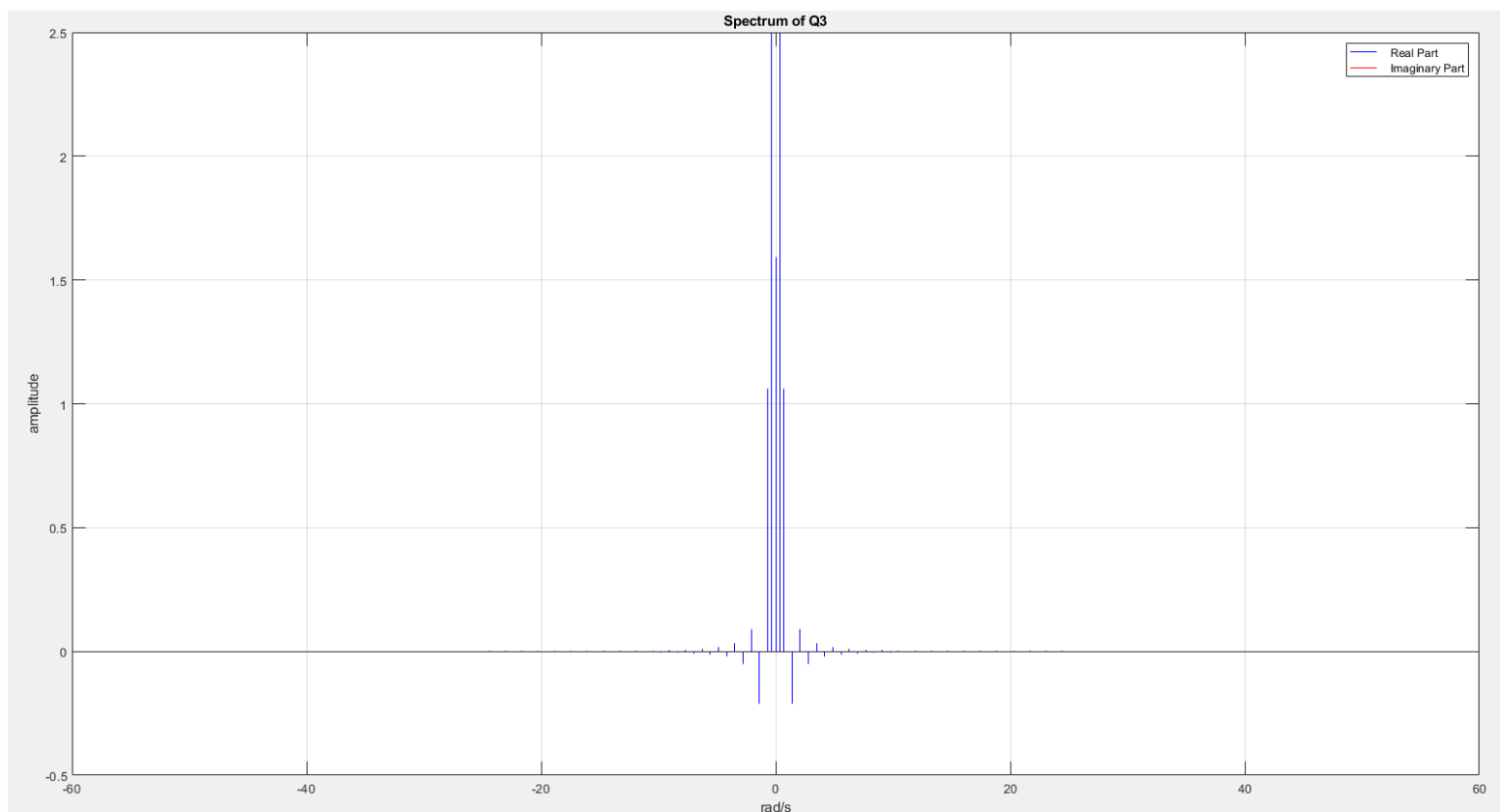
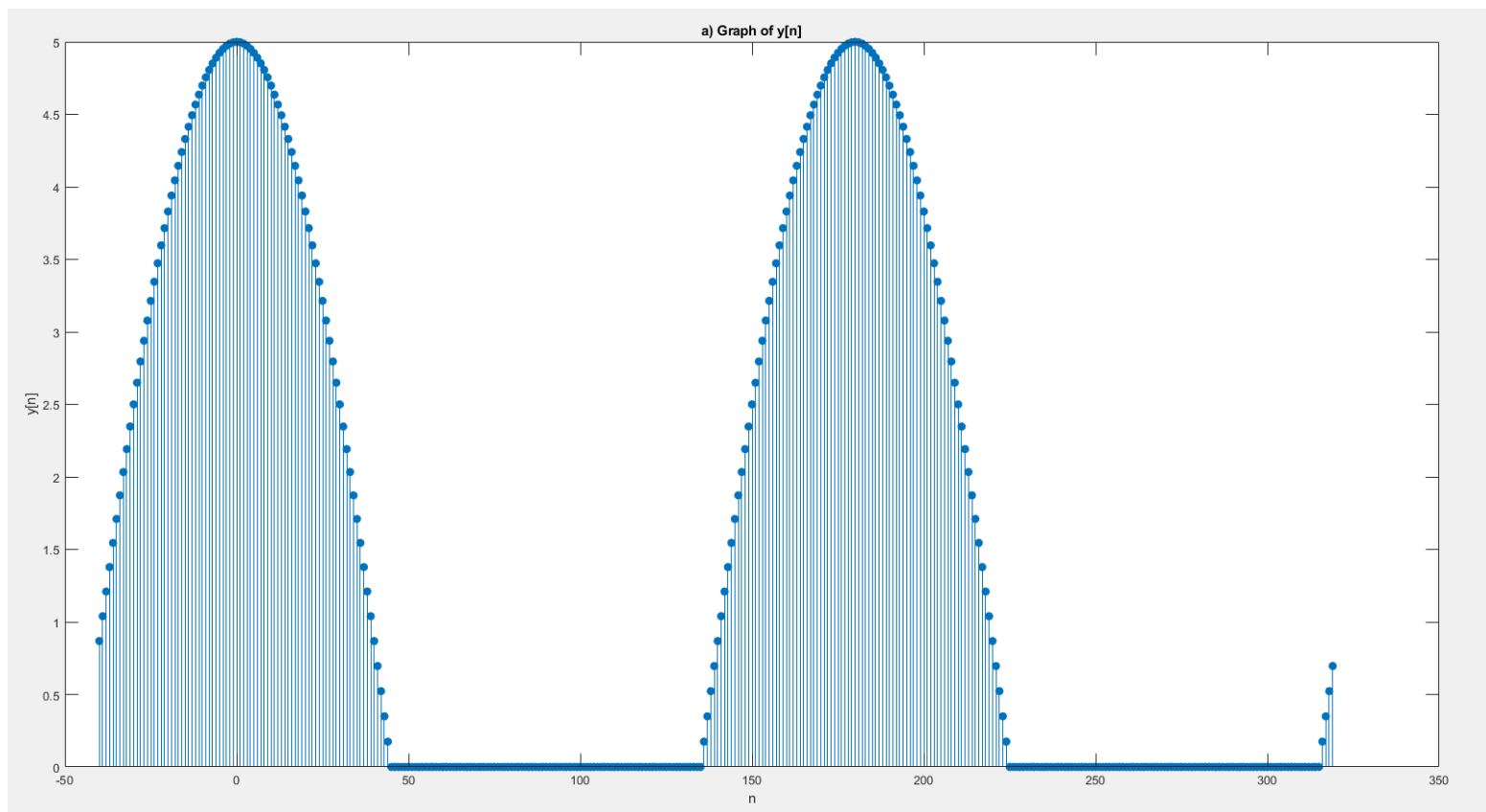
Figures 3,4: Remaining Graphs for Q1



Figures 5,6: First Two Graphs for Q2

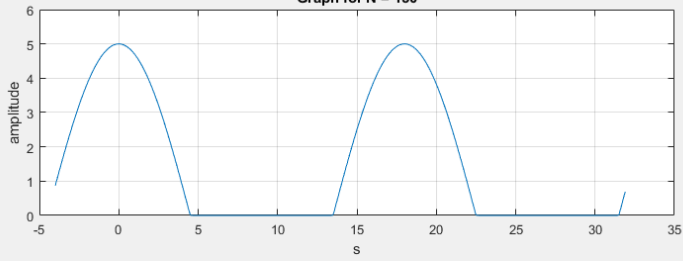


Figures 7,8: Remaining Graphs for Q2

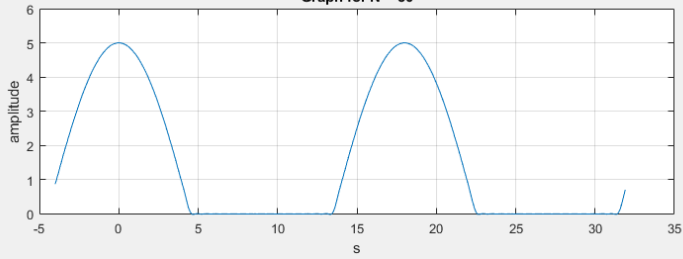


Figures 9,10: First Two Graphs for Q3

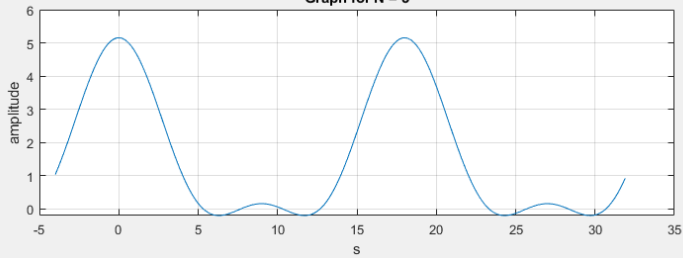
Graph for N = 150



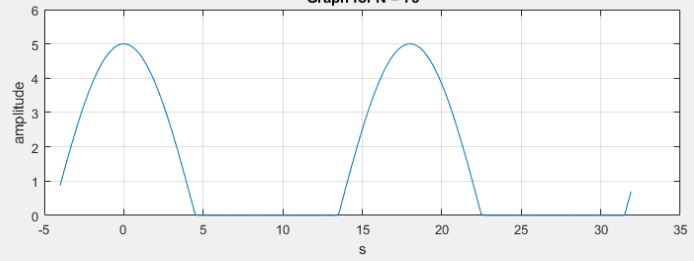
Graph for N = 30



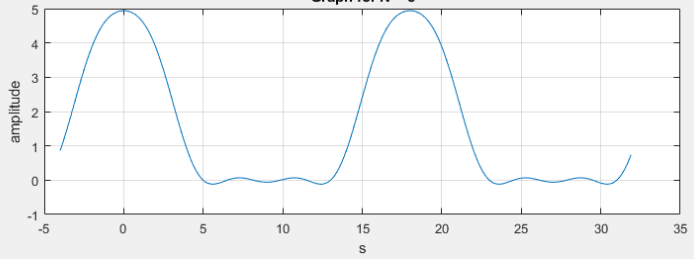
Graph for N = 3



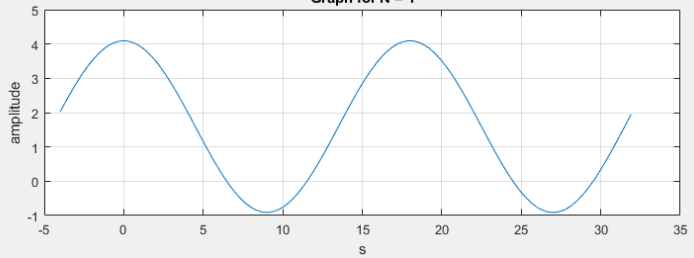
Graph for N = 75



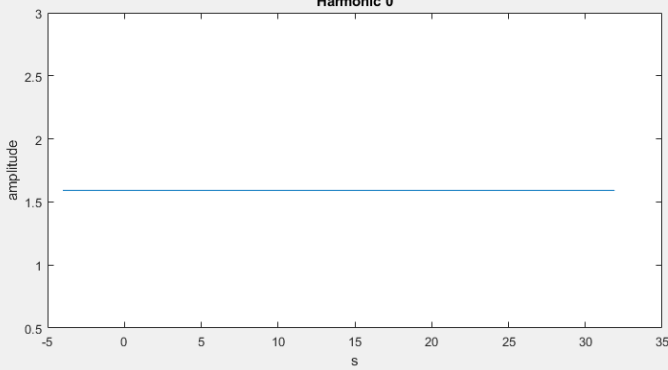
Graph for N = 5



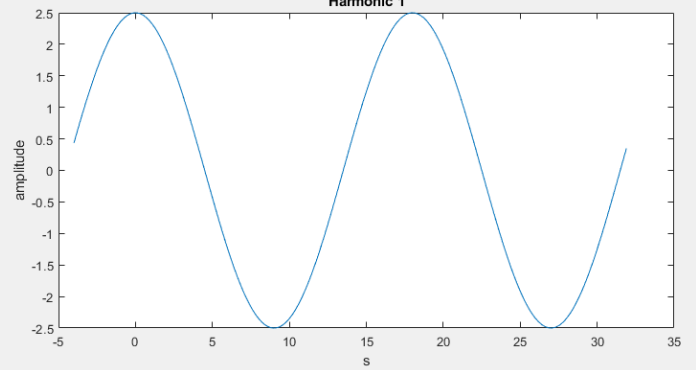
Graph for N = 1



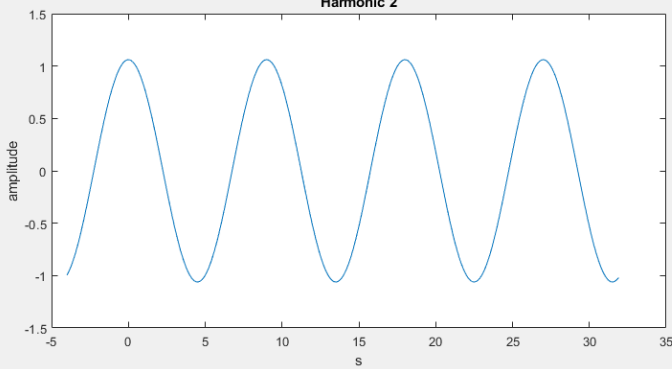
Harmonic 0



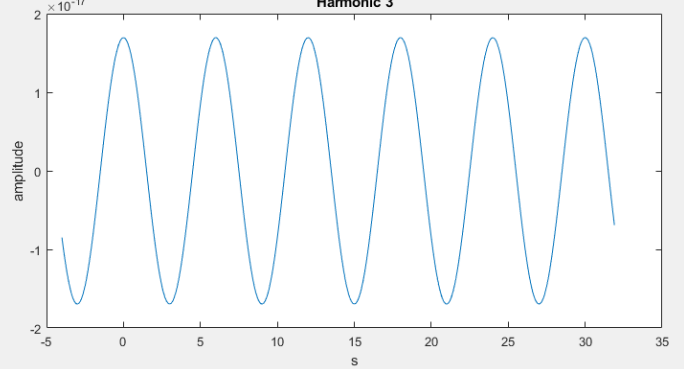
Harmonic 1



Harmonic 2



Harmonic 3



Figures 11,12: Remaining Graphs for Q3

HANDWRITTEN SOLUTIONS

Q1

$$x[n] = \begin{cases} 0, & 0 \leq n < 7 \\ 8, & 7 \leq n < 10 \\ 0, & 10 \leq n < 18 \end{cases} \Rightarrow T=180$$

$$a_0 = \frac{1}{18} \int_0^{18} x_a(t) dt = \int_7^{10} 8 dt \cdot \frac{1}{18} = \frac{1}{18} \cdot 8t \Big|_7^{10} = \frac{24}{18} = \frac{4}{3}$$

$$a_k = \frac{1}{18} \int_0^{18} x_a(t) e^{j \frac{2\pi k t}{18}} dt = \frac{1}{18} \int_7^{10} 8 e^{-j \frac{2\pi k t}{18}} dt = \frac{8}{18} \cdot (-15) \cdot \frac{1}{20k} \left[e^{-j \frac{2\pi k t}{18}} \right]_7^{10}$$

$$= \frac{4}{j\pi k} \left(e^{-j \frac{20\pi k}{18}} - e^{-j \frac{14\pi k}{18}} \right)$$

$$= \frac{4}{j\pi k} \left(e^{-j \frac{10\pi k}{9}} - e^{-j \frac{7\pi k}{9}} \right)$$

$$x_a(t) = \frac{4}{j\pi k} \left(\cos\left(\frac{10\pi k}{9}t\right) - \cos\left(\frac{7\pi k}{9}t\right) \right) + j \left(\sin\left(\frac{7\pi k}{9}t\right) - \sin\left(\frac{10\pi k}{9}t\right) \right)$$

$$k \neq 0$$

$$k=0$$

$$\frac{4}{3}$$

(spectrum)

$$x_a(t) = \frac{4}{3} \sum_{k=-\infty}^{\infty} a_k e^{j \frac{2\pi k t}{18}} = \frac{4}{3} + \sum_{k=1}^{\infty} 2 a_k \cos\left(\frac{2\pi k t}{18}\right)$$

(FSR a.)

$$Z[n] = \frac{4}{3} + \sum_{k=1}^N \frac{2}{\pi k} \left(\cos\left(\frac{10\pi k}{9}\right) - \cos\left(\frac{7\pi k}{9}\right) \right) + j \left(\sin\left(\frac{7\pi k}{9}\right) - \sin\left(\frac{10\pi k}{9}\right) \right)$$

$$= \cos\left(\frac{7\pi k}{9}\right)$$

Figure 13: Solution for Q1

Q2

$$y_a(t) = 5 \cos\left(\frac{\pi}{9}t\right) \quad \omega_0 = 9$$

$$a_0 = \frac{1}{T} \int_0^T x(t) dt = \frac{1}{9} \int_{-4.5}^{4.5} 5 \cos\left(\frac{\pi}{9}t\right) dt = \frac{1}{9} \cdot \frac{9}{\pi} \cdot 5 \left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right) = \frac{10}{\pi}$$

$$a_k = \frac{1}{T} \int_{-4.5}^{4.5} y_a(t) e^{-j \frac{2\pi}{T} kt} dt = \frac{1}{9} \int_{-4.5}^{4.5} 5 \cos\left(\frac{\pi}{9}t\right) e^{-j \frac{2\pi}{9} kt} dt$$

$$= \frac{1}{18} \cdot 5 \int_{-4.5}^{4.5} \left[e^{j \frac{\pi}{9}t + (-j \frac{2\pi}{9} kt)} + e^{-j \frac{\pi}{9}t + (-j \frac{2\pi}{9} kt)} \right] dt$$

$$= \frac{5}{18} \left[\frac{e^{j \frac{\pi}{9}(1-2k)t}}{j \frac{\pi}{9}(1-2k)} + \frac{e^{-j \frac{\pi}{9}(1+2k)t}}{-j \frac{\pi}{9}(1+2k)} \right]_{-4.5}^{4.5}$$

$$= \frac{5}{18} \cdot \frac{9}{\pi} \left(\frac{e^{j \frac{\pi}{2}(1-2k)} - e^{-j \frac{\pi}{2}(1-2k)}}{1-2k} + \frac{e^{-j \frac{\pi}{2}(1+2k)} - e^{j \frac{\pi}{2}(1+2k)}}{1+2k} \right)$$

$$= \frac{5 \sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi - 2k\pi} + \frac{5 \sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi + 2k\pi}$$

$$\Rightarrow \frac{10}{\pi} + \sum_{k=-\infty}^{\infty} a_k e^{j \frac{2\pi}{9} kt} = \frac{10}{\pi} + \sum_{k=1}^{\infty} 2a_k \cos\left(\frac{2\pi}{9} kt\right) = FS E$$

$$a_k = \begin{cases} \frac{5 \sin\left(\frac{\pi}{2}(1-2k)\right)}{\pi - 2k\pi} + \frac{5 \sin\left(\frac{\pi}{2}(1+2k)\right)}{\pi + 2k\pi}, & k \neq 0 \\ \frac{10}{\pi}, & k = 0 \end{cases}$$

(spectrum)

$$y_{az}(t) = \frac{10}{\pi} + \sum_{k=1}^{\infty} \left[\frac{10 \sin\left(\frac{\pi-2k\pi}{2}\right)}{\pi - 2k\pi} + \frac{10 \sin\left(\frac{\pi+2k\pi}{2}\right)}{\pi + 2k\pi} \right] \cos\left(\frac{2\pi}{9} kt\right)$$

(FS E a.)

$$\hat{x}[n] = \frac{10}{\pi} + \sum_{N=1}^N \left[\frac{10 \sin\left(\frac{\pi-2k\pi}{2}\right)}{\pi - 2k\pi} + \frac{10 \sin\left(\frac{\pi+2k\pi}{2}\right)}{\pi + 2k\pi} \right] \cos\left(\frac{2\pi}{9} kt\right)$$

no: 319

Figure 14: Solution for Q2

(from Q2)

$$y_{02}(t) = \frac{y_0(t) + x(t)}{2} \quad \text{Q3}$$

$$x(t) = 5 \cos\left(\frac{2\pi}{18}t\right) = \frac{5}{2} e^{j\frac{\pi}{9}t} + \frac{5}{2} e^{-j\frac{\pi}{9}t}$$

$$\Rightarrow y_2(t) = \left(\frac{10}{2\pi} + \sum_{k=-\infty}^{\infty} \left[\frac{\sin\left(\frac{\pi - 2k\pi}{2}\right)}{\pi - 2k\pi} + \frac{5 \sin\left(\frac{\pi + 2k\pi}{2}\right)}{\pi + 2k\pi} \right] e^{j\frac{2\pi}{9}kt} \right) \cdot \frac{1}{2}$$

* $m = 2n$

$$\Rightarrow y_2(t) = \frac{5}{\pi} + \left[\sum_{k=-\infty}^{\infty} \left(\frac{5}{2} \frac{\sin\left(\frac{\pi - \pi m}{2}\right)}{\pi - \pi m} + \frac{5}{2} \frac{\sin\left(\frac{\pi + \pi m}{2}\right)}{\pi + \pi m} \right) e^{j\frac{\pi}{9}m} \right]$$

$$+ \frac{5}{4} (e^{j\frac{\pi}{9}t} + e^{-j\frac{\pi}{9}t})$$

$$q_1 = a_{-1} = \frac{1}{18} \int_{-9/2}^{9/2} \left[y_{02}(t) + 5 \cos\left(\frac{\pi t}{9}\right) e^{-j\frac{\pi}{9}t} \right] dt$$

$$= \frac{5}{4}$$

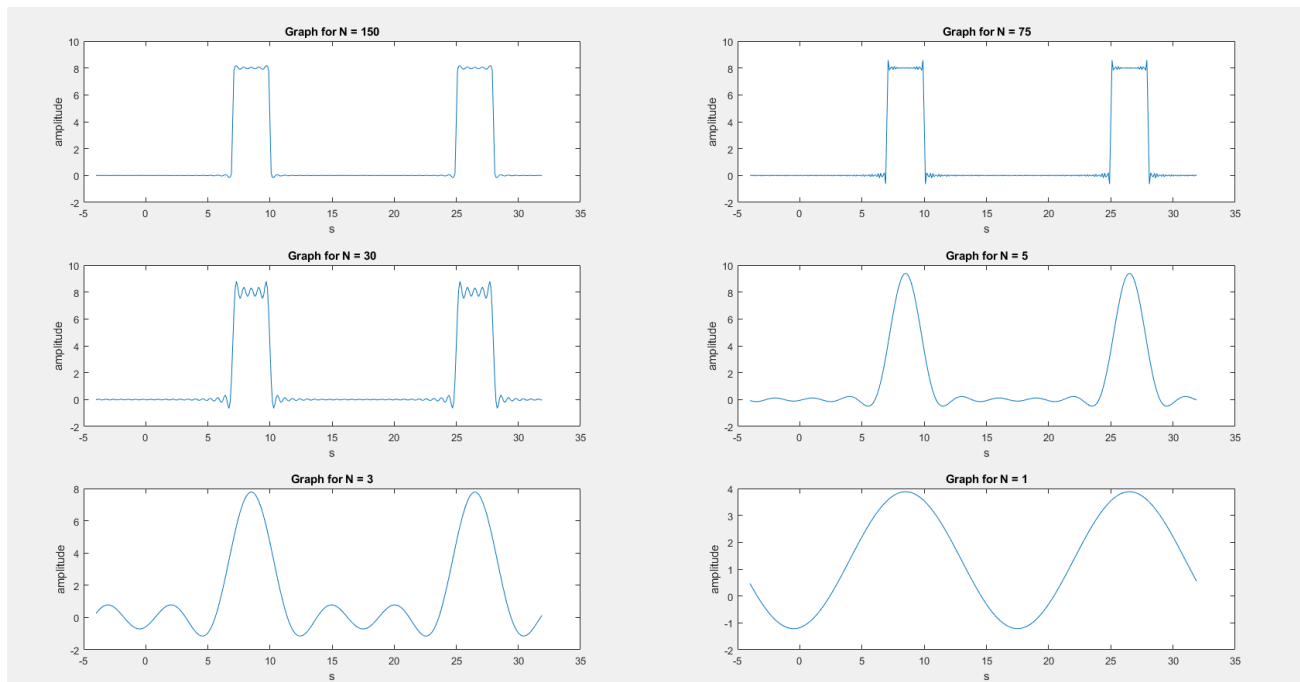
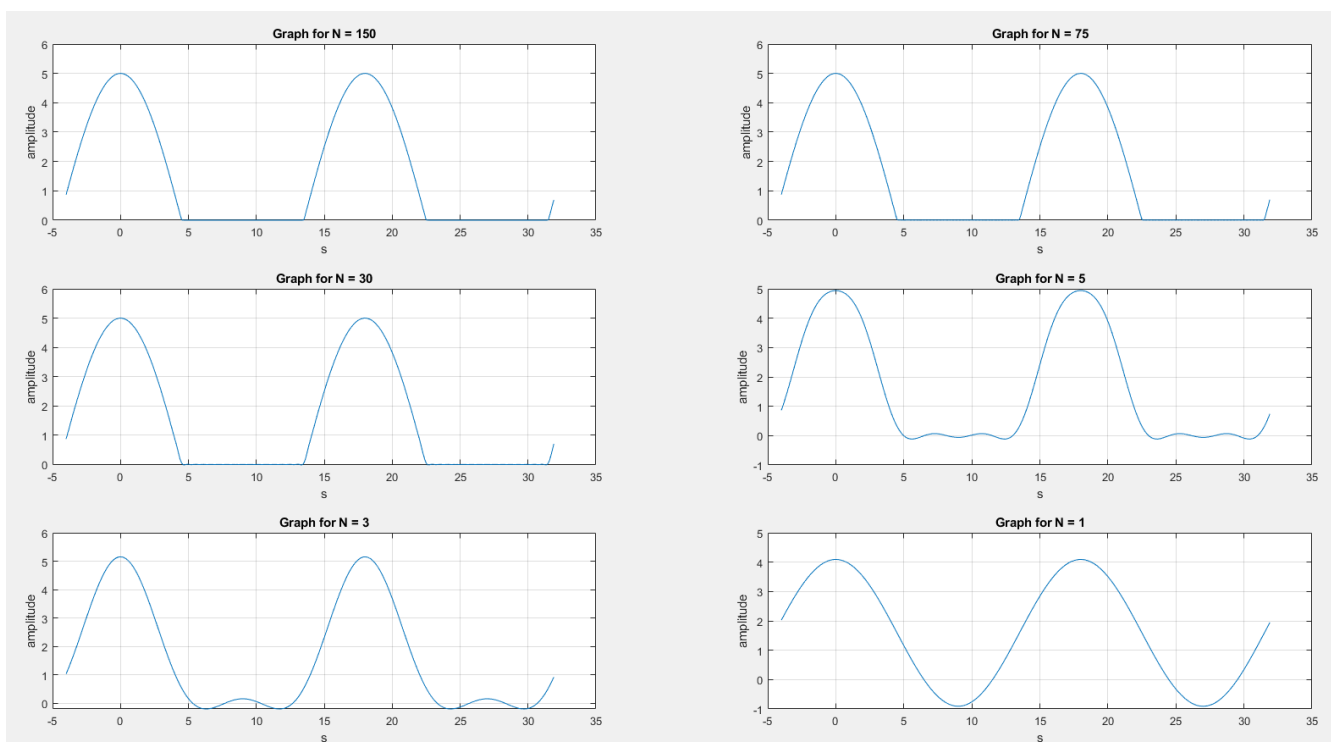
(FSD.)

$$y_{03}(t) = \frac{t}{\pi} + \frac{5}{2} \cos\left(\frac{\pi t}{9}\right) + \left[5 \sum_{m=-\infty}^{\infty} \left(\frac{\sin\left(\frac{\pi - \pi m}{2}\right)}{\pi - \pi m} + \frac{\sin\left(\frac{\pi + \pi m}{2}\right)}{\pi + \pi m} \right) \cdot \cos\left(\frac{\pi t}{9}\right) \right]$$

Figure 15: Solution for Q3

ANSWER TO ASKED COMMENTS

While finding $z[n]$, we combine multiple terms and at some instances observe spikes and oscillations at the corners of graphs. This is due to the Gibbs phenomenon, as can be seen in the graphs for changing N values. The increase of N values also linearizes the shape of the graph as more terms are included. This happens due to discontinuities that exist especially for Q1, which make space for Gibbs phenomenon to occur clearly. These can be seen as below in Figures 16, 17.

Figure 16: Graphs for Changing N Values in Q1Figure 17: Graphs for Changing N Values in Q3

APPENDIX – MATLAB CODE

Q1

```

1  clc;
2  clear;
3  close all;
4
5  % 22201832 Emir A. Bayer EEE321 Lab4 Q1
6
7  fundT = 18;
8  n = -40:319;
9  t = n/10;
10 fundW = 2 * pi / fundT;
11 %the sampling period is 1/10
12
13 ya = zeros(size(t));
14 t_mod = mod(t, fundT);
15 ya((t_mod >= 7) & (t_mod < 10)) = 8;
16 y_n_two_periods = [ya, ya];
17 n_two_periods = [n, n + length(n)];
18
19 figure;
20 stem(n, ya, 'filled');
21 title('a) Graph of y[n]');
22 xlabel('n');
23 ylabel('y[n]');
24 grid on;
25
26 %zeroth harmonic = 4/3 -> DC component, average value over one period
27
28 r_ak = zeros(1, 150);
29 im_ak = zeros(1, 150);
30
31 %f coeffs for 1:150
32 for k = 1:150
33     ak = (4 * 1j / (pi * k)) * (exp(-1j * 10 * pi * k / 9) - exp(-1j * 7 * pi * k / 9));
34     r_ak(k) = real(ak);
35     im_ak(k) = imag(ak);
36 end
37
38 mag_ak = [fliplr(r_ak + 1j * im_ak), 4/3, r_ak + 1j * im_ak];
39 frequencies = fundW * (-150:150);
40
41 %1C spectrum
42 figure;
43
44 stem(frequencies, real(mag_ak), 'b', 'Marker', 'none', 'DisplayName', 'Real');
45 hold on;
46 stem(frequencies, imag(mag_ak), 'r', 'Marker', 'none', 'DisplayName', 'Imaginary');
47 legend;
48 xlabel('rad/s');
49 ylabel('magnitude');
50 title('Spectrum of Q1');
51

```

```

53 %ID
54 N_values = [150, 75, 30, 5, 3, 1];
55 figure;
56 for j = 1:length(N_values)
57     N_val = N_values(j);
58     out_N = 4/3 * ones(size(t));
59
60     for k = 1:N_val
61         out_N = out_N + r_ak(k) * cos(k * fundW * t) - im_ak(k) * sin(k * fundW * t);
62         out_N = out_N + r_ak(k) * cos(-k * fundW * t) - (-im_ak(k)) * sin(-k * fundW * t);
63     end
64
65     subplot(3, 2, j);
66     plot(t, real(out_N));
67     title(['Graph for N = ', num2str(N_val)]);
68     xlabel('s');
69     ylabel('amplitude');
70 end
71
72 figure;
73 %harm0
74 subplot(2, 2, 1);
75 plot(t, 4/3 * ones(size(t)));
76 xlabel('s');
77 ylabel('amplitude');
78 title('Harmonic 0');
79 %harm1
80 subplot(2, 2, 2);
81 harm_1 = 2 * (r_ak(1) * cos(fundW * t) - im_ak(1) * sin(fundW * t));
82 plot(t, harm_1);
83 title('Harmonic 1');
84 xlabel('s');
85 ylabel('amplitude');
86 %harm2
87 subplot(2, 2, 3);
88 harm_2 = 2 * (r_ak(2) * cos(2 * fundW * t) - im_ak(2) * sin(2 * fundW * t));
89 plot(t, harm_2);
90 title('Harmonic 2');
91 xlabel('s');
92 ylabel('amplitude');
93 %harm3
94 subplot(2, 2, 4);
95 harm_3 = 2 * (r_ak(3) * cos(3 * fundW * t) - im_ak(3) * sin(3 * fundW * t));
96 plot(t, harm_3);
97 title('Harmonic 3');
98 xlabel('s');
99 ylabel('amplitude');

```

Q2

```

1  clc
2  clear
3  close all
4  % 22201832 Emir A. Bayer EEE321 Lab4 Q2
5
6
7  fundT = 18;
8  n = -40:319;
9  t = n / 10;
10 fundW = 2 * pi / fundT;
11
12
13 y_a2_t = abs(5 * cos(pi * t / 9));
14 figure;
15 stem(n, y_a2_t, 'filled');
16 title('a) Graph of y[n]');
17 xlabel('n');
18 ylabel('y[n]');
19
20 dcAve = 10 / pi; %dc ave for one per
21
22 r_ak = zeros(1, 150);
23 im_ak = zeros(1, 150);
24
25 %f coeffs for 1:150
26 for k = 1:150
27     r_ak(k) = 10 * (sin(pi/2 * (1 - 2*k)) / (pi * (1 - 2*k)) + sin(pi/2*(1+2*k)) / (pi * (1+2*k))) * cos(2*pi/9 * k * pi/9);
28     im_ak(k) = 0; %symmetry makes it 0
29 end
30
31 mag_ak = [fliplr(r_ak + 1j * im_ak), 4/3, r_ak + 1j * im_ak];
32 frequencies = fundW * (-150:150);
33
34 %2C spectrum
35 figure;
36 stem(frequencies, real(mag_ak), 'b', 'Marker', 'none', 'DisplayName', 'Real');
37 hold on;
38 stem(frequencies, imag(mag_ak), 'r', 'Marker', 'none', 'DisplayName', 'Imaginary');
39 legend;
40 title('Spectrum of Q2');
41 xlabel('rad/s');
42 ylabel('magnitude');
43
44
45 %repeating for new Ns
46 repeats_list = [150, 75, 30, 5, 3, 1];
47 figure;
48 for j = 1:length(repeats_list)
49     Nval = repeats_list(j);
50     outN = dcAve * ones(size(t));

```

```
51
52     %reals first
53     for k = 1:Nval
54         outN = outN + r_ak(k) * cos(k * fundW * t);
55     end
56     subplot(3, 2, j);
57     plot(t, real(outN));
58     title(['Graph for N = ', num2str(Nval)]);
59     xlabel('s');
60     ylabel('amplitude');
61 end
62
63 figure;
64 %harm0
65 subplot(2, 2, 1);
66 plot(t, dcAve * ones(size(t)));
67 title('Harmonic 0');
68 xlabel('s');
69 ylabel('amplitude');
70 %harm1
71 subplot(2, 2, 2);
72 harm1 = r_ak(1) * cos(fundW * t);
73 plot(t, harm1);
74 title('Harmonic 1');
75 xlabel('s');
76 ylabel('amplitude');
77 %harm2
78 subplot(2, 2, 3);
79 harm2 = r_ak(2) * cos(2 * fundW * t);
80 plot(t, harm2);
81 title('Harmonic 2');
82 xlabel('s');
83 ylabel('amplitude');
84 %harm3
85 subplot(2, 2, 4);
86 harm_3 = r_ak(3) * cos(3 * fundW * t);
87 plot(t, harm_3);
88 title('Harmonic 3');
89 xlabel('s');
90 ylabel('amplitude');
```


Q3

```

1  clc;
2  clear;
3  close all;
4
5  % 22201832 Emir A. Bayer EEE321 Lab4 Q3
6
7  % Parameters
8  fundT = 18; % signal per
9  n = -40:319;
10 t = n / 10; % time vector for samples
11 fundW = 2 * pi / fundT;
12 %the sampling period is 1/10
13
14 ya = (5 * cos(pi * t / 9)) .* ((5 * cos(pi * t / 9)) > 0);
15
16 figure;
17 stem(n, ya, 'filled');
18 hold on;
19 title('a) Graph of y[n]');
20 xlabel('n');
21 ylabel('y[n]');
22
23
24 %fourier coefficients
25 r_ak = zeros(1, 150);
26 im_ak = zeros(1, 150);
27
28 %calculate Fourier coefficients analytically for k = 1 to N
29 for k = 1:150
30     r_ak(k) = (5/2) * cos(2*pi/18+pi/9)*(sin((pi/2) * (1-2*k))/(pi*(1+2*k)))/(pi*(1+2*k));
31     im_ak(k) = 0; % Imaginary part is zero due to symmetry
32 end
33
34 %extending for negative frequencies and the zeroth component
35 ak_full = [fliplr(r_ak + 1j * im_ak), a0, r_ak + 1j * im_ak];
36 frequencies = fundW * (-150:150);
37
38 %spectrum
39 figure;
40 stem(frequencies, real(ak_full), 'b', 'filled', 'Marker', 'none', 'DisplayName', 'Real Part');
41 hold on;
42 stem(frequencies, imag(ak_full), 'r', 'filled', 'Marker', 'none', 'DisplayName', 'Imaginary Part');
43 title('Spectrum of Q3');
44 xlabel('rad/s');
45 ylabel('amplitude');
46 legend;
47 grid on;
48

```

```

49 %different Ns (repeats)
50 repeats_list = [150, 75, 30, 5, 3, 1];
51 figure;
52 for j = 1:length(repeats_list)
53     Nval = repeats_list(j);
54     outN = a0 * ones(size(t)); %firstly the DC component
55
56     %adding harmonics up to the current Nval accounting for real part only
57     for k = 1:Nval
58         outN = outN + r_ak(k) * cos(k * fundW * t);
59     end
60
61     subplot(3, 2, j);
62     plot(t, real(outN));
63     title(['Graph for N = ', num2str(Nval)]);
64     xlabel('s');
65     ylabel('amplitude');
66     grid on;
67 end
68
69 figure;
70
71 %harm0
72 subplot(2, 2, 1);
73 plot(t, a0 * ones(size(t)));
74 title('Harmonic 0');
75 xlabel('s');
76 ylabel('amplitude');
77
78 %harm1
79 subplot(2, 2, 2);
80 harm_1 = r_ak(1) * cos(fundW * t);
81 plot(t, harm_1);
82 title('Harmonic 1');
83 xlabel('s');
84 ylabel('amplitude');
85
86 %harm2
87 subplot(2, 2, 3);
88 harm_2 = r_ak(2) * cos(2 * fundW * t);
89 plot(t, harm_2);
90 title('Harmonic 2');
91 xlabel('s');
92 ylabel('amplitude');
93
94 %harm3
95 subplot(2, 2, 4);
96 harm_3 = r_ak(3) * cos(3 * fundW * t);
97 plot(t, harm_3);
98 title('Harmonic 3');
99 xlabel('s');
100 ylabel('amplitude');

```