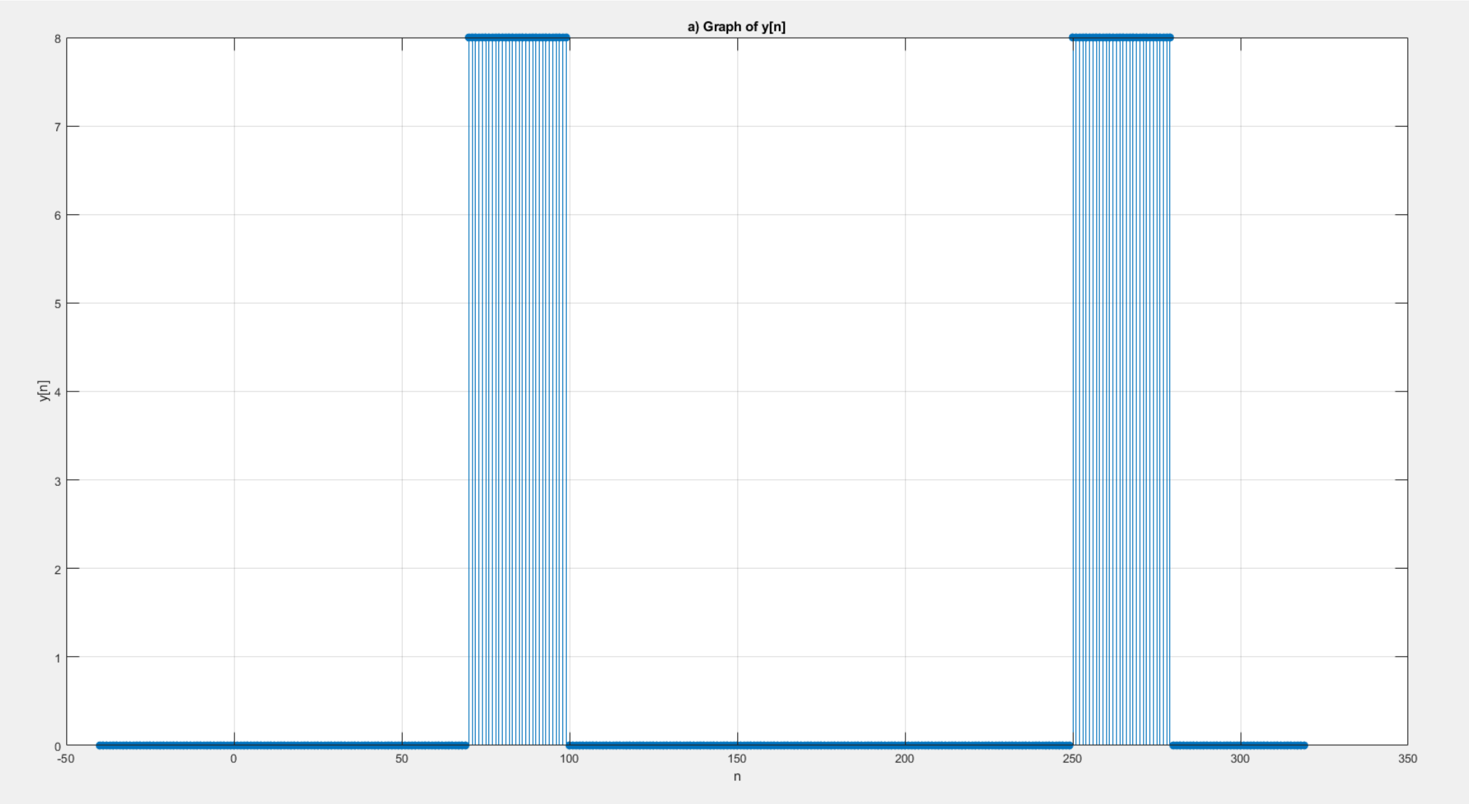
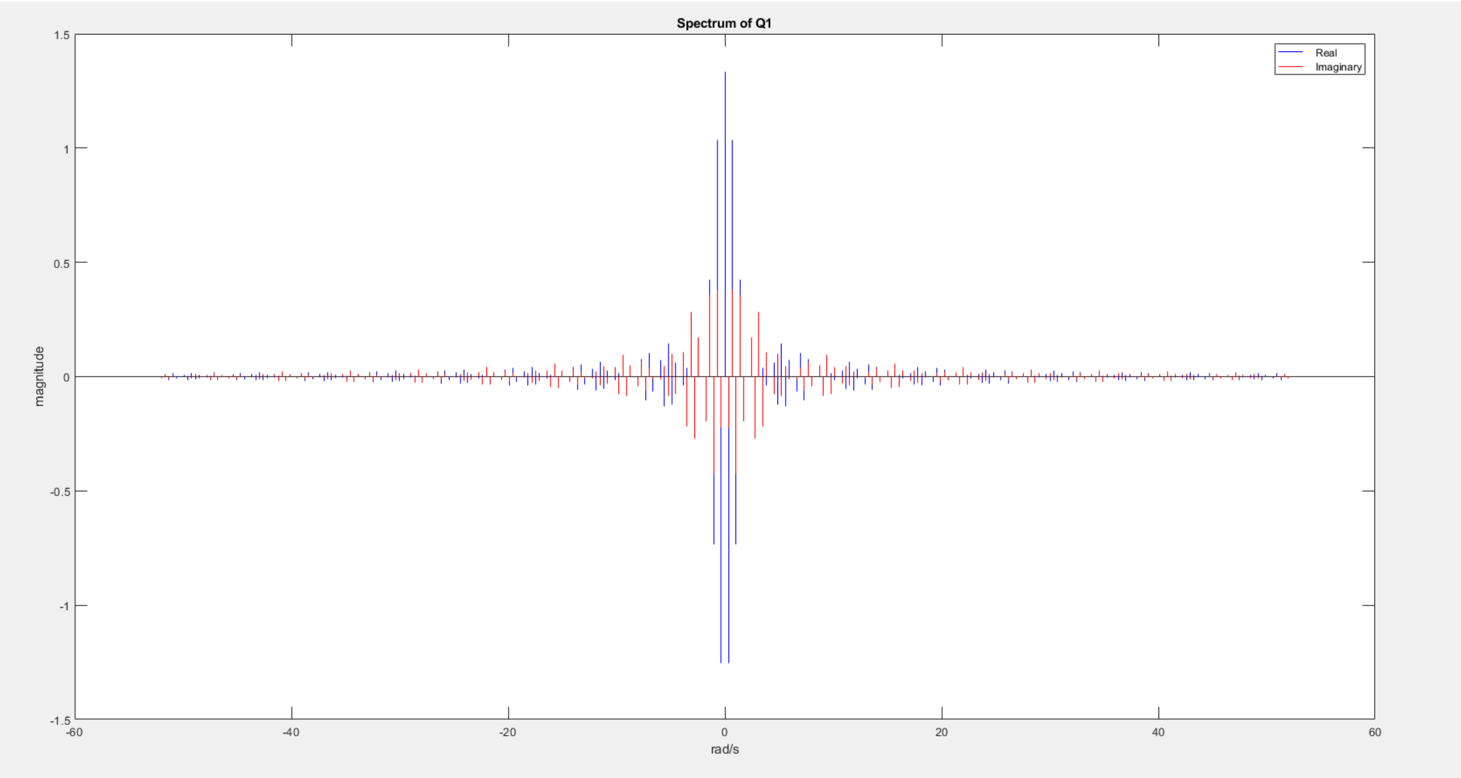
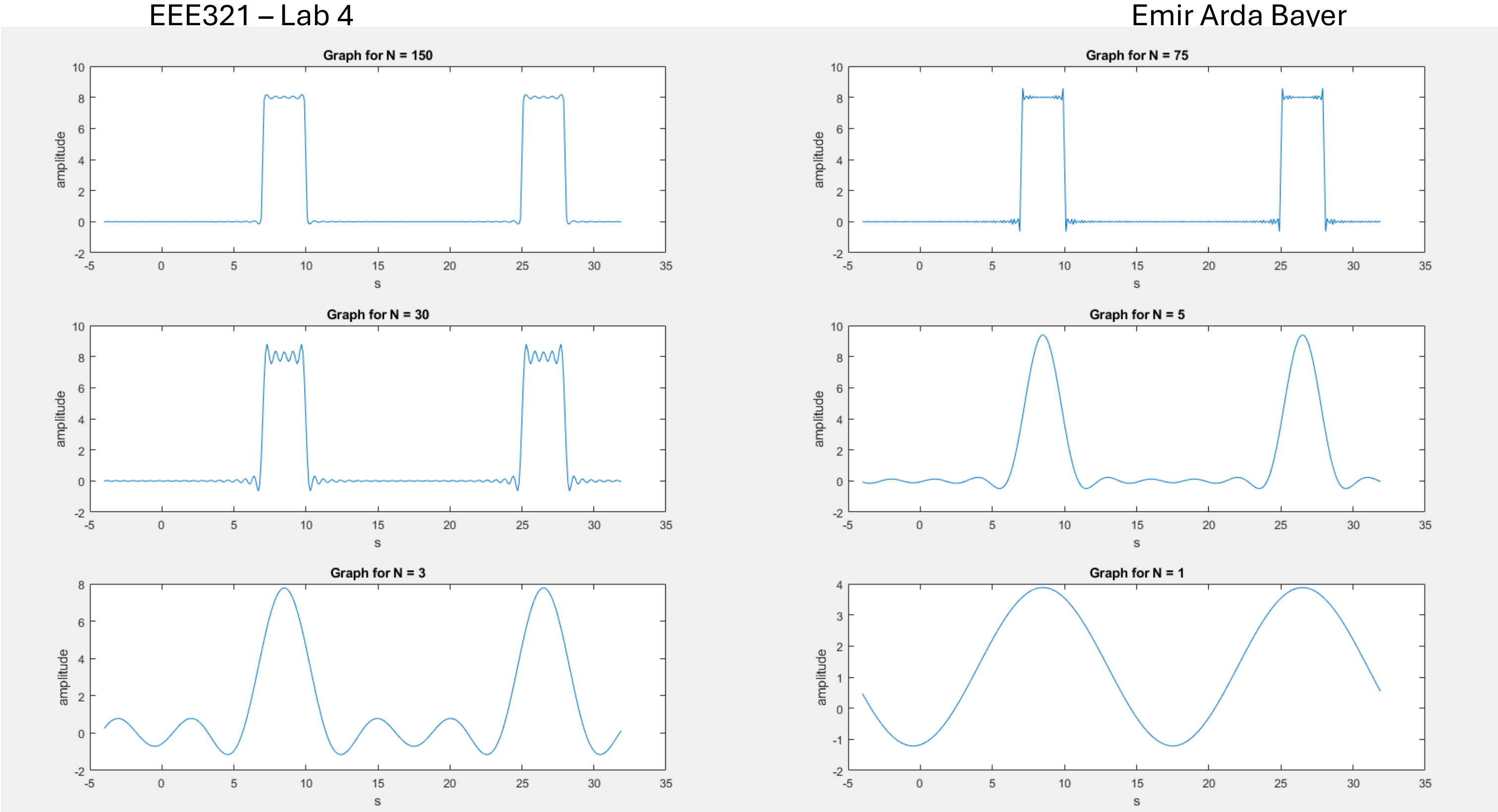
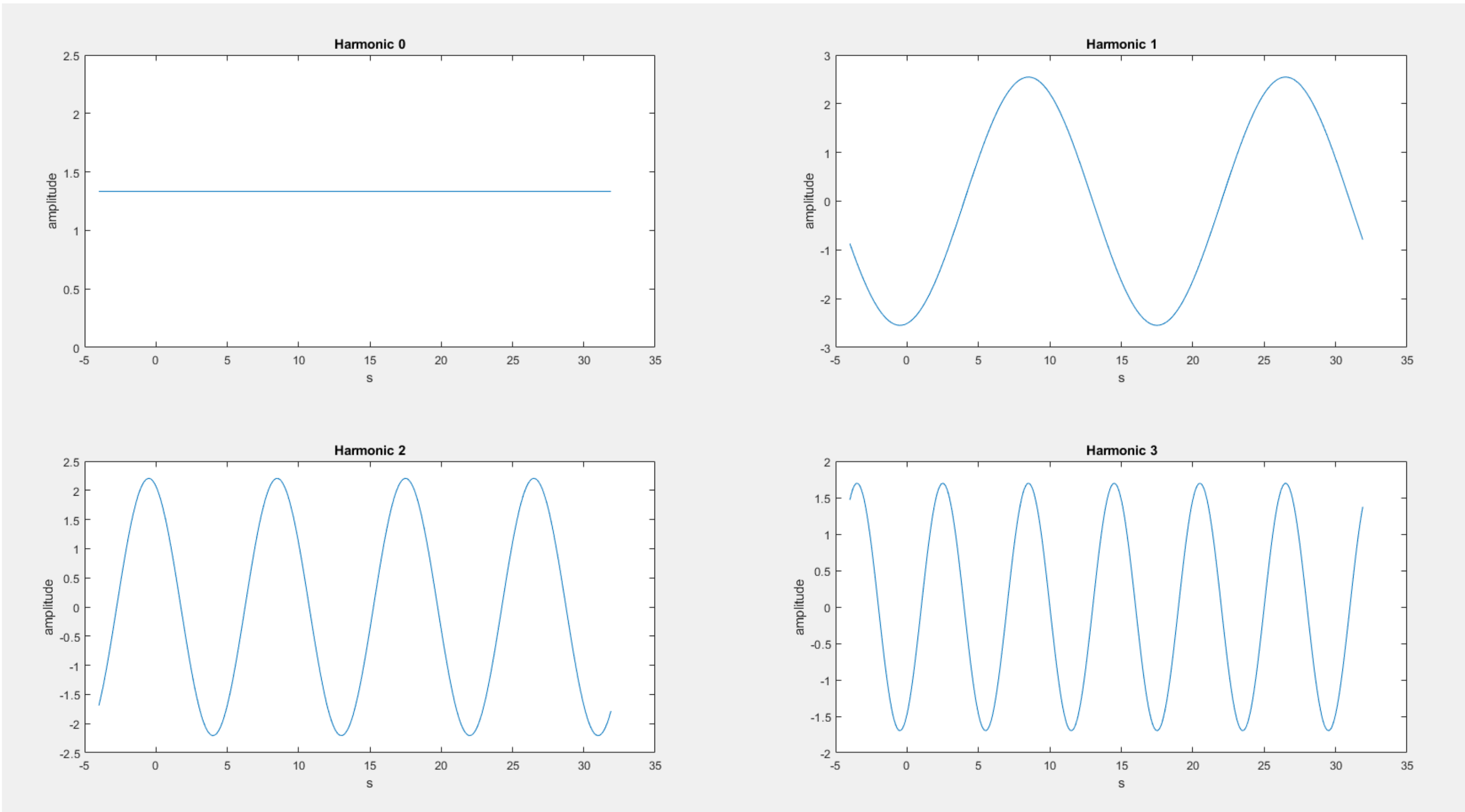
MATLAB PLOTS



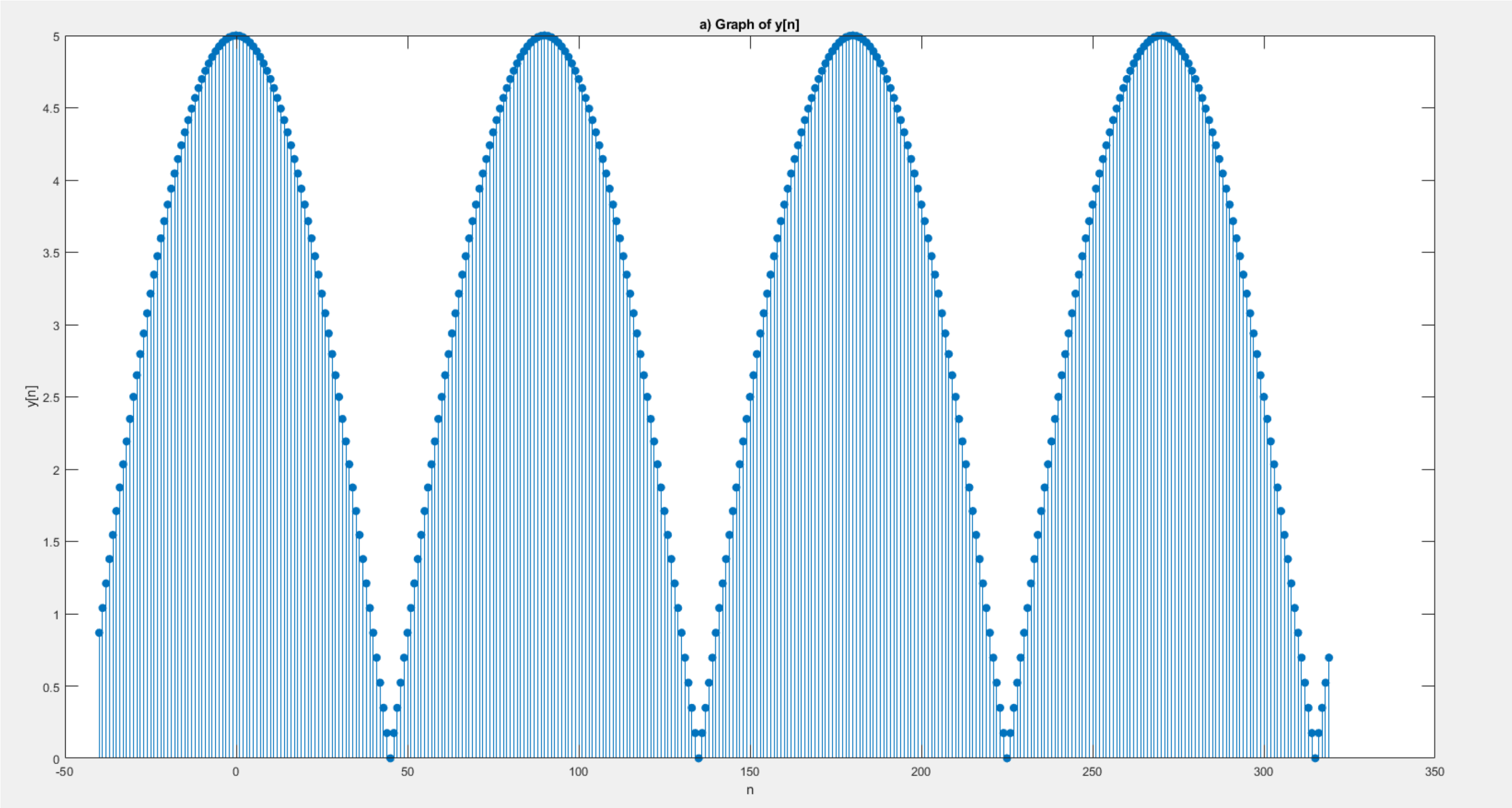


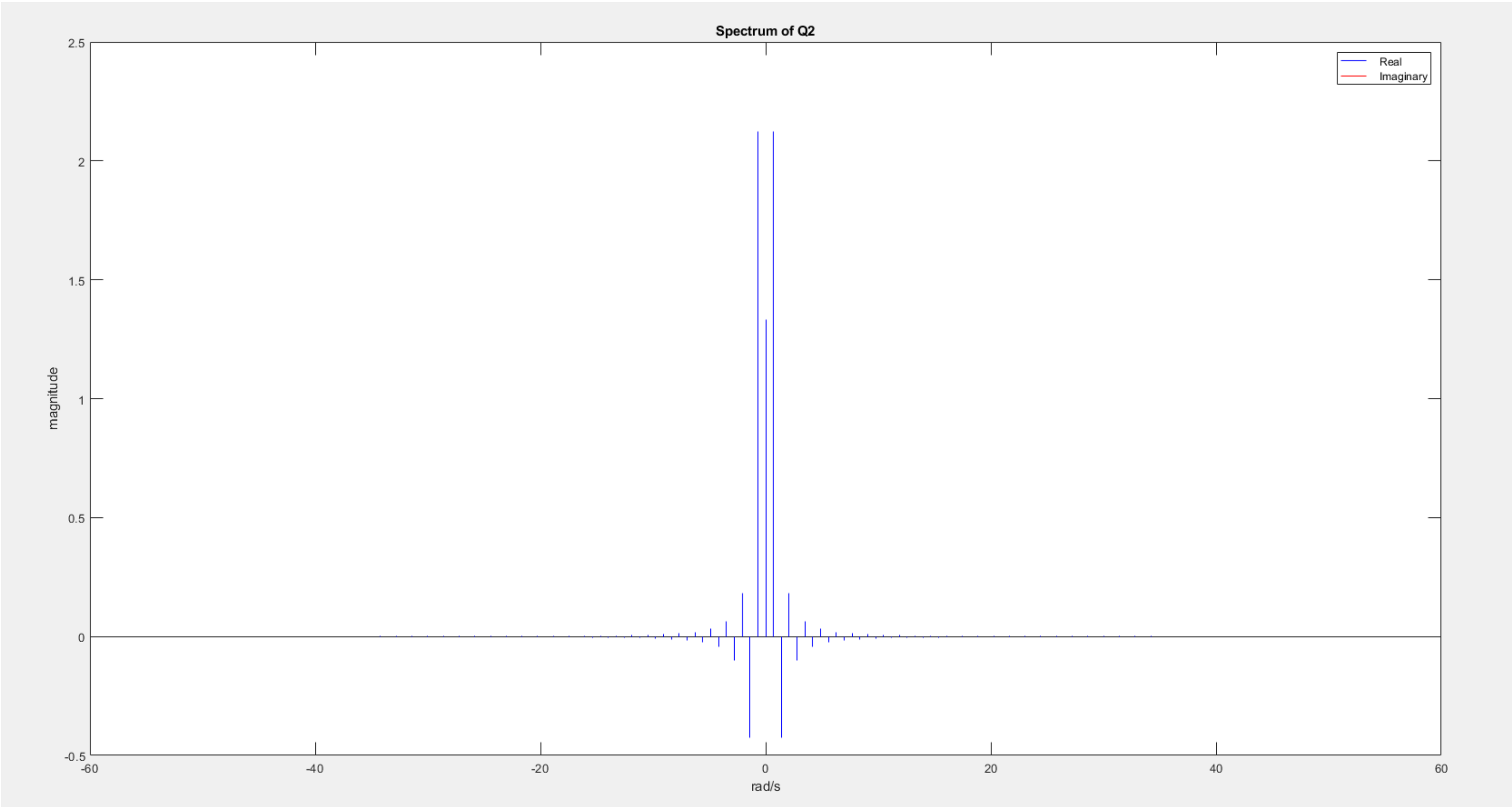
Figures 1, 2: First Two Graphs for Q1



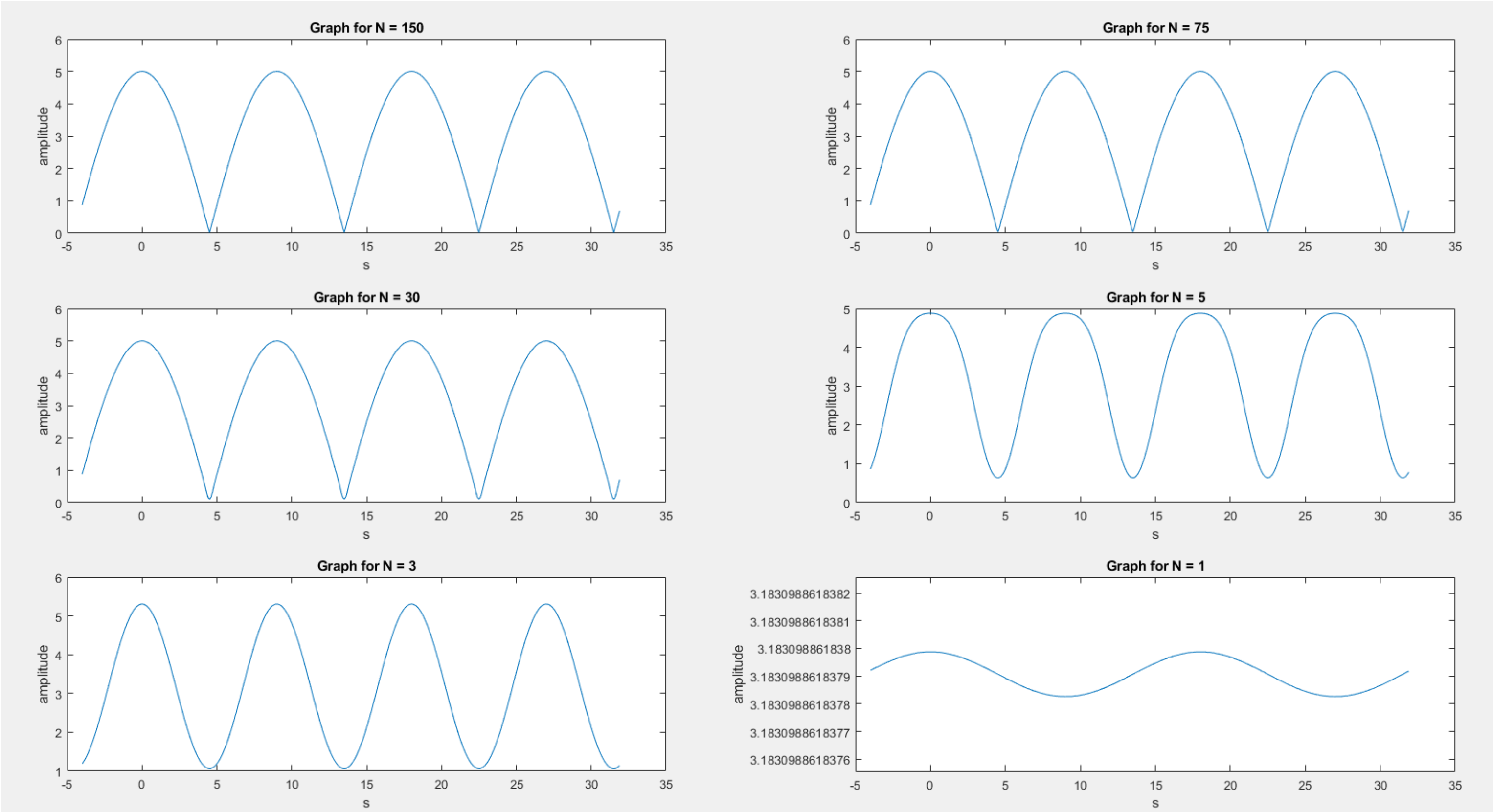


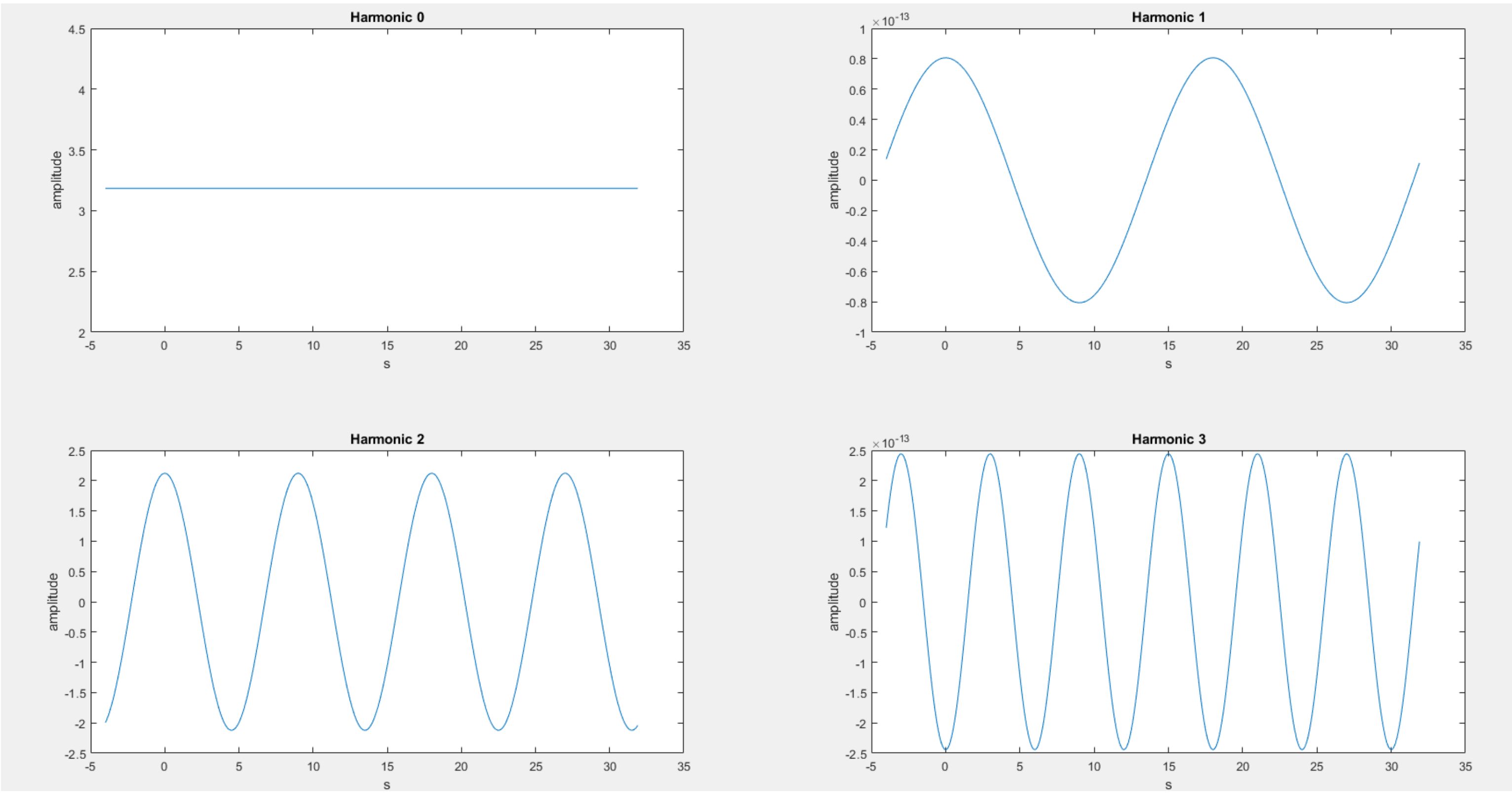
Figures 3,4: Remaining Graphs for Q1



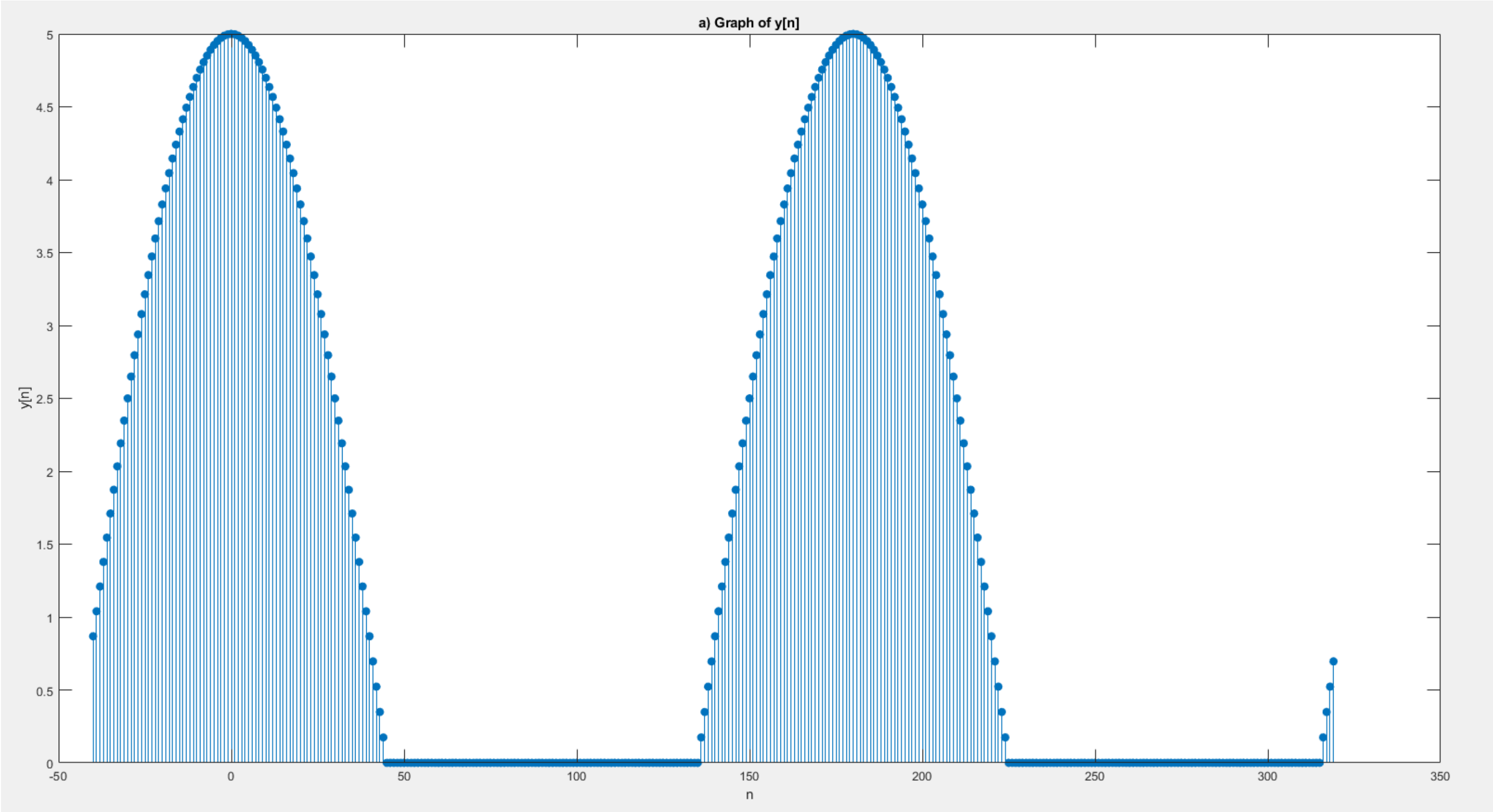


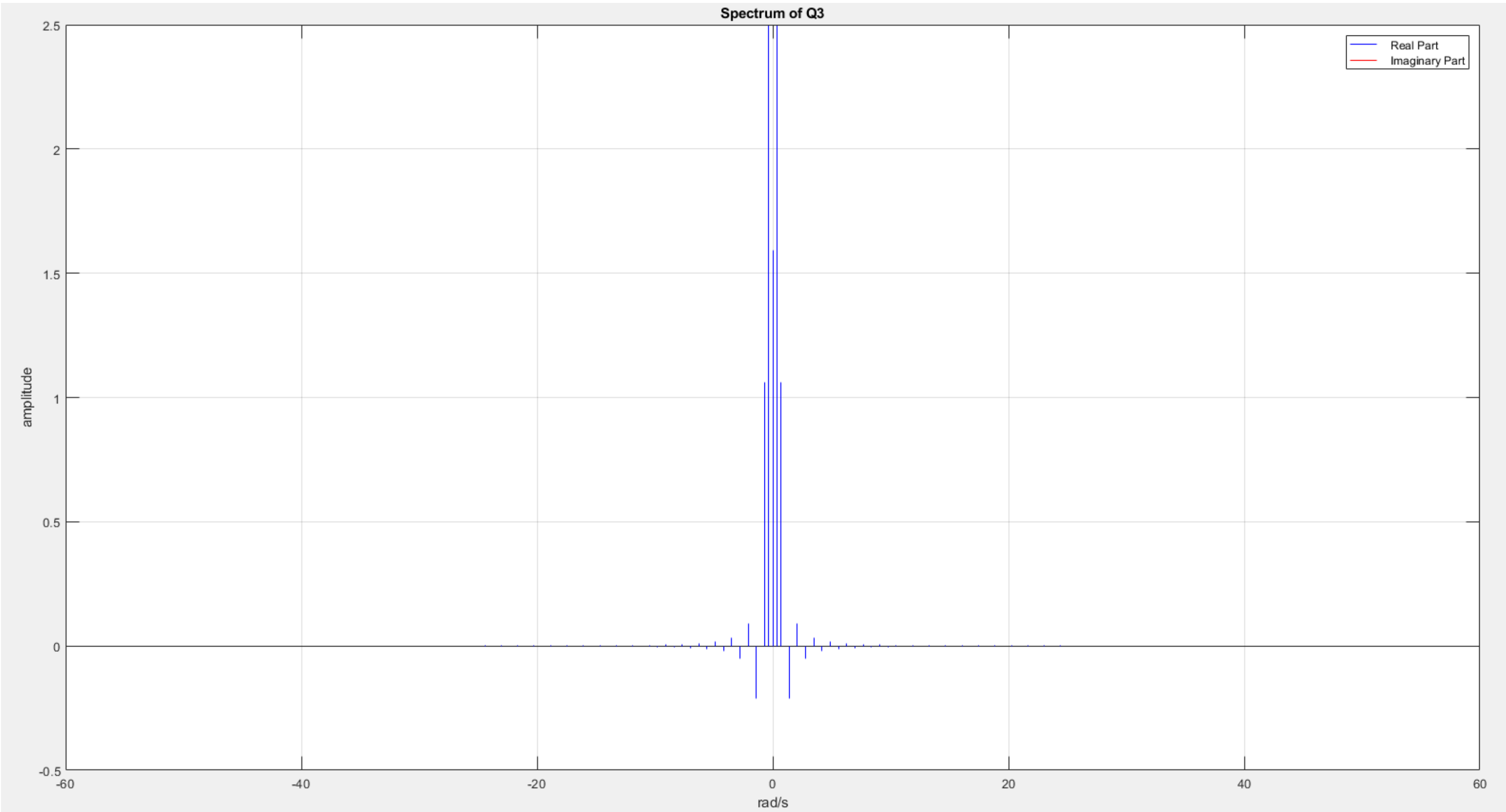
Figures 5,6: First Two Graphs for Q2





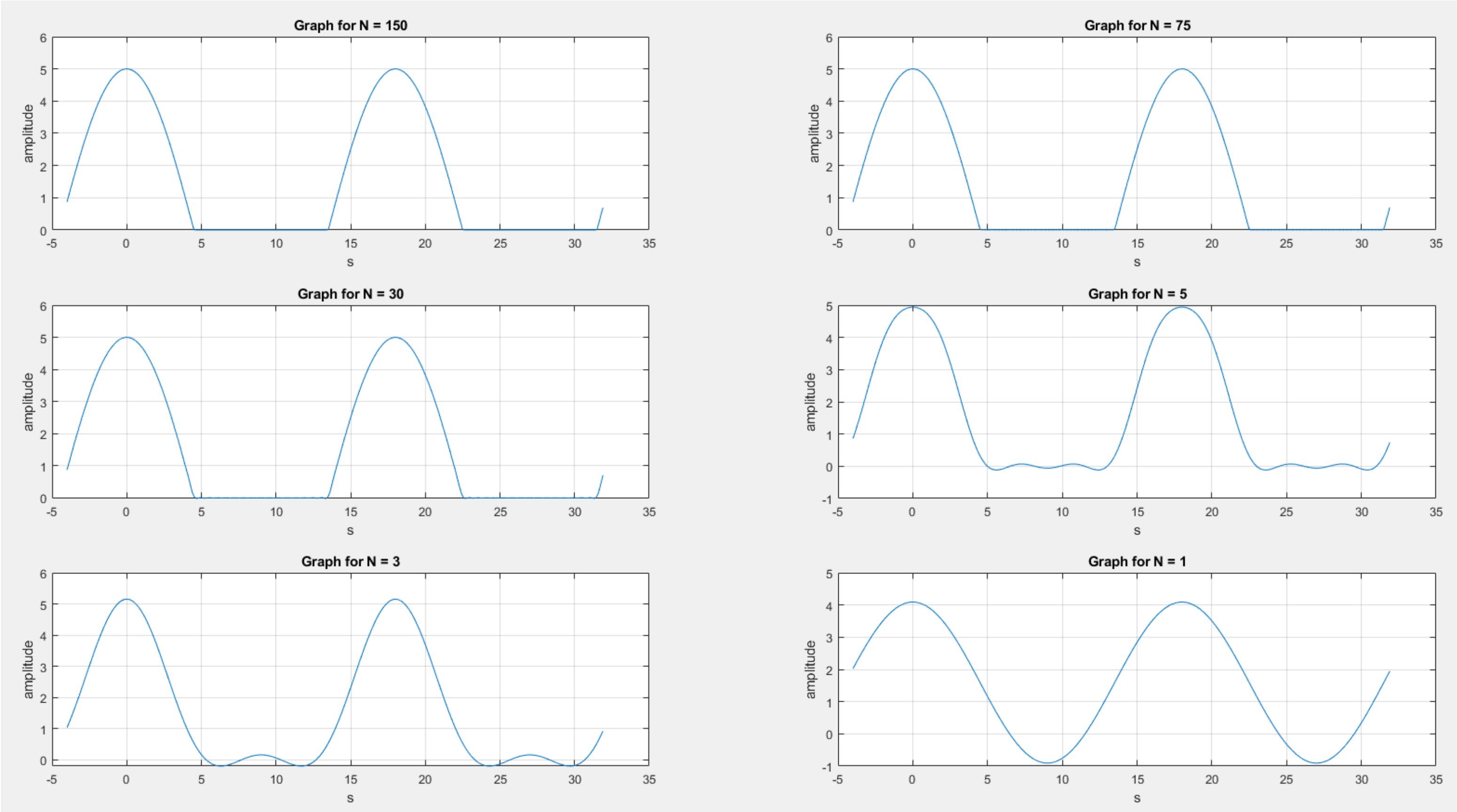
Figures 7,8: Remaining Graphs for Q2

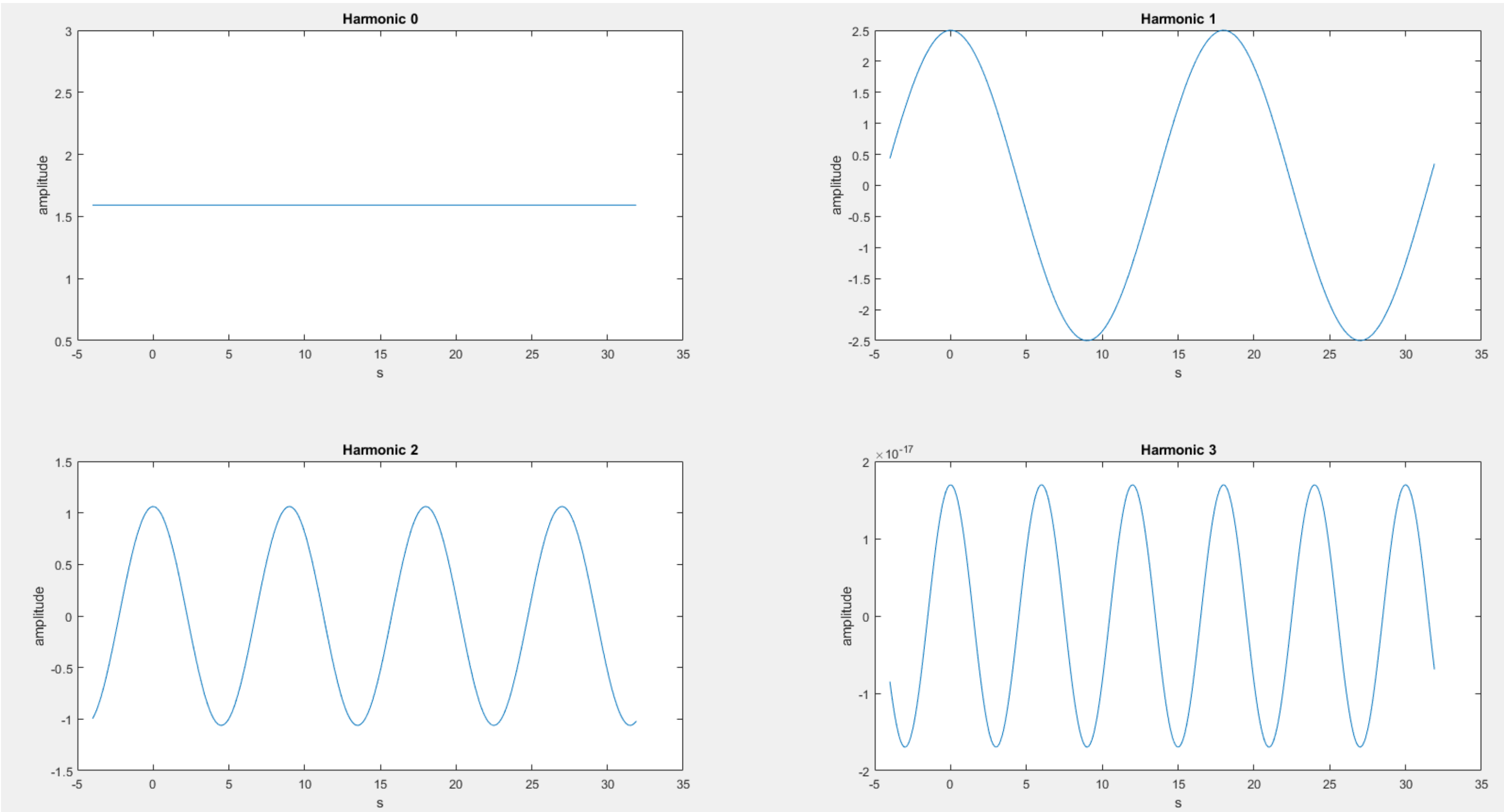




Figures 9,10: First Two Graphs for Q3

Emir Arda Bayer
22201832





Figures 11,12: Remaining Graphs for Q3

HANDWRITTEN SOLUTIONS

Q1

$$y[n] = \begin{cases} 0, & 0 < n < 7 \\ 8, & 7 \leq n < 10 \\ 0, & 10 \leq n < 18 \end{cases} \Rightarrow T = 180$$

$$a_0 = \frac{1}{18}\int_0^{18} y_a(t)\,dt = \left(\int_7^{18} 8\,dt\right)\cdot\frac{1}{18} = \frac{1}{18}\cdot 8t\Big|_7^{10} = \frac{24}{18} = \frac{4}{3}$$

$$a_k = \frac{1}{18}\int_0^{18} y_a(t)\, e^{-j\frac{2\pi k t}{18}}\,dt = \frac{1}{18}\int_7^{10} 8e^{-j\frac{2\pi k t}{18}}\,dt = \frac{8}{18}\cdot(-18)\frac{1}{2\pi k j}\left[e^{-j\frac{2\pi k t}{18}}\right]_7^{10}$$

$$= \frac{4}{j\pi k}\left(e^{-j20\pi k\cdot\frac{1}{18}} - e^{\frac{-j14\pi k}{18}}\right)$$

$$\text{FSE} = y_a(t) = \frac{4}{3} + \sum_{k=-\infty}^{\infty} a_k e^{-j\frac{\pi k t}{9}}$$

$$= \frac{4}{j\pi k}\left(e^{-j\frac{10}{9}\pi k} - e^{-j\frac{7}{9}\pi k}\right)$$

$$a_k = \begin{cases} \frac{4}{k\pi j}\left(\cos\left(\frac{10\pi}{9}k\right) - \cos\left(\frac{7\pi}{9}k\right) + j\left(\sin\left(\frac{7\pi}{9}k\right) - \sin\left(\frac{10\pi}{9}k\right)\right)\right), & k \neq 0 \\ \frac{4}{3}, & k = 0 \end{cases}$$

(spectrum)

$$y_a(t) = \frac{4}{3}\sum_{k=-\infty}^{\infty} a_k e^{j\frac{\pi}{9}kt} = \frac{4}{3} + \sum_{k=1}^{\infty} 2a_k\cos\left(\frac{\pi}{9}kt\right)$$

(FSEa.)

$$z[n] = \frac{4}{3} + \sum_{k=1}^{N}\frac{8}{\pi j k}\left(\cos\left(\frac{10\pi}{9}k\right) - \cos\left(\frac{7\pi}{9}k\right) + j\left(\sin\left(\frac{7\pi}{9}k\right) - \sin\left(\frac{10\pi}{9}k\right)\right)\right)\cdot\cos\left(\frac{\pi n k}{9\cdot 81}\right)$$

Figure 13: Solution for Q1

Q2

$y_a(t) = |5\cos(\frac{\pi}{9}t)|$ $\quad T_0 = 9$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{1}{9}\int_{-4.5}^{4.5} 5\cos(\frac{\pi}{9}t)\,dt = \frac{1}{9}\cdot\frac{9}{\pi}\cdot 5\left(\sin(\frac{\pi}{2}) - \sin(-\frac{\pi}{2})\right) = \frac{10}{\pi}$$

$$a_k = \frac{1}{T}\int_{o.o.p} y_a(t)\, e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{9}\int_{-4.5}^{4.5} 5\cos(\frac{\pi}{9}t)\, e^{-j\frac{2\pi}{9}kt}\,dt$$

$$= \frac{1}{18}\cdot 5\int_{-4.5}^{4.5}\Big[\underbrace{e^{j\frac{\pi}{9}t + (-j\frac{2\pi}{9}kt)}}_{e^{j\frac{\pi}{9}(1-2k)t}} + \underbrace{e^{-j\frac{\pi}{9}t + (-j\frac{2\pi}{9}kt)}}_{e^{-j\frac{\pi}{9}(1+2k)t}}\Big]dt$$

$$= \frac{5}{18}\left[\frac{9e^{j\frac{\pi}{9}(1-2k)t}}{\pi(1-2k)j} - \frac{9e^{-j\frac{\pi}{9}(1+2k)t}}{\pi(1+2k)j}\right]_{-4.5}^{4.5}$$

$$= \frac{5}{18}\cdot\frac{9}{\pi}\left(\frac{e^{j\frac{\pi}{2}(1-2k)} - e^{-j(1-2k)\frac{\pi}{2}}}{1-2k} + \frac{e^{j\frac{\pi}{2}(1+2k)} - e^{-j\frac{\pi}{2}(1+2k)}}{1+2k}\right)$$

$$= \frac{5\sin(\frac{\pi}{2}(1-2k))}{\pi - 2k\pi} + \frac{5\sin(\frac{\pi}{2}(1+2k))}{\pi + 2k\pi}$$

$$\Rightarrow \frac{10}{\pi} + \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi}{9}kt} = \frac{10}{\pi} + \sum_{k=1}^{\infty} 2a_k\cos(\frac{2\pi}{9}kt) = FSE$$

(spectrum)

$$a_k = \begin{cases} \dfrac{5\sin(\frac{\pi}{2}(1-2k))}{\pi - 2k\pi} + \dfrac{5\sin(\frac{\pi}{2}(1+2k))}{\pi + 2k\pi}, & k \neq 0 \\ \dfrac{10}{\pi}, & k = 0 \end{cases}$$

$$y_{a2}(t) = \frac{10}{\pi} + \sum_{k=1}^{\infty}\left[\frac{10\sin(\frac{\pi - 2k\pi}{2})}{\pi - 2k\pi} + \frac{10\sin(\frac{\pi + 2k\pi}{2})}{\pi + 2k\pi}\right]\cos(\frac{2\pi}{9}kt)$$

(FSE a.)

$$y_1[n] = \frac{10}{\pi} + \sum_{N=1}^{N}\left[\frac{10\sin(\frac{\pi - 2k\pi}{2})}{\pi - 2k\pi} + \frac{10\sin(\frac{\pi + 2k\pi}{2})}{\pi + 2k\pi}\right]\cos(\frac{2\pi^2}{81}k)$$

$-40 : 319$

Figure 14: Solution for Q2

(from Q2)

$y_{a3}(t) = \frac{y_a(t)+x(t)}{2}$ Q3

$x(t) = 5\cos\left(\frac{2\pi t}{18}\right) = \frac{5}{2}e^{j\frac{\pi}{9}t} + \frac{5}{2}e^{-j\frac{\pi}{9}t}$

$$\Rightarrow y_3(t) = \left(\frac{10}{\pi} + \sum_{k=-\infty}^{\infty}\left[\frac{5\sin\left(\frac{\pi-2k\pi}{2}\right)}{\pi-2k\pi} + \frac{5\sin\left(\frac{\pi+2k\pi}{2}\right)}{\pi+2\pi k}\right] e^{j\frac{2\pi}{9}kt}\right)\cdot\frac{1}{2}$$

* $m = 2n$

$$\Rightarrow y_3(t) = \frac{5}{\pi} + \left[\sum_{k=-\infty}^{\infty}\left(\frac{5}{2}\frac{\sin\left(\frac{\pi-m\pi}{2}\right)}{\pi-\pi m} + \frac{5}{2}\frac{\sin\left(\frac{\pi+\pi m}{2}\right)}{\pi+\pi m}\right)e^{j\frac{\pi}{9}m}\right] + \frac{5}{4}\left(e^{j\frac{\pi}{9}t} + e^{-j\frac{\pi}{9}t}\right)$$

$$a_1 = a_{-1} = \frac{1}{18}\int_{-9/2}^{9/2}\left[y_{a2}(t) + 5\cos\left(\frac{\pi t}{9}\right)e^{-j\frac{\pi}{9}t}\right]dt$$

$$= \frac{5}{4}$$

(FSEQ.)

$$y_{a3}(t) = \frac{5}{\pi} + \frac{5}{2}\cos\left(\frac{\pi t}{9}\right) + \left[5\sum_{m=-\infty}^{\infty}\left(\frac{\sin\left(\frac{\pi-\pi m}{2}\right)}{\pi-\pi m} + \frac{\sin\left(\frac{\pi+\pi m}{2}\right)}{\pi+\pi m}\right)\cdot\cos\left(\frac{\pi}{9}t\right)\right]$$

Figure 15: Solution for Q3

Emir Arda Bayer
22201832

ANSWER TO ASKED COMMENTS

While finding z[n], we combine multiple terms and at some instances observe spikes and oscillations at the corners of graphs. This is due to the Gibbs phenomenon, as can be seen in the graphs for changing N values. The increase of N values also linearizes the shape of the graph as more terms are included. This happens due to discontinuities that exist especially for Q1, which make space for Gibbs phenomenon to occur clearly. These can be seen as below in Figures 16, 17.

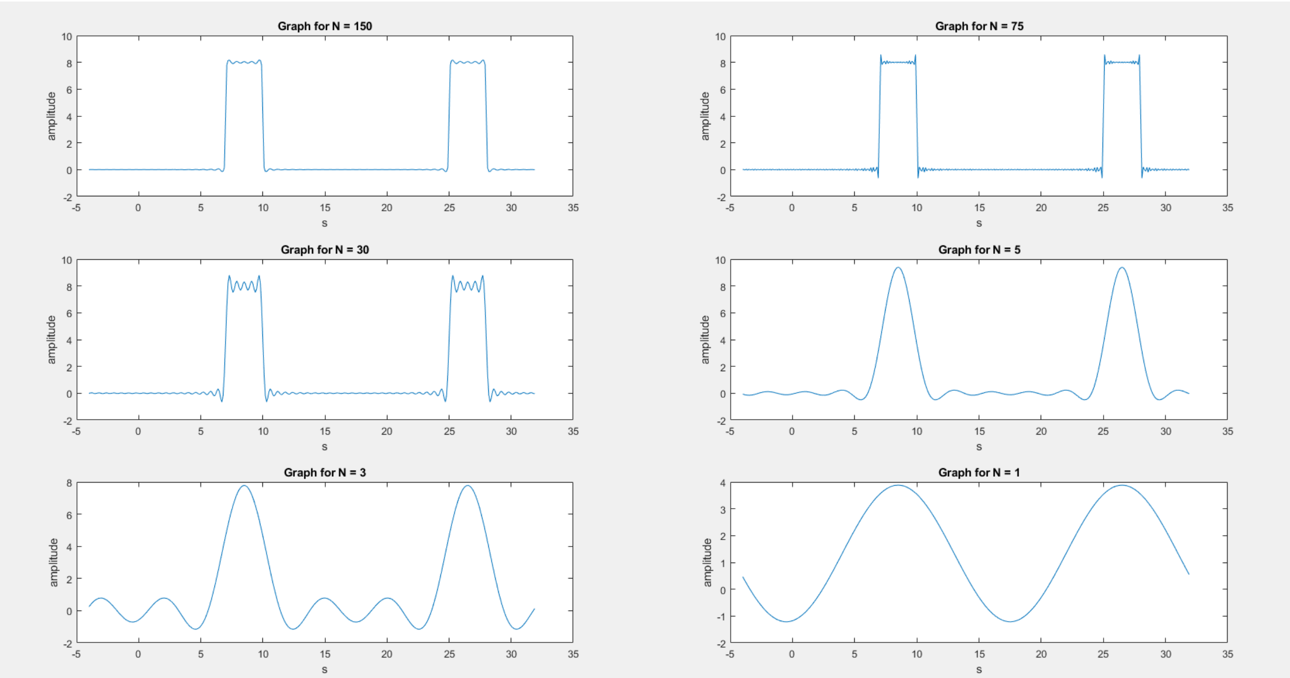


Figure 16: Graphs for Changing N Values in Q1

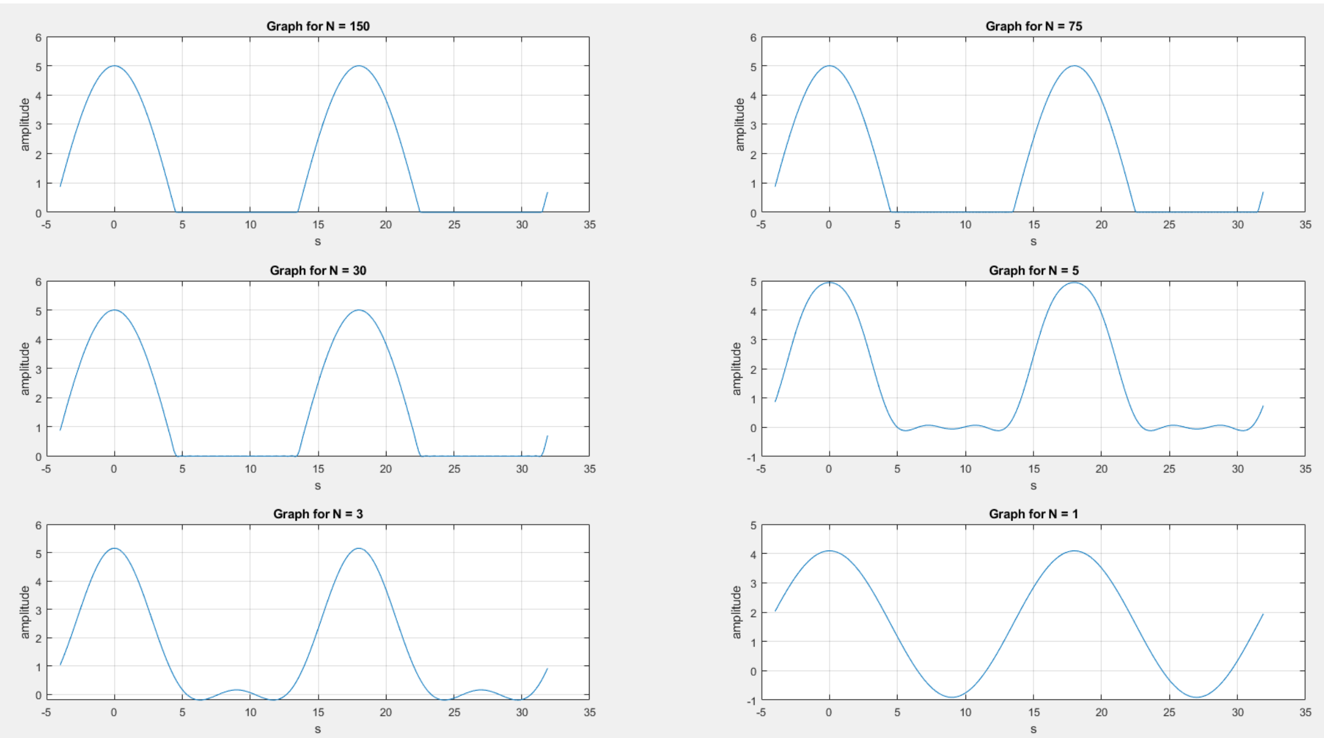


Figure 17: Graphs for Changing N Values in Q3

APPENDIX – MATLAB CODE

Q1

```
clc;
clear;
close all;

% 22201832 Emir A. Bayer EEE321 Lab4 Q1

fundT = 18;
n = -40:319;
t = n/10;
fundW = 2 * pi / fundT;
%the sampling period is 1/10

ya = zeros(size(t));
t_mod = mod(t, fundT);
ya((t_mod >= 7) & (t_mod < 10)) = 8;
y_n_two_periods = [ya, ya];
n_two_periods = [n, n + length(n)];

figure;
stem(n, ya, 'filled');
title('a) Graph of y[n]');
xlabel('n');
ylabel('y[n]');
grid on;

%zeroth harmonic = 4/3 -> DC component, average value over one period

r_ak = zeros(1, 150);
im_ak = zeros(1, 150);

%f coeffs for 1:150
for k = 1:150
    ak = (4 * 1j / (pi * k)) * (exp(-1j * 10 * pi * k / 9) - exp(-1j * 7 * pi * k / 9));
    r_ak(k) = real(ak);
    im_ak(k) = imag(ak);
end

mag_ak = [fliplr(r_ak + 1j * im_ak), 4/3 , r_ak + 1j * im_ak];
frequencies = fundW * (-150:150);

%1C spectrum
figure;

stem(frequencies, real(mag_ak), 'b' , 'Marker', 'none', 'DisplayName', 'Real');
hold on;
stem(frequencies, imag(mag_ak), 'r' , 'Marker', 'none', 'DisplayName', 'Imaginary');
legend;
xlabel('rad/s');
ylabel('magnitude');
title('Spectrum of Q1');

```

```
%1D
N_values = [150, 75, 30, 5, 3, 1];
figure;
for j = 1:length(N_values)
    N_val = N_values(j);
    out_N = 4/3 * ones(size(t));

    for k = 1:N_val
        out_N = out_N + r_ak(k) * cos(k * fundW * t) - im_ak(k) * sin(k * fundW * t);
        out_N = out_N + r_ak(k) * cos(-k * fundW * t) - (-im_ak(k)) * sin(-k * fundW * t);
    end

    subplot(3, 2, j);
    plot(t, real(out_N));
    title(['Graph for N = ', num2str(N_val)]);
    xlabel('s');
    ylabel('amplitude');
end

figure;
%harm0
subplot(2, 2, 1);
plot(t, 4/3 * ones(size(t)));
xlabel('s');
ylabel('amplitude');
title('Harmonic 0');
%harm1
subplot(2, 2, 2);
harm_1 = 2 * (r_ak(1) * cos(fundW * t) - im_ak(1) * sin(fundW * t));
plot(t, harm_1);
title('Harmonic 1');
xlabel('s');
ylabel('amplitude');
%harm2
subplot(2, 2, 3);
harm_2 = 2 * (r_ak(2) * cos(2 * fundW * t) - im_ak(2) * sin(2 * fundW * t));
plot(t, harm_2);
title('Harmonic 2');
xlabel('s');
ylabel('amplitude');
%harm3
subplot(2, 2, 4);
harm_3 = 2 * (r_ak(3) * cos(3 * fundW * t) - im_ak(3) * sin(3 * fundW * t));
plot(t, harm_3);
title('Harmonic 3');
xlabel('s');
ylabel('amplitude');
```

Q2

```
clc
clear
close all
% 22201832 Emir A. Bayer EEE321 Lab4 Q2


fundT = 18;
n = -40:319;
t = n / 10;
fundW = 2 * pi / fundT;


y_a2_t = abs(5 * cos(pi * t / 9));
figure;
stem(n, y_a2_t, 'filled');
title('a) Graph of y[n]');
xlabel('n');
ylabel('y[n]');

dcAve = 10 / pi; %dc ave for one per

r_ak = zeros(1, 150);
im_ak = zeros(1, 150);

%f coeffs for 1:150
for k = 1:150
    r_ak(k) = 10 * (sin(pi/2 * (1 - 2*k)) / (pi * (1- 2*k)) + sin(pi/2*(1+2*k)) / (pi * (1+2*k)))* cos(2*pi/9 *k * pi/9);
    im_ak(k) = 0; %symmetry makes it 0
end

mag_ak = [fliplr(r_ak + 1j * im_ak), 4/3 , r_ak + 1j * im_ak];
frequencies = fundW * (-150:150);

%2C spectrum
figure;
stem(frequencies, real(mag_ak), 'b' , 'Marker', 'none', 'DisplayName', 'Real');
hold on;
stem(frequencies, imag(mag_ak), 'r' , 'Marker', 'none', 'DisplayName', 'Imaginary');
legend;
title('Spectrum of Q2');
xlabel('rad/s');
ylabel('magnitude');


%repeating for new Ns
repeats_list = [150, 75, 30, 5, 3, 1];
figure;
for j = 1:length(repeats_list)
    Nval = repeats_list(j);
    outN = dcAve * ones(size(t));
```

```

    %reals first
    for k = 1:Nval
        outN = outN + r_ak(k) * cos(k * fundW * t);
    end
    subplot(3, 2, j);
    plot(t, real(outN));
    title(['Graph for N = ', num2str(Nval)]);
    xlabel('s');
    ylabel('amplitude');
end

figure;
%harm0
subplot(2, 2, 1);
plot(t, dcAve * ones(size(t)));
title('Harmonic 0');
xlabel('s');
ylabel('amplitude');
%harm1
subplot(2, 2, 2);
harm1 = r_ak(1) * cos(fundW * t);
plot(t, harm1);
title('Harmonic 1');
xlabel('s');
ylabel('amplitude');
%harm2
subplot(2, 2, 3);
harm2 = r_ak(2) * cos(2 * fundW * t);
plot(t, harm2);
title('Harmonic 2');
xlabel('s');
ylabel('amplitude');
%harm3
subplot(2, 2, 4);
harm_3 = r_ak(3) * cos(3 * fundW * t);
plot(t, harm_3);
title('Harmonic 3');
xlabel('s');
ylabel('amplitude');
```

Q3

```
clc;
clear;
close all;

% 22201832 Emir A. Bayer EEE321 Lab4 Q3

% Parameters
fundT = 18; % signal per
n = -40:319;
t = n / 10; % time vector for samples
fundW = 2 * pi / fundT;
%the sampling period is 1/10

ya = (5 * cos(pi * t / 9)) .* ((5 * cos(pi * t / 9)) > 0);

figure;
stem(n, ya, 'filled');
hold on;
title('a) Graph of y[n]');
xlabel('n');
ylabel('y[n]');


%fourier coefficients
r_ak = zeros(1, 150);
im_ak = zeros(1, 150);

%calculate Fourier coefficients analytically for k = 1 to N
for k = 1:150
    r_ak(k) = (5/2) * cos(2*pi/18+pi/9)*(sin((pi/2) * (1-2*k))/(pi*(1+2*k))/(pi*(1+2*k)));
    im_ak(k) = 0; % Imaginary part is zero due to symmetry
end

%extending for negative frequencies and the zeroth component
ak_full = [fliplr(r_ak + 1j * im_ak), a0, r_ak + 1j * im_ak];
frequencies = fundW * (-150:150);

%spectrum
figure;
stem(frequencies, real(ak_full), 'b', 'filled', 'Marker', 'none', 'DisplayName', 'Real Part');
hold on;
stem(frequencies, imag(ak_full), 'r', 'filled', 'Marker', 'none', 'DisplayName', 'Imaginary Part');
title('Spectrum of Q3');
xlabel('rad/s');
ylabel('amplitude');
legend;
grid on;

```

```
%different Ns (repeats)
repeats_list = [150, 75, 30, 5, 3, 1];
figure;
for j = 1:length(repeats_list)
    Nval = repeats_list(j);
    outN = a0 * ones(size(t)); %firstly the DC component

    %adding harmonics up to the current Nval accounting for real part only
    for k = 1:Nval
        outN = outN + r_ak(k) * cos(k * fundW * t);
    end

    subplot(3, 2, j);
    plot(t, real(outN));
    title(['Graph for N = ', num2str(Nval)]);
    xlabel('s');
    ylabel('amplitude');
    grid on;
end

figure;

%harm0
subplot(2, 2, 1);
plot(t, a0 * ones(size(t)));
title('Harmonic 0');
xlabel('s');
ylabel('amplitude');

%harm1
subplot(2, 2, 2);
harm_1 = r_ak(1) * cos(fundW * t);
plot(t, harm_1);
title('Harmonic 1');
xlabel('s');
ylabel('amplitude');

%harm2
subplot(2, 2, 3);
harm_2 = r_ak(2) * cos(2 * fundW * t);
plot(t, harm_2);
title('Harmonic 2');
xlabel('s');
ylabel('amplitude');

%harm3
subplot(2, 2, 4);
harm_3 = r_ak(3) * cos(3 * fundW * t);
plot(t, harm_3);
title('Harmonic 3');
xlabel('s');
ylabel('amplitude');
```