Simulation and Analysis of a Capacitive Displacement System with Spring-Mass Dynamics and Electrical Coupling

Ahmet Arif Tezgel Author MEE, Emir Ayar Author MEE

***Abstract*— This paper presents a simulation analysis of a capacitive displacement system that combines mechanical spring-mass dynamics with electrical circuit behavior. The system consists of a variable parallel plate capacitor whose capacitance changes with displacement, a network of resistors connected to the capacitor and a DC voltage source. Simulations using MATLAB analyze the time-dependent response of the system under realistic physical parameters such as spring constant, capacitance, resistance and input voltage. The results reveal the dynamic behavior of displacement, capacitance, current flow and electrical power dissipation.**

***Index Terms*—Capacitive, displacement, spring-mass dynamics, electrical coupling, sensors**

I. Introduction

Capacitive displacement systems play a critical role in modern industrial automation and precision measurement applications. These systems provide efficient solutions to position measurement needs that require high precision and accuracy. Based on electric field principles, capacitive displacement sensors provide both high reliability and superior performance in harsh operating environments thanks to their non-contact measurement capabilities. In this paper, the basic working principles, application areas and advantages of capacitive displacement systems will be discussed in detail. It will also shed light on the increasing importance of this technology in today's industrial processes and its potential future applications.

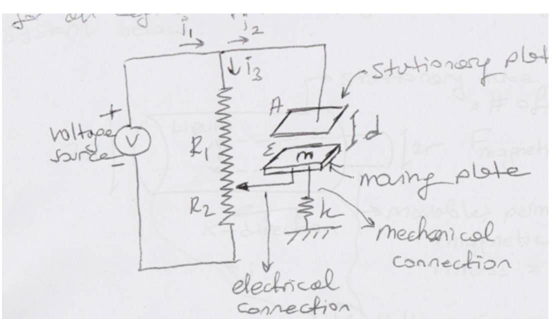


Figure 1. Model on paper of the capacitive displacement system given to us in the reference sheet[1]. The system consists of resistors, a voltage source, moving plates and a spring. The capacitive displacement system in the figure consists of a voltage source (Vin), resistors R1 and R2, a capacitor, the dielectric constant of the capacitor (E), the plate area of the capacitor (A) and the capacity of the capacitor (C), the mass of the moving plate (m), the spring constant of the spring (k) and the position of the moving plate in common with the electrical system. The forces acting on the system are the spring force and the electrostatic force.

The capacitive displacement system can be used in the industry as follows:

A. Sensor Technologies

Capacitive displacement systems are widely used for measuring small mechanical movements. This system is especially preferred in situations where high precision is required. Example areas of use:

Industrial Automation: Sensing and controlling micron-level movements of machine parts.
Robotics: Measuring precise movements of robot arms and providing feedback.
Medical Devices: Motion sensing in magnetic resonance (MRI) devices or surgical robots.

B. Micro-Electro-Mechanical Systems (MEMS)

MEMS devices contain electromechanical systems with small dimensions. Capacitive displacement sensors are commonly used in such devices:

Accelerometers: Accelerometers used in the automotive and aerospace industries.
Pressure Sensors: Systems used to measure air pressure or gas pressure.
Micro-Actuators: Microscale motion mechanisms that require precise position control.

C. Electromechanical Energy Conversion

The energy storage properties of capacitors allow this system to be used in energy conversion:

Piezoelectric Energy Harvesting: Conversion of mechanical vibrations into electrical energy.
Renewable Energy Systems: Capacitive displacement measurement in wind turbines or vibrating energy harvesting systems.

This paper presents the derivation and solution of the equation of motion of the introduced capacitive displacement system and simulates the equations found for this system in MATLAB using current flow, voltage variation, power dissipation, capacitance variation, etc. in a realistic way[3].

II. SYSTEM OVERVIEW AND SYSTEM CALCULATIONS

The capacitive displacement system consists of the following components:

Variable parallel plate capacitor: Its capacitance depends on the displacement. Some of the values we will use in the equations for a variable parallel plate capacitor: the dielectric constant of the material between the plates, plate area, initial plate spacing, displacement.

Spring-mass mechanism: It determines the displacement dynamics and is expressed by Newton's second law: Some of the values we will use to construct the equations of the spring-mass mechanism: mass, spring constant, electrical force.

Electrical circuit: Includes a network of resistors (resistors R1 and R2) and a DC voltage source. Kirchhoff's Voltage Law (KVL) is applied as follows: Here: Voltage across the capacitor, current flowing through R1.

In order to find and simulate the capacitive displacement system, the system needs to be analyzed in 2 parts. These parts can be called the dynamic and electrical parts of the system. The first part to be examined is the dynamic part of the system. The second part to be examined is the electrical part[4].

A. Dynamic Part

To create the equations of the dynamic part, we start by finding the equation of motion of the capacitor. From Newton's 2nd law of motion, the equation of motion of the capacitor is found as follows:

$$F_{cap} = m\ddot{x} + b\,\dot{x} + kx \tag{1}$$

In Newton's 2nd law of motion b refers to the damping force. Since the reference sheet [1] states to ignore the damping force, the equation of motion of the capacitor from Newton's 2nd law of motion can be expressed as follows.

$$F_{cap} = m\ddot{x} + kx \tag{2}$$

In the reference sheet [1] the capacitor force (Fcap) is expressed as follows:

$$F_{cap} = \frac{-V_{cap}^2\,dC}{2d^2\,dx} \tag{3}$$

Kirchhoff second law is used to find the capacitor voltage (Vcap) in Equation (3).

$$R_{total} = R_1 + R_2 \tag{4}$$

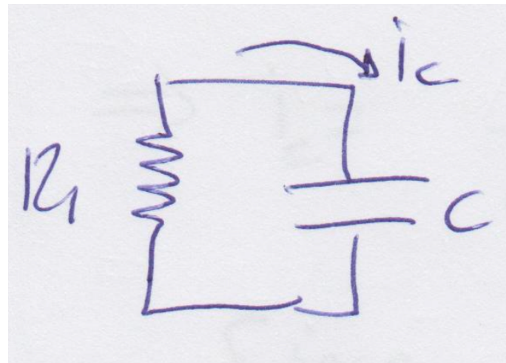


Figure 2. Voltage divider circuit according to Kirchhoff second law in the second reference sheet[2].

As can be seen from the voltage divider circuit in Figure 2, we obtain the equation $V_{R1} = V_c$. From Ohm's law and the voltage divider principle, V_{R1} can be derived with the following formula:

$$V_{R1} = \frac{V_{in}\cdot R_1}{(R_1 + R_2)} \tag{5}$$

In this step we use the capacitor voltage finding formula to find the voltage across the capacitor:

$$C = \left(\frac{\varepsilon\,A}{d+x}\right) \tag{6}$$

$$Vc = \left(\frac{\varepsilon\,A}{d+x}\right)^{-1} \int i_c\,dt \tag{7}$$

Based on the equation $V_{R1} = V_c$ that we found from Figure 2, we can equate the V_c formula from the capacitor equation (Equation (5)) and the V_c formula from the voltage divider formula (Equation (7)).

$$V_c = \left(\frac{\varepsilon\,A}{d+x}\right)^{-1} \int i_c\,dt = \frac{V_{in}\cdot R_1}{(R_1 + R_2)} \tag{8}$$

Using the formula for the capacitor force (Fcap) given in the reference sheet[1] (Equation (3)), we equate it to the formula obtained from Newton's 2nd law of motion (Equation (2)).

$$\frac{-V_{cap}^2\,dC}{2d^2\,dx} = m\ddot{x} + kx \tag{9}$$

Using Equation (9) we obtain the equation for finding $\ddot{x}$.

$$\ddot{x} = \frac{\left(\frac{V_{in}\cdot R_1}{(R_1 + R_2)}\right)^2 \varepsilon\,A}{2\,m\,(d+x)^4} - \frac{k}{m\,x} \tag{10}$$

From this Equation (10), the acceleration equation of the capacitive displacement system is obtained.

A. Electrical Part

From Kirchhoff's Voltage Law, the voltage equation of the capacitive displacement system can be found.

$$-V_{in} + R_1\, i_3 + R2\, \tilde{i_3} = 0 \tag{11}$$

From the nodal analysis according to the capacitive displacement system, the equality of the currents i_1, i_2, i_3 can be provided as follows.

$$i_1 = i_2 + i_3 \tag{12}$$

According to nodal analysis, the current passing through the capacitor i_c can be called i_2. Therefore, from now on, we will refer to i_c instead of i_2 in the equation.

From the capacitor current equation, the equation of the current flowing through the capacitor can be found as follows.

$$i_c = \frac{C\, dV}{dt} \tag{13}$$

The current through the capacitor equation and the capacitor equation found when examining the equation of motion are used to find the current through the capacitor. We recalculate the i_c equation.

$$C = \left(\frac{\mathcal{E}\,\mathrm{A}}{d+x}\right) \tag{14}$$

$$Vc = \left(\frac{\mathcal{E}\,\mathrm{A}}{d+x}\right)^{-1} \int i_c\, dt \tag{15}$$

$$i_c = \left(\frac{\mathcal{E}\,\mathrm{A}}{d+x}\right) \frac{d\left(\frac{V_{in}\,.\,R_1}{(R_1 + R_2)}\right)}{dt} \tag{16}$$

The i_c obtained from nodal analysis can be recalculated as follows.

$$i_1 - i_2 = i_3 \tag{17}$$

Here we can find $\tilde{i_3}$ as follows. $\tilde{i_3} = i_1$

Again, we use Kirchhoff's Voltage Law and derive the voltage analysis equation of the system.

$$-V_{in} + (R_1 + R_2)\, i_1 - R_1\, i_c = 0 \tag{18}$$

Here are all the formulas we will use to simulate a capacitive displacement system. Thanks to these calculations we will be able to measure how a capacitive displacement system will behave.

III. PARAMETERS

Capacitive displacement is generally used in Sensor Technologies in industry. The values to be selected will be consistent values for Sensor Technologies.

A. Spring

Silicon and stainless steel are generally used in the spring. We will use stainless steel for this system. Stainless Steel is widely used in industry because of its higher hardness, good mechanical properties, corrosion resistance.

Values of spring :

The G value of Stainless Steel is usually between 74 - 81 GPa. In this simulation, we take the G value to be 80 GPa.

TABLE I
SPRING VALUES

Parameter	Value	Unit
G (Stainless Steel)	80	GPa
Initial distance between plates (d)	0.18	m
D	0.47	m
Number of turn (n)	5	

With the given values we can find the spring constant. Spring constant formula to find the spring constant:

$$k = \frac{G\, d^4}{8\, D^3 n} \tag{19}$$

From this formula k is 19.9 Nm

B. Capacitor

TABLE II
CAPACITOR VALUES

Parameter	Value	Unit
Density (Nickel)	8912	kg/m³
Thickness	25	µm
Area	0.0001	m^2
Dielectric Constant $\mathcal{E}$ (Air)	$8.854\ 10^{-12}$	F m

We use the weight equation to calculate the plate weight of the capacitor.

$$m = \text{Density} + \text{Volume} \tag{20}$$

$$Volume = \mathrm{A.t} \tag{21}$$

To find the weight in this equation, we first calculate the volume and then multiply it by the density. As a result of these calculations, the mass of capacitor plate turns out to be 2.228 10^{-5} kg.

C. R1, R2

The resistors in this system will be taken as R1 = 0.002, R2 = 0.001.

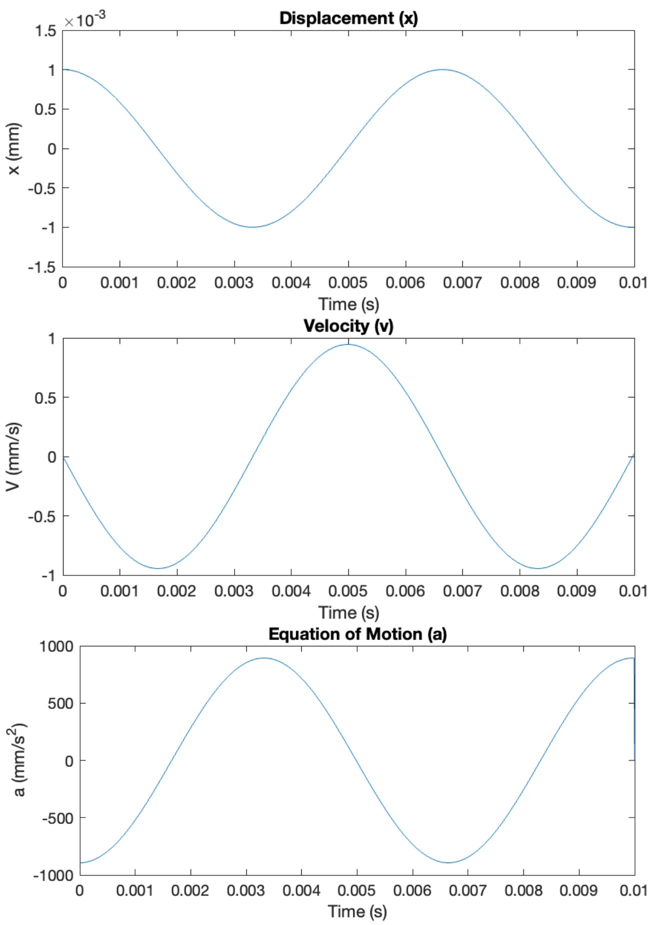
D. Vin

In capacitive displacement systems, V is usually taken between 5 and 20. In this system, V will be taken as 12.

IV. CONCLUSION

This paper introduces a method to model and simulate how capacitive displacement systems behave with realistic physical parameters. The dynamic interactions of the mechanical and electrical components are analyzed in detail and the effect of these interactions on the overall performance of the system is presented.

The capacitive displacement system is analyzed using a plate model that exhibits a harmonic motion. In this model, a mechanical spring resistor and an electrical voltage source are used. The displacement and velocity data of the system are analyzed as a function of time and important conclusions about the dynamic properties of the system are obtained from these data. The study provides valuable insights into the design and performance evaluation of capacitive sensors and electromechanical systems.



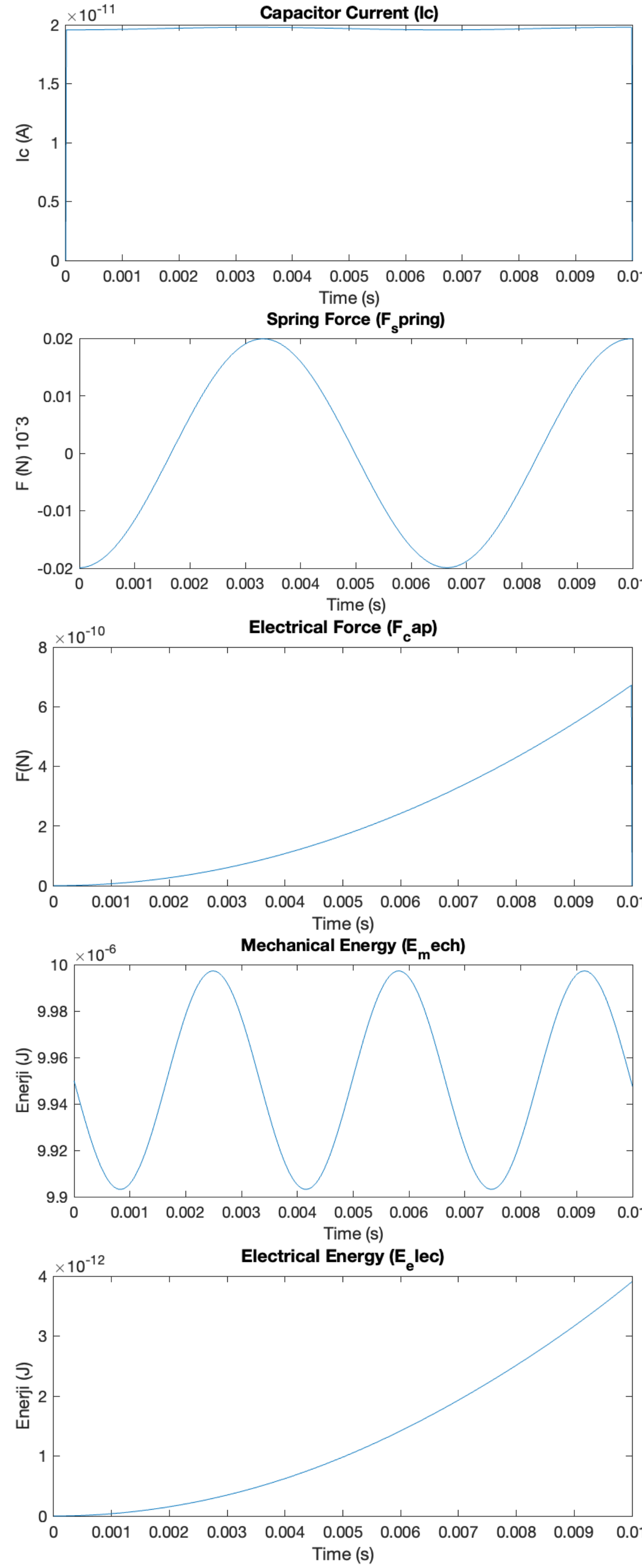


Figure 3. The figure contains tables showing the variation of parameters such as displacement, velocity, motion, capacitor current, spring force, electrical force, mechanical energy, electrical energy, displacement, velocity, motion, capacitor current, spring force, electrical force, mechanical energy, electrical energy with respect to time, created using MATLAB for the capacitive displacement system whose values and formulas are found in the article. [5].

Acknowledgment

We would you like the express our sincere appreciation to Assoc. Prof. Ahmet Fatih Tabak for supporting and directing us while we were doing this project.

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