

FINDING A PATTERN

PROBLEM 1:

- Find the sum of the first 50 odd numbers?

STEP 1: Understand the problem

- **The problem asks us what is the solution if we sum the first 50 odd numbers.**

STEP 2: Making a Plan

- You can use *Finding a Pattern Strategy* while solving this problem.
- Try to find a pattern.

STEP 3: Carry Out the Plan

- Examine this problem by looking for a pattern.

| <u>Addends</u> | <u>Number of Addends</u> | <u>Sum</u> |
|----------------|--------------------------|------------|
| 1 | 1 | 1 |
| 1+3 | 2 | 4 |
| 1+3+5 | 3 | 9 |
| 1+3+5+7 | 4 | 16 |
| 1+3+5+7+9 | 5 | 25 |
| 1+3+5+7+9+11 | 6 | 36 |
| . | . | . |
| . | . | . |

Carry Out the Plan

- The table shows us there is a relation between the sum and the number of addends.
- It is clearly seen that the sum of the first n odd numbers is n^2 .
- In this problem $n(\text{number of addends})=50$
- The answer is $50^2=2500$

STEP 4: Check the Solution

- Another method rather than *Finding a Pattern*:

You can solve this problem by writing out all of the odd numbers from 1 through 59 and adding them.

$$\begin{array}{r} 1 + 3 + 5 + 7 + 9 + \dots + 91 + 93 + 95 + 97 + 99 \\ + \quad 99 + 97 + 95 + 93 + 91 + \dots + 9 + 7 + 5 + 3 + 1 \\ \hline 100 + 100 + 100 + 100 + 100 + \dots + 100 + 100 + 100 + 100 \end{array}$$

$100 \times 50 / 2 = 2500$ (2 times the sum of the first 50 odd numbers)

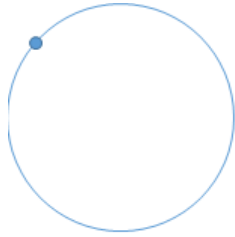
So divide 5000 to 2 to find the solution.

$$5000 / 2 = 2500$$

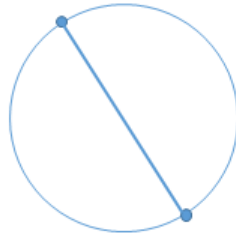
STEP 5 : Extend the Problem

- **Find the sum of the first 200 even numbers.**

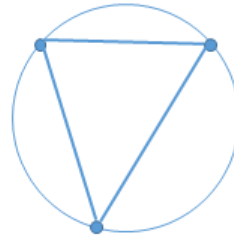
PROBLEM 2



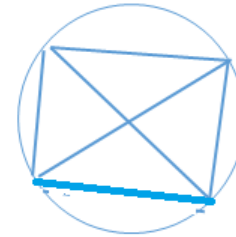
I. Step



II. Step



III. Step



IV. Step

In the figure above, we choose point(s) on a circle and connect them to form distinct, no overlapping regions.

What is number of regions obtained by connecting n -points?

UNDERSTAND THE PROBLEM:

What is given?

- If there is 1 point on the circle, this point divides the circle to the one region.
- If there are 2 points on the circle, these points divide the circle to no overlapping 2 regions.
- If there are 3 points on the circle, these points divide the circle to no overlapping 4 regions.
- If there are 4 points on the circle, these points divide the circle to no overlapping 8 regions.

What is asked?

- What is number of regions obtained by connecting n points?

MAKE A PLAN:

- We can use “Finding a Pattern Strategy”
- By starting to construct a table showing number of points and maximum number of regions to recognize a pattern.

CARRY OUT THE PLAN:

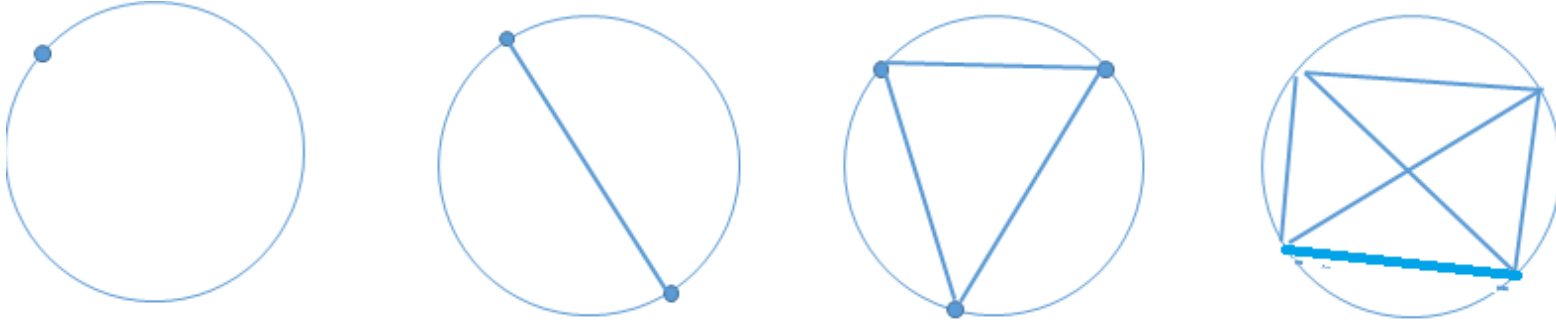
| Number of points | 1 | 2 | 3 | 4 | 5 | | n |
|-----------------------|---|---|---|---|---|------|---|
| Max number of regions | 1 | 2 | 4 | 8 | | | ? |

2^{n-1} ?????? for n
points

Check/Look Back

- $n = 6$ number of regions: 31
- $n = 6$ is $2^{n-1} = 2^5 = 32$????
- Number of Regions for n Points $= \frac{1}{24} (n^4 - 6n^3 + 23n^2 - 18n + 24)$

EXTENSION



In the figure above, we choose point(s) on a circle and connect them to form distinct, no overlapping regions.

Is there any relationship between number of lines obtained by connecting n -points and regions obtained by line segments/chords?

REFERENCES

- <http://www.teachervision.fen.com/problem-solving/teaching-methods/48900.html>
- www.eduplace.com/math/mw/practice/3/problems/15_5.pdf
- <http://www.nzetc.org/tm/scholarly/name-411876.html>
- http://mathforum.org/library/topics/problem_solving/