CASE STUDY II

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GROUP 42

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1 Introduction

MMWP is a strategic approach to workforce planning that uses mathematical models to optimize planning and balance various factors. It helps organizations achieve their long-term goals by identifying workforce requirements, nurturing talent, enhancing employee satisfaction, and optimizing costs.

By 2023, Mercedes-Dantes Toys plans to adapt to toy production trends but faces reduced demand due to the increasing use of tablets and smartphones. The company must fulfill the workforce requirements stated in Table 1. There are 1,150, 1,700, and 2,400 workers available for Types A, B, and C, respectively, at the beginning of 2024. The company may hire, promote, degrade, idle, outsource, or hire part-time workers. Estimated resignation rates for employees A, B, and C are 5%, 5%, and 10%, respectively. The company decides how many employees to hire for each type at the beginning of each year, with an upper limit of 600, 900, and 500 for Types A, B, and C, respectively. Newly hired employees cannot resign in their first year. The company can train employees and promote up to 300 Type C workers to Type B at a cost of $400/person. After training, promoting B to A costs $300. Performance reviews may result in the degradation of employees, with 50% resigning. Idle time costs $500/person, $500/person, and $200/person for Types A, B, and C, respectively, for a year. Outsourcing is $1,200, $1,100, and $1,000/person/year, respectively. The company can hire up to 80 part-time workers/year at costs of $550/person for Type A, $400/person for Type B, and $350/person for Type C. Outsourced and part-time employees are seasonal. The objective of the management of Mercedes-Dantes Toys is to minimize the total cost and number of idle workers. To assist in determining a workforce plan for the next three years, the management of Mercedes-Dantes Toys has hired our team as consultants. We used the pyomo library to solve this problem, and we will be expressing the different results produced by our model according to two objectives in certain parts of the report.

2 Formulation of the Linear Programming Problem

2.1 Sets & Indices :

i : Worker types, $i \epsilon I = \{1(\text{A}), 2(\text{B}), 3(\text{C})\}$

j : Years, $j \epsilon J = \{1(2024, 2(2025), 3(2026)\}$

2.2 Parameters:

W_{i0} : Number of i type workers at the beginning of 2024

r_i : Resignation percentages in a year for i type of workers

M : Maximum number of C type workers can be promoted to type B workers in a year (300)

k_3 : Cost of promoting a type C worker can be promoted to type B ($400)

k_2 : Cost of promoting a type B worker can be promoted to type A ($300)

L : Maximum ratio of educated type B workers to type B workers of the previous year (20%)

G : Resigning workers ratio among degraded workers (50%)

n_i : Number of i type workers can be hired each year

s_i : Cost of idleness of a worker for a year for i type workers

U : Maximum number of workers can be outsourced in all worker types (175)

w_i : Cost of outsourcing i type workers

H: Total number of part-time workers can be hired in a year (80)

e_i : Cost of i type part-time workers

y : Constant ratio showing that part-time workers work half the time of a regular worker (0.5)

Z_{ij} : Active workforce requirements for i type worker in year j

c_i : Cost of hiring a new i type worker

a_i : Cost of outsourcing i type worker

v_i : Cost of i type part-time worker

2.3 Decision Variables:

x_{ij} : Number of i type workers hired in year j

p_{ij}^{+} : Number of i type workers promoted in year j

p_{ij}^{-} : Number of i type workers degraded in year j

l_{ij} : Number of i type idle workers in year j

o_{ij} : Number of outsourced i type workers in year j

t_{ij} : Number of type i part-time workers hired in year j

W_{ij} : Workforce of i type workers in year j

2.4 Constraints:

$$W_{1(j-1)} + x_{1j} + p_{2j}^{+} - p_{1j}^{-} = W_{1j} \qquad (1.1)$$

$$W_{1(j-1)} - w_{1(j-1)}r_1 + x_{1j} + p_{2j}^{+} - p_{1j}^{-} = W_{1j} \qquad (1.2)$$

$$W_{1(j-1)} - w_{1(j-1)}r_1 + x_{1j} + p_{2j}^{+} - p_{1j}^{-} = W_{1j} \qquad (1.3)$$

$$W_{2(j-1)} + x_{2j} + p_{3j}^{+} - p_{2j}^{-} + p_{1j}^{-}G - p_{2j}^{-} = W_{2j} \qquad (2.1)$$

$$W_{2(j-1)} - w_{2(j-1)}r_2 + x_{2j} + p_{3j}^{+} - p_{2j}^{-} + p_{1j}^{-}G - p_{2j}^{-} = W_{2j} \qquad (2.2)$$

$$W_{2(j-1)} - w_{2(j-1)}r_2 + x_{2j} + p_{3j}^{+} - p_{2j}^{-} + p_{1j}^{-}G - p_{2j}^{-} = W_{2j} \qquad (2.3)$$

$$W_{3(j-1)} + x_{3j} + +p_{2j}^{-}G - p_{3j}^{+} = W_{3j} \qquad (3.1)$$

$$W_{3(j-1)} - w_{3(j-1)}r_3 + x_{3j} + +p_{2j}^{-}G - p_{3j}^{+} = W_{3j} \qquad (3.2)$$

$$W_{3(j-1)} - w_{3(j-1)}r_3 + x_{3j} + +p_{2j}^{-}G - p_{3j}^{+} = W_{3j} \qquad (3.3)$$

$$W_{ij} - l_{ij} + o_{ij} + t_{ij}y = Z_{ij} \quad \forall \; i \in I, \; \forall \; j \in J \qquad (4)$$

$$x_{ij} \leq n_i \qquad \forall \; i \in I, \; \forall \; j \in J \qquad (5)$$

$p_{3j}^{+} \leq M \qquad \forall \ j \in J \quad (6)$

$p_{2j}^{+} \leq W_{2(j-1)}L \qquad \forall \ j \in J \quad (7)$

$\sum_{i\in \mathrm{I}} o_{ij} \leq U \qquad \forall \ j \in J \quad (8)$

$\sum_{i\in \mathrm{I}} t_{ij} \leq H \qquad \forall \ j \in J \quad (9)$

$x_{ij}, p_{ij}^{+}, p_{ij}^{-}, l_{ij}, o_{ij}, \ \ t_{ij} \geq 0 \qquad (10)$

2.5.1 Objective Function 1:

$$Minimize \ \ z_1 = \sum_{j\in \mathrm{J}} k_3 p_{3j}^{+} + \sum_{j\in \mathrm{J}} k_2 p_{2j}^{+} + \sum_{i\in \mathrm{I}} \sum_{j\in \mathrm{J}} l_{ij} s_i + \sum_{i\in \mathrm{I}} \sum_{j\in \mathrm{J}} o_{ij} w_i + \sum_{i\in \mathrm{I}} \sum_{j\in \mathrm{J}} t_{ij}\, e_i + \sum_{i\in \mathrm{I}} \sum_{j\in \mathrm{J}} x_{ij} c_{ij} \quad (1)$$

2.5.2 Objective Function 2:

$$Minimize \ \ z_2 = \sum_{i\in \mathrm{I}} \sum_{j\in \mathrm{J}} l_{ij} \quad (2)$$

2.6 Description of Constraints and Objective Functions in LP:

In this section, we'll explain the constraints and objective functions in the math model. We aim to help you understand how they align with the problem's goals and decision-making process. This will enable managers to make better-informed decisions.

Constraints (1), (2), and (3) represent the workforce balance equation for workers of type A, B, and C respectively. They are used to calculate the workforce of each worker type every year while taking into account factors like new hires, resignations, promotions, and degradations. Constraint (4) ensures that the net workforce is equal to the workforce requirements, while Constraint (5) sets the maximum number of workers that can be hired each year. Constraints (6) and (7) set upper limits on the number of promoted C and B type workers respectively. Constraints (8) and (9) limit the total number of outsourced and part-time workers respectively. Finally, Constraint (10) specifies that the number of workers cannot be negative.

The company aims to minimize total cost by considering various expenses such as promoting costs, idleness costs, outsourcing costs, part-time worker costs, and hiring costs for different types of workers (A, B, and C).

The company's second objective is to reduce worker idleness to ensure effective resource utilization. They must keep track of idle workers and identify the root cause to minimize idleness. Minimizing idleness can increase productivity, efficiency, and profitability.

2.7 Assumptions:

- At the beginning of 2024, none of the workers have resigned. Resignations began in 2025.
- Recruitment and degradation of workers, outsource worker recruitment and part-time worker recruitment take place at the beginning of the year. However, the number of workers to be promoted is determined at the beginning of the year and the promotion of workers continues until the last day of the year. The requested number of workers is valid for the last day of the given year. Annual cost is calculated on the last day of the year.

2.8 Solution Methods:

After analyzing all the available solution methods and the targets assigned to us, we have come to the conclusion that minimizing the total cost and idleness won't have a significant impact, regardless of whether we are in the best or worst-case scenario. However, the priorities or weights assigned may affect the outcome.

2.8.1 Weighted Sum Method:

In the weighted sum method, four different weight variables were used; these weights are 1-0, 0.6-0.4, 0.6-0.4, and 0-1 for both values, respectively. The mathematical model created and the data obtained from the program are given below.

$$Minimize \quad w_1 z_1(x) + w_2 z_2(x)$$

$$Subject \quad to$$

$$x \;\epsilon\; X$$

$$w_1 + \quad w_2 = 1$$

$$w_1, w_2 \geq 0$$

Values encountered at certain weights:

	w1 = 1 w2 = 0	w1 = 0.6 w2 = 0.4	w1 = 0.4 w2 = 0.6	w1 = 0 w2 = 1
Total Cost	$2,455,450	$2,455,450	$2,530,150	$2,591,400
Total Idle	3,171	3,171	3,051	3,051

Results for weighted sum method

Based on the data presented above, it can be inferred that there is not much variation in the total number of idle workers. This means that changing the weights within a certain range, such as w1 = 1-0.5, does not affect the results significantly. However, when w1 is assigned a value between 0.5 and 0, it is possible to lower the total cost without altering the total idleness. This indicates that both demands can be met at certain rates. Moreover, while creating the mathematical model, the worst-case scenario value was used for scaling in both objectives.

2.8.2 e-constraint Method:

In the e-constraint method, various results can be obtained with different constraints. The objective is to minimize total cost while imposing limits on the total number of idle workers. Five different outcomes were generated using this method.

$$Minimize \quad z_1$$

$$Subject \quad to$$

$$z_2 \leq e$$

	Total Idle <= 3,051	Total Idle <= 3,081	Total Idle <= 3,111	Total Idle <= 3,141	Total Idle <= 3171
Total Cost	$2,530,150	$2,511,475	$2,492,800	$2,474,125	$2,455,450

Results for e-constraint method

Different results were obtained for each restriction using this method, leading to varied values compared to other methods. Since our model operates within narrow ranges, the e-constraint method can be considered appropriate for generating diverse results. Moreover, the e-constraint method helps prevent additional costs that would otherwise occur in Total Cost. For instance, with the e-constraint method, you won't observe the extra cost of $61,250 in the weighted sum when w1 = 0. The result table of the nondominated values generated using the ε-constraint method is on Appendix 1.

2.8.3 Weighted Goal Programming Approach:

Weighted goal programming involves problems that are similar to those in the weighted sum method. The resulting model has only two extreme points, which means that we get two different results in total from five different solutions. The same scaling method is used for both objectives to determine the worst-case scenario value.

The company has set certain targets for us to achieve, which include a maximum total cost of $3,200,000 and a maximum total idle time of 2,200. However, the values we calculated for these objectives are different from the targets. The maximum total cost can be at most $2,614,618, and the minimum total idle time can be 3,051. As a result, there will always be a deviation of 0 for the first objective, while the deviation for the second objective can never be 0.

$$Minimize \;\; w_1 d_1^+ + w_2 d_2^+$$

$$Subject \;\; to$$

$$Total \;\; cost \leq 3.200.000 + d_1^+$$

$$Total \;\; Idle \leq 2.200 + d_2^+$$

$$d_1^+, d_2^+ \geq 0$$

	w1 = 1 w2 = 0	w1 = 0.75 w2 = 0.25	w1 = 0.5 w2 = 0.5	w1 = 0.25 w2 = 0.75	w1 = 0 w2 = 1
Overachievement for Total Cost	0	0	0	0	0
Overachievement for Total Idle	971	851	851	851	851
Total Cost	$2,539,918	$2,614,618	$2,614,618	$2,614,618	$2,614,618
Total Idle	3,171	3,051	3,051	3,051	3,051

Results for weighted goal programming method

As previously stated, when using goal programming methods, total cost overachievement will always be 0, making it insufficient to produce different and optimal values. Moreover, the values obtained through this approach tend to be higher than those obtained with other methods. For instance, using the weighted sum method can reduce the total cost to $2,530,150 while maintaining the number of workers at 3,051. On the other hand, using weighted goal programming costs approximately a hundred thousand dollars more.

2.8.4 Preemptive Goal Programming Approach:

Similar problems arise in pre-emptive goal programming. Here, in the first model, the total cost was minimized, and then the resulting value was added as a constraint and tried to be minimized with total idle.

$$Model\ \ 1$$

$$Minimize\ \ d_1^+$$

$$Subject\ \ to$$

$$(Total\ \ Cost)^* \leq 3200000 + d_1^+$$

$$d_1^+ \geq 0$$

$$Part \ \ 2$$

$$Minimize \ \ d_2^+$$

$$Subject \ \ to$$

$$Total \ \ Cost = \ \ (Total \ \ Cost)^*$$

$$Total \ \ Idle \ \ \leq 2200 + d_1^+$$

$$d_2^+ \geq 0$$

The second model aims to minimize the total idle workers, add the resulting value as a constraint, and minimize the total cost. However, since total idle can never fall below the given target, the values resulting from this model are not optimal.

$$Model \ \ 2$$

$$Minimize \ \ d_2^+$$

$$Subject \ \ to$$

$$(Total \ \ Idle)^* \leq 2200 + d_2^+$$

$$d_2^+ \geq 0$$

$$Part \ \ 2$$

$$Minimize \ \ d_1^+$$

$$Subject \ \ to$$

$$Total \ \ Idle = (Total \ \ Idle)^*$$

$$Total \ \ Cost \leq 3200000 + d_1^+$$

$$d_1^+ \geq 0$$

Values generated in two different models are given below:

	Model 1	Model 2
Overachievement for Total Cost	0	0
Overachievement for Total Idle	971	851
Total Cost	$2,455,450	$2,530,150
Total Idle	3,171	3,051

Results for pre-emptive goal programming method

2.9 Comparison of the Results:

The e-constraint approach may be the most appropriate in comparing the various solution methods due to its ability to produce diverse results. Moreover, the weighted sum method produced two different extreme points due to programming. However, since this method only creates extreme points, obtaining different results is impossible. Nevertheless, the extreme values obtained from the method's results will be used when discussing sensitivity analysis or engaging in a discussion. Conversely, Weighted Goal Programming and Pre-emptive Goal Programming methods failed to produce optimal results due to deviations.

3 Discussion

3.1 General Discussion:

The total number of idle workers has not changed much since there have been no resignations. The minimum and maximum value range from 3,051 to 3,171 respectively, indicating that there is not much difference between the values produced. However, this still remains well above the company's target value of 2,200 for total idleness. Additionally, even in the worst-case scenario, the maximum total cost value of $3,200,000 could not be exceeded. Therefore, in the sensitivity analysis (e-constraint method t=0), an extreme point with a total idle value of 3,051 (minimum) and a total cost value of $2,530,150 was used.

The initial quantities of Type A and Type B workers were similar to the requested amounts, but the same cannot be said for Type C workers. This has led to degradation in certain years for Type A and Type B workers. In some years, the degradation has reached excessive levels to meet demands and prevent idle payments for Type A and Type B workers. However, this situation has increased the idle worker count for Type C workers, and the desired idle target has not been achieved.

3.2 Sensitivity Analysis:

It seems that the annual promotions for workers in category C are insufficient to meet the minimization target. In all years, the entire promotion capacity is being utilized. To tackle this issue, it is suggested to improve the education system and increase training capacity. This can prove to be highly profitable in reducing costs in future years, as a substantial part of the cost is incurred in recruiting workers. Increasing the training capacity from C to B can result in a cost reduction of $1,228 in 2024, $1,712 in 2025, and $972 in 2026. With the reduced cost in 2024, training capacity can be further increased in 2025 and 2026. Alternatively, an agreement can be made with other companies to provide training, although it may increase the cost.

If it's not feasible to increase the training capacity, then some of the type C workers may have to be laid off. In case of a layoff, every type C worker would provide a profit of $2,296 in 2024, $1,562 in 2025, and $822 in 2026. If laying off workers is not an option, then the demand for C-type workers can be increased annually. Alternatively, a job field can be created specifically for C-type workers and promoted regularly in the coming years. Another possibility is to outsource C-type workers or provide them part-time to other companies in need.

The cost of outsourcing and part-time employment is significantly higher compared to hiring or promoting employees. For this reason, outsourcing or part-time work has not been utilized in any of the previous years. However, based on our analysis, it appears that it will be challenging to fulfill the required number of workers by 2025. For instance, offering a discount of approximately 6% on the outsourcing price for type A workers in 2025 ($1,128) could help meet the annual demands needed that year. This discount might be offered in cases like collective labor agreements or when hiring a large number of workers. By doing this, a certain portion of the required workforce can be fulfilled through outsourcing, which can ultimately reduce the cost per unit by $1,157. Similarly, a discount of 3% on the outsourcing price for type B workers in 2025 ($1,067) can decrease the cost per unit by $1,072. However, offering high discounts for outsourcing might not be feasible in other years. In 2025, the same situation applies to workers who can be employed part-time. If a 5% discount is offered for type A workers ($522.5), it can help reduce the cost per unit by $528. Similarly, a 4% discount on type B workers in the same year ($384) can decrease the cost per unit by $386.

In 2025 and 2026, the company's entire hiring capacity for type A workers is already being utilized. To address this issue, two possible solutions can be implemented. First, increasing the hiring capacity will result in a cost reduction of $200 per unit over the next two years. Second, the company can outsource or hire type A workers part-time at a discounted rate during specific periods.

4 Conclusion

In conclusion, it was observed that while the total cost targets for the company were met, the overall cost target was higher than it should have been. To rectify this situation, there are a few potential solutions. Firstly, the training capacity for C-type workers can be increased. Secondly, C-type workers can be laid off or outsourced to other companies or made part-time employees. It's possible that reducing costs may increase the total cost in some cases, but since we are currently well below the expected target, there shouldn't be any issues. Additionally, there have been some difficulties in meeting the demand for type A and B workers in certain years. To address this issue, part-time and outsourcing prices could be reduced during those years or hiring capacity could be increased.

5 References

- IE 251 2023-2024 Fall Recitation 9 , Tolga Karabaş , Hasan Taş
- IE 251 2023-2024 Fall Recitation 10, Tolga Karabaş , Hasan Taş

6 Appendix

Yearly Workforce Requirement	2024	2025	2026
A	1250	1800	2500
B	1600	2250	3000
C	1200	750	0

Table 1 Workforce Requirements

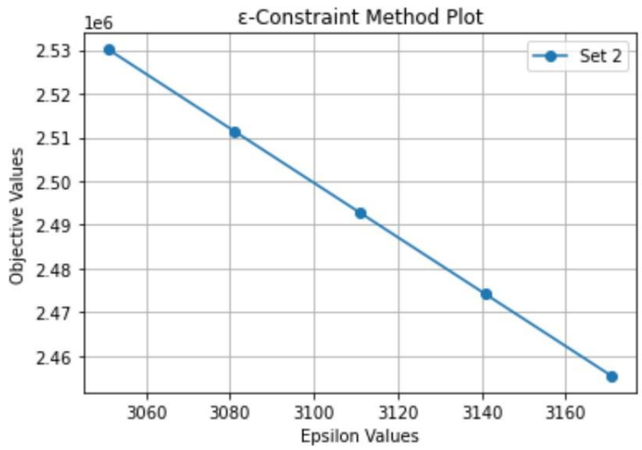

Table 2 Plot of e-constraint Method Results

Column name	St	Activity
Hiring amount Of A-type workers in 2024	NL	.
Hiring amount Of A-type workers in 2025	BS	600.00000
Hiring amount Of A-type workers in 2026	BS	600.00000
Hiring amount Of B-type workers in 2024	NL	.
Hiring amount Of B-type workers in 2025	BS	271.50000
Hiring amount Of B-type workers in 2026	BS	752.50000
Hiring amount Of C-type workers in 2024	NL	.
Hiring amount Of C-type workers in 2025	NL	.
Hiring amount Of C-type workers in 2026	NL	.

Table 3.1 Hiring Amount of Each type of Worker

		Activity
Promoting B type workers in 2024	BS	340.00000
Promoting B type workers in 2025	BS	12.50000
Promoting B type workers in 2026	BS	190.00000
Promoting C type workers in 2024	BS	300.00000
Promoting C type workers in 2025	BS	300.00000
Promoting C type workers in 2026	BS	300.00000

Table 3.2 Promoting of B and C type Workers

		Activity
A type idle workers in 2024	NL	.
A type idle workers in 2025	NL	.
A type idle workers in 2026	NL	.
B type idle workers in 2024	BS	180.00000
B type idle workers in 2025	NL	.
B type idle workers in 2026	NL	.
C type idle workers in 2024	BS	900.00000
C type idle workers in 2025	BS	840.00000
C type idle workers in 2026	BS	1131.00000

Table 3.3 Amount of Idle Workers

		Activity
Degrading A 2024	BS	240.00000
Degrading A 2025	BS	1250.00000
Degrading A 2026	NL	.
Degrading B 2024	BS	1800.00000
Degrading B 2025	NL	.
Degrading B 2026	BS	2500.00000

Table 3.4 Amount of Degraded Workers

		Activity
Hiring A 2024	BS	.
Hiring A 2025	NU	600.00000
Hiring A 2026	NU	600.00000

Table 4.1 Hiring Constraint for A type Worker

		Activity
Hiring B 2024	BS	.
Hiring B 2025	BS	271.50000
Hiring B 2026	BS	752.50000

Table 4.2 Hiring Constraint for B type Worker