1. a. $f(u,v) = Au \cdot Au = A(U_1 \cdot V_1) + A(U_2 \cdot V_2) + A(U_3 \cdot V_3)$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 2 & 0 & 4 \end{bmatrix} (U_1 \cdot V_1) + \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 2 & 0 & 4 \end{bmatrix} (U_2 \cdot U_2) + \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 2 & 0 & 4 \end{bmatrix} (U_3 \cdot V_3)$$

$f(v,u) = Av \cdot Au = A(V_1 \cdot U_1) + A(V_2 \cdot U_2) + A(V_3 \cdot U_3)$

$$\rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 2 & 0 & 4 \end{bmatrix} (V_1 \cdot U_1) + \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 2 & 0 & 4 \end{bmatrix} (V_2 \cdot U_2) + \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 1 \\ 2 & 0 & 4 \end{bmatrix} (V_3 \cdot U_3)$$

maka terbukti fungsi $f(u,v) = Au \cdot Av$ adalah hasil kali dalam pada R^3

c. Tidak, karena jika bobot yang dimiliki adalah 0 maka tidak memenuhi aksioma keempat dan jika bobot yang dimiliki adalah negatif bisa dimisalkan $f(u,v) = 3u_1v_1 - 7u_2v_2 + 4u_3v_3$, $V = (1,1,1)$ maka $f(v,v) = 0$. sehingga tidak memenuhi aksioma keempat.

b. $\langle \vec{u}, \vec{v} \rangle = u_1v_1 + 2u_2v_2 + 3u_3v_3$

- Aksioma Pertama $\langle \vec{u}, \vec{v} \rangle = \langle \vec{v}, \vec{u} \rangle$

$$\begin{aligned} \langle \vec{u}, \vec{v} \rangle &= u_1V_1 + 2u_2V_2 + 3u_3V_3 \\ &= V_1u_1 + 2V_2u_2 + 3V_3u_3 \\ &= \langle \vec{v}, \vec{u} \rangle \end{aligned}$$

- Aksioma Kedua $\langle (\vec{u} + \vec{v}), \vec{w} \rangle = \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle$

$$\begin{aligned} \langle (\vec{u} + \vec{v}), \vec{w} \rangle &= \langle ((u_1+v_1), (u_2+b_2), (u_3+v_3)), (w_1, w_2, w_3) \rangle \\ &= (u_1+v_1)w_1 + 2(u_2+v_2)w_2 + 3(u_3+v_3)w_3 \\ &= \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle \end{aligned}$$

• > Aksioma Ketiga $\langle k\vec{u}, \vec{v} \rangle = k\langle \vec{u}, \vec{v} \rangle$

$$\langle k\vec{u}, \vec{v} \rangle = \langle (ku_1, ku_2, ku_3), (v_1, v_2, v_3) \rangle$$
$$= (ku_1v_1 + 2ku_2v_2 + 3ku_3v_3)$$
$$= k(u_1v_1 + 2u_2v_2 + 3u_3v_3)$$
$$= k\langle \vec{u}, \vec{v} \rangle$$

• > Aksioma Keempat

$\langle \vec{u}, \vec{v} \rangle = u_1^2 + 2u_2^2 + 3u_3^2$ sehingga selalu bernilai ≥ 0

untuk setiap $\vec{u}$ dan $\langle \vec{u}, \vec{u} \rangle = 0$ hanya jika $\vec{u} = 0$

2.a. $\langle \bar{p}, \bar{q} \rangle = 2a_0b_0 + 2a_1b_1 + 3a_2b_2 + a_3b_3$

$\Rightarrow$ Aksioma Pertama $\langle \vec{p}, \bar{q} \rangle = \langle \bar{q}, \bar{p} \rangle$

$\langle p, q \rangle = 2a_0b_0 + 2a_1b_1 + 3a_2b_2 + a_3b_3$

$= 2b_0a_0 + 2b_1a_1 + 3b_2a_2 + b_3a_3 = \langle q, p \rangle$

$\Rightarrow$ Aksioma Kedua $\langle (\vec{p}+\vec{q}), \vec{r} \rangle = \langle \vec{p}, \vec{r} \rangle + \langle \vec{q}, \vec{r} \rangle$

$\langle (\vec{p}+\vec{q}), \vec{r} \rangle = \langle ((a_0+b_0), (a_1+b_1), (a_2+b_2), (a_3+b_3)),$

$(c_0, c_1, c_2, c_3) \rangle$

$= 2(a_0+b_0)c_0 + 2(a_1+b_1)c_1 + 3(a_2+b_2)c_2$

$+ (a_3+b_3)c_3$

$= (2a_0c_0 + 2a_1c_1 + 3a_2c_2 + a_3c_3) + (2b_0c_0$

$+ 2b_1c_1 + 3b_2c_2 + b_3c_3)$

$= \langle \vec{p}, \vec{r} \rangle + \langle \vec{q}, \vec{r} \rangle$

$\Rightarrow$ Aksioma Ketiga $\langle k\vec{p}, \vec{q} \rangle = k \langle \vec{p}, \vec{q} \rangle$

$\langle k\vec{p}, \vec{q} \rangle = \langle (ka_0, ka_1, ka_2, ka_3) . (b_0, b_1, b_2, b_3) \rangle$

$= \langle (2ka_0b_0 + 2ka_1b_1 + 3ka_2b_2 + ka_3b_3) \rangle$

$= k \langle \vec{p}, \vec{q} \rangle$

$\Rightarrow$ Aksioma Keempat $\langle q, q \rangle$

$\langle \vec{q}, \vec{q} \rangle = 2b_0b_0 + 2b_1b_1 + 3b_2b_2 + b_3b_3 \rightarrow$ positif

b. Bisa, apabila w_1 diubah menjadi -2 maka

$\langle \vec{p}, \vec{q} \rangle = 2a_0b_0 - 2a_1b_1 + 3a_2b_2 + a_3b_3$

$q \Rightarrow (1,1,1,1) \rightarrow \cancel{f}(q,q) = 4 \rightarrow$ positif sehingga

aksioma keempat terpenuhi dan bobot dapat didefenisikan

suatu hasil kali dalam berbobot pada P^3

2c. $a = (1, 0, 0, 0) \rightarrow p(x) = 1$

$F(p(x); p(x)) = \int_{-2}^{-1} p(x)\, p(x)\, dx = \int_{-2}^{-1} 1 \cdot 1\, dx$

$= [x + C]_{-2}^{-1}$

$= 1 + C$

Aksioma ke empat $\rightarrow \int_{-2}^{-1} p(x)^2\, dx \geq 0$ jika dan hanya jika $C \geq 0$

3. $\langle \vec{a}, \vec{b} \rangle = a_{11}b_{11} + 2a_{12}b_{12} + a_{21}b_{21} + 3a_{22}b_{22} + a_{31}b_{31} + 4a_{32}b_{32}$

- Aksioma Pertama

$\langle \vec{a}, \vec{b} \rangle = a_{11}b_{11} + 2a_{12}b_{12} + a_{21}b_{21} + 3a_{22}b_{22} + a_{31}b_{31} + 4a_{32}b_{32}$

$= b_{11}a_{11} + 2b_{12}a_{12} + b_{21}a_{21} + 3b_{22}a_{22} + b_{31}a_{31} + 4b_{32}a_{32}$

$= \langle \vec{b}, \vec{a} \rangle$

- Aksioma Kedua

$$\langle (\vec{a} + \vec{b}), \vec{c} \rangle = \left\langle \begin{vmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \\ a_{31}+b_{31} & a_{32}+b_{32} \end{vmatrix}, \vec{c} \right\rangle$$

$= (a_{11}+b_{11})c_{11} + 2(a_{12}+b_{12})c_{12} + (a_{21}+b_{21})c_{21}$
$+ 3(a_{22}+b_{22})c_{22} + (a_{31}+b_{31})c_{31} +$
$4(a_{32}+b_{32})c_{32}$

$= a_{11}c_{11} + b_{11}c_{11} + 2a_{12}c_{12} + 2b_{12}c_{12} +$
$a_{21}c_{21} + b_{21}c_{21} + 3a_{22}c_{22} + 3b_{22}c_{22}$
$+ a_{31}c_{31} + b_{31}c_{31} + 4a_{32}c_{32} + 4b_{32}c_{32}$

$= \langle \vec{a}, \vec{c} \rangle + \langle \vec{b}, \vec{c} \rangle$

- Aksioma Ketiga

$\langle k\vec{a}, \vec{b} \rangle = ka_{11}b_{11} + k2a_{12}b_{12} + ka_{21}b_{21} + k3a_{22}$
$b_{22} + ka_{31}b_{31} + k4a_{32}b_{32}$

$= k \langle \vec{a}, \vec{b} \rangle$

- Aksioma Keempat

$\langle \vec{a}, \vec{a} \rangle = a_{11}^2 + 2a_{12}^2 + a_{21}^2 + 3a_{22}^2 + a_{31}^2 + 4_{32}^2$

sehingga selalu berniai ≥ 0 untuk $\langle \vec{a}, \vec{a} \rangle = 0$
jika dan hanya jika $\vec{a} = 0$

4. $\langle \vec{p} \cdot \vec{q} \rangle = a_0 b_0 + a_1 b_1 + a_2 b_2 + a_3 b_3$

$p = 2 - 3x + 6x^2 - 2x^3$

$q = 1 - 2x - 3x^2 + 5x^3$

$\langle \vec{p} \cdot \vec{q} \rangle = 2 \cdot 1 + -3 \cdot -2 + 6(-3) + -2 \cdot 5$

$= 2 + 6 + (-18) + 10$

$= 0 \rightarrow$ tidak memenuhi aksioma 4

5. a. $\|\vec{u}\| = \sqrt{u \cdot u} = \sqrt{3u_1^2 + 2u_2^2}$

b. $d(u, v) = \|\vec{u} - \vec{v}\| = \sqrt{3(u_1 - v_1)^2 + 2(u_2 - v_2)^2}$

c. $\cos(\alpha) = \dfrac{u \cdot v}{\|u\|\|v\|} = \dfrac{3u_1 \cdot v_1 + 2u_2 v_2}{\left(\sqrt{(3u_1)^2 + (2u_2)^2}\right) \cdot \left(\sqrt{(3v_1)^2 + (2v_2)^2}\right)}$

Date. /

6. a. $\langle \vec{f}, \vec{g} \rangle = \int_1^{10} f(x)\, g(x)\, dx$

→ Aksioma Pertama $\langle \vec{f} \cdot \vec{g} \rangle = \langle \vec{g} \cdot \vec{f} \rangle$

$\rightarrow \int_1^{10} f(x) \cdot g(x)\, dx = \int_1^{10} g(x)\, f(x)\, dx)$

$\rightarrow \langle \vec{g}, \vec{f} \rangle$

→ Aksioma Kedua $\langle (\vec{f} + \vec{g}), \vec{h} \rangle = \langle \vec{f}, \vec{h} \rangle + \langle \vec{g}, \vec{h} \rangle$

$\rightarrow \int_1^{10} (f+g)(x)\, h(x)\, dx = \int_1^{10} f(x)\, h(x)\, dx)$

$+$

$\langle \vec{f}, \vec{h} \rangle + \langle \vec{g}, \vec{h} \rangle = \int_1^{10} g(x)\, h(x)\, d(x))$

→ Aksioma Ketiga $\langle k\vec{f} \cdot \vec{g} \rangle = k \langle \vec{f} \cdot \vec{g} \rangle$

$\rightarrow \int_1^{10} k\, f(x)\, g(x)\, dx = k \int_1^{10} f(x)\, g(x)\, dx$

$\rightarrow k \langle \vec{f}, \vec{g} \rangle$

→ Aksioma Keempat $\langle \vec{f}, \vec{f} \rangle$

$\rightarrow \int_1^{10} f^2(x)\, dx \geq 0$ apabila $\langle \vec{f}, \vec{f} \rangle = 0$ jika dan hanya jika $\vec{f} = 0$

b. $\vec{f} = f(x) = x + 3$

$\|\vec{f}\| = \sqrt{\langle \vec{f}, \vec{f} \rangle} = \sqrt{\int_1^{10} f(x)\, f(x)\, dx} = \sqrt{\int_1^{10} f^2(x)\, dx}$

$= \sqrt{\int_1^{10} (x+3)^2\, dx} = \sqrt{\int_1^{10} x^2 + 6x + 9\, dx}$

$= \sqrt{\left[\frac{1}{3}x^3 + 3x^2 + 9x\right]_1^{10}}$

$= \sqrt{711} = 26{,}6$

Date. /

c. $\vec{f} = f(x) = x+5$, $\vec{g} = g(x) = x+2$

$\vec{f} - \vec{g} = x+5-(x+\cancel{3}2) = 2$

$d(\vec{f},\vec{g}) = \sqrt{\langle \vec{f}-\vec{g}, \vec{f}-\vec{g}\rangle}$

$= \sqrt{\int_1^{10} (f(x)-g(x))^2\,dx} = \sqrt{\int_1^{10} 2^2\,dx}$

$= \sqrt{4x\,]_1^{10}} = \sqrt{40-4} = \sqrt{36} = 6$

d. $f(x) = 0$, $g(x) = 0$

$\downarrow \qquad \downarrow$

$\vec{f} \qquad \vec{g}$

$\langle \vec{f}, \vec{g} \rangle = 0 \rightarrow$ ortogonal

$\hookrightarrow \langle f(x), g(x) \rangle = \int_1^{10} f(x) g(x)\,dx$

$= \int_1^{10} 0 \cdot 0 = 0$

Date. /

7. a. hasil kali dalam antara dua vektor tersebut sama dengan 0

b. $u = (1, 0, 0, -1)$

$b = (0, 1, -1, 0)$

$\langle a, b \rangle = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$

$= (1.0) + (0.1) + (0.-1) + (-1.0) = 0$

c. $a = 2 + 3x + x^2 - x^3 + 5x^4$

$b = 2 + 3x + x^2 + 9x^3 - x^4$

$\langle a ; b \rangle = (2.2) + (3.3) + (1.1) + (-1.9) + (5.(-1))$

$= 4 + 9 + 1 + (-9) + (-5)$

$= 0$

d. Iya, berdasarkan definisi ortoghonal dan aksioma positif

HKD $\rightarrow (\bar{u}, \bar{u}) = 0$ jhj u vektor nol

e. Iya, $k0 = 0$ untuk k bilangan riil apapun dan berdasarkan konsep bahwa kelipatan skalar sebuat vektor akan menghasillkan vektor yang sejajar dengan dirinya sendiri