Tackling Informality and Inequality: A Directed Search Model

Emiliano Rojas Eng*

This Version: September 11, 2023

Click here for the latest version

PRELIMINARY please do not cite or circulate

Abstract

Existing models of informality often assume a single, market-clearing wage per sector, thus leaving questions regarding its relationship with inequality unanswered. Who bears the costs and who reaps the benefits of tackling informality? In this model, workers with different levels of income, wealth, and ability choose whether to search for a wage in the formal labor market or face the uncertainty of income in the informal labor market. Low wealth or ability leads to being trapped in the informal sector, while higher levels of either make it easier to secure a high-paying formal job. Tackling informality reduces income inequality, increases wealth inequality and increases consumption inequality. Notably, most policies primarily benefit workers already in the formal sector and workers with lower income and wealth.

JEL: E21, E24, E26, J24, J64, O17.

Keywords: Informality, Inequality, Directed search, Uncertainty, Barriers to entry, Unemployment Insurance, Universal Basic Income.

*Email: emirojaseng@gmail.com. I am grateful with Gustavo Leyva for his comments on an earlier version of this paper.

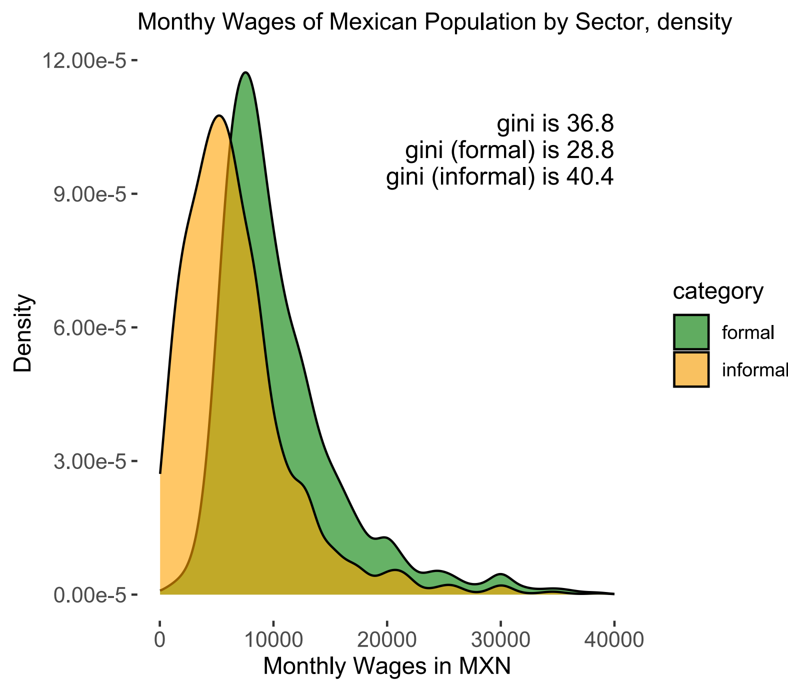
1 Introduction

Informality, an important barrier to economic growth in developing countries, remains a pressing concern globally. Numerous policies have been proposed to mitigate its impacts, but understanding these policies' impact on inequality has been obscured by limitations in existing modeling techniques. This paper bridges this knowledge gap by exploring the intersection of informality and inequality, and answers the question of who bears the costs and who reaps the benefits of tackling informality.

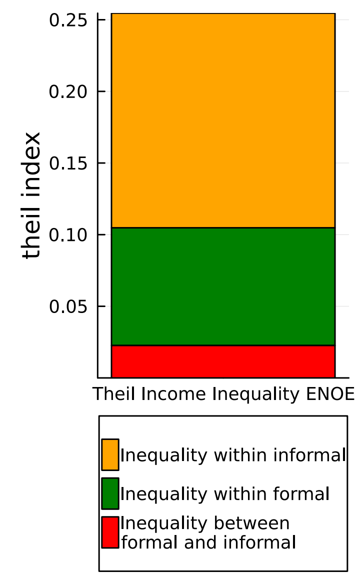
The key innovations that allows this model to generate inequality of income, wealth, and consumption in an informal economy are the following: (i) Agents are risk-averse and have different levels of ability, wealth and labor income. They choose which sector to enter, which wage to pursue, how much to save and consume, and can search on-the-job for better opportunities. (ii) In the formal labor market, a spectrum of wage possibilities exist for each worker type. Firms competitively enter each submarket and match with workers that search for their preferred wage level. (iii) The informal labor market, although more accessible, is characterized by uncertainty of earnings. (iv) Matching in the labor market is done through directed search, inducing a block recursive equilibrium which reduces the complexity of the model.

Reducing informality, meaning non-compliance with the relevant fiscal and labor laws, is a policy goal around the world. Informal firms are on average smaller, pay lower wages, are run by less educated individuals, employ less educated workers, and earn lower profits than formal firms (Ulyssea, 2020). Informal firms are the least productive, especially informal firms with non-salaried workers (whose income is uncertain). Despite their lower productivity, informal firms constitute a significant proportion of the economy — in Mexico, they employ 55% of the labor force and use 40% of the capital (Levy, 2016).

The model presented here can explain the empirical data obtained from the Mexican distributions of earnings, as shown in Figure 1. Figure 1a illustrates that average formal earnings are greater than average informal earnings. However there is a considerable overlap in earnings, contradicting the modeling approach of other papers. There is neither a single wage (as in Leal-Ordoñez, 2014; Lopez-Martin, 2019; Franjo et al., 2022), nor a single cutoff point in earnings that distinctly separates the two sectors (as in Granda & Hamann, 2018; Dell'Anno, 2018; Leyva & Urrutia, 2020), and since the overlap persists even when controlling for education level, distributions generated using wage per unit of effective labor (as in Ulyssea, 2019; Esteban-Pretel & Kitao, 2021) do not provide satisfactory explanations. In the present model, the overlap across the entire wage spectrum is generated by directed



(a) Wages, distribution by sector.



(b) Theil index of

Figure 1: Distribution of wages and inequality indices for Mexican workers in the formal and informal labor market. Density plot. The x-axis does not show the full range of the sample to enhance visualization. Source: National Occupation and Employment Survey (ENOE) Q1-2023

search and on-the-job labor search by heterogeneous agents in the formal sector, coupled with a simple informal sector characterized by uncertain earnings.

Income inequality is driven mostly by inequality within the informal sector. In this paper I discuss inequality calculated with the help of two indices, the Gini index and the Theil index. The primary advantage of the Gini index lies in its ease of interpretation; it serves as a standard measure of income inequality, ranging from 0 to 1, where 0 signifies perfect equality and 1 signifies perfect inequality. The Theil index's key advantage is its decomposability, meaning that the inequality measure for the entire population can be disaggregated into the (weighted) sum of the inequality measures within the formal and informal sectors, plus the inequality between both sectors. For a detailed description of both indices see Appendix C. As shown in Figure 1a, the Gini index of the informal sector is much larger than the Gini index of the informal income. Figure 1b, with the Theil index and its components, confirms that inequality within the informal sector is the main component of income inequality. This raises the question, how does tackling informality affect the inequality of income? I find, using policy experiments, that a decrease in informality also decreases income inequality.

This model is most closely related to Chaumont and Shi (2022). Their model has directed search for their single labor market, with agents that are heterogeneous in earnings and wealth. They find that on-the-job search and wealth accumulation generate large frictional wage dispersion, and together with directed search provide self-insurance against earnings risk. This model is tractable and easily computable because it has a Block Recursive Equilibrium (Menzio & Shi, 2010). This means that since labor search is directed, the agents' decisions are influenced only by their individual characteristics and the aggregate labor market conditions, not by the choices made by other agents or the distribution of other agents across states. This contrasts with models featuring random matching like Burdett and Mortensen (1998), Postel-Vinay and Robin (2002) and Burdett and Coles (2003), which require storing the full wage distribution as a state variable, making it very hard computationally.

In my model, both greater ability and wealth lead to higher income in the formal sector. This is because more lucrative wages are available for high-ability individuals, and having wealth mitigates the risk involved in seeking good job opportunities. Conversely, individuals lacking either ability or wealth are more likely to enter the informal sector, which serves as an alternative to the lowest-paying jobs in the formal sector. The simulation reveals substantial overlaps in income, wealth, consumption, and ability among workers in both sectors. According to the generated distributions, income inequality is greater within the informal sector, consistent with the empirical findings of Binelli (2016) and Elgin et al.

(2021), and Figure 1. In contrast, the main component of wealth inequality is inequality within the formal sector, while consumption inequality is more prevalent between sectors and within the formal sector.

This paper studies four types of policies that are popular in the literature: fiscal incentives, punitive measures, reducing barriers to entry of formal firms, and increasing social security benefits. I highlight the following primary insights. First, formal workers often reap the majority of benefits from policies aimed at addressing informality. These policies typically function by either enhancing the attractiveness of formality or reducing the appeal of informality. This outcome occurs even when policies disproportionately benefit those with lower wealth and income. Second, a decrease in informality results in a reduction of income inequality given the greater disparity of income within the informal sector. Third, a decrease in informality results in an increase of wealth inequality, driven by an increase in inequality within the formal sector. Fourth, a decrease in informality results in an increase of consumption inequality, driven by an increase of inequality within the formal sector. Fifth, two policies that increase tax revenue while decreasing informality are reducing the cost of entry to the formal sector, thus increasing the number of formal sector jobs; and limiting the possibility of entering the informal sector by increasing enforcement. However, the welfare implications of these approaches are starkly contrasting.

Section 2 is a literature review, section 3 presents the model, section 4 presents the results for the baseline model, section 5 presents the policy experiments, section 6 concludes.

2 Literature Review

This section reviews the literature on the relationship between inequality and informality. The first type of papers model informality and inequality by proposing an initial exogenous distribution of income. Chong and Gradstein (2007) has a two-period model in which the first period income is exogenous and the second period income depends on investment and sector-choice decisions. They find that greater inequality and lower institutional quality leads to higher informality. Dell'Anno (2018) presents a two-period overlapping generations model with identical agents who receive an inheritance, choose a sector subject to entry costs and borrowing constraints, and pass on their inheritance at the end of their life. The author finds that higher credit constraints, higher income taxes, and higher barriers of entry increase informality, while any initial wealth distribution collapses to two points: low-wealth for informal workers and high-wealth for formal workers.

The second type of papers have models with two wages, one for the formal sector, and one for the informal sector. Granda and Hamann (2015) present an occupational choice model, in which formal workers earn the minimum wage, which is higher than the market-clearing wage, and the rest of the workers are forced to informality; there is no earnings inequality within sectors. In their model wealth inequality is driven by precautionary savings of informal sector workers, thus reducing worker informality reduces wealth inequality among workers. Antón and Levy (2012) model informality in Mexico with two intermediate and two final goods in a dual system of Social Insurance. They find that by shifting the financing of social insurance from labor taxes to consumption taxes formality increases, workers have more complete coverage against risks, and income inequality decreases. Leyva and Urrutia (2020) propose a business cycle model of informality with endogenous participation, and study the business cycle properties of informality; the model has two wages, one for each sector and a representative agent. They find that making formality more attractive reduces informality, increases labor participation, while increasing the volatility of output. In a slightly different setup, Docquier and Iftikhar (2019) have a continuous time model with random matching, and with both high skill and low skilled in both formal and informal work, in equilibrium there are two wages determined through nash-bargaining: the high skill wage and the low skill wage, with workers of both type of abilities preferring to work in the formal sector, but becoming informal if unsuccessful in getting a formal job.

The third type of papers have models with a single market-clearing wage per efficient unit of labor, which leads to distributions of income linearly dependent on the distribution of human capital. Esteban-Pretel and Kitao (2021) present an overlapping generations model where workers receive offers exogenously from both sectors and cannot search on-the-job. Workers in this model can accumulate wealth for retirement and accumulate human capital while employed. There is a single market-clearing wage per efficient unit of labor, and the amount of efficient units of labor depends on the worker's human capital and idiosyncratic productivity, the latter ultimately drives the wage gap between formal and informal labor. Ulyssea (2018) proposes a model with entry of firms with different productivity into formality (the extensive margin) and in which formal firms can decide to hire informal workers (the intensive margin); this model has a single wage per efficient unit of work and a representative agent. The author shows that increasing enforcement reduces informality but also reduces welfare, while reducing formal sector entry cost increases welfare but its impact on informality is limited, since the intensive margin of informality remains active. Lopez-Martin (2019) has a model in which all workers earn the same wage and can accumulate wealth to be able to be an entrepreneur either in the formal or informal sector, reducing financing constrains improves welfare and reduces informality, while size-dependent distortions reduce the effectiveness of financial development.

To my knowledge the only other paper to successfully model an endogenous distribution of wages is Meghir et al. (2015), which features a model of random matching, but has workers who are identical in productivity and hold no wealth. In this model workers continuously maximize their expected income stream, choose which sector to work in, and can search on the job. Firms have different levels of productivity and enter the formal or informal labor market until their profits are equal at each level of productivity. The model has an endogenous distribution of formal and informal wages from which workers sample, but the distribution is assumed to be parametric for calibration. Formal and informal jobs coexist only within a range of intermediate wages.

Turning our attention to the empirical evidence, there is evidence for a positive relationship of informality and inequality. Binelli (2016) shows that informality within the informal sector is the most important component of wage inequality, and gives causal evidence that an increase in informality increases inequality. Elgin et al. (2021) establish a statistical relationship in which an increase in informality increases income inequality, and an increase in the rate of profit also increases inequality, but informality reduces the effect of the rate of profit on inequality since informality increases the income of lower income groups. Gutiérrez-Romero (2021), taking a long-run perspective, shows through three centuries of data that higher inequality in the past persistently increases present-day informality across various countries.

This paper seeks to fill the gaps in the literature by proposing a model that generates an endogenous distribution of income, wealth, and consumption, using a directed search and matching model. The proposed model allows for a nuanced analysis of policy impacts, which can provide valuable insights for policy makers aiming to reduce informality and income inequality. In the following sections, the model and its results will be presented.

3 Model

3.1 Deciding consumption and savings

Workers are heterogeneous in their ability z, income w, wealth a, and employment status $\epsilon \in \{f, i, u\}$, which stands for employed in the formal sector, employed in the informal sector and unemployed respectively.

At the beginning of each period, the agent, characterized by (z, w, a) and employment status

ϵ, chooses consumption (c) and savings ($\hat{a} - a$). Here, $\hat{a}$ denotes the agent's wealth in the next period. The agent makes the consumption and savings decisions by solving the problem defined by equation (1):

$$\begin{aligned} V_\epsilon(z, w, a) = \quad & \max_{(c,\hat{a})} [u(c) + \beta R_\epsilon(z, w, \hat{a})] \\ \text{s.t.} \quad & c + \frac{\hat{a}}{1+r} = I_u + w + a, \\ & \hat{a} \geq \underline{a} \end{aligned} \tag{1}$$

In the above equation, $V_\epsilon(z, w, a)$ denotes the value function, $u(c)$ is the utility of consumption, and β is the discount factor. The term $R_\epsilon(z, w, \hat{a})$ is the future reward associated with choosing wealth $\hat{a}$ for the next period. This implies entering the labor market as an agent with characteristics $(z, w, \hat{a})$ and employment status ϵ. The interest rate is represented by r, I_u is an unconditional transfer that is set near zero (but increased when studying the effects of implementing a universal basic income), and $\underline{a}$ is the borrowing constraint. It's important to note that the employment status ϵ influences the expected rewards of saving $R_\epsilon(z, w, \hat{a})$, leading to divergent decisions.

3.2 Searching in the labor market

The next step involves labor market search. First, the agent decides which labor market to search in. This decision is based on comparing the rewards of entering either the formal or informal labor market, and the agent chooses the one that provides higher expected utility. On-the-job search is possible, and there are no associated costs with job switching or transitioning between formality and informality. If the labor search is successful, the agent begins work in the next period.

Consider an agent[1] that enters the labor market with characteristics (z, w, a) and employment status ϵ. This agent can choose to search either in the formal sector, with reward $R_{\epsilon f}(z, w, a)$, or in the informal sector, with reward $R_{\epsilon i}(z, w, a)$. The total reward $R_\epsilon(z, w, a)$ is the maximum of both. Equation (2) details the trade-offs of searching in each labor market.

[1]For simplicity of notation, this agent is not the same as the one in the previous subsection, which entered the labor market as $(z, w, \hat{a})$ with labor status ϵ

$$
\begin{aligned}
R_\epsilon(z,w,a) = \quad & \max\{R_{\epsilon f}(z,w,a), R_{\epsilon i}(z,w,a)\} \\
& \text{where:} \\
R_{\epsilon f}(z,w,a) = & \max_{\{\hat{w}\}}[\lambda_\epsilon p(\theta(z,\hat{w},a))V_f(z,\hat{w},a) + (1-\lambda_\epsilon p(\theta(z,\hat{w},a)))\bar{V}_\epsilon(z,w,a)] \\
R_{\epsilon i}(z,w,a) = & \lambda_\epsilon p_{inf} E_w(V_i(z,w,a)) + (1-\lambda_\epsilon p_{inf})\bar{V}_\epsilon(z,w,a)
\end{aligned}
\tag{2}
$$

First, the agent gets the opportunity to search in any labor market with probability λ_ϵ. The second line of equation (2) describes the expected reward from searching the formal labor market. The third line of equation (2) describes the expected reward from searching the informal labor market.

When searching in the formal labor market, the agent looks for jobs that pay a wage $\hat{w}$, given his ability z and wealth a. The probability of finding such a job depends on the labor market tightness $\theta(z,\hat{w},a)$. The probability of successfully entering the submarket $(z,\hat{w},a)$ is denoted by $p(\theta(z,\hat{w},a))$. If the agent is successful in finding a job, his Value Function in the next period is $V_f(z,\hat{w},a)$.

When the agent searches in the informal labor market, there is no differentiation between skills, wages, or wealth. Instead, the probability of entry is given by p_{inf}, and if successful in each period labor income w is randomly drawn from a known probability distribution $F(w)$. Thus, successful entry into the informal labor market has an expected utility of $E_w(V_i(z,w,a)) = \int_0^\infty V_i(z,w,a)\,dF(w)$.

In all cases, if the agent does not secure a new job, he receives an expected value of maintaining his current state $\bar{V}_\epsilon(z,w,a)$. This state could either be remaining employed or becoming unemployed, which is resolved in the next stage.

3.3 Remaining employed or becoming unemployed

The expected utility of remaining in the same position after failing to find a new job is denoted as $\bar{V}_\epsilon(z,w,a)$. Equation (3) elaborates on how this is determined.

$$
\begin{aligned}
\bar{V}_f(z,w,a) &= \delta_f V_u(z,\rho w,a) + (1-\delta_f)V_f(z,w,a) \\
\bar{V}_i(z,w,a) &= \delta_i V_u(z,0,a) + (1-\delta_i)E_w(V_i(z,w,a)) \\
\bar{V}_u(z,w,a) &= \chi V_u(z,0,a) + (1-\chi)V_u(z,w,a)
\end{aligned}
\tag{3}
$$

For a formal worker, there exists an exogenous probability δ_f of becoming unemployed, with probability $(1-\delta_f)$ the formal worker stays the same. Similarly, an informal worker faces an exogenous probability δ_i of becoming unemployed, with probability $(1-\delta_i)$ the informal worker stays the same, and thus draws an informal income for next period.

The parameters ρ and χ are used when studying the implementation of unemployment insurance. In the baseline scenario, ρ is set to zero and χ is set to one, implying that both formal and informal workers exit the labor market with zero income, relying solely on their savings. An unemployed agent has no income, and if the agent does not find a new job this period, he remains unemployed with no income. In the policy scenario of an implementation of unemployment insurance ρ is positive, indicating that formal workers exit the labor market with unemployment insurance amounting to $\rho * 100\%$ of their last wage. An unemployed agent with unemployment insurance will continue to receive his current unemployment insurance w with a probability of $(1-\chi)$, and with probability χ the unemployment insurance expires.

This concludes the characterization of the worker's problem in its recursive form. The remaining parameter to specify is the market tightness of the formal labor market for each submarket (z, w, a), denoted as $\theta(z, w, a)$.

3.4 Firms and labor market tightness

Consider a formal firm that employs a worker with ability z and wealth a and compensates the worker with wage w, this will be the formal submarket (z, w, a). The firm has a constant returns to scale technology, with production output $(y+z)$, which comprises an exogenous productivity y, and a productivity dependent on the worker's ability z. The firm pays a profit tax τ and a wage tax τ_w. Thus, the net profit earned by the firm is $\pi(z, w) = (1-\tau)(y+z-(1+\tau_w)w)$. The firm continues its operations next period unless the worker is separated exogenously with probability δ_f or finds another job with probability $\lambda_f \hat{p}$, where $\hat{p}$ depends on the worker's decisions. Therefore, the value function of an operating firm in the sub-market (z, w, a) can be given by equation (4):

$$J(z,w,a) = \begin{cases} \text{if agent } (z,w,a) \text{ wants to remain formal and } \pi(z,w) \geq 0\text{:} \\ \quad \pi(z,w) + \beta(1-\delta_f)(1-\lambda_f p(\theta(z,\hat{w},\hat{a})))J(z,w,\hat{a}) \\ \text{if agent } (z,w,a) \text{ wants to transition to informality and } \pi(z,w) \geq 0\text{:} \\ \quad \pi(z,w) + \beta(1-\delta_f)(1-\lambda_f p_{inf})J(z,w,\hat{a}) \\ \text{if } \pi(z,w) < 0\text{:} \\ \quad 0 \end{cases} \tag{4}$$

In the above equation, $\hat{a}$ and $\hat{w}$ represent the optimal wealth accumulation and wage search policies of an employee starting the period in (z, w, a) with employment status f. The corresponding optimal labor market sought by such employees is $\theta(z, \hat{w}, \hat{a})$. The constraint that $J(z, w, a) \geq 0$ plays no role, since any firm value less than or equal to 0 leads to the submarket to being empty (market tightness 0) as we will see in Equation 5, the only reason for this constraint is to make the proof of convergence easier.

Now, let's examine a new firm aiming to enter the formal sub-market (z, w, a). Initially, the firm must incur a fixed entry cost denoted by k_f, which is uniform across all firms and represents the cost of establishing a firm in the formal sector. After paying the fixed cost, the candidate firm searches for a worker in the labor market and finds a match with probability $q(\theta(z, w, a))$. If successfully matched, the firm starts production in the next period. Alternatively, each submarket can be though of as being composed of multiple vacancies, with each vacancy incurring a fixed recruiting cost of k_f, recruiting successfully with probability $q(\theta(z, w, a))$, and yielding an expected value of J. The expected value for a firm entering the formal market is given by the expected benefit of successful recruitment minus the cost of vacancy posting:

$$q(\theta(z, w, a))\beta J(z, w, a) - k_f$$

We define the matching probability functions ($p(\theta)$ for workers and $q(\theta)$ for firms) such that $p, q : [0, \infty] \to [0, 1]$, $p'(\theta) > 0$, $q'(\theta) < 0$ and $\theta = \frac{p(\theta)}{q(\theta)}$. This means that higher market tightness θ leads to lower probability of finding an employee for a firm, and a higher probability of finding a job for an agent (and vice-versa).

Assuming free entry, vacancies are posted in all sub-markets until the expected entry value zeroes out. If the expected value of entry is always negative, no firm will want to enter. Thus, the equation for the market tightness is given by equation (5):

$$\theta(z, w, a) = \begin{cases} q^{-1}\left(\frac{k_f}{\beta J(z, w, a)}\right), & \text{if } \beta J(z, w, a) \geq k_f, \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

The primary characteristic distinguishing the informal labor market is the income uncertainty in each period. In this model, informal productivity is not *a priori* inferior to the formal labor market, and the income may sometimes exceed that in the formal sector. The probability of successfully entering the informal labor market is set at P_{inf}. The primary parameter to define is the exogenous income distribution in the informal sector.

3.5 Equilibrium

The equilibrium defined for this model is a Block Recursive Equilibrium. The main characteristic of this type of equilibrium is that the value and policy functions for all economic agents do not depend on the distribution of agents across states. Put simply, workers only need to be aware of the labor markets relevant to them, rather than tracking the position of every other individual in the economy.

Definition 1: An equilibrium consists of the value function of workers V_ϵ and reward functions R_ϵ, $R_{\epsilon f}$, $R_{\epsilon i}$ for $\epsilon \in \{f, i, u\}$, the firm value function J, the market tightness function θ policy functions c_ϵ, $\hat{a}_\epsilon$, $\hat{w}_\epsilon$, for $\epsilon \in \{f, i, u\}$ and the transition function of the aggregate state $\mathcal{T}$, that satisfy the requirements (i)-(v) below:

(i) The value function of workers, $V_\epsilon : Z \times W \times A \to \mathbb{R}$, satisfies the consumption and savings decision (1), with policy functions given by c_ϵ and $\hat{a}_\epsilon$.

(ii) The reward functions R_ϵ, $R_{\epsilon f}$, $R_{\epsilon i}$ satisfy (2) and (3), and the corresponding optimal policy function is given by $\hat{w}_\epsilon$

(iii) The firm's value function, $J : Z \times W \times A \to \mathbb{R}$, satisfies (4)

(iv) The market tightness function $\theta : Z \times W \times A \to [0, \infty]$ satisfies (5)

(v) The transition function of the aggregate state $\mathcal{T}$ is consistent with the policy functions and induces the aggregate state in the next period.

The proof of existence of equilibrium is given in Appendix A. Notice that neither the policy or value functions depend directly on the aggregate distribution of agents across states ψ; the distribution of agents only affects the labor market tightness θ. This is possible with directed search since agents segment themselves across employment submarkets given their characteristics. The end result is that agents in each submarket are identical.

3.6 Calibration

Following Chaumont and Shi (2022), the functional forms for consumption and match probability are defined in (6)

$$
\begin{aligned}
u(c) =& \frac{c^{1-\sigma}}{1-\sigma} \\
p(\theta) =& (1+\theta^{-\gamma})^{-\frac{1}{\gamma}} \\
q(\theta) =& \frac{p(\theta)}{\theta} = (1+\theta^{\gamma})^{-\frac{1}{\gamma}} \Rightarrow \\
q^{-1}(q) =& (q^{-\gamma}-1)^{\frac{1}{\gamma}}
\end{aligned} \tag{6}
$$

The parameters used in the baseline model and their interpretation are listed in Table 1. This section describes the procedure used to calibrate the parameters.

The parameters are calibrated to Mexican data with a monthly frequency. The externally set parameters: β is set to 0.99, r is set to 4% annual, and σ is set to 2.0. Note that $\beta(1+r)$ is less than one. γ is set to 0.65, as in Chaumont and Shi (2022). The unconditional transfer I_u is set to the lowest value of 0.01. y_f is set to represent an average capital share of production of 0.30.

The exogenous separation rate for informal workers, δ_i, is set to be 2.73 times higher than the exogenous separation rate for formal workers, δ_f. This is taken from the 2005-2022 average in the ENOE (National Labor Market Survey). δ_f is set to be 0.0112, which gives an involuntary separation rate for both types of workers of 1.94% as in the ENOE (2005-2022). ρ is set to 0 and χ is set to 1, which means there is no unemployment insurance in the baseline scenario. The profit tax τ is set to 0.30 as in Mexican law. The labor tax τ_w is set to 0.196, equal to the tax wedge for a single worker earning the average wage as reported by the OECD in *taxing wages*.

The domain of possible wages is set to be $[0, 1]$. The distribution of informal labor income is set to be a beta distribution, with parameters beta_a and beta_b. The distribution of ability is also set to take values of $[0, 1]$ from the same probability distribution. In principle the distribution of ability can be of any form without affecting the calibration of the model, since the equilibrium market tightness θ is independent of the number of persons with each ability levels. For example, an increase in the number of persons with ability z^* would make entry more profitable, increasing the number of firms, so that in the end θ remains constant. Since any distribution of ability can be used, to discipline the model I assume that it is exactly the same as the distribution of informal labor income. This can be interpreted as informal labor income being secondary draws of labor productivity, applicable while on the informal sector. The borrowing limit $\underline{a}$ is set to -3.753 so that 31.47% of agents are in net debt as in the 2019 Household Finance Survey (ENFIH). The parameters λ_f, λ_i, λ_u, P_{inf}, k_f, beta_a,

Parameter	Baseline	Meaning
β	0.99	discount parameter
r	0.327%	interest rate, 4% annual
σ	2.0	risk aversion
γ	0.65	elasticity of p given θ
δ_f	0.0112	probability of formal $\rightarrow$ unemployment
δ_i	0.0306	probability of informal $\rightarrow$ unemployment
χ	1.00	probability of insurance $\rightarrow$ no insurance
ρ	0.00	replacement rate of unemployment insurance
λ_f	0.3824	probability of searching given formal status
λ_i	0.1942	probability of searching given informal status
λ_u	0.70	probability of searching given unemployed status
P_{inf}	0.889	probability of finding a job in the informal sector
k_f	2.86	fixed cost of entering the formal market
y_f	0.30	non-labor production of the formal firm
τ	0.30	profit tax
τ_w	0.196	labor tax
I_u	0.01	minimum income
beta_a	0.67	parameter 1 of informal income / productivity
beta_b	2.02	parameter 2 of informal income / productivity
$\underline{a}$	-3.753	borrowing limit

Table 1: Parameters used in the model.

beta_b are calibrated to match the formality rate of 57.86% and an unemployment rate of 4.20% which comes from the ENOE (2005-2022).

Using these parameters we solve for equilibrium using a nested fixed point algorithm and get the policy functions, value functions, and market tightness. For an outline of the algorithm please see Appendix B. The code, written in Julia, is also available on GitHub at https://github.com/emirojaseng/tacklinginformality

4 Results

4.1 The agents' choices: sector choice, and wage searched

The first step in understanding the nature of the model is to understand the agent's choices. The optimal choice always depends on the agent's current situation: what is his employment status, wealth, income, and ability. This is plotted in Figure 2.

Figure 2a presents how unemployed agents make decisions when facing the labor market, from which we can draw two main points.

First, low ability individuals choose to enter the informal labor market. The reason is that

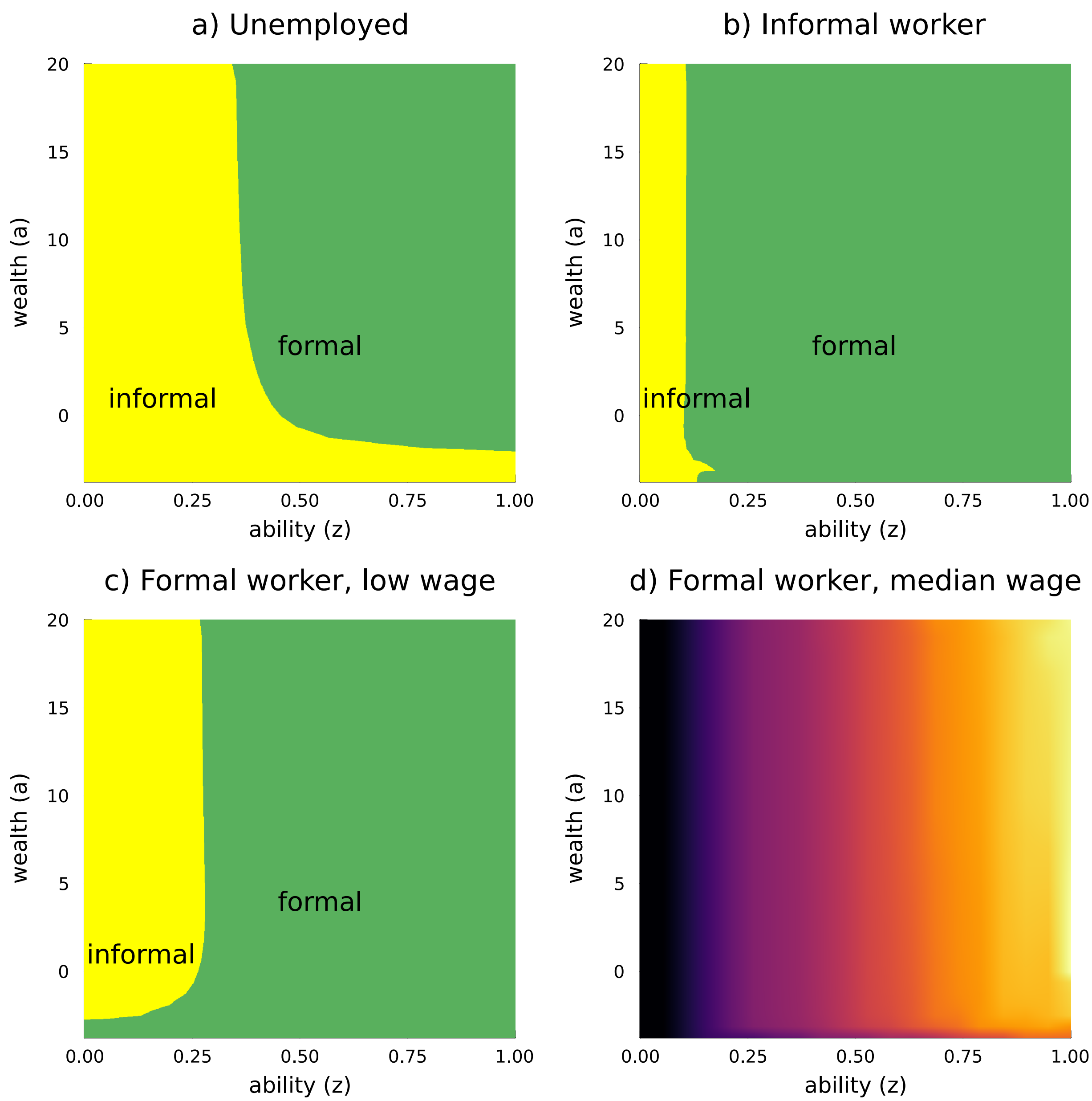


Figure 2: The agent's choices of sector and wage. All plots show the choice made by an agent with ability in the x-axis and wealth in the y-axis. The color yellow represents a choice to enter the informal labor market. The color green represents a choice to search in the formal labor market. The color gradient shows the wage searched by a formal worker with median wage, where warmer colors represent higher wage searched. a) is the sector choice of an unemployed worker, b) is the sector choice of an informal worker, c) is the sector choice of a formal worker with a low wage ($w = 0.183$), d) is the wage choice of a formal worker with the median wage (in the simulation, $w = 0.395$).

the informal labor market offers the possibility of higher earnings; if they were to enter the formal labor market their low productivity means that only the lowest wages are available to them. Thus, the informal labor market becomes an easily accessible alternative that offers the possibility of better labor income.

Second, low wealth forces even highly productive agents to enter the informal labor market. The reason is that there is a harsh consequence of failing to enter the formal labor market when there is no income and no self-insurance. Thus, the informal labor market becomes an easily accessible safety line.

Figure 2b shows decision-making of workers currently in the informal sector. It reveals that agents with the lowest ability invariably opt to remain in this sector. Interestingly, many agents who previously did not consider formal employment while unemployed now express a desire to transition into the formal sector. Unfortunately, being in the informal sector has the lowest probability of being in the labor market ($\lambda_i < \lambda_f < \lambda_u$). This situation effectively creates a trap, with many stuck in the informal sector while earnestly seeking opportunities to enter the formal sector.

Figure 2c shows the decisions of workers currently in the informal sector, but with a wage that is too low (specifically, half the expected value of informal income). Under these circumstances, the dynamics shift: agents with low wealth and ability are drawn towards the relative security provided by low-paying jobs in the formal sector. Conversely, possessing more wealth tends to encourage these agents to transition towards informality. Meanwhile, the majority of agents actively search for better-paying formal employment while still on the job.

Figure 2d portrays a more typical scenario within the formal sector, specifically, formal workers earning a median wage. In this context, all agents opt to stay in the formal sector, actively seeking better-paying formal jobs while still employed. The color gradient shows the wage searched, to understand how the agent's characteristics change their searching behavior.

Having more wealth and higher ability increases the wage the agent aims for. Greater wealth alleviates the risk associated with aiming for a higher wage and possibly not finding the desired job; increased ability, on the other hand, leads to higher output, making the agent more valuable to firms. *Ceteris paribus*, an agent with either higher wealth or higher ability has a better chance of landing the job he desires. Although it is not shown, the same pattern applies every time the agent chooses to enter search for a job in the formal sector (when unemployed, when transitioning from informality, and when searching from a lower wage). That is, higher wealth and higher ability result in better outcomes in the formal sector.

Let us now turn our attention to characterizing the relationship between the value function of firms, denoted as J, and labor market tightness, denoted as θ. Figure 3 illustrates the close relationship between J (on the left side) and θ (on the right side, represented as the probability of successfully obtaining a job $p(\theta)$ for ease of interpretation). The figure features two wealth submarkets: the top row corresponds to a firm with a high-wealth employee, while the bottom row corresponds to a firm with a low-wealth employee. Three key insights can be obtained from Figure 3.

First, for a firm to operate in the labor market at wage w, it must be profitable post-entry; and the higher the profit margin, the tighter the labor market becomes. This is evident from the symmetry between firm value and labor market tightness. Consequently, firms find it more beneficial to pay lower wages and hire more skilled employees, which leads to increased labor market tightness.

Second, submarkets that hire high-wealth individuals are tighter. The primary reason for this lies in reduced job turnover: similar to the findings of Chaumont and Shi (2022), the security provided by higher wealth encourages agents to aim for better-paying jobs. As an unintended consequence, these individuals transition out of jobs less frequently than their lower-wealth counterparts. Since firms know that the high rotation of the low-paying vacancies reduces the expected benefit of recruiting, the association of low wealth and worse labor market outcomes is self-reinforcing.

Third, the existence of an informal labor market limits how low the wages can be. A high-wealth agent will not accept a wage that is too low, given that he has the option to enter the informal market where his wealth provides a buffer against uncertain income. This finding contrasts with Chaumont and Shi (2022), where the absence of an alternative income source means that the lowest wages are accepted.

4.2 Distributions of workers and inequality

The economy is simulated using the agents' choices and the equilibrium labor market tightness. In this subsection, we will analyze the steady state distributions of income, wealth, ability, consumption, separated by employment status (formal, informal, and unemployed). Using these endogenous distributions we calculate the inequality of income, wealth and con-

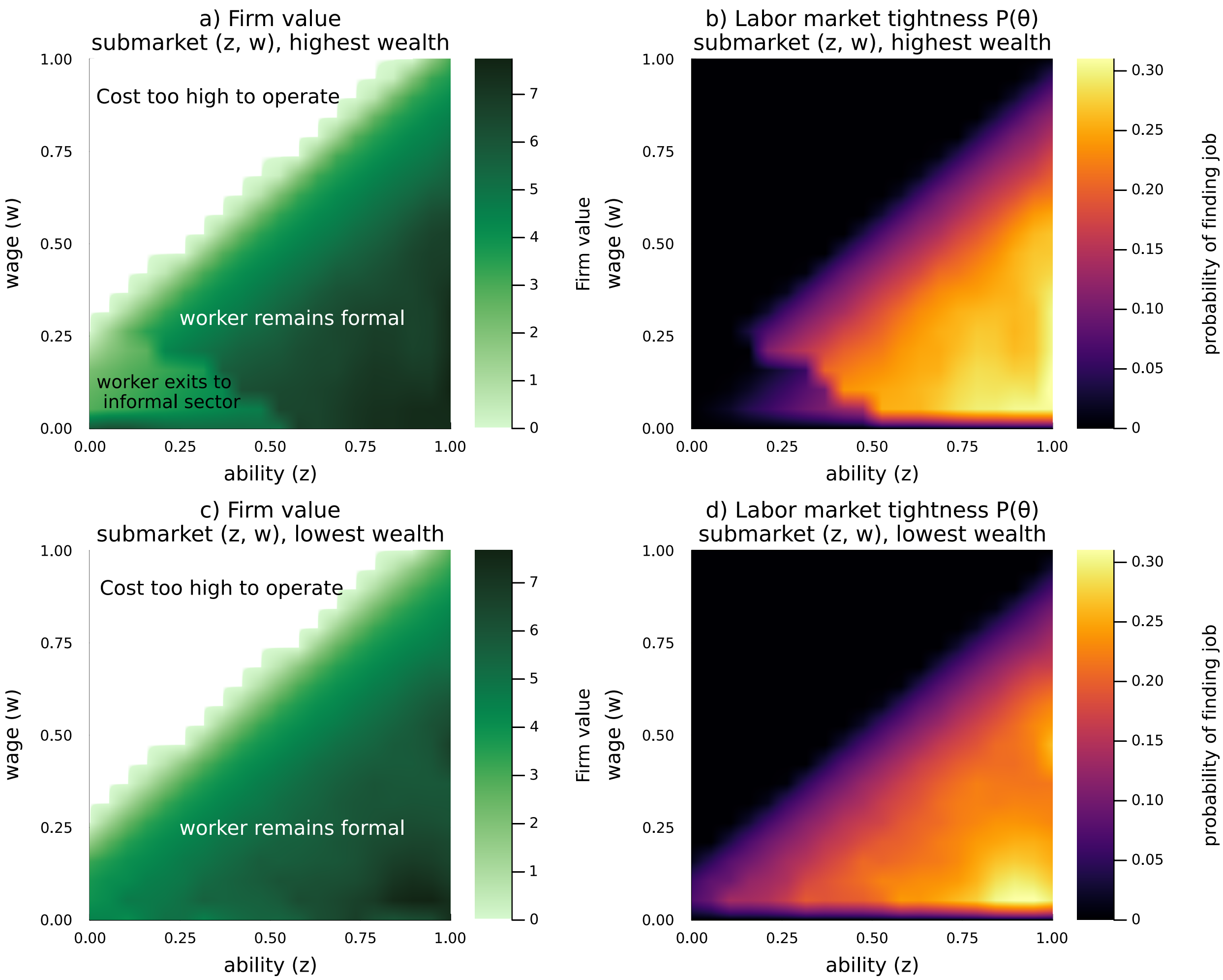


Figure 3: Firm value and formal labor market tightness. The y-axis is how much the firm is paying and the x-axis is the ability requested by the firm a) shows the firm value J for a firm (z,w) with a high wealth employee (a = 20), b) is the equilibrium labor market tightness $p(\theta)$ for the submarket (z, w) for workers with high wealth (a = 20), c) shows the firm value J for a firm (z,w) with a high wealth employee (a = a_min), d) is the equilibrium labor market tightness $p(\theta)$ for the submarket (z, w) for workers with low wealth (a = a_min))

sumption in the economy.

The simulation is done by generating 100,000 agents with their ability drawn from a Beta probability distribution with parameters beta_a and beta_b, and wealth initialized at zero. We then use the policy functions, probabilities, and the equilibrium functions, as outlined in sections 3.1–4.1, to simulate the economy one period ahead. We repeat this for 1,200 periods to obtain the steady state vector of agents, with individual employment status, ability, income, and accumulated wealth, from which any statistic can be calculated.

To facilitate the interpretation of each graph, I compute two types of inequality measures: the Gini Index and the Theil Index. As said in the introduction, the primary advantage of the Gini index is its ease of interpretation; it serves as a standard measure of income inequality, ranging from 0 to 1, where 0 signifies perfect equality and 1 signifies perfect inequality. The primary advantage of the Theil index is its decomposability, meaning that the inequality measure for the entire population can be disaggregated into the (weighted) sum of the inequality measures within the formal and informal sectors, plus the inequality between both sectors. For more information on the calculation of each index, see Appendix C. The Gini index is annotated in the following charts, while the Theil indices are shown in a consolidated bar chart in Figure 8.

The distribution of income for each employment status is plotted in Figure 4. Income in the formal sector is higher on average than in the informal sector. However, the informal sector encompasses a wide range of incomes. There is no reason for the income of a particular informal worker to be lower than the income of a particular formal worker, income is only lower on average.

Working in the informal sector comes with a wide range of potential incomes. Indeed, inequality of income is driven mostly by inequality within the informal sector. This is evidenced by a higher Gini index in the informal sector than in the formal sector. The Theil Index, in Figure 8, supports this conclusion, since the inequality of income is mostly driven by inequality within the informal sector, followed by the inequality between both sectors. Inequality within the formal sector plays a lesser role. An important implication is that a decrease in informality decreases income inequality while increasing average income; this is confirmed in section 5.

The distribution of wealth for each employment status is plotted in Figure 5. The wealth accumulated in the formal sector has a higher dispersion than in the informal sector. On the one hand, the formal worker has a higher amount of wealth saved, since a higher wealth makes

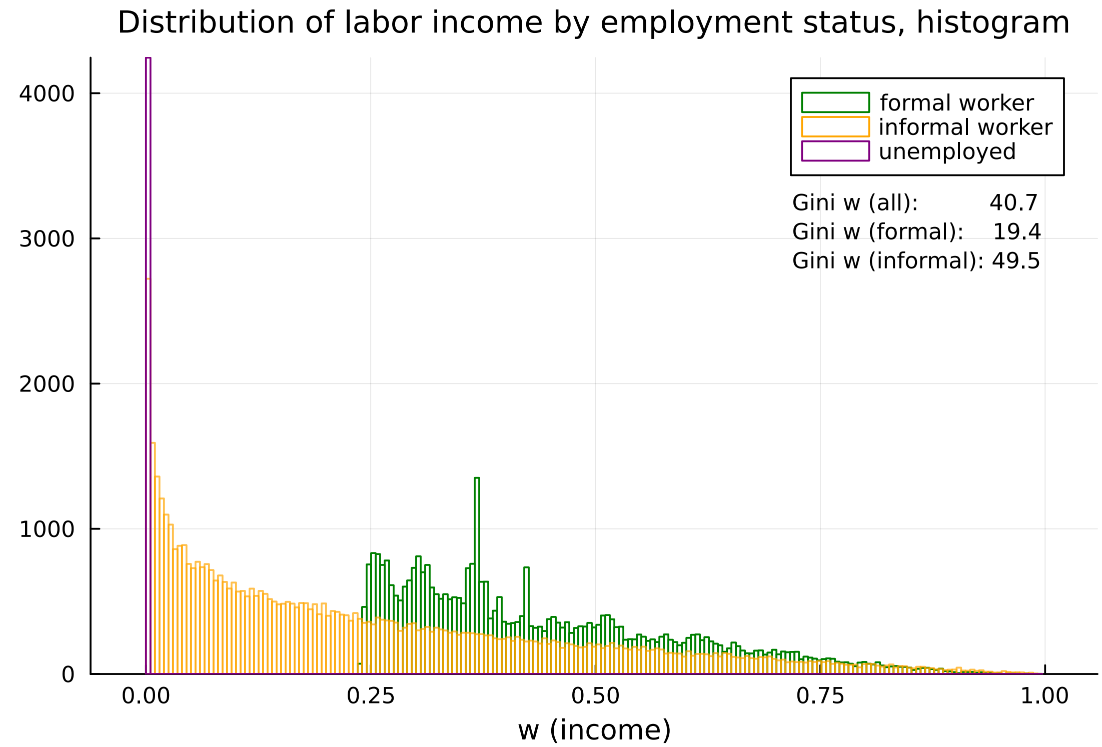


Figure 4: Overlapped histograms of income for each employment status. x-axis is the agent's income (w), y-axis is the histogram count.

Figure 5: Overlapped histograms of wealth for each employment status. The x-axis is the agent's wealth (a), The y-axis is the histogram count.

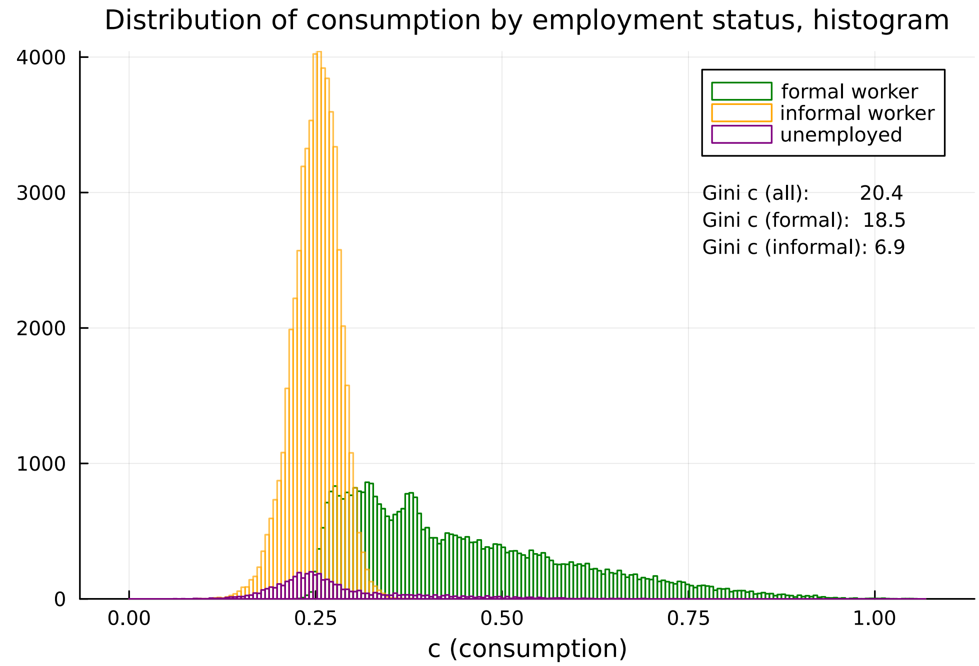


Figure 6: Overlapped histograms of consumption for each employment status. x-axis is the agent's consumption (c), y-axis is the histogram count.

reentering the formal sector easier when facing unemployment, and the lower rotation of high-wealth individuals increases their labor market tightness. Empirically, there is evidence in Kabas and Roszbach (2021), that lower leverage enables workers to search in the labor market for longer, leading to better labor market outcomes.

On the other hand, informal workers cannot accumulate as much wealth since they face higher uncertainty of income. Indeed, the priority of the informal worker is to smooth consumption, which is accomplished by using up savings in bad times and accumulating them in good times. As a result of the inability to accumulate greater amounts of wealth, a vicious circle is created: low wealth makes it difficult to search in the formal sector, which leads to being trapped in the informal sector with low wealth.

The inequality of wealth composition is the opposite of the inequality of income. In Figure 8 inequality is mostly driven by inequality within the formal sector, while inequality within the informal sector and between the formal and informal sector is smaller by comparison. An implication, explored in section 5, is that a policy that reduces informality increases wealth inequality. It is unsatisfactory that total inequality of wealth is smaller than the total inequality of income, the reason is that in this model there is no further incentive to accumulate enormous amounts of wealth, other than self-insurance and easier job search.

Figure 7: Overlapped histograms of ability for each employment status (except unemployed). The x-axis is the agent's ability (z), the y-axis is the histogram count.

The distribution of consumption for each employment status is plotted in Figure 6. Consumption is higher for most of the formal sector employees than informal sector employees. Informal workers exhibit a narrower distribution of consumption compared to the other distributions in this section; this results from smoothing consumption against their income shocks. The formal sector, on the contrary, can enjoy greater consumption, which depends on their characteristics.

The total inequality of consumption is the lowest when comparing it to wealth or income inequality, as shown in Figure 8, since consumption can be smoothed against income shocks. The inequality of consumption is mostly driven by inequality between both sectors and inequality within the formal sector. An implication is that reducing informality increases inequality of consumption. This is explored in section 5.

Let's consider the differences in ability of agents who end up in the formal sector as compared to those who end up in the informal sector. As shown in Figure 7, the informal sector primarily comprises individuals with lower ability, a phenomenon which we interpret as voluntary informality, since an agent with lower ability might expect lower wages in the formal sector, making their decision to remain informal as an optimal choice.

Similarly, a worker with high ability has a lower likelihood of being in the informal sector;

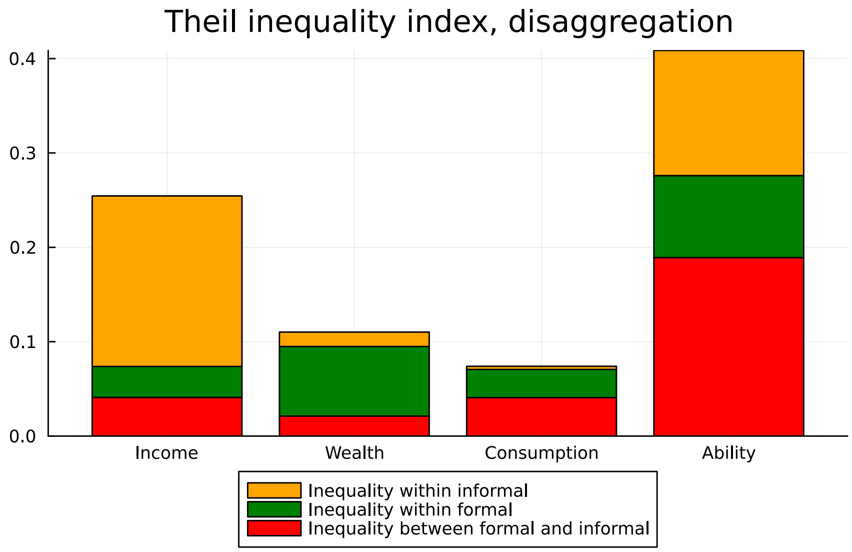


Figure 8: Theil index for the distribution of income (w), wealth (a), consumption (c) and ability (z). Each Theil index is disaggregated by components. Unemployed agents are excluded from all calculations.

however, this probability is never zero. We can think of this as involuntary informality, since two agents with the same ability can be in different sectors because one had a sufficient amount of wealth to risk the formal labor market, and the other didn't.

The nature of informality — whether it is a voluntary or involuntary choice — is a subject of debate. Informality is voluntary when workers choose to be informal because it is more desirable given their characteristics than being formal. Informality is involuntary when workers would prefer to be formal, but there are barriers to formality that prevent them from doing so. In the baseline model 60.4% of the informal workers will never choose to be formal since they have a low level of ability, which means that only the lowest paying formal jobs are within their reach. The other 39.6% of the informal workers are trapped because, when they were unemployed, their wealth was too low and the risk of failing in the formal labor market was too high; they are only waiting for an opportunity to transition to formality. For comparison, Alcaraz et al. (2015) estimates that in Mexico, between 73% and 88% of informal workers are voluntary based on their attributes, including their educational level. This interpretation is further supported by figure 8, which illustrates that the majority of the inequality in ability lies between the informal and formal sectors.

5 Welfare and distributional effects of tackling informality

Economists have proposed various policies to tackle informality. However, due to the limitations of previous models, little can be said about the impact of such policies on inequality. With the understanding of the mechanisms of this model, we can assess the impact of the most common policy recommendations on welfare, informality and inequality.

We classify policies into four types: fiscal incentives, punitive measures, reducing barriers to entry of formal firms, and increasing social security benefits. For each type of policy, the details of its implementation are important for its effectiveness in reducing informality and on its impact on inequality.

The impact of each policy on the economy is determined by simulation. This is done by starting with the baseline economy described in section 4 and following the baseline 100,000 agents in the economy as they adjust to the new policy over a 10-year period. The main results are presented in 4 plots and 2 tables for each policy, which give a comprehensive picture of its effect on the economy. Although the effect of the policy can be described in words, the graphs act as supporting evidence.

In order to streamline the discussion, I will briefly describe the content of the Figures 9–14. For each policy, the four supporting graphs show the following: first, the percentages of formal, informal and unemployed agents before and after the policy; second, the formal labor market tightness (conditional on wage searched) before and after the policy; third, the evolution of mean income, mean wealth, and mean consumption — for the whole population, and conditional on the evolution of their labor status — expressed as a percentage change; fourth, the evolution in inequality of income, wealth and consumption — including its components, inequality within sectors and inequality between sectors — changes in the Theil index and its subindexes before and after the policy. Then two tables are shown: the first table shows the Equivalent Consumption of each policy, which I will describe in a moment; the second table shows the changes in average duration of unemployment in months, the percentage change in tax revenue (net of unemployment insurance payments), and the percentage of informal workers that are involuntary (i.e. those that are above the threshold of ability to enter the formal sector). The second table is obtained using bootstrapping to get standard errors (shown in parenthesis).

To measure the policy's impact on welfare appropriately, the Equivalent Consumption is calculated. The Equivalent Consumption is the additional consumption (in percentage) all the agents in the baseline economy would have to get to obtain the same aggregate utility as in the economy with the implemented policy. That is, if $\{c^0_{1:t}\}_{i=1}^{N\_ind}$ are the consumption paths of the agents in the baseline economy $\{c^1_{1:t}\}_{i=1}^{N\_ind}$ are the consumption paths of the agents in the economy with the policy. Then the Equivalent Consumption $EC \in \mathbb{R}$ is such that the following equation holds:

$$\sum_{i=1}^{N\_ind}\sum_{t=0}^{\infty}\beta^t u(c^0_{it}(1+EC)) = \sum_{i=1}^{N\_ind}\sum_{t=0}^{\infty}\beta^t u(c^1_{it})$$

By tracking the full evolution of consumption under the policy, the Equivalent Consumption includes all transition costs from baseline to the new steady state, thus, it is the appropriate measure to evaluate the welfare implications of the policies. By partitioning agents by their baseline employment status, we can calculate the Equivalent Consumption of formal workers and informal workers. With these measures we know how a policy will affect differently the welfare of (current) formal and informal workers.

To investigate in more detail to whom the benefits of the policies are being passed on, we do one last estimation. First, calculate the individual Equivalent Consumptions: for agent i his individual Equivalent Consumption $EC_i \in \mathbb{R}$ is such that the following equation holds:

$$\sum_{t=0}^{\infty}\beta^t u(c^0_{it}(1+EC_i)) = \sum_{t=0}^{\infty}\beta^t u(c^1_{it})$$

A linear regression is then estimated for the individual Equivalent Consumption, with individual characteristics in the baseline as explanatory variables:

$$EC_i = \alpha_0 + \alpha_z z^0_{i0} + \alpha_w w^0_{i0} + \alpha_a a^0_{i0}$$

The interpretation of the coefficients tells us how the policies affect different parts of the distribution of ability, income and wealth. A negative α_z means that the policy favors those with less ability; a negative α_w means that the policy favors those with less labor income; a negative α_a means that the policy favors the less wealthy. The results of these regressions are in Table 2

Finally, to summarize the effect of the policies on inequality, Figure 15 collects all the policy effect (changes in Theil indices) of Figures 9-14.

5.1 Fiscal Incentives

Consider a reduction in the payroll tax τ_w. In the baseline model, this tax was set at 19.6%, comparable to Mexico's tax wedge, calculated by the OECD, which includes income tax, employers' social security contributions, and employees' social security contributions. What would happen if the costs of hiring formal employees were reduced? What if, for example, social security were financed by general taxes instead of being a specific tax on labor income? To answer this question, the first fiscal incentive experiment is a reduction of the payroll tax to 9.3%, which reduces the percentage of informality by 5 percentage points. For comparison, the mexican tax wedge that includes only the income tax is 7.94% according to the same OECD report. The results are shown in Figure 9.

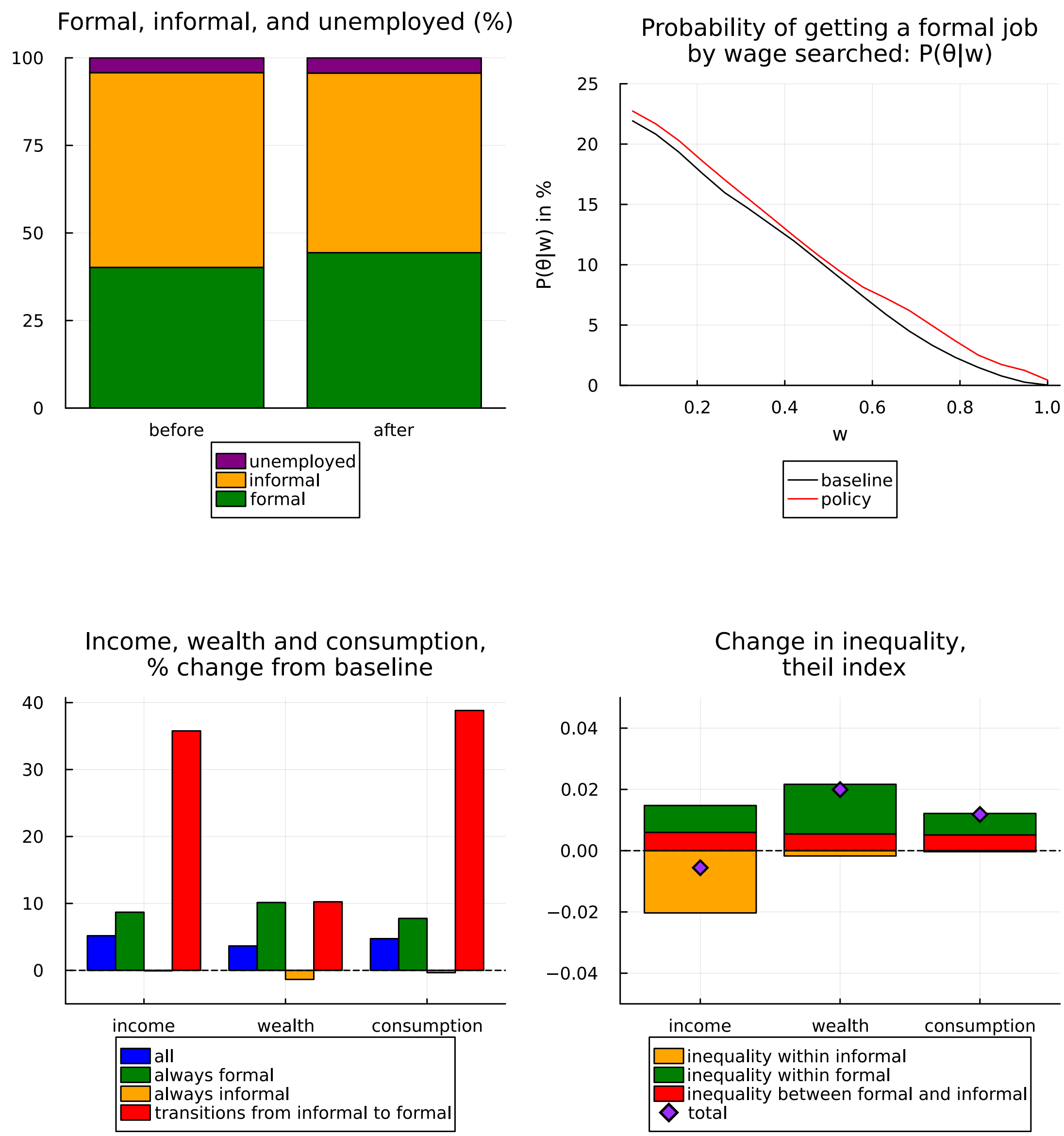
The reduction in labor taxes makes firms more profitable at all wage levels, with a stronger effect for the firms that pay the highest wages. Higher profitability leads to more firms entering the formal sector, which increases the probability of getting formal work, especially high-paying jobs. As it becomes more attractive to enter the formal sector, workers who didn't have sufficient ability and wealth before the policy, now decide to enter, which increases formality. The greater attractiveness of formal work reduces mainly voluntary informality; however, this policy has a heavy cost, since there is a dramatic decline in tax collection.

As a result of the policy, income inequality decreases slightly, even as income increases across the board. This directly proportional relationship —lower informality leading to lower income inequality— appears across all policies. Greater wealth accumulation within the formal sector leads to greater wealth inequality. Consumption increases, especially for those workers that were able to make the transition from informal to formal, and consumption inequality increases.

Next, consider a reduction in the profit tax τ. In the baseline model, this tax was set at 30% as in Mexico. What would happen if the profit tax were slashed? for comparison with the previous policy, the profit tax τ was set to 22.58%, which reduces informality by 5 percentage points as well. As a reference in the United States the corporate profit tax is 21%. Figure 10 shows the results.

The reduction in profit tax increases profits, however, this benefits more those firms with higher profit margins, which are those that pay the lowest wages. Indeed, in this model the firms that pay the highest wages have razor-thin margins and see less benefit from the policy incentive. As a result, there is more entry by low-paying high-margin firms, which increases the probability of finding a formal job at the lower end of the wage spectrum.

Fiscal Incentive: Reduce payroll tax τ_w



Welfare: Equivalent Consumptions

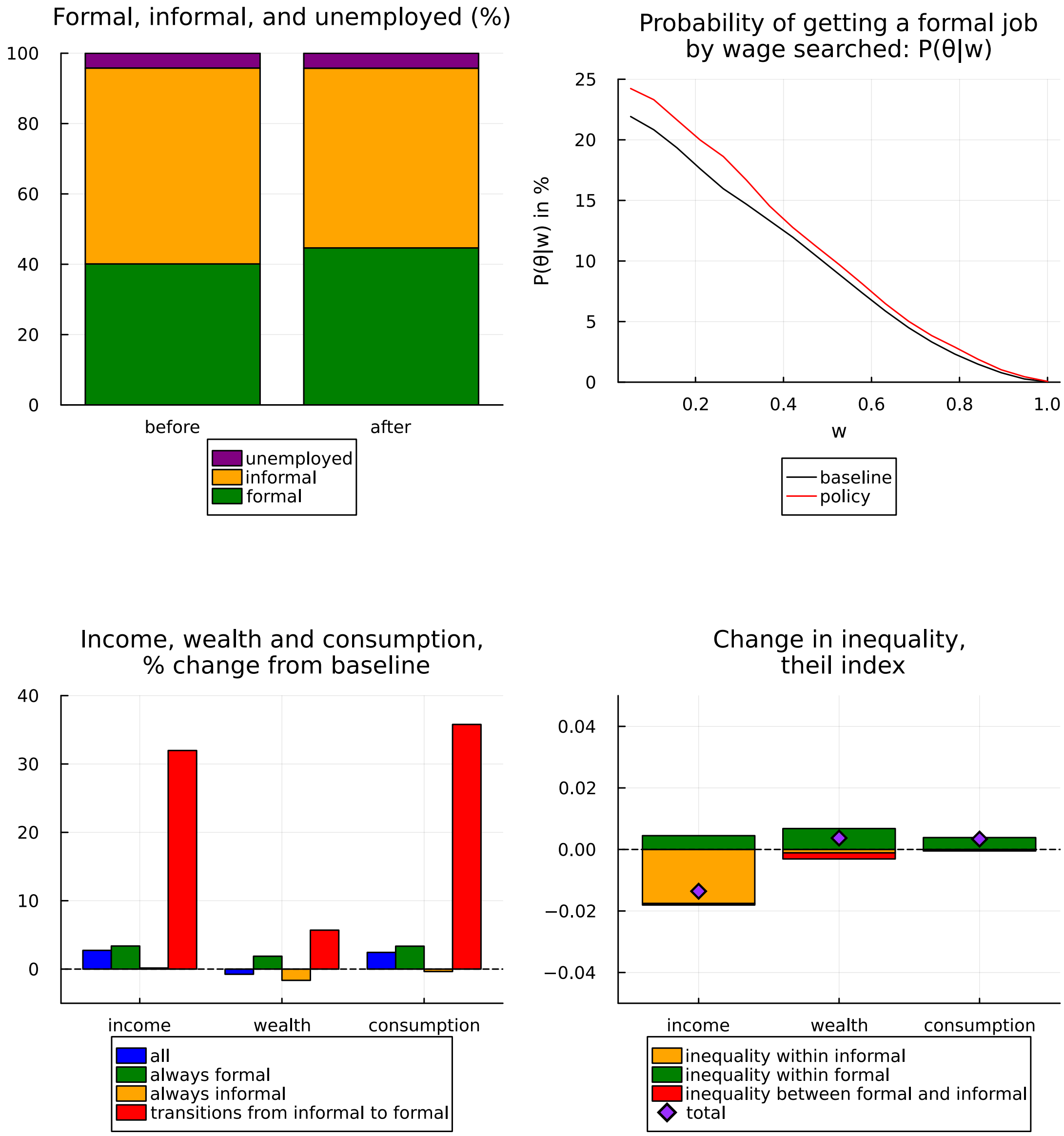
EC All	2.1%
EC Formal (in t=0)	4.1%
EC Informal (in t=0)	1.1%

Additional statistics

Change in unemployment duration (# months)	0.26 (0.07)
Change in tax revenue (net, %change)	-27.32% (0.22)
Involuntary informality (as % of informal workers)	51.49% (0.11)

Figure 9

Fiscal Incentive: Reduce profit tax τ



Welfare: Equivalent Consumptions

EC All	1.7%
EC Formal (in t=0)	2.3%
EC Informal (in t=0)	0.8%

Additional statistics

Change in unemployment duration (# months)	0.09 (0.06)
Change in tax revenue (net, %change)	-4.81% (0.27)
Involuntary informality (as % of informal workers)	47.87% (0.11)

Figure 10

Since informality offers an option against the lowest wages, an increase in the probability of low-paying jobs has a dampened effect, thus this policy benefits mostly those that are already in the formal sector.

As a result of reducing profit tax, income inequality decreases with the decrease of informal workers, wealth inequality increases with the increase in formal workers, and consumption inequality increases, driven by an increase in consumption inequality within the formal sector.

Comparing the two fiscal policies, it is clear that their details matter. The reduction in the payroll tax reduced informality by increasing the possibility of earning higher wages, thereby increasing the benefits of formality. The profit tax cut increases the probability of finding a low-paid formal job, which means that those that are already formal don't get as much benefit. The benefits of both fiscal policies accrue mostly to those already in the formal sector. The regressions in Table 2 show that both fiscal policies benefit agents with less wealth and less income, but higher ability.

Policies interact with informality in unexpected ways. Without informality a reduction of the profit tax, which increases the number of low paying jobs, the benefits would be even greater for the low-skilled and unemployed workers. Without informality a decrease in the payroll tax, which favors the highest paying jobs, would increase income inequality.

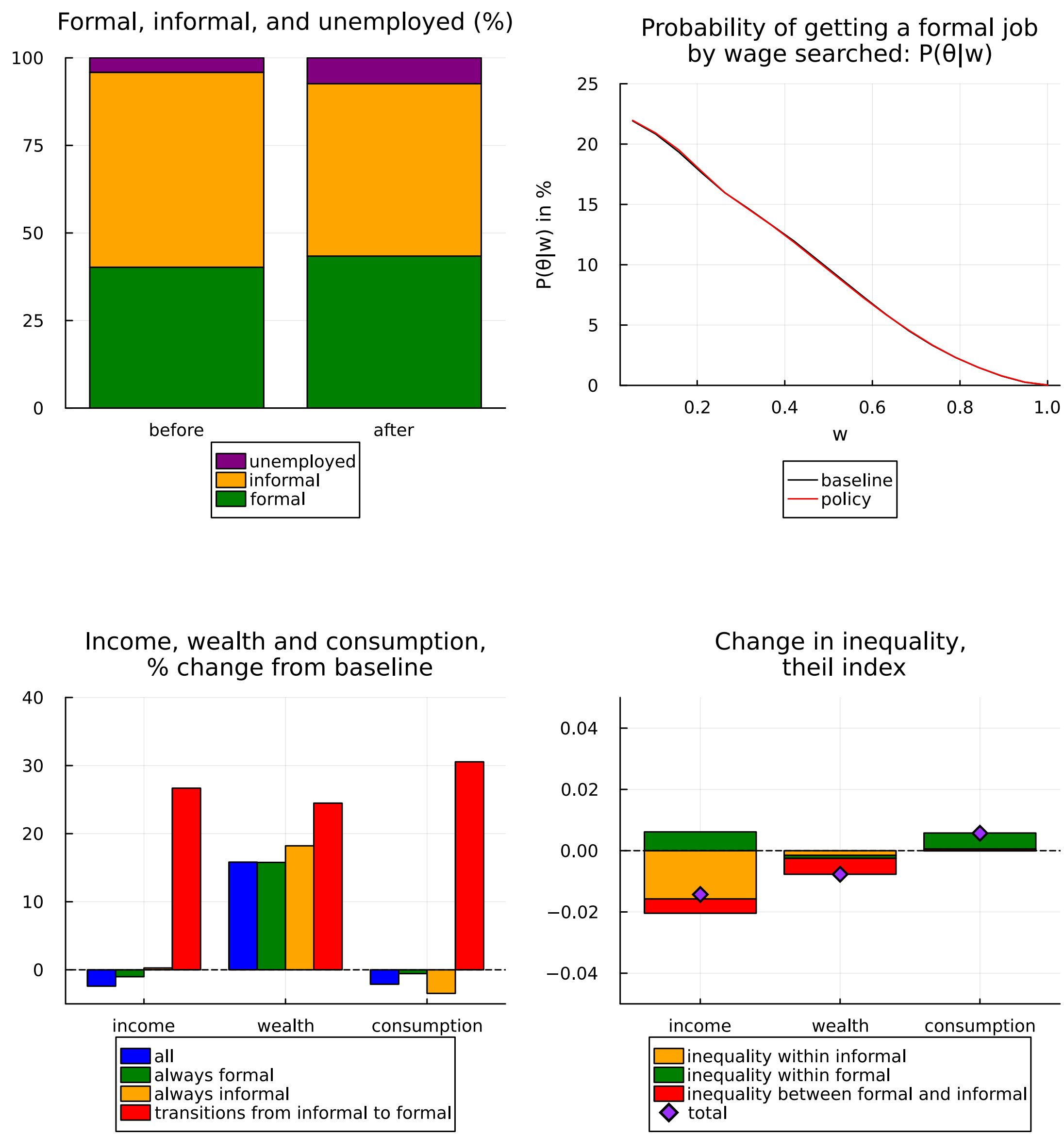
5.2 Punitive Measures

Informality acts upon the worker's decisions by offering an alternative to low-paying jobs. Consider now a policy that increases inspections and punishment of informal work, thus, reducing the probability of entering informality P_{inf}. In the baseline model, the probability of entering informality is 88.9% in each period; effectively, anyone that wants to enter informality will probably enter within a month. Let us consider a reduction of the probability of finding an informal job to 43.1%, which also results in a reduction of the percentage of informality of 5 percentage points. The results are shown Figure 11.

The increased difficulty of finding an informal job, without a corresponding increase in formal jobs, forces those that remain informal to face longer periods of unemployment, and to increase their savings accordingly. Many prefer to take a formal job at any wage, which reduces the population income.

As a result of this policy, informality decreases at the cost of heavy welfare losses for informal workers. There is a sharp increase in unemployment and unemployment duration. Tax revenue increases due to the increase in formal employment.

Punitive: Decrease probability of entering informality P_{inf}



Welfare: Equivalent Consumptions

EC All	-5.9%
EC Formal (in t=0)	-3.9%
EC Informal (in t=0)	-6.8%

Additional statistics

Change in unemployment duration (# months)	1.09 (0.05)
Change in tax revenue (net, %change)	8.21% (0.33)
Involuntary informality (as % of informal workers)	44.99% (0.16)

Figure 11

Income inequality decreases as a result of less informality, even if there is also an increase in inequality within the formal sector due to more low-paying formal sector jobs. Wealth inequality decreases, as everyone is forced to increase their wealth levels, since the safety net provided by the informal sector is reduced. Consumption drops significantly for those agents that were informal before the policy, and consumption inequality increases.

It is clear that punishing the informal sector is effective in reducing informality, but places a heavy burden on those who are currently informal. The Equivalent Consumption is negative, with most of the burden being placed on the informal sector. The regressions in Table 2 show that the negative consequences of this policy falls mainly on the low-skilled and low-paid.

It is surprising that formal workers are also affected by the destruction of informal jobs. The reason is that if they were to face a series of negative shocks, such as facing a long unemployment spell, there is no informal sector that can act as a safety net. This forces formal workers to hold even more precautionary savings to be able to search in the formal sector, which affects their consumption paths.

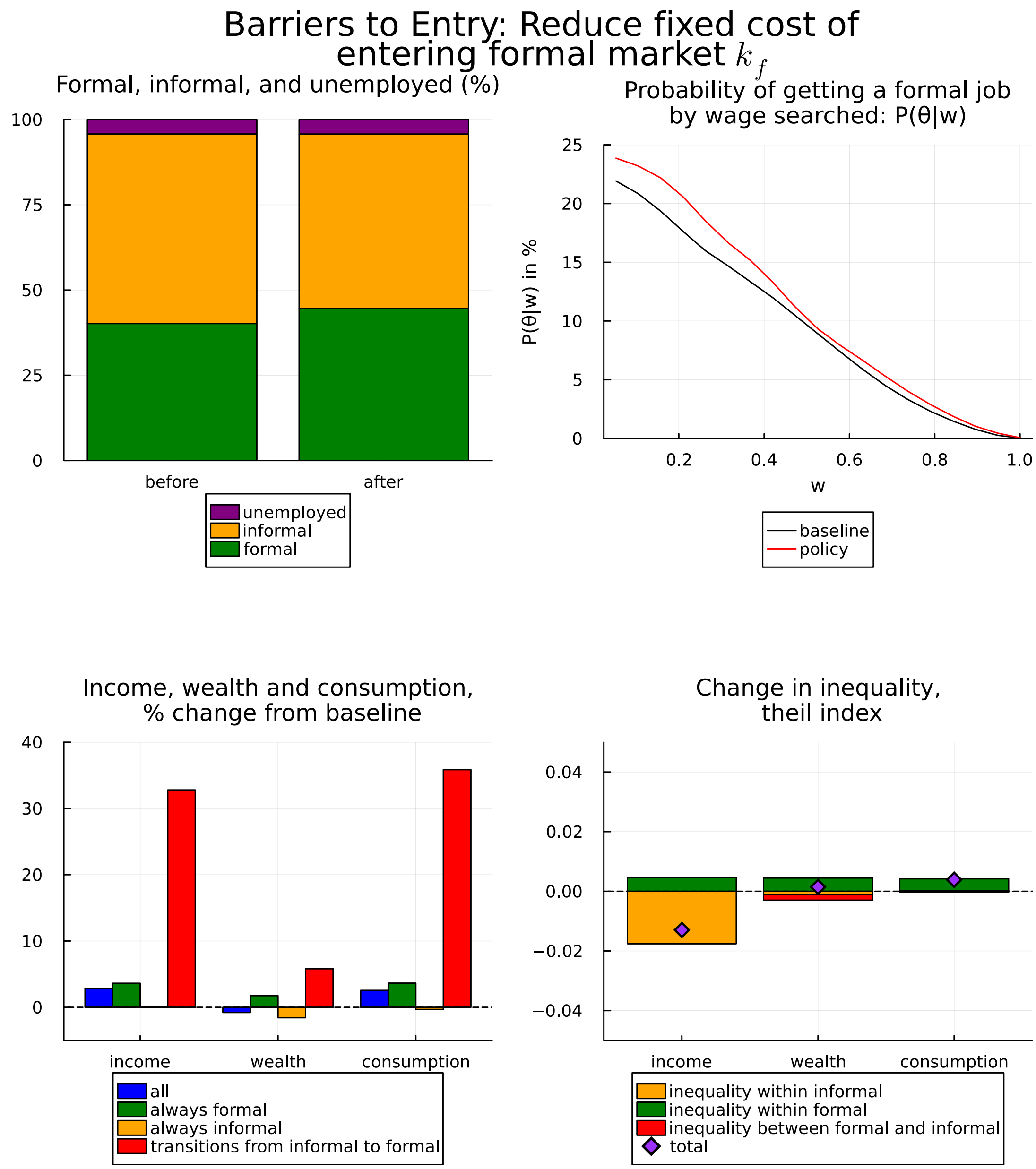
When designing sanctions against informality, policy makers need to consider the full implementation of their strategy: Will there be measures to ensure that workers expelled from informality find a living wage in the formal sector? The increased tax revenue derived from these policies could potentially be used to offset the welfare losses they incur.

5.3 Reducing Barriers to Entry

Barriers to entry impose a large deadweight loss on society by deterring firms from even attempting to enter the formal market. What would happen if the barriers to entry were eliminated?

In this model I defined the barrier to entry to the formal sector as the price that is paid to post a vacancy k_f. Consider a reduction of k_f from 2.86 in the baseline model to 2.529, which reduced informality by 5 percentage points. The results are shown in Figure 12.

As a result of this policy, more firms enter the formal labor market at every wage level, which increases the probability of finding a formal job, with greater effect on the lower-wage jobs (since $q(\theta)$ is decreasing and convex, this policy benefits more the tightest labor markets). As it becomes easier to enter the formal sector, workers who now have sufficient ability and wealth decide to enter, reducing informality. Tax revenue increases with the increase in formality.



Welfare: Equivalent Consumptions

EC All	1.8%
EC Formal (in t=0)	2.5%
EC Informal (in t=0)	0.9%

Additional statistics

Change in unemployment duration (# months)	0.09 (0.07)
Change in tax revenue (net, %change)	6.86% (0.32)
Involuntary informality (as % of informal workers)	47.54% (0.13)

Figure 12

Average income increases and income inequality decreases as a result of less informality and more availability of better-paying formal jobs. Wealth inequality increases slightly, as the highest levels of wealth become easier to reach for formal workers. Consumption and consumption inequality increases.

Removing barriers to entry is an effective way to increase formality, tax revenue, reduce income inequality, and at the same time to benefit all of the members of the economy. Reducing general barriers to entry benefits the labor markets that are already tight more, allowing more people to move from informality to entry-level formal jobs. It also allows for higher wages and more job transitions. Reducing barriers to entry benefits mostly those already in the formal sector. The regressions in Table 2 show the policy benefits those workers with less wealth and less income, but higher ability.

5.4 Increasing Social Security

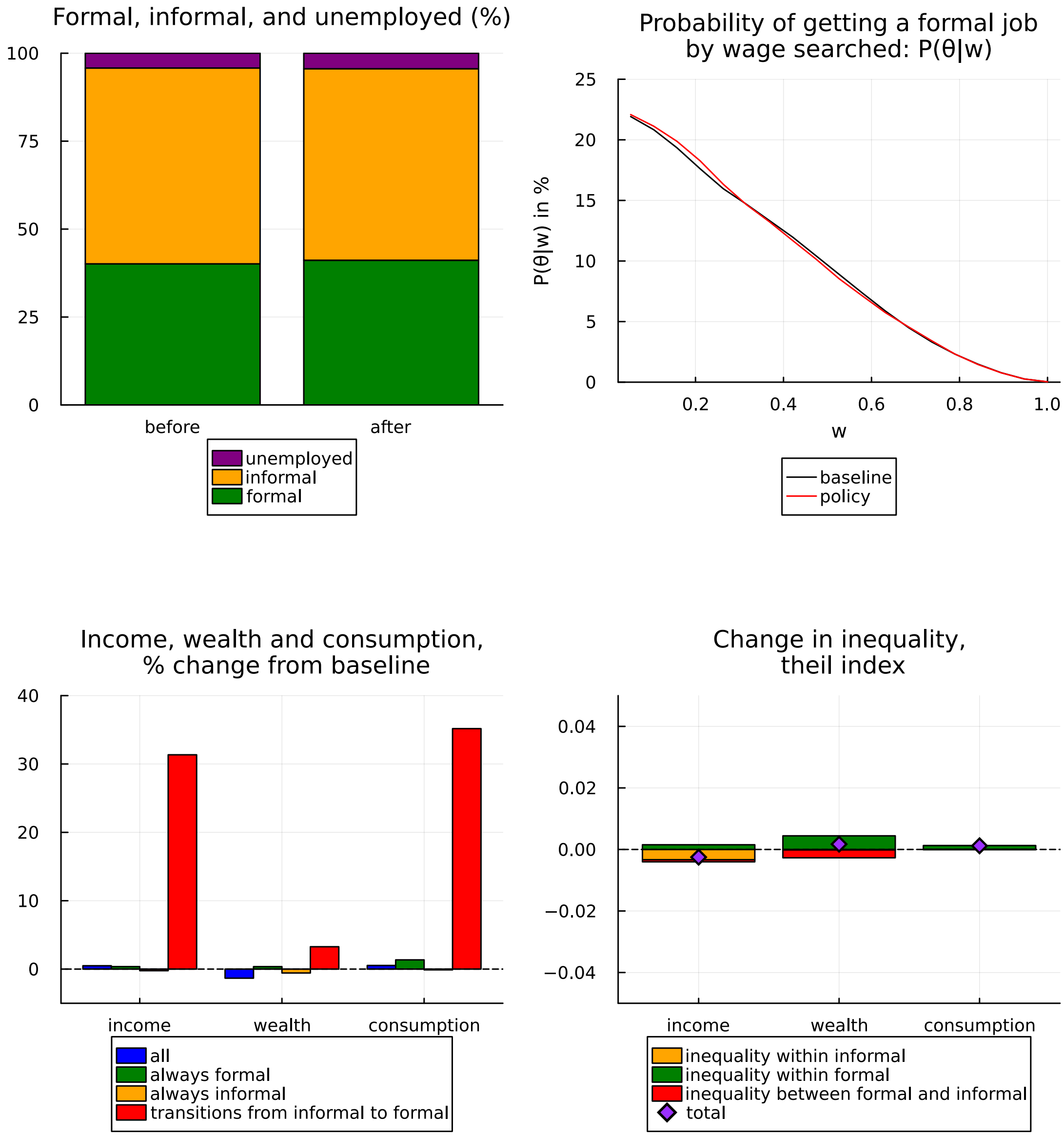
This section considers two policies that increase social security: implementing unemployment insurance for formal workers and implementing universal basic income. As noted in section 5.1 changing fiscal incentives has significant effects on informality, thus, the full effects of redesigning social security depend on who pays for these. However, in order to get clearer information on the policy effect itself, in this section it is assumed that the government pays for all additional costs without changing its fiscal policies.

Consider an implementation of Unemployment Insurance (UI). This is represented in this model by changing replacement rate of income for those laid-off from formal work (ρ, see Equation 3) from 0% in the baseline model (no UI) to 59% of the last wage, which is the same as the average net replacement rate in unemployment of the OECD as of 2022 (Single person without children, average wage). The average duration of this unemployment insurance is set to be three months ($\chi = 0.50$, see Equation 3). This results in a decrease in informality of 1 percentage point. Figure 13 shows the results.

Unemployment Insurance provides those laid off from formality with an income that allows them to continue looking for work in the formal sector, this reduces the cost of informality of formal workers, which leads to more wealth and consumption. UI increases the desirability of formality; thus, the implementation of UI increases formality.

The main beneficiaries of this policy are those that are already formal workers, however, those that transition to formality end up getting higher income and higher consumption on average. The duration of unemployment increases marginally.

Social Security: Unemployment Insurance



Welfare: Equivalent Consumptions

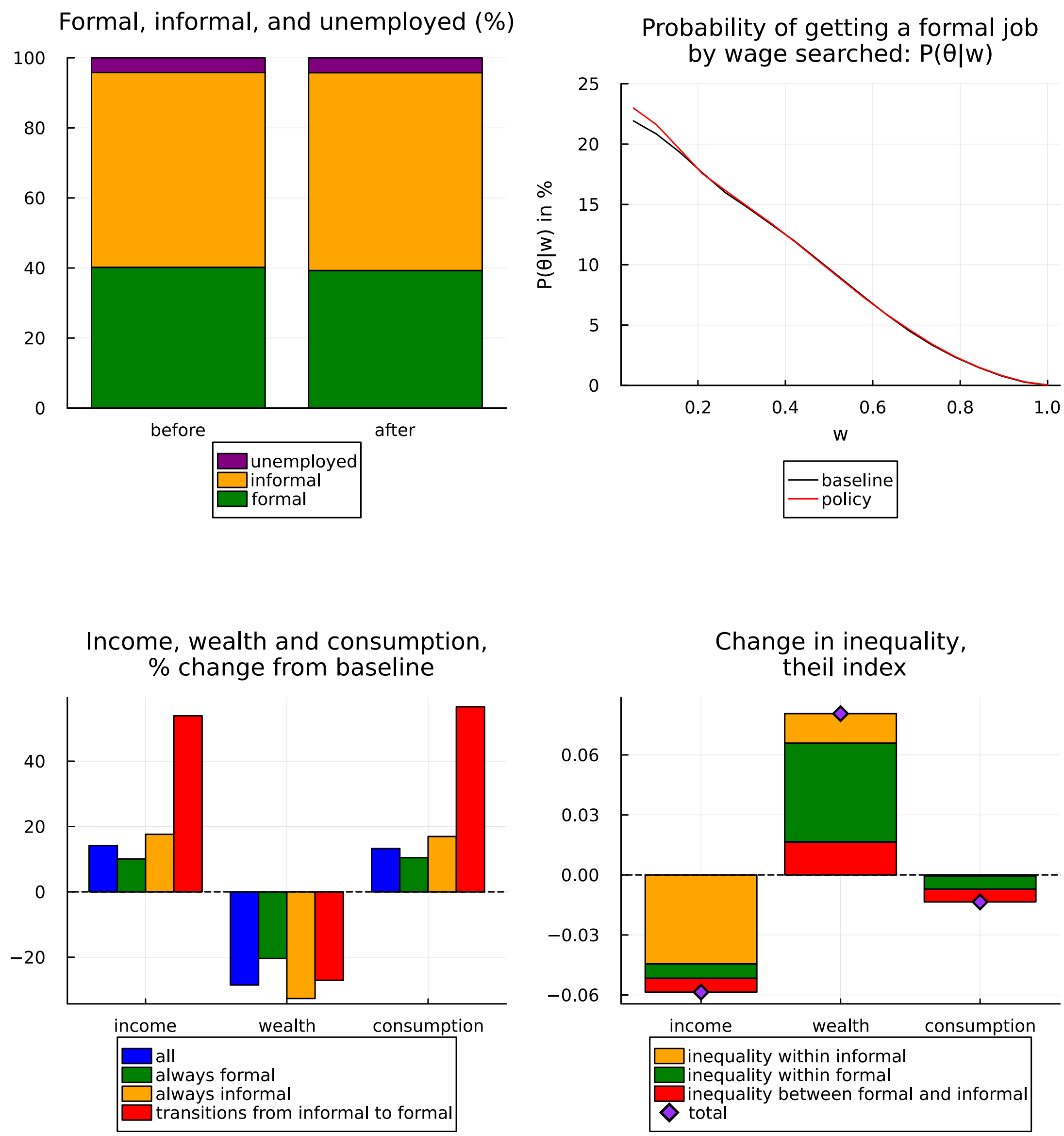
EC All	1.0%
EC Formal (in t=0)	1.1%
EC Informal (in t=0)	0.1%

Additional statistics

Change in unemployment duration (# months)	0.26 (0.07)
Change in tax revenue (net, %change)	-1.84% (0.41)
Involuntary informality (as % of informal workers)	41.08% (0.13)

Figure 13

Social Security: Universal Basic Income I_u



Welfare: Equivalent Consumptions

EC All	19.5%
EC Formal (in t=0)	15.5%
EC Informal (in t=0)	20.7%

Additional statistics

Change in unemployment duration (# months)	0.04 (0.06)
Change in tax revenue (net, %change)	-93.62% (0.28)
Involuntary informality (as % of informal workers)	41.23% (0.34)

Figure 14

The policy reduces income inequality, increases wealth inequality and increases consumption inequality. An important assumption behind these results is that agents in this model do not have the option of exiting the labor market, delay the search for a formal job, or work informally while keeping their UI income.

Now consider an implementation of Universal Basic Income (UBI) in this economy. This is done by increasing the amount of unconditional cash transfers I_u from 0.01 to 0.055, which is the maximum the government can offer such that all tax revenue is spent on UBI. This represents 13.8% of the median earnings for formal workers and 30.0% of the median earnings for informal workers. The effect on informality is an increase of 1 percentage point. Figure 14 shows the results.

This policy has large effects on inequality. Since UBI counts towards income, income increases for everyone, especially for those who can now transition to formal work; income inequality decreases as a result. Wealth holdings fall for everyone, since there is less need to self-insure against negative shocks; as a result wealth inequality rises. Consumption increases for everyone, especially, again, for those that transition to formal work; as a result consumption inequality matters.

It is worthy to note that of all policies considered, UBI is the only one that benefits more those with less ability (Table 2) and those that are already informal. Indeed, the main mechanism through which UBI acts is by shielding against the very worst outcomes (zero income and zero wealth). However, UBI is an expensive policy, and coupling it with a change in fiscal policy can be even more detrimental to informality.

Both UI and UBI are policies designed to benefit the least well-off in society. However, their effectiveness on informality is limited. UI increases the attractiveness of being formal, but since it doesn't affect the tightness of the labor market, the ultimate benefits go to those already in the formal sector. UBI makes the search for work in the formal sector less risky, but also reduces the income uncertainty in the informal sector, with the end result being an increase in informality; in addition, UBI is expensive to implement. The regressions in Table 2 confirm that both UI and UBI policies benefit those workers with less wealth, and less income. However, UBI has the distinction of being the only policy of all considered that benefits more those with the least ability and informal workers.

Figure 15 collects all the policy changes in inequality. When tackling informality, income inequality is reduced, since income inequality is driven mostly by inequality within the informal sector. Similarly, when tackling informality, wealth inequality increases, since wealth

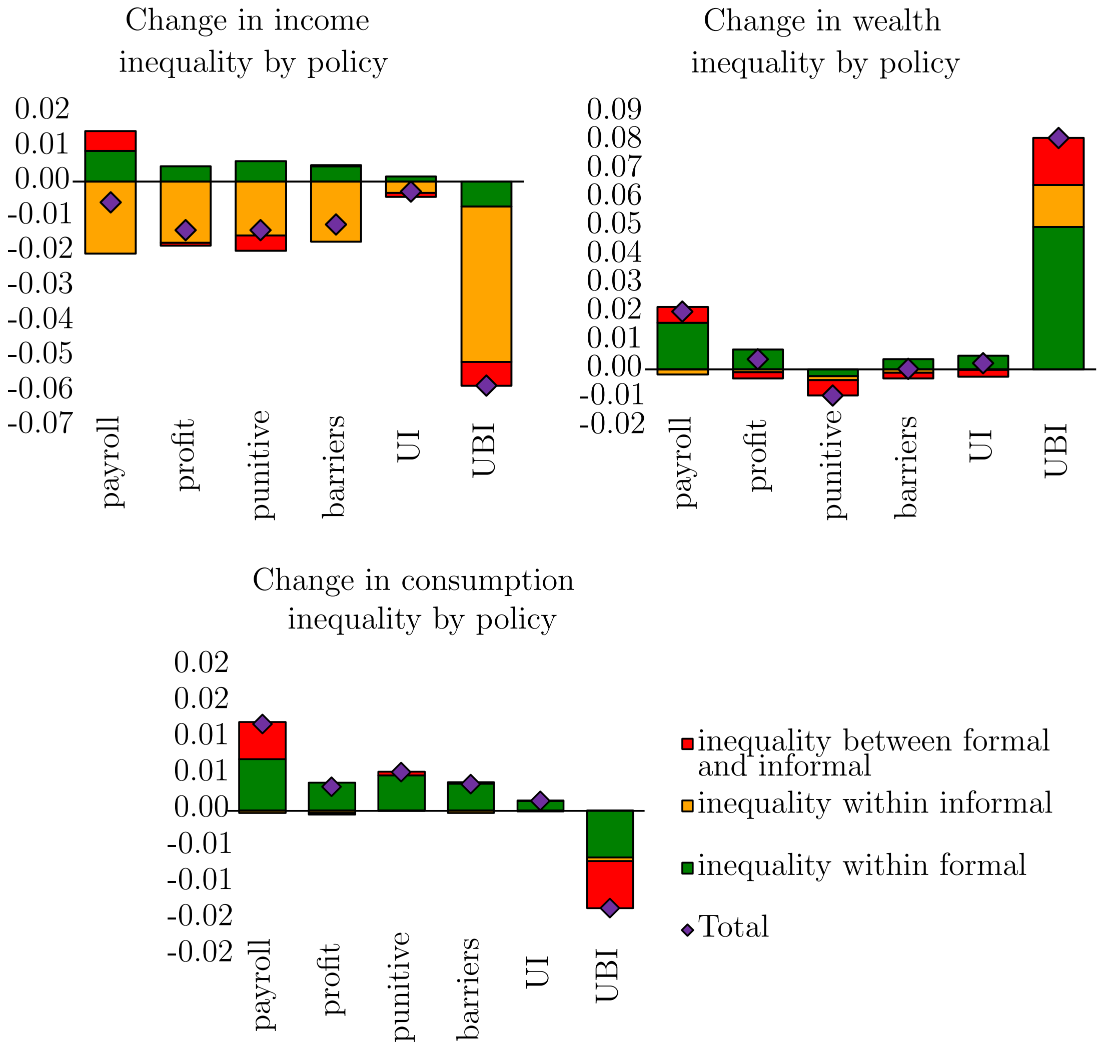


Figure 15: All the policy effects on inequality of income, wealth and consumption. These are the changes in the Theil index presented in Figures 9-14

inequality is driven mostly by inequality within the formal sector, the only exception is the punitive policy, since it forces informal workers to save a lot to face long spells of unemployment. Again, when tackling informality consumption inequality increases, consumption inequality is driven mostly by inequality between sectors and within the formal sector; the only exception is implementing UBI, since it increases more the consumption of those less well-off.

	Individual EC					
	Fiscal Incentives		Punitive Measure	Barriers to Entry	Social Security	
	τ_w	τ	P_{inf}	k_f	UI	UBI
(Intercept)	0.474***	0.660***	-7.108***	0.671***	-0.038	22.424***
	(0.088)	(0.084)	(0.085)	(0.084)	(0.081)	(0.089)
z^0	15.267***	9.947***	9.327***	10.652***	6.049***	-7.682***
	(0.280)	(0.265)	(0.265)	(0.267)	(0.254)	(0.281)
w^0	-0.497*	-0.626**	1.715***	-0.635**	0.398	-2.276***
	(0.250)	(0.238)	(0.240)	(0.239)	(0.229)	(0.253)
a^0	-0.308***	-0.303***	0.065**	-0.340***	-0.114***	-0.591***
	(0.023)	(0.022)	(0.022)	(0.022)	(0.021)	(0.023)
Estimator	OLS	OLS	OLS	OLS	OLS	OLS
N	100,000	100,000	100,000	100,000	100,000	100,000
R^2	0.043	0.016	0.027	0.017	0.008	0.046

Table 2: Regressions of individual Equivalent Consumption given individual characteristics before the change in policy. UI stands for Unemployment Insurance, UBI stands for Universal Basic Income

6 Conclusion

What is the impact of tackling informality on inequality? What are the mechanisms? Who bears the costs and who reaps the benefits of tackling informality? These questions are at the heart of this paper.

The model is characterized by a formal sector with matching and direct wage search, and an informal sector, that is easier to get into, but that has uncertain earnings. Workers are differentiated by their levels of ability, income, and wealth. They choose which sector to work in, which wage to pursue, and how much to save and consume.

Either a low level of wealth or a low level of ability can trap workers in the informal sector. Conversely, higher wealth and higher ability increase the probability of getting a highly paid formal job. The informal sector is preferred to working the lowest-paying formal jobs. However, there remains a wide overlap of formal and informal labor income, including at the top of the distribution.

The model predicts that, as a result, income inequality is greater within the informal sector, wealth inequality is greater within the formal sector, and consumption inequality is greater between sectors and within the formal sector. Reducing informality leads to lower income inequality, since inequality within the informal sector accounts for the largest share of income inequality. Reducing informality increases wealth inequality, since inequality within the formal sector is the largest share of income inequality. Reducing informality increases consumption inequality.

Formal workers often reap the majority of benefits from policies that tackle informality, since these policies typically function by either increasing the attractiveness of formality or reducing the appeal of informality. This happens even when policies disproportionately benefit those with lower wealth and income.

Fiscal incentives place their burden on the government by reducing the amount of taxes collected, the benefits accrue to workers that are already formal and those with higher ability. Since informality is an option to escape the lowest-paying jobs means that policies that increase the availability of low-paying jobs are less beneficial in the presence of informality.

Punishing informality places the burden on informal workers, with negative spillover effects on formal workers as this additional safety net disappears; the government reaps the benefits as the tax base increases. It is an effective policy at decreasing informality, but has large welfare losses.

Lowering formal sector entry barriers benefits all, particularly high-ability individuals. The only cost is the implementation of the policy itself. By massively increasing the availability of all formal jobs this policy is effective at reducing informality with large welfare gains.

Increasing social security places the burden on whoever ends up paying for it. Implementing unemployment insurance benefits formal workers. An increase of unconditional payments, such as Universal Basic Income, is the only policy that benefits more the informal workers and those with the lowest ability.

Increasing formality has other benefits for the economy that are not included in this model, such as allowing for greater investment, greater diffusion of knowledge, and thus greater productivity growth. The results could be further strengthened by explicitly modeling the incentives of informal firms. A key assumption in this model is that higher profits lead to greater firm entry, a premise that may not hold as strongly when policies are implemented in reality. Future research can be done on the interaction of informality and inequality in the presence of market power.

If we are to challenge the pervasiveness of informality in developing economies, we need to know the full impact of the policies we propose. This model allows us to evaluate our proposals more carefully than ever before, showing the mechanisms and assumptions under which they operate. This paper shows that there are great gains in designing policy appropriately — and that we can tackle both informality and inequality.

7 References

[1] Alcaraz, C., Chiquiar, D., & Salcedo, A. (2015). Informality and segmentation in the Mexican labor market (No. 2015-25). Working Papers.

[2] Antón, A., Trillo, F. H., & Levy, S. (2012). The end of informality in México?: fiscal reform for universal social insurance (Vol. 1300). Washington, DC: Inter-American Development Bank.

[3] Bezanson, J., Edelman, A., Karpinski, S., & Shah, V. B. (2017). Julia: A fresh approach to numerical computing. SIAM review, 59(1), 65-98.

[4] Binelli, C. (2016). Wage inequality and informality: evidence from Mexico. IZA Journal of Labor & Development, 5, 1-18.

[5] Burdett, K., & Coles, M. (2003). Equilibrium wage-tenure contracts. Econometrica,

71(5), 1377-1404.

[6] Burdett, K., & Mortensen, D. T. (1998). Wage differentials, employer size, and unemployment. International Economic Review, 257-273.

[7] Cerna, L. (2013). The nature of policy change and implementation: A review of different theoretical approaches. Organisation for Economic Cooperation and Development (OECD) report, 492-502.

[8] Chancel, L., Piketty, T., Saez, E., & Zucman, G. (Eds.). (2022). World inequality report 2022. Harvard University Press.

[9] Chaumont, G., & Shi, S. (2022). Wealth accumulation, on-the-job search and inequality. Journal of Monetary Economics, 128, 51-71.

[10] Chong, A., & Gradstein, M. (2007). Inequality and informality. Journal of public Economics, 91(1-2), 159-179.

[11] Dell'Anno, R. (2018). Inequality, informality, and credit market imperfections. Macroeconomic Dynamics, 22(5), 1184-1206.

[12] Docquier, F., & Iftikhar, Z. (2019). Brain drain, informality and inequality: A search-and-matching model for sub-Saharan Africa. Journal of International Economics, 120, 109-125.

[13] Elgin, C., Elveren, A. Y., & Bourgeois, J. (2021). Informality, inequality and profit rate. Applied Economics Letters, 28(12), 1017-1020.

[14] Fernández-Villaverde, J., & Valencia, D. Z. (2018). A practical guide to parallelization in economics (No. w24561). National Bureau of Economic Research.

[15] Franjo, L., Pouokam, N., & Turino, F. (2022). Financial frictions and firm informality: a general equilibrium perspective. The Economic Journal, 132(645), 1790-1823.

[16] Granda, C., & Hamann, F. (2018). Informality, Saving and Wealth Inequality in Colombia (No. 6815). Inter-American Development Bank.

[17] Gutiérrez-Romero, R. (2021). Inequality, persistence of the informal economy, and club convergence. World Development, 139, 105211.

[18] Kabas, G., & Roszbach, K. (2021). Household Leverage and Labor Market Outcomes Evidence from a Macroprudential Mortgage Restriction. Available at SSRN 3835232.

[19] Leal-Ordoñez, J. C. (2014). Tax collection, the informal sector, and productivity. Review of Economic Dynamics, 17(2), 262-286.

[20] Levy Algazi, S. (2018). Esfuerzos mal recompensados: la elusiva búsqueda de la prosperidad en México. Washington, DC: Banco Interamericano de Desarrollo.

[21] Leyva, G., & Urrutia, C. (2020). Informality, labor regulation, and the business cycle. Journal of International Economics, 126, 103340.

[22] Lopez-Martin, B. (2019). Informal sector misallocation. Macroeconomic Dynamics, 23(8), 3065-3098.

[23] Meghir, C., Narita, R., & Robin, J. M. (2015). Wages and informality in developing countries. American Economic Review, 105(4), 1509-46.

[24] Menzio, G., & Shi, S. (2010). Block recursive equilibria for stochastic models of search on the job. Journal of Economic Theory, 145(4), 1453-1494.

[25] Meyer, B. D., & Sullivan, J. X. (2017). Consumption and Income Inequality in the US Since the 1960s (No. w23655). National Bureau of Economic Research.

[26] OECD (2022). Taxing Wages 2022, OECD Publishing, Paris, https://doi.org/10.1787/f7f1e68a-en

[27] Plata-Pérez, L., Sánchez-Pérez, J., & Sánchez-Sánchez, F. (2015). An elementary characterization of the Gini index. Mathematical Social Sciences, 74, 79-83.

[28] Postel-Vinay, F., & Robin, J. M. (2002). The distribution of earnings in an equilibrium search model with state-dependent offers and counteroffers. International Economic Review, 43(4), 989-1016.

[29] Rogerson, R., Shimer, R., & Wright, R. (2005). Search-theoretic models of the labor market: A survey. Journal of economic literature, 43(4), 959-988.

[30] Sargent, T. J., & Ljungqvist, L. (2000). Recursive macroeconomic theory. Massachusetts Institute of Technology.

[31] Shorrocks, A. F. (1980). The class of additively decomposable inequality measures. Econometrica: Journal of the Econometric Society, 613-625.

[32] Stokey, N. L., & Lucas Jr, R. E. (1989). Recursive Methods in Economic Dynamics. Harvard University Press.

[33] Ulyssea, G. (2018). Firms, informality, and development: Theory and evidence from Brazil. American Economic Review, 108(8), 2015-2047.

[34] Ulyssea, G. (2020). Informality: Causes and consequences for development. Annual Review of Economics.

A Proof of existence of equilibrium

This section proves the existence of equilibrium for the model. To do this we use the Schauder Fixed-Point Theorem (taken from Stokey and Lucas, 1989, Theorem 17.4). This proof is inspired by Menzio and Shi (2010).

Theorem A.1 (Schauder Fixed-Point Theorem) *Let X be a bounded subset of $\mathbb{R}^n$, and let $C(X)$ be the space of bounded continuous functions on X, with the sup norm. Let $F \subset C(X)$ be nonempty, closed, bounded, and convex. If the mapping $T : F \to F$ is continuous and the family $T(F)$ is equicontinuous, then T has a fixed point in F*

Let $Z = [0, \bar{Z}]$, $W = [0, \bar{w}]$, $A = [\underline{a}, \bar{a}]$. Then $X = Z \times W \times A$ is a bounded subset of R^3. Let $C(X)$ be the space of bounded continuous functions on X, with the sup norm.

Let $\pi(z, w) = (1-\tau)(y+z-(1+\tau_w)w)$ be the net profit of the firm that hires ability z and pays wage w. All parameters are fixed and known. To make the proof easier I partition X in X^+ and X^- defined as $(z, w, a) \in X^+ \iff \pi(z, w) \geq 0, \qquad (z, w, a) \in X^- \iff \pi(z, w) < 0.$

Define $\mathcal{J} \subset C(X)$ as the subset of bounded lipschitz continuous functions such that the following three conditions are met:

1. $\forall (z, w, a) \in X^+, \quad J(z, w, a) \geq \pi(z, w)$
2. $\forall (z, w, a) \in X^+, \quad J(z, w, a) \leq \frac{\pi(z,w)}{1-\beta(1-\delta_f)}$
3. $\forall (z, w, a) \in X^-, \quad J(z, w, a) = 0$

In words, this means that $\mathcal{J}$ is the set of all lipschitz continuous functions in the domain such that the functions are greater than or equal to the value of a firm that operates just a single period, and less than or equal to the value of a firm that operates forever (minus the exogenous separation rate). If the firm generates negative profit the total value is 0.

Lemma A.2 *The set of functions $\mathcal{J}$ is nonempty, bounded, closed, and convex*

Proof

Nonempty: Trivially, the function defined by $J(z,w,a) = \pi(z,w)$ if $\pi(z,w) \geq 0$ and $J(z,w,a) = 0$ if $\pi(z,w) < 0$ exists, is bounded and lipschitz continuous in the domain, and fulfills conditions 1, 2, and 3 to be in the set $\mathcal{J}$

Bounded: Let J be an element of $\mathcal{J}$. Then $\forall (z,w,a) \in X^+, \quad 0 \leq \pi(z,w) \leq J(z,w,a) \leq \frac{\pi(z,w)}{1-\beta(1-\delta_f)} \leq \pi(\bar{z},0)$. In addition $\forall (z,w,a) \in X^-, \quad J(z,w,a) = 0$ Thus $\mathcal{J}$ is bounded.

Closed: Suppose that $\{J_n\}_{n=1}^{\infty}$, where $\forall n \; J_n \in \mathcal{J}$, converges uniformly to function J. I will prove that $J \in \mathcal{J}$.

Suppose that J does not fulfill condition 1, then $\exists (z,w,a) \in X^+$ such that $J(z,w,a) < \pi(z,w) \Rightarrow 0 < \pi(z,w) - J(z,w,a) = c$. Since $\forall n, J_n \in \mathcal{J}$ it follows that $\forall n, J(z,w,a) < \pi(z,w) \leq J_n(z,w,a) \Rightarrow \forall n, 0 < c \leq J_n(z,w,a) - J(z,w,a) \Rightarrow \forall n, \|J_n - J\| \geq c$ which contradicts that $\{J_n\}_{n=1}^{\infty}$ converges to J.

Similarly, now suppose that J does not fulfill condition 2, then $\exists (z,w,a) \in X^+$ such that $J(z,w,a) > \frac{\pi(z,w)}{1-\beta(1-\delta_f)} \Rightarrow 0 > \frac{\pi(z,w)}{1-\beta(1-\delta_f)} - J(z,w,a) = -c$. Since $\forall n, J_n \in \mathcal{J}$ it follows that $\forall n, J_n(z,w,a) \leq \frac{\pi(z,w)}{1-\beta(1-\delta_f)} < J(z,w,a) \Rightarrow \forall n, J_n(z,w,a) - J(z,w,a) \leq -c < 0 \Rightarrow \forall n, \|J_n - J\| \geq c$ which contradicts that $\{J_n\}_{n=1}^{\infty}$ converges to J.

Now suppose that J does not fulfill condition 3, then $\exists (z,w,a) \in X^-$ such that $J(z,w,a) = c \neq 0$. Since $\forall n, J_n \in \mathcal{J}$ it follows that $\forall n, J(z,w,a) - J_n(z,w,a) = c \neq 0 \Rightarrow \forall n, \|J - J_n\| \geq |c| > 0$, which contradicts that $\{J_n\}_{n=1}^{\infty}$ converges to J.

In addition, since $\{J_n\}_{n=1}^{\infty}$ converges uniformly to J and all $J_n \in \mathcal{J}$ are lipschitz continuous and bounded, then J is lipschitz continuous and bounded. Thus $J \in \mathcal{J}$, and the set is closed.

Convex: Let $J_1, J_2 \in \mathcal{J}$ and $\alpha \in [0,1]$. Now, let $J = \alpha J_1 + (1-\alpha) J_2$. I will prove that $J \in \mathcal{J}$. Since $J_1, J_2 \in \mathcal{J}$ are bounded and lipschitz continuous functions, their convex combination is bounded and lipschitz continuous. Take $(z,w,a) \in X^+$ then $J(z,w,a) = \alpha J_1(z,w,a) + (1-\alpha) J_2(z,w,a) \geq \alpha \pi(z,w) + (1-\alpha)\pi(z,w) = \pi(z,w)$. Thus J fulfills condition 1. Similarly, take $(z,w,a) \in X^+$, then $J(z,w,a) = \alpha J_1(z,w,a) + (1-\alpha) J_2(z,w,a) \leq \alpha \frac{\pi(z,w)}{1-\beta(1-\delta_f)} + (1-\alpha) \frac{\pi(z,w)}{1-\beta(1-\delta_f)} = \frac{\pi(z,w)}{1-\beta(1-\delta_f)}$ Thus J fulfills condition 2. Lastly, take $(z,w,a) \in X^-$ then $J(z,w,a) = \alpha J_1(z,w,a) + (1-\alpha) J_2(z,w,a) = \alpha 0 + (1-\alpha) 0 = 0$. Thus J fulfills condition 3. Thus $J \in \mathcal{J}$ and the set is convex.

In conclusion, $\mathcal{J}$ as defined is a nonempty, bounded, closed and convex set of lipschitz continuous functions. ■

To continue, let's provide some further results. Given any function J we can apply Equation

5 to obtain an updated value of θ, which I name θ_J. Given any θ_J we can use (1), (2), and (3) to compute V_f, V_i, V_u and their respective policy functions. We know these policy and value functions exists since for any θ the problem satisfies Blackwell's sufficient conditions for being a contraction (Stokey and Lucas, 1989, Theorem 3.3).

Define T as the function $T : \mathcal{J} \to C(X)$ which maps each J to a TJ defined as follows:

$$TJ(z,w,a) = \begin{cases} \pi(z,w) + \beta(1-\delta_f)(1-\lambda_f p(\theta(z,\hat{w},\hat{a})))J(z,w,\hat{a}), & \text{if } (z,w,a) \in X^+ \\ 0, & \text{if } (z,w,a) \in X^- \end{cases} \tag{7}$$

Where $\hat{w}$ and $\hat{a}$ are the policy functions that solve the agent's problem (Equations (1), (2), and (3) given $\theta(J)$ (Equation 5). This is a constrained model, since the formal firm does not internalize the possibility of a formal employee searching an informal job.

Lemma A.3 *T maps the set $\mathcal{J}$ into itself*

Proof
TJ satisfies condition 1 if and only if $\forall (z,w,a) \in X^+$,
$\beta(1-\delta_f)(1-\lambda_f p(\theta_J(z,\hat{w},\hat{a})))J(z,w,\hat{a}) \geq 0$. Since all the terms of the product are always greater than or equal to 0, the expression is true. Hence, TJ satisfies condition 1.

TJ satisfies condition 2 if and only if $\forall (z,w,a) \in X^+$,

$$\begin{aligned} \pi(z,w) + \beta(1-\delta_f)(1-\lambda_f p(\theta_J(z,\hat{w},\hat{a})))J(z,w,\hat{a}) &\leq \frac{\pi(z,w)}{1-\beta(1-\delta_f)} \iff \\ \beta(1-\delta_f)(1-\lambda_f p(\theta_J(z,\hat{w},\hat{a})))J(z,w,\hat{a}) &\leq \frac{\beta(1-\delta_f)\pi(z,w)}{1-\beta(1-\delta_f)} \iff \\ (1-\lambda_f p(\theta_J(z,\hat{w},\hat{a})))J(z,w,\hat{a}) &\leq \frac{\pi(z,w)}{1-\beta(1-\delta_f)} \iff \\ J(z,w,\hat{a}) &\leq \frac{\pi(z,w)}{1-\beta(1-\delta_f)}. \end{aligned}$$

Which is true, since $J \in \mathcal{J}$ and it satisfies condition 2. Therefore TJ satisfies condition 2.

TJ satisfies condition 3 if and only if $\forall (z,w,a) \in X^-$, $TJ(z,w,a) = 0$ which is true by definition of TJ. Thus, TJ satisfies condition 3.

Since $\pi(z,w), J(z,w,a)$ and $p(\theta_J)$ are all lipschitz continuous and bounded functions, it follows that TJ is a lipschitz continuous and bounded function. In conclusion, $TJ \in \mathcal{J}$ which means that $T : \mathcal{J} \to \mathcal{J}$ ■

Lemma A.4 *T is a lipschitz continuous mapping*

Proof

Let $J_n, J_r \in J$, then $\|TJ_n - TJ_r\| = \beta(1-\delta_f)\|(1-\lambda_f p(\theta_{Jn}))J_n - (1-\lambda_f p(\theta_{Jr}))J_r\|$.

Take $(z, w, a) \in X$, and without loss in generality assume $J_n(z, w, \hat{a}_{J_n}) \geq J_r(z, w, \hat{a}_{J_r})$, from Equation 5 it follows that $\theta_{Jn}(z, w, \hat{a}_{J_n}) \geq \theta_{Jr}(z, w, \hat{a}_{J_r})$. Since θ is a continuous function, increasing in a, decreasing in w, and the agent is risk averse, it follows that this means that the policy functions $\hat{w}_J(z, w, a)$ and $\hat{a}_J(z, w, a)$ of the identical agent facing a tighter labor market cannot be such that it risks a lower probability of entry for a higher wage, thus $\theta_{Jn}(z, \hat{w}_{Jn}, \hat{a}_{Jn}) \geq \theta_{Jr}(z, \hat{w}_{Jr}, \hat{a}_{Jr}) \iff p(\theta_{Jn}(z, \hat{w}_{Jn}, \hat{a}_{Jn})) \geq p(\theta_{Jr}(z, \hat{w}_{Jr}, \hat{a}_{Jr})) \iff p_{Jn}(z, w, a) > p_{Jr}(z, w, a)$

To simplify the notation I will ignore the dependency on (z, w, a) of p_{Jn}, p_{Jr}, J_n, and J_r. I want to prove that $|(1-\lambda_f p_{Jn})J_n - (1-\lambda_f p_{Jr})J_r| < J_n - J_r$

Suppose that the inside of the norm is positive, then $(1-\lambda_f p_{Jn})J_n - (1-\lambda_f p_{Jr})J_r < J_n - J_r \iff -p_{Jn}J_n + p_{Jr}J_r \leq 0 \iff p_{Jr}J_r \leq p_{Jn}J_n$, which is true since $p_{Jr} \leq p_{Jn}$ and $J_r \leq J_n$

Suppose that the inside of the norm is negative, then $-(1-\lambda_f p_{Jn})J_n + (1-\lambda_f p_{Jr})J_r \geq 0 \iff (1-\lambda_f p_{Jr})J_r \geq (1-\lambda_f p_{Jn})J_n \iff J_r - J_n \geq \lambda_f p_{Jr}J_r - \lambda_f p_{Jn}J_n \iff J_n - J_r \leq \lambda_f p_{Jn}J_n - \lambda_f p_{Jr}J_r$. We use this to get the following inequality $-(1-\lambda_f p_{Jn})J_n + (1-\lambda_f p_{Jr})J_r \leq J_n - J_r \leq \lambda_f p_{Jn}J_n - \lambda_f p_{Jr}J_r \iff -J_n + J_r \leq 0 \iff J_r \leq J_n$ which is true.

Thus, it is proved that for any $(z, w, a) \in X$ $|(1-\lambda_f p_{Jn})J_n - (1-\lambda_f p_{Jr})J_r| < |J_n - J_r| \Rightarrow \|(1-\lambda_f p(\theta_{Jn}))J_n - (1-\lambda_f p(\theta_{Jr}))J_r\| \leq \|J_n - J_r\|$.

So, $\|TJ_n - TJ_r\| = \beta(1-\delta_f)\|(1-\lambda_f p(\theta_{Jn}))J_n - (1-\lambda_f p(\theta_{Jr}))J_r\| \leq \beta(1-\delta_f)\|J_n - J_r\|$. Thus T is a Lipschitz continuous mapping ■.

Lemma A.5 *The family of functions $T(\mathcal{J})$ is equicontinuous. That is, for every $\epsilon > 0$, $\exists \delta > 0$ such that for every $(z_1, w_1, a_1), (z_2, w_2, a_2) \in X$ such that $\|(z_2, w_2, a_2) - (z_1, w_1, a_1)\| < \delta \Rightarrow \|TJ(z_2, w_2, a_2) - TJ(z_1, w_1, a_1)\| < \epsilon \quad \forall TJ \in T(\mathcal{J})$*

Proof

Let $\epsilon > 0$. Select $\delta_z = \frac{\epsilon}{3(1+\tau)}$. Select $\delta_w = \frac{\epsilon}{3(1+\tau)(1+\tau_w)}$. Select δ_J such that $\delta_J = \frac{\epsilon}{3L\beta(1-\delta_f)}$

where L is the supremum of the set of Lipschitz constants of all $J \in \mathcal{J}$, we know L is bounded since if it were not, then the set $\mathcal{J}$ wouldn't be bounded. Now, select $\delta = min\{\delta_z, \delta_w, \delta_J\}$.

Let $(z_1, w_1, a_1), (z_2, w_2, a_2) \in X$ such that $\|(z_2, w_2, a_2) - (z_1, w_1, a_1)\| < \delta \Rightarrow \|TJ(z_2, w_2, a_2) - TJ(z_1, w_1, a_1)\| = \|\pi(z_2, w_2) - \pi(z_1, w_1)\| + \beta(1 - \delta_f)\|(1 - \lambda_f p(\theta_J(z_2, \hat{w}_2, \hat{a}_2)))J(z_2, w_2, \hat{a}_2) - (1 - \lambda_f p(\theta_J(z_1, \hat{w}_1, \hat{a}_1)))J(z_1, w_1, \hat{a}_1)\| \leq (1 - \tau)|z_2 - z_1| + (1 - \tau)(1 + \tau_w)|w_2 - w_1| + \beta(1 - \delta_f)\|J(z_2, w_2, \hat{a}_2) - J(z_1, w_1, \hat{a}_1)\|$ The last expression comes from a similar argument than in the proof of the last lemma, since $J(z_2, w_2, \hat{a}_2) \leq J(z_1, w_1, \hat{a}_1) \iff p(\theta_J(z_2, \hat{w}_2, \hat{a}_2)) \leq p(\theta_J(z_1, \hat{w}_1, \hat{a}_1))$.

Therefore $\|TJ(z_2, w_2, a_2) - TJ(z_1, w_1, a_1)\| < (1 - \tau)\delta_z + (1 - \tau)(1 + \tau_w)\delta_w + \beta(1 - \delta_f)L\delta_J = (1 - \tau)\frac{\epsilon}{3(1+\tau)} + (1 - \tau)(1 + \tau_w)\frac{\epsilon}{3(1+\tau)(1+\tau_w)} + \beta(1 - \delta_f)L\frac{\epsilon}{L\beta(1-\delta_f)} = \epsilon$ ■

Theorem A.6 (Existence of equilibrium) *There exists J^* such that $J^* = TJ^*$ with associated θ_{J*}, V^*_ϵ R^*_ϵ, $R^*_{\epsilon f}$, $R^*_{\epsilon i}$ that satisfy Equations (1-5)*

Proof

From Lemmas A.2 to A.5 we have that $\mathcal{J}$ and T as defined previously satisfy the conditions of Theorem A.1. Thus T has a fixed point, that is, there exists J^* such that $J^* = TJ^*$. This has an associated θ_{J*} computed using Equation 5. Given θ_{J*}, as discussed previously, the agent's problem is a contraction mapping, thus there exists V^*_ϵ R^*_ϵ, $R^*_{\epsilon f}$, $R^*_{\epsilon i}$ that satisfy Equations (1-5). This is a Block Recursive Equilibrium ■

The mapping T represents the constrained equilibrium, where a formal firm that is already in operation does not incorporate the possibility of its employee switching to the informal sector. For the unconstrained model, if all employees that already chose to be formal, have no incentive to transition back to informality, then V^*_ϵ R^*_ϵ, $R^*_{\epsilon f}$, $R^*_{\epsilon i}$ are also the equilibrium of the unconstrained model, and in the submarkets where the measure of formal agents is positive J^* and θ_{J*} are also an equilibrium of the unconstrained model. For the submarkets where no agent chose to be formal, there is a discontinuity in J^* and θ^* (see Figure 3), but the discontinuity plays no role in equilibrium, since no one chooses to enter those submarkets. Thus, for the calibration and simulations the restricted model was used. The unrestricted model is only used to illustrate the equilibrium J and θ.

B Algorithm and simulation outline

This section describes the algorithm used to estimate the model. The same steps are taken for both the baseline model and for all the policy experiments.

A model is defined with the following 6 objects: *params*, V_f, V_i, V_u, J_f, θ.

The collection of all parameters in the model, *params*, include the limits and sizes of the grids for ability, income and wealth. Apart from the parameters discussed in the main document, ability is set to be in $z \in [0, 1]$, income is set to be $w \in [0, 1]$, wealth is set to be in $a \in [-\bar{a}, 20]$. V_f, V_i, and V_u are the worker's value functions for all labor statuses and states. J_f is the formal firm's value function for all states and θ is the labor market tightness for all states. $V_f, V_i, V_u, J_f, \theta \in \mathbb{R}^{N_z \times N_w \times N_a}$.

When defining the parameters for the first time a guess is used to start the algorithm. Given the values of V_f, V_i, V_u, J_f, θ and the parameters of the model *params*, the algorithm does the following:

1. Given θ solve the worker's optimization problem and compute optimal savings, consumption, and job search decisions. This step is done by iterating on the worker's value function and reward functions, until convergence. This returns new values of V_f, V_i, and V_u and the policy functions implied.

2. Given the worker's policy functions in 1. Compute the formal firm's value function J_f.

3. Given the formal firm's value function J_f, compute the market tightness θ.

4. Repeat until convergence of θ.

Given a model (*params*, V_f, V_i, V_u, J_f, θ), we can get its associated policy functions and simulate the economy with $N = 100,000$ workers and $T = 1,200$ periods (months) to get the steady-state distributions.

To simulate the transition from baseline to policy, first define the series to be tracked at the end of each period t, then compute the steady-state distribution for baseline as described in the previous paragraph, recording 24 periods after steady-state is reached for comparison. Next, replace all model parameters with those associated with the new policy. Finally, simulate the economy 120 periods into the future and track the results.

All code is written in Julia and is available on GitHub at `https://github.com/emirojaseng/tacklinginformality`

C Inequality Measures

Let $x = \{x_1, x_2, \dots, x_n\}$ be a vector where $x_i > 0 \quad \forall x_i \in x$. The formula for the Theil inequality index for the distribution of x is given by

$$T(x) = \frac{1}{n} \sum_{i=1}^{n} \frac{x_i}{\bar{x}} log(\frac{x_i}{\bar{x}})$$

Where $\bar{x}$ is the average of vector x i.e. $\frac{1}{n}\sum_{i=1}^{n} x_i$. The Theil index can be interpreted as a measure of entropy of x, and the main advantage of using the Theil index is that it is additively decomposable, which means that if we partition it into G non-empty subgroups, the Theil index of x can be decomposed into a weighted average of the Theil index of each group in G (within inequality), plus the Theil index of the means of each group (between).

The main example used in this work is separating income inequality of the employed population by employment status: informal or formal. Let the vector of N incomes x be partitioned into a vector x^F of length N^F of formal workers' income, and a vector x^I of length N^I of informal workers' income ($N = N^F + N^I$). Let the vector x^{Between} be the vector of incomes that would result if everyone received the same income as their group's average, i.e. $x^{\text{Between}} = [\underbrace{\bar{x}^F, \bar{x}^F, ..., \bar{x}^F}_{N^F \text{ times}}, \underbrace{\bar{x}^I, \bar{x}^I, ..., \bar{x}^I}_{N^I \text{ times}}]$. Let $w^F = \frac{N^F \bar{x}^F}{N\bar{x}}$ and $w^I = \frac{N^I \bar{x}^I}{N\bar{x}}$ be the share of income taken by the formal and informal workers respectively ($w^F + w^I = 1$). Then the Theil Index is decomposed as follows:

$$T(x) = \underbrace{w^F T(x^F)}_{\text{within formal}} + \underbrace{w^I T(x^I)}_{\text{within informal}} + \underbrace{T(x^{\text{Between}})}_{\text{between formal and informal}}$$

The Theil Index has the following properties (Shorrocks, 1980):

- Continous and symmetric in x. (Order doesn't matter)
- $T(x) \geq 0$ with equality holding if and only if everyone earns the same
- Continuous first and second partial derivatives
- Additive separability.
- If a transfer is made from a richer person to a poorer person inequality decreases.
- Population replication. That is, if r groups, each containing N individuals and having an identical distribution x, are aggregated into a single population of rn individuals, then aggregate inequality is the same as in each of the constituent groups

- Scale Independence. That is, if all values are multiplied by a constant, the inequality index stays the same: $T(\lambda x) = T(x)$
- Transfers between incomes impact inequality depending only on their relative difference. That is, a transfer to A from B impacts inequality proportional to $log(\frac{x_A}{x_B})$.

The Gini Index formula used in this work is a standard one. Given the vector x the Gini index $G(x)$ is given by:

$$G(x) = \frac{1}{n}\left[n + 1 - 2\frac{\sum_{i=1}^{n}(n+1-i)x_i^*}{\sum_{i=1}^{n} x_i^*}\right]$$

Where x^* is the vector x reordered from lowest to highest. The Gini index has the following properties (Plata-Pérez et. al., 2015):

- Symmetry. Indeed, order doesn't matter, since the vector is ordered as part of the formula.
- Scale independence. That is, if all values are multiplied by a constant, the inequality index stays the same: $G(\lambda x) = G(x)$
- Standarization. This is, if there is 1 unit of income, which is shared equally by $n-k$ individuals, and k individuals have nothing, then: $G(x) = \frac{k}{n}$
- Co-monotone separability. This is, if there are two distributions x_A and x_B are such that it ranks individuals in the same order, then $G(\beta x_A + (1-\beta)x_B) = \beta G(x_A) + (1-\beta)G(x_B)$

In this paper, we measured the inequality of vectors w, a, c, and z. In particular, w, c, and z always take strictly positive values for employed workers. The vector of wealth a can take negative values; thus, to calculate the inequality of wealth I substracted the minimum wealth value to all agents before calculation, that is, I used the vector $a - \underline{a}$ to calculate inequality.