$$\boxed{3(a)}$$

$$x_j^{t+1} = x_j^t + \alpha_j^t \frac{2}{n} \sum_{i:\|x_i - x_j^t\| < 1} (x_i - x_j^t) \quad \text{— gradient ascent} \quad (1)$$

$$\text{local mean} \quad \tilde{x}_j^{t+1} = \frac{\sum_{i:\|x_i - x_j^t\| < 1} x_i}{\sum_{i:\|x_i - x_j^t\| < 1} 1}$$

$$\frac{\sum_{i:\|x_i - x_j^t\| < 1} x_i}{\sum_{i:\|x_i - x_j^t\| < 1} 1} = x_j^t + \alpha_j^t \frac{2}{n} \left(\sum_{i:\|x_i - x_j^t\| < 1} x_i - x_j^t \cdot \sum_{i:\|x_i - x_j^t\| < 1} 1 \right)$$

$$\alpha_j^t = \frac{\cancel{\sum_{i:\|x_i - x_j^t\| < 1} x_i - x_j^t \cdot \sum_{i:\|x_i - x_j^t\| < 1} 1}}{\sum_{i:\|x_i - x_j^t\| < 1} 1} \cdot \frac{1}{\cancel{\sum_{i:\|x_i - x_j^t\| < 1} x_i - x_j^t \sum_{i:\|x_i - x_j^t\| < 1} 1}} \cdot \frac{n}{2}$$

$$\alpha_j^t = \frac{n}{2 \cdot \sum_{i:\|x_i - x_j^t\| < 1} 1}$$

When substituting α_j^t with this fraction, the last term of (1) looks like this:

$$\frac{\sum_{i:\|x_i - x_j^t\| < 1} (x_i - x_j^t)}{\sum_{i:\|x_i - x_j^t\| < 1} 1}$$

which is the <u>mean</u> vector of the neighbourhood — a sensible choice