RIGA TECHNICAL UNIVERSITY

FACULTY OF COMPUTER SCIENCE AND INFORMATION TECHNOLOGY

INSTITUTE OF APPLIED COMPUTER SYSTEMS

Introduction to Operations Research
Assignment 3
Simplex Method

Name: Emir

Surname: Oguz

Student Number: 230ADB011

Question

Solve the following problem using the Simplex Method

- $Maximize\ Z = x1 + 2x2$

Subject to:

- $x1 + 3x2 \leq 8$
- $x1 + x2 \leq 4$
- $x1 \geq 0,\ x2 \geq 0$

Answer

To solve this problem using the Simplex Method, we first convert the problem to its standard form:

- Maximize: Z = x1 + 2x2

Subject to:

- x1 + 3x2 + s1 = 8
- x1 + x2 + s2 = 4
- x1 ≥ 0, x2 ≥ 0, s1 ≥ 0, s2 ≥ 0

Where s1 and s2 are slack variables.

Objective Function:

- \- x1 - 2x2 + Z = 0

The initial simplex table is:

	x1	**x2**	**s1**	**s2**	**Z**	**Result**
s1	1	3	1	0	0	8
s2	1	1	0	1	0	4
Z	-1	-2	0	0	1	0

- The column with the most negative number in the bottom row is designated as the pivot column.
  - Pivot Column:

x2
3
1
-2

- One of the positive numbers in the rows above the bottom row in the 'column x2' is selected as the pivot element.

- The number with the smaller division result is chosen as the pivot element.
  - 8/3 < 4/1
  - So the pivot element is 3 in column x2.

- The selected pivot element is set to 1 with a required operation.
    - ***1/3 x Row1 → Row1***
    - New version of the table:

	x1	x2	s1	s2	Z	Result
?	1/3	1	1/3	0	0	8/3
?	1	1	0	1	0	4
?	-1	-2	0	0	1	0

- Other rows in the column where the pivot element is located are set to 0 by a required operation.
    - ***- Row1 + Row2 → Row2***
    - ***2 x Row1 + Row3 → Row3***
    - New version of the table:

	x1	x2	s1	s2	Z	Result
x2	1/3	1	1/3	0	0	8/3
s2	2/3	0	-1/3	1	0	4/3
Z	-1/3	0	2/3	0	1	16/3

    - The x2 column is now the column with the pivot element, so s1 is replaced by x2 in the row labels.

- The column with the most negative value in the bottom row is made the pivot column.
    - Pivot Column:

x1
1/3
2/3
-1/3

- One of the positive numbers in the rows above the bottom row in the 'column x1' is selected as the pivot element.

- The number with the smaller division result is chosen as the pivot element.
    - 4/3 **/** 2/3 **<** 8/3 **/** 1/3
    - So the pivot element is 2/3 in column x1.

- The selected pivot element is set to 1 with a required operation.
    - ***3/2 x Row2 → Row2***
    - New version of the table:

	x1	x2	s1	s2	Z	Result
?	1/3	1	1/3	0	0	8/3
?	1	0	-1/2	3/2	0	2
?	-1/3	0	2/3	0	1	16/3

- Other rows in the column where the pivot element is located are set to 0 by a required operation.
    - ***– 1/3 x Row2 + Row1 → Row1***
    - ***1/3 x Row2 + Row3 → Row3***
    - New version of the table:

	x1	**x2**	**s1**	**s2**	**Z**	**Result**
x2	0	1	1/2	-1/2	0	2
x1	1	0	-1/2	3/2	0	2
Z	0	0	1/2	1/2	1	6

- The table is now in its final form, and the optimal solution is **Z = 6**, **x1 = 2**, and **x2 = 2**. The slack variables s1 and s2 are both equal to 0, which means that the constraints are satisfied with equality.

Conclusion:

- To maximize the Z = x1 + 2x2 objective function, the values **x1 = 2** and **x2 = 2** should be used. So the maximum value will be **Z = 6**.