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28 points is 10, 17 points is 4 (minimum positive grade)

1. Gateway, Inc. has to decide on the number of 250GB and 500GB HDD sets to be produced at one of its factories. Market research indicates that at most 60 of the 500GB sets and 30 of the 250GB sets can be sold per month. The maximum number of work-hours available is 1200 per month. A 500GB set requires 15 work-hours, and a 250GB set requires 12 work-hours. Each 500GB set sold produces a profit of €70, and each 250GB set produces a profit of €40. A wholesaler has agreed to purchase all the HDD sets produced if the numbers do not exceed the maxima indicated by the market research. Formulate a linear programming model for this problem. (5 pt)

- Let **x** be the number of 500GB sets produced and **y** be the number of 250GB sets produced.
- Each 500GB set sold produces a profit of €70, and each 250GB set produces a profit of €40. Based on these data, we will try to maximize the total profit with the equation **Z = 70x + 40y**.
- The constraints are:
    - **x <= 60** (Market research indicates that at most 60 of the 500GB sets can be sold per month.)
    - **y <= 30** (Market research indicates that at most 30 of the 2500GB sets can be sold per month.)
    - **15x + 12y <= 1200**
    - **x >= 0, y >= 0** (The number of sets produced must be non-negative.)
- Therefore, the linear programming model for this problem is:
    - Maximize:
        - **Z = 70x + 40y (Profit)**
    - Subject to:
        - **0 <= x <= 60**
        - **0 <= y <= 30**
        - **15x + 12y <= 1200**
- Solution:
    - **x = 60** (500 GB)
    - **y = 25** (250 GB)
    - **Profit = (70*60) + (40*25) = 5200**

2. The GlassX produces glass doors and windows. The company has three plants: Plant 1, Plant 2, and Plant 3. For each door to produce requires 2 capacity unit at Plant 1 and 4 units at Plant 3. For each window requires 4 capacity unit at Plant 2 and 1 unit at Plant 3. Plant capacity is for Plant 1 equals to 4, for Plant 2 equals to 12, and for Plant 3 equals to 18 units. Profit per door is 6 units; profit per window is 9 units. Solve using the Simplex Method. (5 pt)

- x1: number of doors produced
- x2: number of windows produced
- The constraints are:
    - Plant 1 capacity: **2x1 <= 4 (x1 <= 2)**
    - Plant 2 capacity: **4x2 <= 12 (x2 <= 3)**
    - Plant 3 capacity: **4x1 + x2 <= 18**
    - Non-negativity: **x1, x2 >= 0**
- To solve this problem using the Simplex Method, we first convert the problem to its standard form:
    - Maximize: **Z = 6x1 + 9x2**

Subject to:
- $2x1 + s1 = 4$
- $4x2 + s2 = 12$
- $4x1 + x2 + s3 = 18$
- $x1 \geq 0, x2 \geq 0, s1 \geq 0, s2 \geq 0, s3 \geq 0$

Where s1 and s2 are slack variables.

Objective Function:
$-6x1 - 9x2 + Z = 0$

- The initial simplex table is:

	x1	**x2**	**s1**	**s2**	**s3**	**Z**	**Result**
s1	2	0	1	0	0	0	4
s2	0	4	0	1	0	0	12
s3	4	1	0	0	1	0	18
Z	-6	-9	0	0	0	1	0

- The column with the most negative number in the bottom row is designated as the pivot column.
  - Pivot Column:

x2
0
4
1
-9

- One of the positive numbers in the rows above the bottom row in the 'column x2' is selected as the pivot element.
- The number with the smaller division result is chosen as the pivot element.
  - 12/4 < 18/1
  - So the pivot element is 4 in column x2.
- The selected pivot element is set to 1 with a required operation.
  - ***1/4 x Row2 → Row2***
  - New version of the table:

	x1	**x2**	**s1**	**s2**	**s3**	**Z**	**Result**
?	2	0	1	0	0	0	4
?	0	1	0	1/4	0	0	3
?	4	1	0	0	1	0	18
?	-6	-9	0	0	0	1	0

- Other rows in the column where the pivot element is located are set to 0 by a required operation.
  - ***– Row2 + Row3 → Row3***
  - ***9Row2 + Row4 → Row4***
  - New version of the table:

	x1	**x2**	**s1**	**s2**	**s3**	**Z**	**Result**
s1	2	0	1	0	0	0	4
x2	0	1	0	1/4	0	0	3
s3	4	0	0	-1/4	1	0	15
Z	-6	0	0	9/4	0	1	27

  - The x2 column is now the column with the pivot element, so s1 is replaced by x2 in the row labels.

- The column with the most negative number in the bottom row is designated as the pivot column.
  - Pivot Column:

x1
2
0
4
-6

- One of the positive numbers in the rows above the bottom row in the ‘column x1’ is selected as the pivot element.
- The number with the smaller division result is chosen as the pivot element.
  - 4/2 < 15/4
  - So the pivot element is 2 in column x1.
- The selected pivot element is set to 1 with a required operation.
  - ***1/2 x Row1 → Row1***
  - New version of the table:

	x1	x2	s1	s2	s3	Z	Result
?	1	0	1/2	0	0	0	2
?	0	1	0	1/4	0	0	3
?	4	0	0	-1/4	1	0	15
?	-6	0	0	9/4	0	1	27

- Other rows in the column where the pivot element is located are set to 0 by a required operation.
  - ***– 4Row1 + Row3 → Row3***
  - ***6Row1 + Row4 → Row4***
  - New version of the table:

	x1	x2	s1	s2	s3	Z	Result
x1	1	0	1/2	0	0	0	2
x2	0	1	0	1/4	0	0	3
s3	0	0	-2	-1/4	1	0	7
Z	0	0	3	9/4	0	1	39

- The table is now in its final form, and the optimal solution is **Z = 39**, **x1 = 2**, and **x2 = 3**. The slack variables s1 and s2 are both equal to 0, which means that the constraints are satisfied with equality.

Conclusion:

- To maximize the Z = 6x1 + 9x2 objective function, the values **x1 = 2** and **x2 = 3** should be used. So the maximum value will be **Z = 39**.

3. Correct mistakes and explain each line (7.5pt):

dvar int+ UrgentCount;

- There is no mistake in this line.

dvan int NormalCount;

- There is a typo in this line. "dvan" should be corrected to "**dvar**".
- Since the NormalCount variable cannot be a negative value, the expression "int" must be written as "**int +**".

maxinize

- "maxinize" should be corrected to "**maximize**" to specify the objective function.

*3 * UrgentCount _ 1 * NormalCount;*

- In the objective function, it is necessary to sum the terms correctly, so "_" should be corrected to "+". So the equation should be written as "**3 * UrgentCount + 1 * NormalCount;**"

subject to [

- The parentheses for the "subject to" section are incorrect. "**{**" should be used instead of "[".

analystAvailabilityMax:

- There is no mistake in this line.

1 UrgentCount + 2 * NormalCount <= 10;*

- There is no mistake in this line.

developerAvailabilityMax:

- There is no mistake in this line.

*3 * UrgentCount + 2 * NormalCount <== 12;*

- "<==" should be corrected to "<=" in the constraint for "developerAvailabilityMax".

}

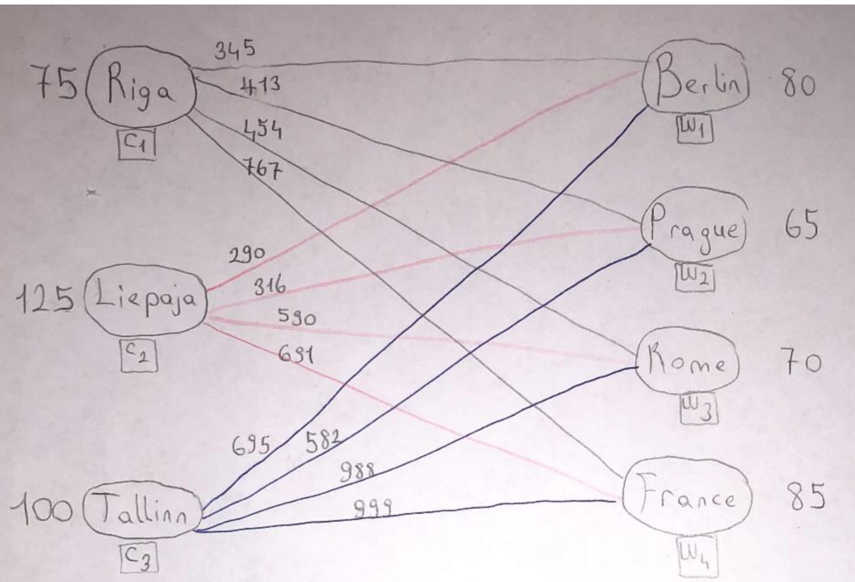
<u>The edited version of the code:</u>

```
dvar int+ UrgentCount;
dvar int+ NormalCount;

maximize
    3 * UrgentCount + 1 * NormalCount;

subject to {
    analystAvailabilityMax:
        1 * UrgentCount + 2 * NormalCount <= 10;
    developerAvailabilityMax:
        3 * UrgentCount + 2 * NormalCount <= 12;
}
```

4. Z&N is a small family-owned business that processes fresh juice drinks in glass bottles and then distributes them for eventual sale. These drinks are processed in: Riga, Latvia; Liepaja, Latvia; and Tallinn, Estonia. The bottles are shipped to Berlin, DE; Prague, CZ; Rome, IT and Paris, FR.

	Warehouse				
Bottle filling	*Berlin*	*Prague*	*Rome*	*France*	***Supply***
Riga	€345	€413	€454	€767	*75*
Liepaja	€290	€316	€590	€691	*125*
Tallinn	€695	€582	€988	€999	*100*
Demand	*80*	*65*	*70*	*85*	

Solve with Network Presentation of Z&N Problem and Mathematical Model for Z&N Transportation Problem. (7.5 pt)



→ Minimize:

$$Z = 345x_{11} + 413x_{12} + 454x_{13} + 767x_{14} + 290x_{21} + 316x_{22} + 590x_{23} + 691x_{24} + 695x_{31} + 582x_{32} + 988x_{33} + 999x_{34}$$

→ Subject to:

$$\begin{cases} x_{11} + x_{12} + x_{13} + x_{14} = 75 \\ x_{21} + x_{22} + x_{23} + x_{24} = 125 \\ x_{31} + x_{32} + x_{33} + x_{34} = 100 \end{cases}$$

$$\begin{cases} x_{11} + x_{21} + x_{31} = 80 \\ x_{12} + x_{22} + x_{32} = 65 \\ x_{13} + x_{23} + x_{33} = 70 \\ x_{14} + x_{24} + x_{34} = 85 \end{cases}$$

$$x_{ij} \geqslant 0 \quad (i = 1,2,3 \,;\ j = 1,2,3,4)$$

- **Code**

```
/*********************************************
 * OPL 22.1.1.0 Model
 * Author: Emir
 * Creation Date: 16 Mar 2023 at 11:48:30
 *********************************************/

dvar int+ x11;
dvar int+ x12;
dvar int+ x13;
dvar int+ x14;
dvar int+ x21;
dvar int+ x22;
dvar int+ x23;
dvar int+ x24;
dvar int+ x31;
dvar int+ x32;
dvar int+ x33;
dvar int+ x34;

minimize
    345 * x11 + 413 * x12 + 454 * x13 + 767 * x14 + 290 * x21 + 316 * x22 + 590 * x23 + 691 * x24 + 695 * x31 + 582 * x32 + 988 * x33 + 999 * x34;

subject to {
    x11 + x12 + x13 + x14 == 75;
    x21 + x22 + x23 + x24 == 125;
    x31 + x32 + x33 + x34 == 100;
    x11 + x21 + x31 == 80;
    x12 + x22 + x32 == 65;
    x13 + x23 + x33 == 70;
    x14 + x24 + x34 == 85;
}
```

- **Result**

Solution with objective 162,600

Name	Value
Decision variables (12)	
x11	5
x12	0
x13	70
x14	0
x21	75
x22	0
x23	0
x24	50
x31	0
x32	65
x33	0
x34	35

5. Explain the following concepts: Operations Research, Deterministic models, optimal solution. (3pt)

- **Operations Research**: Operations research is an interdisciplinary science that uses scientific methods such as mathematical modeling, statistics, and algorithms to generate ideas for real complex problems of the world related to the coordination and execution of operations within an organization.

- **Deterministic Models**: The deterministic model is a method in which all parameters and variables associated with a data are known and optimizations are made accordingly.

- **Optimal Solution:** The optimum solution is a feasible solution in which the objective function reaches its maximum (or minimum) value according to the desired situation. The *optimal solution* for the model is a set of decision variable values that produce the results that are mathematically best.