



## LAB WEEK 10

### 1. Euclidean Algorithm

Euclidean Algorithm is an algorithm to calculate greatest common divisor (gcd) of two integers. Suppose that we want to calculate gcd of two integers  $x$  and  $y$  where  $x > y$ .

$\text{gcd}(x, y) = \text{gcd}(y, x \% y)$  where  $x \% y$  is the remainder

Here is an example:

$\text{gcd}(219, 93) = \text{gcd}(93, 33)$  since  $219 \% 93 = 33$

$= \text{gcd}(33, 27)$  since  $93 \% 33 = 27$

$= \text{gcd}(27, 6)$  since  $33 \% 27 = 6$

$= \text{gcd}(6, 3)$  since  $27 \% 6 = 3$

$= \text{gcd}(3, 0)$  since  $6 \% 3 = 0$  **STOP**

You should stop if the remainder is 0. gcd is the last nonzero remainder.

Write a recursive method which takes as parameter two integers and returns their gcd.

### 2. Population growth

Assume that a city with currently 1 million people has a population growth rate of 1% per year, and it also receives 1 thousand immigrants per year. Find its population in 10 years from now by writing a recursive method called `population` which takes the year  $n$  as its parameter and returns the population after  $n$  years.

### 3. Staircase - Tribonacci Numbers

Suppose you want to walk up a staircase, and you can take 1, 2 or 3 steps at a time. How many ways are there to walk the 6 steps?

For instance, you can take 1-1-1-1-1-1, 3-3, 3-1-1-1.

Let  $S_n$  denote the number of ways to walk  $n$  steps. Write a recursive relation by thinking of the last step you are going to take. (There are 3 possibilities.)

Using the recurrence relation you have found write a recursive algorithm to solve the problem.

4. Write a function using recursion to check if a number  $n$  is prime (you have to check whether  $n$  is divisible by any number below  $n$ ).
5. Write a function using recursion that takes in a string and returns a reversed copy of the string. The only string operation you are allowed to use is string concatenation
6. Write a recursive function that converts a number from the 10th number system to binary. The function returns the result as a number. For instance, 13 will be converted to 1101.