Measuring the Topological Dimension of Economic Systems: A Gauss's Law Approach

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The dimensionality of the space in which economic interactions occur is often implicitly assumed to be either zero (point-mass models) or three (physical reality). In this letter, we propose a method to measure the effective topological dimension (d) of economic systems using the inverse application of Gauss's Law. By analyzing the distance decay exponent (β) in the empirical Gravity Model of Trade, where $F \propto r^{-\beta}$, we derive the relationship $\beta = d - 1$. Extensive empirical literature confirms $\beta \approx 1$, which rigorously implies $d \approx 2$. Furthermore, the linearity of velocity in the Fisher Equation ($MV = PY$) contradicts the kinetic energy scaling (v^2) of 3D free particles, supporting the hypothesis of a constrained 2D fluid manifold. This dimensionality constraint provides a first-principles explanation for the long-range nature of economic influence (Logarithmic Potential) and the stability of monopolies (Inverse Energy Cascades in 2D turbulence).

I. INTRODUCTION

In physics, the dimensionality of space dictates the form of fundamental laws. Gravity scales as $1/r^2$ in 3D, but as $1/r$ in 2D. In economics, the spatial structure of trade and interaction is often treated as a secondary variable or modeled via ad-hoc "friction" parameters.

However, Econophysics demands that we define the geometry of the "phase space" before writing dynamic equations. Is the economy a 3D gas, a 2D fluid, or a high-dimensional fractal?

We propose a direct measurement method based on three converging lines of evidence: the physical constraints of the planetary surface, the flux decay in trade gravity models, and the dimensional analysis of the equation of exchange.

II. EVIDENCE I: THE PHYSICAL HARD CONSTRAINT

The first evidence is geometric. Human economic activity is physically constrained to the lithosphere of the Earth. Let R be the radius of the Earth (≈ 6371 km) and h be the maximum vertical scale of economic activity (skyscrapers, mines, airspace ≈ 20 km). The aspect ratio of the economic system is:

$$\frac{h}{R} \approx \frac{20}{6371} \ll 1. \tag{1}$$

In fluid dynamics, when the vertical scale is negligible compared to the horizontal scale, the system is mathematically treated as a **Thin Film** or a **2D Manifold** (specifically, a sphere S^2).

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Consequently, the phase space of economic agents is topologically compressed. Conservation laws for mass and momentum must be integrated over an **Area** ($\int \rho \mathbf{v} \cdot d\mathbf{l}$), not a Volume.

III. EVIDENCE II: THE GRAVITY MODEL OF TRADE

The Gravity Model of Trade, first proposed by Tinbergen [1], is one of the most robust empirical findings in macroeconomics. It states that the trade volume F_{ij} between two economies is proportional to their sizes (M) and inversely proportional to distance (r):

$$F_{ij} = G\frac{M_i M_j}{r_{ij}^{\beta}}. \tag{2}$$

We can derive the dimensionality d from the exponent β using **Gauss's Law for Flux**. Consider a source emitting economic influence (flux Φ). The flux density (force F) at a distance r is determined by the surface area of the enclosing boundary $S_d(r)$ in d dimensions:

$$S_d(r) \propto r^{d-1}. \tag{3}$$

Thus, the force scales as:

$$F(r) \propto \frac{1}{r^{d-1}}. \tag{4}$$

Comparing this to the trade equation ($F \propto r^{-\beta}$), we obtain:

$$\beta = d - 1. \tag{5}$$

Decades of meta-analysis on trade data [2, 3] consistently place the value of β in the range of **0.8 to 1.2**, with a mean very close to **1.0**. Substituting $\beta \approx 1$:

$$d \approx 1 + 1 = 2. \tag{6}$$

If the economy were a 3D system (like gas molecules in a box), flux would decay as $1/r^2$ ($\beta = 2$). The empirical fact that $\beta = 1$ is a "smoking gun" proving that economic interactions are constrained to a **2D Manifold**.

IV. EVIDENCE III: THE FISHER EQUATION COUNTER-PROOF

A third line of evidence comes from the dimensional analysis of the Fisher Equation ($MV = PY$) [4].

If the economy were modeled as a 3D Ideal Gas, the "Temperature" (Economic Activity) would be proportional to the average kinetic energy of the particles:

$$T_{3D} \propto \frac{1}{2}mv^2. \tag{7}$$

This would imply that output or price levels should scale with the **square** of velocity (V^2), similar to dynamic pressure in Bernoulli's principle ($q = \frac{1}{2}\rho v^2$).

However, the Fisher Equation shows a linear relationship: $PY \propto V$. In fluid mechanics [5], a linear relationship between Flux and Velocity ($J = \rho v$) arises only from the **Continuity Equation** in a constrained channel (Pipe Flow or 2D Sheet Flow).

The linearity of V in macroeconomics confirms that money behaves as a **Mass Flux** on a manifold, not as **Kinetic Energy** in free space.

V. IMPLICATIONS OF $d = 2$

The determination that $d = 2$ has profound consequences for economic theory, formalizing the **EMIS Framework**:

A. Logarithmic Potential (Confinement)

In 2D, the potential energy $V(r)$ is the integral of force $F \propto 1/r$:

$$V(r) \propto -\int \frac{1}{r}dr \propto \ln(r). \tag{8}$$

Unlike 3D potential ($1/r$), 2D logarithmic potential diverges at infinity. This means economic gravity is a **long-range force** that does not vanish. This provides a geometric explanation for "Global Extraction", where capital centers exert significant influence over infinite distances.

B. Inverse Cascades

Fluid dynamics on 2D manifolds exhibit **Inverse Energy Cascades** (Kraichnan-Leith-Batchelor theory). Energy flows from small scales to large scales, leading to the spontaneous formation of large, stable vortices. In economics, this provides a thermodynamic proof for the inevitability of **Monopoly** and oligopolistic structures.

VI. CONCLUSION

By triangulating evidence from physical constraints ($h \ll R$), empirical trade gravity ($\beta \approx 1$), and the linearity of the Fisher Equation, we conclude that the economic system operates on a topological dimension of $d \approx 2$. The economy behaves as a **2D Fluid Manifold**. This dimensionality constraint is not an assumption but a measurable physical fact, serving as the root cause of long-range inequality and structural consolidation.

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