Deriving the Fisher Equation from First Principles: The Economy as a 2D Fluid Manifold

Fei-Yun Wang*

OS-Question Research Lab,

Open Source Quantitative Science Initiative.

Beijing, China

(Dated: January 28, 2026)

The Equation of Exchange, $MV = PY$, proposed by Irving Fisher [1], serves as the foundation of monetarism. However, it is traditionally treated as an accounting identity rather than a derived physical law. A persistent dimensional anomaly exists: if money represents energy, why does the equation scale linearly with velocity (v), whereas kinetic energy scales with v^2? In this letter, we resolve this anomaly using the EMIS framework. We postulate that social systems operate on a **2D topological manifold** (networks), distinct from 3D Euclidean space [2]. By applying fluid dynamics conservation laws to this restricted geometry, we demonstrate that the Fisher Equation is an emergent property of the **Continuity Equation** for mass flux, rather than kinetic energy transport. This derivation provides a rigorous physical grounding for macroeconomics and explains why economic activity is governed by flow constraints rather than inertial impact.

I. INTRODUCTION

In macroeconomics, the relationship between money supply (M), velocity of circulation (V), price level (P), and output (Y) is encapsulated in the Fisher Equation:

$$MV = PY. \tag{1}$$

While empirically robust, this equation is axiomatic in economics. From a physics perspective, it presents a dimensional puzzle.

If we accept the Econophysics hypothesis that Money maps to Energy [3], the linear dependence on velocity (V) is non-trivial. In classical mechanics, kinetic energy is proportional to the square of velocity ($E_k \propto v^2$). In wave mechanics, energy flux is proportional to v^2 (or amplitude squared). A linear v dependence typically implies **Momentum** ($p = mv$) or **Mass Flux** ($\Phi = \rho v$).

Why does the economy behave like a momentum or flux system rather than an energy system? We propose that the answer lies in the **dimensionality** of the substrate.

II. THE DIMENSIONALITY HYPOTHESIS

We posit that economic interactions do not occur in a 3D volume but on a **2D Manifold** (representing the surface of the earth or the topology of a social graph).

In 3D space, degrees of freedom allow for volumetric dissipation and inertial kinetic energy storage (v^2). However, in a constrained 2D manifold (like a pipe or a surface network), the dominant conservation law governing system throughput is the **Continuity Equation**.

* wehelpwe@outlook.com

III. DERIVATION

Consider the economy as a fluid of "Value" flowing through a network. Let ρ be the density of money per unit area (or per node), and $\mathbf{v}$ be the velocity vector of transaction flow.

The conservation of monetary mass (assuming no central bank injection) is given by the Continuity Equation [4]:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0. \tag{2}$$

For a steady-state economy ($\frac{\partial \rho}{\partial t} = 0$), the divergence of flux is zero. We integrate the flux $\mathbf{J} = \rho \mathbf{v}$ over the entire economic cross-section (Market Area A):

$$\text{Total Flux} = \int_A \rho \mathbf{v} \cdot d\mathbf{a}. \tag{3}$$

We map the physical variables to economic variables:

- Total Mass in system $\int \rho dA \rightarrow$ Money Supply (M).
- Average Drift Velocity $\langle \mathbf{v} \rangle \rightarrow$ Velocity of Money (V).
- Total Flux output $\rightarrow$ Nominal GDP (PY).

Substituting these mappings into the flux integral yields:

$$M \cdot V = P \cdot Y. \tag{4}$$

IV. DISCUSSION

This derivation reveals that the Fisher Equation describes the **Conservation of Monetary Flux** (analogous to Mass Flux) in a lower-dimensional system.

A. Why not V^2?

Models that attempt to define "Economic Temperature" based on kinetic energy ($T \propto v^2$) often fail to reconcile with macroeconomic data because they assume a 3D gas model. In a 2D network, the primary constraint is **throughput** (bandwidth), which is linear in velocity. The "Energy" of the economy is effectively its **Mass Flux** (Throughput).

B. The 2D Manifold Consequence

This finding supports the broader EMIS hypothesis that social systems behave as 2D fluids. This dimensionality constraint also explains:

1. **Inverse Energy Cascades**: Why monopolies (large structures) are stable in 2D flows.
2. **Logarithmic Potentials**: Why economic influence decays slowly ($1/r$) rather than quickly ($1/r^2$).

V. CONCLUSION

We have shown that the linearity of the Fisher Equation is a direct consequence of the 2D topological constraint of social systems. Economics follows the laws of **Fluid Continuity** rather than **Particle Kinetics**. This suggests that future macroeconomic models should utilize the Navier-Stokes equations on 2D manifolds rather than ideal gas laws.

ACKNOWLEDGMENTS

This work is part of the EMIS Framework project.

[1] I. Fisher, *The Purchasing Power of Money: Its Determination and Relation to Credit, Interest and Crises* (Macmillan, New York, 1911).

[2] B. Metcalfe, Metcalfe's law after 40 years of the ethernet, Computer **46**, 26 (2013).

[3] A. Dragulescu and V. M. Yakovenko, Statistical mechanics of money, The European Physical Journal B **17**, 723 (2000).

[4] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics: Course of Theoretical Physics, Vol. 6* (Pergamon Press, 1987).